Astronomy. — "*Investigation of a galactic cloud in Aquila.*" By Dr. A. PANNEKOEK. (Communicated by Prof. W. DE SITTER).

(Communicated in the meeting of March 29, 1919).

A communication to the Meeting of this Academy on Dec. 8, 1911, described how, by means of some photographs, it is possible to obtain data about the increase of star-density with decreasing limiting magnitude. There it was stated already that Prof. HERTZSPRUNG of Potsdam, by means of the Zeisstriplet of the Astrophysical Observatory, had made some photographs (of the galactic cloud N.W. of γ Aquilae), in order to test the method. Various circumstances, however, prevented a final discussion of these plates until quite recently.

The plates are 20 × 20 cm., the centre lies near χ Aquilae, and the region that was photographed is 6° square. The plates immediately used for this purpose are:

Nr. 328	Sept. 2 1910	Expos. 600, 600, 190, 60, 19, 6, 2 sec.	(plate A)
Nr. 329	Sept. 2 1910	Expos. 1900, 1900 sec.	(plate B)
Nr. 1260	Aug. 24 1911	Expos. 40^m, Halbgitter North	(plate C_1)
Nr. 1261	Aug. 24 1911	Expos. 45^m, Halbgitter South	(plate C_2)

As for the counting of plates A and B no reseau was printed on the plates themselves, a reseau of $6^2/_3$ mm. interval was photographed on a separate glassplate instead, which reseau-plate, during the counting, was firmly clasped to the counting-plates.

1. *The countings.* On plate A were counted in each square firstly the numbers of stars with only 2 equal images, secondly those with moreover a 3rd image, (190^s exp.) thirdly those with 4 images, (a still visible image therefore for 60^s), those with 5, and with 6 images. The respective limiting magnitudes differ about 1 magnitude; according to some provisional comparisons with a photograph of the North polar region they amounted to 13,0 ..... 9,0. The uncertainty and the subjective differences of conception so common in star-counting, the faintest star-images not being discernible from casual spots in the plate, are practically done away with here, as every star must present two equal images at a known distance, or as a faint image must present itself at a given spot near the brighter images. Yet this does not do away entirely with the uncertainty in counting;

TABLE I. Number of stars.

	I										
	121 35 18 1 1 0	98 36 15 5 2 0	123 44 19 8 5 1	97 27 17 7 5 1	83 24 11 5 1 0	113 44 17 7 0 0	100 33 18 7 2 0	112 46 27 11 3 1	127 51 22 13 6 2	118 34 15 9 1 0	
	135 40 17 7 1 0	88 34 14 3 1 0	64 28 17 8 3 0	96 38 13 6 4 0	101 36 13 5 2 2	96 28 13 5 1 0	84 24 12 8 5 2	107 46 14 6 4 2	85 31 20 5 2 0	94 37 13 2 1 0	
	83 27 14 6 2 0	96 33 15 6 1 1	97 27 12 5 4 0	123 36 20 7 3 0	115 40 16 8 3 2	91 28 15 6 4 1	90 34 20 11 5 1	90 35 21 7 5 2	77 26 13 7 3 1	111 44 21 8 1 0	
III	76 25 13 4 0 0	76 23 12 4 1 0	102 49 25 8 5 2	105 32 14 6 2 1	77 35 19 8 3 1	87 49 19 8 3 1	95 35 15 6 0 0	120 35 14 6 1 0	144 47 16 5 0 0	117 53 25 13 4 1	II
	52 27 18 8 3 0	87 30 12 4 2 0	76 23 9 3 0 0	94 44 19 8 2 1	59 30 18 10 4 3	69 30 13 7 1 0	92 34 21 8 6 1	103 40 18 8 4 1	97 46 21 12 3 1	127 39 18 9 5 0	
	52 18 9 3 1 0	78 25 8 4 2 0	92 IV 31 9 3 2 0	72 22 10 5 1 0	84 33 19 6 1 0	82 39 20 7 2 1	111 32 17 7 3 2	110 33 21 5 2 0	84 34 16 6 2 0	118 40 11 3 0 0	
	55 13 8 3 2 0	73 21 11 2 1 0	88 30 10 4 1 0	55 16 4 1 0 0	58 21 13 7 3 0	99 35 16 9 5 2	95 49 25 10 4 1	117 41 15 5 2 0	106 55 24 9 4 2	88 36 17 8 2 1	
	45 20 9 6 2 1	52 25 10 2 1 0	54 25 12 5 2 0	19 12 6 0 0 0	28 10 5 0 0 0	72 24 12 6 3 0	89 29 15 5 3 0	88 34 20 6 2 0	87 32 10 2 1 0	94 27 12 3 2 1	
	53 16 9 3 3 2	51 10 6 3 2 2	38 10 6 5 1 0	38 9 5 2 1 0	21 5 2 0 0 0	29 9 5 1 1 0	84 26 14 8 2 2	63 35 16 9 5 0	61 33 15 7 4 1	71 30 14 6 5 2	IV
V	39 19 12 1 0 0	24 13 8 2 1 0	35 8 6 4 1 0	53 15 8 2 0 0	34 11 6 3 2 1	39 15 10 4 2 1	39 18 8 1 1 0	60 30 15 8 2 0	46 20 14 5 2 1	61 31 17 9 3 0	

