

# BULLETIN OF THE ASTRONOMICAL INSTITUTES OF THE NETHERLANDS.

1922 July 7

No. 19.

## COMMUNICATION FROM THE ASTRONOMICAL INSTITUTE OF THE UNIVERSITY OF AMSTERDAM.

### Ionization in stellar atmospheres, by *A. Pannekoek*.

Spectral analysis has disclosed the chemical constitution of stellar atmospheres by the lines visible in their spectra. As to their physical state we may infer the temperature from these spectra also, as the series of spectral types, at least from *B* to *M*, corresponds to a series of decreasing temperatures. But this temperature is not deduced directly from the spectral lines — though they act as indicator of the types — but from the distribution of energy over the spectrum, i. e. the colour of the star. In some single cases only a spectral line (e. g. the *Mg* line 4481) has been used as indicator of temperature directly.

The deduction of the physical conditions in stellar atmospheres from the lines of their spectra has now become possible by the application of the theory of chemical equilibrium on partly ionized gases by Dr. MEGH NAD SAHA (*Philos. Mag.* 1920, 472; 1921, 267; *Proc. Roy. Soc. A* 99, 135). By assuming that free electrons may be treated as atoms of a gas with atomic weight 1/1847 he derived the formula of equilibrium

$$\log p \frac{x^2}{1-x^2} = -\frac{U}{4.571 T} + 2.5 \log T + 6.49 = K.$$

where  $x$  is the ionized,  $1-x$  the non-ionized fraction of the gas,  $p$  its pressure,  $T$  its absolute temperature, and  $U$  the amount of energy (in calories) required to ionize a gram-atom of the gas. This energy may be computed from the ionization potential, which has been determined experimentally for several elements (*H, He, N, O, A, Na, Mg, K, Ca, Zn, Cd, Rb, Cs, Hg*) and which for elements with spectral series may also be deduced by the relation  $Ve = h\nu$  from the frequency of the limit of the principal series. \*) For a mixture

\*) Though for most elements the ionization potential is not yet known, we may say that in each horizontal row of the periodic system of elements it increases (by the increasing stability of the outer ring of electrons) for the higher atomic numbers, while in each vertical column, by the increasing dimensions, it decreases for the higher atomic numbers.

of several gases this formula, as has been lately shown by H. N. RUSSELL (*Aph. J.* 55. 121 1921—22), must be replaced by:

$$K_1 = \log p \frac{x_1}{1-x_1} \frac{x_0}{1+x_0}; K_2 = \log p \frac{x_2}{1-x_2} \frac{x_0}{1+x_0}; \text{ etc.}$$

where  $x_0$  denotes the „average ionization” for all the elements present and  $p$  is the total pressure.

By means of his formula Dr. SAHA has computed  $x$ , the amount of ionization for some elements for different temperatures and pressures and compared it with the relative strength of the enhanced lines (which are emitted by ionized atoms) and the ordinary arc lines. So he was able to establish the scale of temperatures corresponding to the series of spectral classes. Such results, however, are necessarily somewhat vague, because we do not know what pressure should be adopted for the part of the stellar atmosphere causing the spectral lines.

#### *The ionization curve.*

The relations between the different quantities will be more easily shown by making use of a diagram. Putting

$$\log T = \tau; \log \frac{x}{1-x} \frac{x_0}{1+x_0} = \xi; \log \frac{U}{4.571} = 3.702 + \log V = z,$$

the formula takes the form

$$\log p + \xi = -10^z - \tau + 2.5 \tau - 6.49$$

Taking  $\tau$  and  $-\log p$  (high pressure below, corresponding to the deepest layers) as coordinates, this formula represents a curve of constant ionization. As for different values of  $\xi$ , denoting different degrees of ionization, the curve is simply displaced vertically, it is sufficient to draw it for  $\xi=0$ . The only parameter  $z$  depends on the ionization potential of the element. The figure of these ionization curves is iden-

tical for all elements; an increase of 0.1 in  $\kappa$  shifts them 0.1 to the right and 0.25 down. For  $\kappa = 4.5$  the curve is determined by the points:

$\tau =$	3.3	3.4	3.5	3.6	3.7	3.8	3.9
$-\log p =$	14.1	10.6	7.74	5.43	3.55	2.00	0.72
$\tau =$	4.0	4.1	4.2	4.3	4.4	4.5	
$-\log p =$	-0.35	-1.25	-2.02	-2.68	-3.25	-3.76	

Above this curve is the region of ionized atoms, below is the region of the neutral ones. For increasing  $\kappa$  the ionization curve glides along the dotted line, which has a scale of  $\kappa$ . The values of  $V$  and  $\kappa$  used are:

	<i>K</i>	<i>Na</i>	<i>Ba</i>	<i>Sr</i>	<i>Ca</i>	<i>Mg</i>	<i>Zn</i>
$V =$	4.32	5.11	5.19	5.67	6.09	7.61	9.35
$\kappa =$	4.34	4.41	4.42	4.46	4.49	4.58	4.67
	<i>H</i>	<i>He</i>	<i>Ba +</i>	<i>Sr +</i>	<i>Ca +</i>	<i>He +</i>	
$V =$	13.54	25.2	9.86	10.70	11.86	53.4	
$\kappa =$	4.83	5.10	4.70	4.73	4.78	5.43	

#### The atmospheric curve.

We suppose the stars, according to the ideas of EDDINGTON, to be gaseous spheres in radiative equilibrium. Moreover we suppose that all heat is coming from the interior of the star, the outer layers, that are considered here, contributing no appreciable amount to it. Then each volume-element of these layers will emit just as much energy as it absorbs from all the rays traversing it in every direction. Putting  $\rho$  the density and  $k$  the coefficient of absorption for  $\rho = 1$ , the emission will be  $k\rho cT^4 dv$  ( $cT^4$  denoting the black body emission), where  $T$  depends only on  $z$ , the vertical ordinate, counted positive inwards, and the absorption of a ray  $A$  will be  $k\rho A ds$ . By putting  $k\rho dz = dy$  the differences of density are eliminated,  $y$  representing the „homogeneous depth” in the atmosphere.

A first approximation has been given by SCHWARZSCHILD (*Gött. Nachr.* 1906, 41). He supposes only two streams of energy flowing inward and outward and he finds the simple formula

$$T^4/T_0^4 = 1 + y \quad T_r^4 = 2 T_0^4$$

where  $T_0$  is the temperature of the outermost layers and  $T_r$  is the effective temperature, deduced from the emerging stream of heat.

The homogeneous depth  $y$  has a simple relation to the pressure:

$$dp/dz = g\rho \quad \text{or} \quad dp/dy = g/k; \quad \text{thus} \quad p = (g/k)y$$

for the layers, for which  $k$  and  $g$  may be taken constant. Thus we find

$$(k/g)p = T^4/T_0^4 - 1.$$

In this relation, connecting temperature and pressure

in a stellar atmosphere, the vertical distribution of density is eliminated.

In an exact treatment of the radiative equilibrium streams of energy flowing in all directions must be considered; their intensity  $A(\vartheta) d\omega$  is a function of the depth  $z$  and the angle  $\vartheta$  with the radius. For each stream the variation by absorption and emission is given by

$$\cos \vartheta \frac{dA}{dz} = -k\rho A + k\rho I$$

$I d\omega$  being the emission of a volume element. The condition of equilibrium is found by integrating  $d\omega$  over the whole sphere:

$$\int k\rho A d\omega = \int k\rho I d\omega = 4\pi k\rho cT^4.$$

These formulae lead to an integral equation for  $cT^4$  containing integral logarithms; it has been treated in different forms by KING, SCHWARZSCHILD and MILNE\*). The solution can only be found by successive approximations; in the best solution, computed by MILNE, the emission  $cT^4$  does not much deviate from a linear function  $a + 2by$  [with  $a = 1.362b$  and the resulting outward flow of energy, determining the effective temperature  $8/3 b = 1/2 (a + b) T_r^4/T_0^4$ ]. It is given by his Fig. 2 and his Table I (*l. c.* p. 370–371); from this table we take the following values of  $\log T/T_r = \tau - \tau_r$  as a function of  $\log y$ .