on plate B some of the denser galactic regions are as it were dotted with scarcely perceptible spots, so that it often seems entirely arbitrary whether some of them are to be considered as belonging together, and to be counted as stars. In deciding whether a scarcely visible 3^{rd} or 4^{th} image existed alongside of the brighter images, the subjective certainty was considerably greater.

In the case of the brighter stars another uncertainty presented itself. It sometimes happened that of some star, which in the large images was decidedly fainter than another, more faint images could nevertheless be discerned, as here the fainter images were small and sharp, and with the others they were large and diffuse. This is caused by the uncommon achromatisation of the Zeisstriplet [1]), which renders the yellow stars large and hazy, and the white stars bright and small. This circumstance, which may be of use in deciding the colour of such weak stars, often rendered the counting troublesome, as a rule the visibility of the weakest image was taken as a criterion for the classifying.

The region counted comprises 100 squares (in AR. of —7 to +3, in decl. of +5 to —5). The centre of the plate lies on 279°30′ + + 11°30′, the side of each square is 15′,28, so the surface is $0{,}0649 = {}^1/_{15{,}41}$ square degrees. The cornerpoints of the region exploied are situated at

277°40′6+12°44′,8, 277°41′,6 + 10°12′,4, 280°17′,0 + 12°46′,3,
280°16′,5 + 10°13′,6

The countings have been executed by means of the microscope of the Repsold-apparatus for rectangular coordinates at the Leyden observatory, fitted out with the weakest ocular, the enlargement was tenfold, rather too strong for the purpose. The results of the countings have been collected in Table I, each square contains successively the number on plate B, the number on plate A, the numbers on A with at least 3 and 4 images, and the numbers on A with at least 5 and 6 images.

2. *The scale of magnitudes.* In order to find the limiting magnitudes for which these numbers stand, the magnitude of a number of stars had to be ascertained. This part of the investigation presented the greatest difficulties, as it had to be effected with somewhat

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1) The focal distance is minimum for 394 $\mu\mu$ (HERTZSPRUNG A. N. 4951. Vol. **207** 88).

primitive means. To obtain a comparison-scale a portion of a photograph of Coma Berenices was cut out, containing side by side exposures of 12, 15, 19, 24, 30, 38, 48, 60, 76, 95, and 120 seconds, which means 11 images of every star, increasing $0^m,2$ in magnitude. By pressing this plate to the back of plate *A* or *B*, film against film, and comparing with an ocular enlarging 5 times, each star on *A* or *B* could be inserted by means of eye-estimate between the terms of the scale The numbers of the scale-values represent the approximate magnitudes of stars that would have the same images on plate *B*.

By means of this scale in a number of regularly distributed squares the magnitude of all the stars distinctly visible on plate *B* was estimated and the like on *A* for all clearly visible and measurable images. Thus can be found the differences in magnitude between the various exposures, expressed in the provisional scale. To express the unity of this provisional scale in the absolute scale of magnitudes, two strips, North and South, were measured on either plate *C* in such a way as to leave each strip on the one plate entirely covered by the Halbgitter, and on the other quite free. By deducing from this the difference in magnitude of the images with and without the Halbgitter in the provisional scale and comparing it with the known absorption-coefficient of the Gitter, one can find the reduction to absolute scale. By means of a few stars of known magnitude the absolute magnitude can then be deduced.

The execution and reduction of the measurements showed that in case of the more brilliant stars with large images there existed systematic differences, that rendered a further use of them undesirable. With the fainter stars of the scale other errors presented themselves. The smaller images showed as somewhat irregular spots, and neither did these always differ $0^m,2$ in magnitude. This may be caused partly by local differences of sensitiveness and a not wholly regular spreading of the silver-grains, which influence the look of these small faint spots, partly in the accidental coinciding of scale-images with images of other invisible stars. It proved necessary therefore, to ascertain separately the magnitude of all images of the scale that were often used. This was done by estimating them between the images on a polar plate, likewise following each other with a theoretical interval of $0^m,2$; as each scale-image was inserted in various polar-star series, the errors of these series passed into the magnitudes of the scale to only a very slight extent. Thus for the magnitude of the faintest (0) up to the brightest image (10) of the stars *w*, *s* and *r* we found:

	0	1	2	3	4	5	6	7	8	9	10
w		invisible			14,4	14,15	14,0	13,8	13,6	13,4	13,2
s	14,2	14,05	13,9	13,85	13,45	13,15	13,0	12,85	12,7	12,4	12,25
r	13,25	13,0	12,8	12,5	12,25	11,9	11,75	11,75	11,5	11,35	11,15

These values were made use of in order to deduce the magnitude of the star-images in the squares on plate B and A: the shorter exposures give magnitudes decreasing by about 1^m, from which the difference in magnitude of the successive exposures B, A_1, A_2, A_3, may be deduced.

Classifying these differences according to magnitude, we find:

B	A_1	B-A_1	corrected	A_1	A_2	A_1-A_2	corrected
11,40	12,32	0.92(18)	0.98	11,46	12,40	0,94(5)	1,01
11,88	12,76	0,88(17)	0,96	11,85	12,81	0,96(11)	1,04
12,41	13,35	0,94(30)	0,91	12,30	13,49	1,19(17)	1,15
12,70	13,84	1,14(18)	1,00	12,76	13,95	1,19(17)	1,03
			0,95(83)				1,07(50)

A_2	A_3	A_2-A_3	corrected
12,28	13,35	1,07(8)	1.05
12,81	14,02	1,21(11)	1,04
			1,04(19)

The differences are not merely accidental; the fact that with all of them the last value is the greatest, proves that the scale is not yet wholly homogeneous. By successive approximations the following deviations from an evenly running scale were found -

11,42—12,32	—0,06	12,36—13,35	+0,03
11,87—12,73	—0,10	12,75—13,86	+0,15

These are accounted for by the following corrections to the scale:

11,2—11,8	0	13,0	+ 0,07
12,0	+ 0,02	13,2	+ 05
12,2	+ 04	13,4	+ 03
12,4	+ 07	13,6	00
12,6	+ 08	13,8	— 04
12,8	+ 09	14,0	— 08

By introducing these corrections, we get for the difference in magnitude $B_1 - A = 0,95$; $A_1 - A_2 = 1,07$; $A_2 - A_3 = 1,04$. For the shorter exposures only the brighter stars could be used; they gave the result of $A_3 - A_4 = 1,16(7)$; $A_4 - A_5 = 1,09(14)$. The mean error of 1 determination of magnitude is $0^m,14$.

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To this same scale were compared a number of stars in the N. and S.-strip on the Halbgitter-plates C_1 and C_2. Here the result was:

S.-strip: ordinary image C_1 — weakened image $C_2 =$
$= 13{,}78 - 11{,}63 = 2{,}15$ (75)

N.-strip: ordinary image C_2 — weakened image $C_1 =$
$= 13{,}78 - 11{,}48 = 2{,}30$ (38)

This gives for the absorption of the Halbgitter in unities of the provisional scale 2,22. In absolute scale according to the statement of Prof. HERTZSPRUNG at Potsdam there was found for this absorption 1,963 magn. All the intervals deduced here must therefore be multiplied by the factor 0,884, in order to express them in magnitudes (this means that a 10 times larger exposure gives a gain of 1,77 magn.). Then they are:

$$B - A_1 = 0^m{,}84;\ A_1 - A_2 = 0^m{,}95;\ A_2 - A_3 = 0^m{,}92;$$
$$A_3 - A_4 = 1^m{,}02;\ A_4 - A_5 = 0^m{,}96.$$

In order to express also the magnitudes themselves in absolute scale, 16 of the most brilliant stars were used, which are contained in the "Göttinger Aktinometrie"; from the magnitude of their 5^{th} and 6^{th} image was found:

$$m - 11{,}55 = 0{,}884 \text{ (prov. } m - 11{,}55).$$

§ 3. *The limiting magnitude.* The difference in limiting magnitude will be equal to the differences in magnitude found here for the same stars at various exposures, provided the conditions under which the observations are made be absolutely identical. On the plates B and A_1 each star presents two equal images; all the double images therefore that are at all discernible are counted. With regard to the exposures A_2 A_3 A_4 and A_5 on the other hand, a faint, scarcely distinguishable image must be looked for, in a given spot by the side of brighter images. If the chance that by the fluctuations in the conditions a star-image near the limit of visibility can be just discerned $= a$, then the chance that two equal images are both visible $= a^2$; in this case therefore more stars remain invisible. With such counting as on B and A_1 therefore fewer will be counted, systematically, than with the method employed for A_2 etc. For the difference in limiting magnitude $A_1 - A'_2$ the difference in magnitude found above can therefore not be used.

In order to find this difference during the counting of the plates A charts had already been drawn of those squares, where later on the magnitude of all clearly visible stars was to be ascertained, on which charts were indicated all the stars showing 2, 3, 4, 5 and 6 images. We must now find what magnitude, measured on B, forms the limit between the stars that are visible on A and those that are invisible; this is the limiting magnitude for A_1. In the like manner we find out what magnitude forms the limit between the stars with 2 and with 3 images on A; this is limiting magnitude for A_2. From the first follows, with the difference $B—A_1$, the limiting magnitude B, from the second follows, in the same manner, the limiting magnitude for A_3, A_4, and A_5.

In the application this method proved to involve many difficulties as yet, as the magnitudes of the stars visible and invisible on A, as well as those of the stars with 2 and 3 images, extend far the one over the other, and are moreover irregularly distributed. If m_1 is the magnitude measured on B, differing from the real magnitude m by the unequal sensibility of the plate and by errors in the counting, and if the magnitude on the counting plate, likewise diverging from m, is m_2, then the star will be visible or invisible, according to whether $m_2 <$ or $> m_0$, the limiting magnitude. If the differences $m_1—m$ and $m_2—m$ follow the law of errors and if the stars are divided regularly over the various magnitudes, there are two criteria for the ascertaining of m_0:

1. for $m_1 \gtrless m_0$ the number of invisible stars is $\gtrless$ the number of visible ones; m_0 therefore is that value of m_1, for which 50 °/₀ of the stars is visible, 50 °/₀ invisible;

2. for $m_1 \gtrless m_0$ the total number of brighter, invisible stars is $\gtrless$ the total number of fainter, visible stars; m_0 therefore is that value of m_1 above which appear a number of visible stars, equal to the number of invisible ones below.

Now the number of stars for greater m increases; the average m, corresponding with a measured m_1, will consequently be somewhat larger than this latter; the limiting magnitude found according to the first criterion, needs a positive correction, which is somewhat diminished, however, by the differences m_2-m. On the other hand by means of the 2nd criterion the correct limiting magnitude is found if the number of stars is a linear function of the magnitude m [1]).

[1]) This can be proved in the following manner. The number of stars of real magnitude m that is measured on the one plate in magnitude m_1, and likewise the number that on the other plate shows the magnitude m_2; is respectively

$$f(m)\ \text{exp.}\left(-h_1^2(m_1-m)^2\right) dm\, dm_1 \qquad \text{and} \qquad f(m)\ \text{exp.}\left(-h_2^2(m_2-m_1)^2\right) dm\, dm_2$$

in which $f(m)$ represents the number of stars of the magnitude m; this $f(m)$ has the form $a+bm$.

If we pose:

$$\frac{h_1^2 m_1 + h_2^2 m_2}{h_1^2 + h_2^2} = m_3 \qquad \frac{h_1^2 h_2^2}{h_1^2 + h_2^2} = h^2$$

And if this function and the module of accuracy for magnitudes diverging 1^m may be considered as equal, the correction for both limiting magnitudes is equal with the 1st criterion, so that the difference in magnitude A_1-A_2 is correctly found also in this way.

In table II the 2nd and 3rd columns contain the numbers of stars with 0, with 2, with 3 images (n_1 n_2 n_3). In order to smooth the very considerable, accidental irregularities of these numbers, the total of every 3 consecutive numbers have been placed in the following columns (n_1^0 n_2^0 n_3^0). Column p shows how many percentages n_2^0 is of the sum total; where in the increase this amounts to 50 %, the limit lies between invisibility and two images; where in the decrease it amounts to 50 %, the limit lies between two and three images, according to the 1st criterion. Next to that stand the sums-total s of the fainter, visible, and the brighter invisible stars, the limiting magnitude, according to the 2nd criterion lies where these become equal.

From the values p_2 we find as limiting magnitude 13,67 and 12,54; to this must be added the corrections of page 1327, so that they become 13,65 and 12,62. From the 2nd criterion we likewise find

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the number of stars having on the one plate the magnitude m_1, on the other m_2 becomes

$$f(m_3)\ \exp\cdot(-h^2(m_1-m_2)^2)\ dm_1\ dm_2.$$

If m_0 is the limiting magnitude, so that $m_2 \gtrless m_0$ means invisibility or visibility, then the number of invisible and the number of visible stars of the magnitude m_1 is given by:

$$dm_1 \int_{m_0}^{\infty} f(m_3)\ \exp.(-h^2(m_1-m_2)^2)\ dm_2 \quad \text{en} \quad dm_1 \int_{-\infty}^{m_0} f(m_3)\ \exp.(-h^2(m_1-m_2)^2) dm_2.$$

For $m_1 = m_0$ these two are not equal, in consequence of the factor $f(m_3) = a + b\,\dfrac{h_1^2 m_1 + h_2^2 m_2}{h_1^2 + h_2^2}$.

The number of bright invisible stars, that have therefore $m_1 < m_0$, $m_2 > m_0$ and the number of faint visible stars that have $m_1 > m_0$ and $m_2 < m_0$ is

$$\int_{-\infty}^{m_0} dm_1 \int_{m_0}^{+\infty} dm_2\, f(m_3)\ \exp.(-h^2(m_1-m_2)^2)$$

and

$$\int_{m_0}^{\infty} dm_1 \int_{-\infty}^{m_0} dm_2\, f(m_3)\ \exp.(-h^2(m_1-m_2)).$$

These two double-integrals are equal, m_1 and m_2 being completely interchangeable here.

13,72 and 12,63 or, corrected, 13,70 and 12,71. The difference in the limiting magnitudes B_1 and A_2 according to the first criterion is 1,03, according to the second 0,99; this, as we expected, is less than the difference in magnitude 1,07; yet they do not differ as much as might have been expected. The good concurrence of these two values is no proof for their accuracy, as they have been arrived at by means

TABLE II.

m	n_1	n_2	n_1^0	n_2^0	p_2	s_2	s_1
14.5	2		2				
14.45	0		6				
14.4	4		5		0.		
14.35			5	1	17		
						1	
14.3	0	1	6	2	25		
						3	
14.25	5	1	8	3	27		
						6	
14.2	3	1	13	2	13		
						8	
14.15	5	0	16	1	6		
						9	
14 1	8	0	24	0	0		
						9	
14.05	11	0	36	3	8		
						12	
14.0	17	3	38	9	19		
						21	
13.95	10	6	48	18	27		
						39	
13.9	21	9	37	17	32		
						56	171
13.85	6	2	29	12	29		
						68	142
13.8	2	1	21	13	38		
						81	121
13.75	13	10	18	11	38		
						92	103
13.7	3	0	30	27	47		
						119	73
13.65	14	17	19	20	51		
						139	54
13.6	2	3	21	30	59		
						169	33
13.55	5	10	9	13	59		
							24
13.5	2	0	9	26	74		
							15
13.45	2	16	4	17	81		
							11
13.4	0	1	3	21	88		
							8
13.35	1	4	3	16	84		
							5
13.3	2	11	3	16	84		
							2
13.25		1	2	18	90		

m	n_2	n_3	n_2^0	n_3^0	p_2	s_3	s_2
13.2	6		12		100		
13.15	5		21		100		
13.1	10		24		100		
13.05	9		27	1	96		
						1	
13.0	8	1	31	5	86		
						6	
12.95	14	4	27	7	79		
						13	
12.9	5	2	26	8	76		
						21	
12.85	7	2	15	5	75		
						26	112
12.8	3	1	16	4	80		
						30	96
12.75	6	1	14	4	78		
						34	82
12.7	5	2	17	7	71		
						41	65
12.65	6	4	13	12	52		
						53	52
12.6	2	6	11	12	48		
						65	41
12.55	3	2	6	9	40		
						74	35
12 5	1	1	8	7	53		
						81	27
12.45	4	4	6	5	55		
						86	21
12.4	1	0	8	8	50		
						94	13
12.35	3	4	5	13	28		
						107	8
12.3	1	9	5	17	23		
							3
12.25	1	4	2	14	12		
							1
12.2		1	1	15	6		
12.15		10		16	0		
12.1		5		15	0		
12.05		0		5			
12.0		0					

of cognate methods. The irregular course of the numbers $n_1^0\ n_2^0\ n_3^0$ that make up our material, renders it doubtful whether the value found is accurate up to 0,1. If we take the average 1,01, and for A_1 and A_2 13,71 and 12,70, we find for all limiting magnitudes (expressed in the provisional scale):

B 14,66; A_1 13.71; A_2 12,70; A_3 11,66; A_4 10,50; A_5 9,41.

When reduced to the real magnitudes, the limiting magnitude is therefore:

B 14,30; A_1 13,46; A_2 12,57; A_3 11,65; A_4 10,62; A_5 9,66

and the differences in limiting magnitude become:

0,84 0,89 0,92 1,03 0,96 magn.

4. *Results*. In the square that was examined the stars are not regularly distributed. The greatest density is found on the N. and W. sides; it seems as if two star-clouds, one from above, and one from the right, stretch into this region, divided by a region of less density, reaching towards the S.E. Below lies a triangular, very poor region. Herein as a kind of core, lies the three-armed void, which in the photographs of the Galaxy taken by MAX WOLF and BARNARD, shows like a black spot or hole[1]). On dividing the field into 5 regions of equal size, each of 20 squares, (the outlines of which have been indicated on Table I by means of thicker lines), so that I and II comprise the densest, III and IV the medium, and V the poorest region, we find for the numbers of stars:

	I	II	III	IV	V	Sum total	$\log N$	m	$\frac{d \log N}{dm}$
B	2169	2100	1571	1513	801	8154	3.099	14.30	
									0.52
A_1	746	787	601	584	279	2997	2.665	13.46	
									0.36
A_2	336	360	297	283	145	1421	2.341	12.57	
									0.43
A_3	136	142	127	116	48	569	1.943	11.65	
									0.39
A_4	55	51	49	47	22	224	1.538	10.62	
									0.63
A_5	14	13	12	9	7	55	0.928	9.66	

The values resulting herefrom for *log N*, the amount per square degree, and for the gradient, are to be found for the entire square in the last columns, for the five minor regions in the following list.

The gradients for the entire region present a few irregularities. The differences between the last 3 values can be attributed to acci-

[1]) Compare e.g. MAX WOLF, Die Milchstrasse, Fig. 33 and 34.

log N					$\frac{d \log N}{dm}$				
I	II	III	IV	V	I	II	III	IV	V
3.22	3.21	3.08	3.07	2.79					
					0.55	0.51	0.49	0.50	0.55
2.76	2.78	2.67	2.65	2.33					
					0.39	0.37	0.35	0.35	0.32
2.41	2.44	2.36	2.34	2.05					
					0.41	0.42	0 39	0.41	0.52
2.02	2.04	1.99	1.95	1.57					
					0.38	0.44	0 40	0.38	0.33
1.63	1.59	1.58	1.56	1.23					

dental irregularities or to errors, not so however in the case of the two former ones. This is proved by the fact that in all five regions the 2^{nd} gradient is smaller, and the first larger than the others. To all probability the reason for the smallness of the second gradient must be attributed to the fact that the real difference in the limiting magnitudes is smaller still — so that the influence of overlooking the faintest stars on S and A_1 is yet stronger — than was ascertained and accepted above. On account of this therefore all the second gradients should be somewhat larger. The interval $B—A_1$, the first gradient, does not change in this case, as the countings on B and A_1 are absolutely similar.

The first gradient is larger than the others. Here then is manifest *the influence of the distant galactic condensations, which therefore is perceptible in the gradients only after the* $13{,}5^{th}$ *magnitude.*

The fact that the gradients in region V do not essentially differ from those of the other regions, allows us to draw some important conclusions. This region must be considered as a weakened extension of the tripartite dark hole that forms its core. The cause of the lacking of stars in this hole, extends gradually weakening, over a wider region. As a first explanation we may admit that this cause consists in a local diminished space-density of the stars, so that there is an actual hole between and in the dense star-clouds that constitute the galaxy. In this case the nearer stars are not influenced thereby, so they must show no thinning, the brighter stars will be relatively more numerous than the faint ones, and the gradient must be smaller than in the denser regions. Of this the numbers show nothing; the stars from the 10^{th} to the 14^{th} magnitude are all diminished to an equal rate. This would imply that these brighter stars for the greater part belong to the galactic clouds themselves and are situated at the same great distance. This supposition, however, is excluded by the value of the gradients between the 10th and the 13th magnitude.

A second explanation is the admittance of absorbing nebulous masses. If such nebulous matter should exist in the regions of the galactic condensations, only the more distant stars would be dimmed, and the phenomena would be the same as in the former case, a relative excess of brilliant stars. From the numbers found, it therefore becomes evident, *that the absorbing dark nebulous mass causing the tipartite hole, is so near as to dim also the majority of the stars of the 10th and 11th magnitude It stands in no organic connection to the galactic clouds, being only accidentally projected against that clear background.*

5. *Comparison with other results.* Our former investigations [1]) stated for the galactic region in Aquila a strong increase of the gradient up to far over 0,6. These results however cannot be immediately compared with the present ones, another scale of magnitudes having been used. The scale of Groningen Publ 18 that was employed there, needs increasing corrections to reduce them to the visual Harvard scale, in order to obtainthe photographic magnitudes, belonging to *log N*, still increasing positive corrections have to be added, as the average colour-index increases for the fainter stars. [2]) With these corrections we get:

	(*m* Gr. 18)	*m* vis.	*m* phot.	*log N*	$\frac{d \log N}{dm}$ vis		$\frac{d \log N}{dm}$ phot.
1. B D.	9 24	9.35	9.76	0.898			
2. C. d. C. Catal.	11.73	11.97	12.52	2.249	1—3	0.47	0.44
3. EPSTEIN.	12 51	12.89	13.51	2.557	2—4 3—4	0.54 0.76	0.51 0.72
4. C. d. C. Chrt	13.20	13.74	14.41	3.205	3—5 4 5	0.79 0.82	0 74 0.76
5. HERSCHEL.	13.90	14.65	15.39	3.948			

Here an increase of the gradient from the 13th magnitude (phot.) upward is perceptible; this therefore corresponds to the results now obtained. But the values of the gradients now obtained are consi-

[1]) Researches into the structure of the Galaxy. Proceedings R. A. S., Amsterdam, June 25th 1910.

[2]) P. J. VAN RHIJN. On the number of stars of each photographic magnitude. Publ. Groningen N°. 27.

derably less than those resulting from the former investigation (now 0,52 from 13,5 to 14,3, then 0.72 from 13,5 to 14,4) which clearly pointed to the presence of large, more distant star-condensations. Now the photographic scales are absolutely independent of one another, and therefore they are not perhaps immediately comparable. If e.g. the reduction-factor employed, 0,884, were somewhat too large, (so that a tenfold exposure would mean a gain of a little under $2 \times 0^m,884$) in the present investigation all the m's and all their differences would become smaller, and the gradients larger. In how far this is actually the case cannot be ascertained with accuracy. In any case by means of these triplet-photographs we penetrated less deeply into the fainter stars than at the former investigation. When the project for these photographs was made it did not seem really difficult to penetrate further than HERSCHEL's gauges, the limiting magnitude of which then was found to be 13,9. On account of the scale-reductions since obtained, this purpose, as has now become evident, has not yet been accomplished. It would have required an instrument with a larger opening, or a far greater time of exposure.

In order to advance further, Prof. HERTZSPRUNG, at my request, made a few more photographs with the 80 cm. refractor at Potsdam. To immediately fix the scale of magnitudes on the plate, a coarse grating was placed before the objective. so that the central image is weakened $0,^m748$ and the 1st and the 2nd diffraction-image become $2,^m242$ and $3,^m317$ fainter than the central image. On a plate with the centre on 46 Aquila ($19^h37,^m5 + 11^\circ57'$) on 0,343 square degrees 858 stars were counted, of which 101 present the first, and 24 the second diffraction-image. From this we find for

	m_0	m_0—2,24	m_0—3,32
$\log N$	3,398	2,469	1,845

from which result the gradients $\frac{d \log N}{dm} = 0{,}41$ and 0,58. This plate penetrates somewhat further than the Triplet, for from the comparison with the numbers round about 46 Aquilae that were found in Table I, there results $m_0 = 14{,}8$. Here the gradient from 12,6 to 14,8 proves to be only 0,41. The smallness of this amount is probably due to the fact that far fainter side-images were included than principal images, their place being accurately known. Here again it is evident, how easily, through dissimilarity of conditions systematic differences may occur in the amounts of stars counted, which render them useless for the deduction of gradients.

Another means of penetrating still further are the plates taken by FRANKLIN-ADAMS; according to CHAPMAN and MELOTTE they go below the 17^{th} magnitude (photogr.), which is also proved by the numbers they give. For the galactic zone the number N for the magnitude 15,0 16,0 17,0 according to their original statements was 650, 1300, 2050; these numbers, on account of erroneous formation of mean values are too small, and later on Dr. CHAPMAN gave for the two former 840 and 1700 ($log\ N$ 2,92 and 3,23)[1]) that is 29°/₀ and 31°/₀ more: if therefore we take the latter 33°/₀ larger, the number for 17,0 becomes $N = 2800$. The table of VAN RHIJN for these N gives the photographic limiting magnitudes 15,2 16,2 and 16,9, which proves that the stars up to 17,0 have been incompletely counted. From the values of N, deducted in Gron. Publ. 18 for HERSCHEL viz. $N = 175{,}6$ 373 1023 for the 3 zones 40—90, 20—40 and 0—20 galactic latitude, follows the photographic limiting magnitude for HERSCHEL 15,30 15,18 and 15,17. The countings therefore, indicated by CHAPMAN and MELOTTE with 17,0, penetrate $1\frac{1}{2}$ magnitude further into the faint stars than HERSCHEL's gauges.

The separate countings on plate 136 (A.R. 20^h,0; decl. 15°) containing the region of Aquila, have been kindly put at my disposal by Prof. DYSON. For this plate the limiting magnitudes have not been determined photometrically, so that $\frac{d\,log\,N}{dm}$ cannot be strictly deduced. If for the m the average values are taken, then we find (as the average of 6 regions, situated in the Galaxy in Aquila and Sagitta)

$m =$	14,4	15,3	16,3	17,0
$N =$	965	3445	11883	14310
$\frac{d\,log\,N}{dm} =$	0,61	0,53	0,12	

This last difference once more proves that CHAPMAN and MELOTTE have counted the faintest stars very incompletely, in these dense galactic regions even more so than elsewhere. Also in the other differences little is to be detected of the strong gradient that might have been expected from HERSCHEL's numbers. CHAPMAN has treated also the densest parts of the galactic zone separately, and finds for it:

for $m =$	13	14	15	16
$log\ N =$	2,63	3,07	3,37	3,60

[1]) S. CHAPMAN. The number and galactic distribution of the stars. Table A *Monthly Notices* 78. p. 70.

Thus the gradients become 0,44 0,30 and 0,23. These numbers again show no trace of a spacial-condensation in distant galactic clouds.

The contradiction that appears in all these results, and that has repeatedly disappointed the hope of penetrating further than HERSCHEL, can be summarised thus: *in the bright galactic clouds the Franklin-Adams plates show hardly any greater amount of stars than did the gauges of* HERSCHEL, although, as far as the average numbers are concerned, they go far deeper. On the region of plate 136 the countings of CHAPMAN and MELOTTE give 9340 stars per square degree, and HERSCHEL 7500, whereas the average of the entire galactic zone with the one surpasses 2800, and with the other only amounts to 1023.

It is not immediately clear what may be the cause hereof. The most plausible explanation is, that the countings of the faintest stars on the Franklin-Adams plates in the densest regions are far more incomplete than in other regions. Another explanation would be, that in the bright, dense galactic clouds the colour-index is higher, so that there the average of the stars would be redder than in the average of the galactic zone. In this case with countings on photographic plates, no matter how complete, we advance less than with visual countings by means of a telescope with a wide opening. So far therefore we cannot penetrate further into the depths of the galactic clouds than HERSCHEL did; our material reaches hardly any further than that collected by WILLIAM HERSCHEL more than a century ago. That nothing has been done during the whole of the 19th century to complete and correct his work, is doubtlessly due to the fact that the photographic method with regard to the counting of stars promised so much more, but has failed as yet to fulfill its promise. The numerous systematic differences which the photographic method involves, — the decrease of star-density towards the borders, the greater influence of atmospheric absorption, the variation in limiting magnitude — all this renders it extremely difficult, to deduce a homogeneous material from a photographic survey of the sky. If we consider, moreover, that the faintest stars, the main object of investigation, as an average have a higher colour-index, it becomes yet more evident how desirable visual countings with instruments of high power are for the study of the galatic condensations.