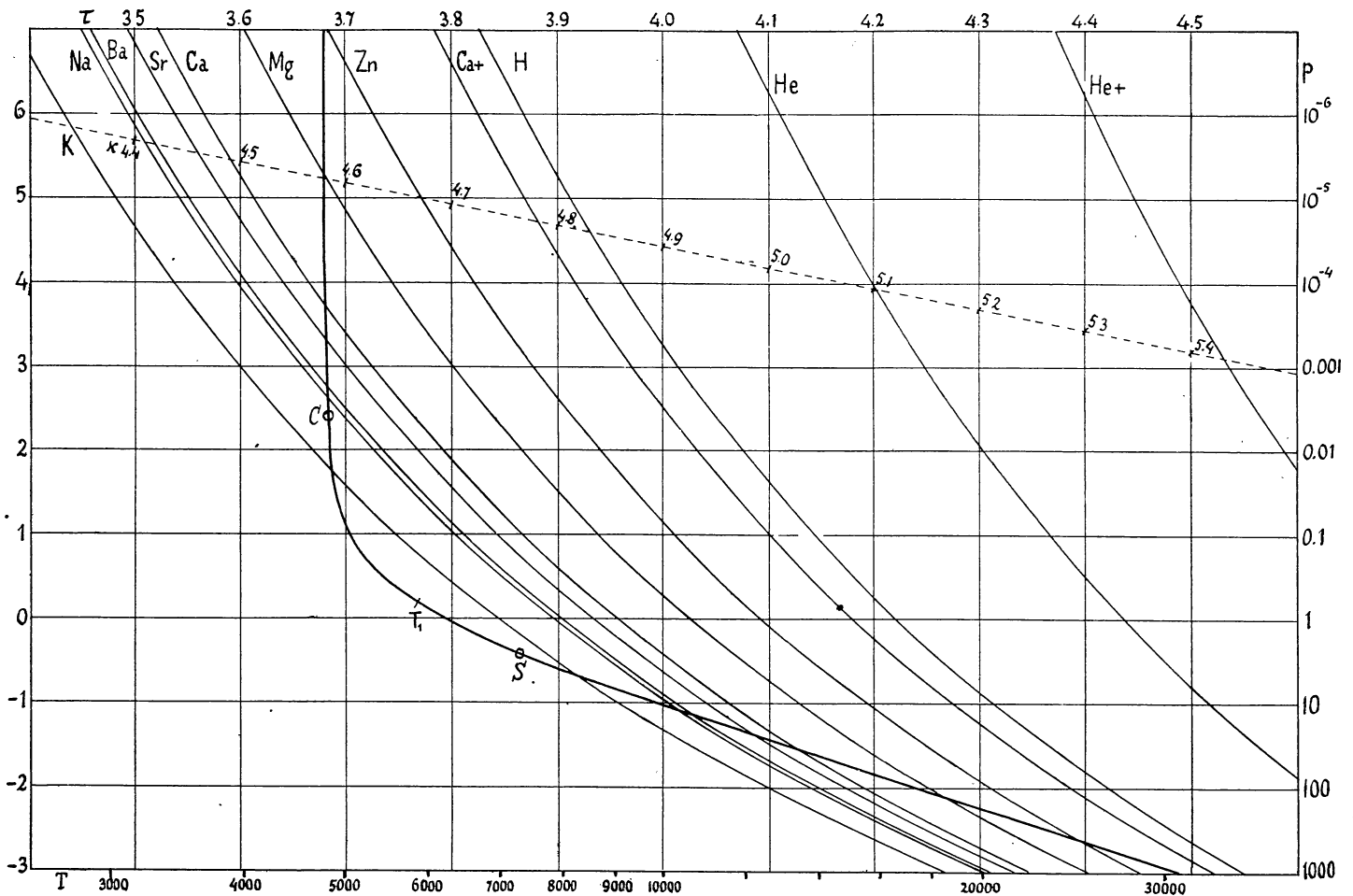
$\log y$	$\tau - \tau_r$	$\log y$	$\tau - \tau_r$
$-\infty$	-0.088	+0.20	+0.058
-1.70	-0.81	+0.30	+0.076
-1.30	-0.74	+0.60	+0.136
-1.00	-0.63	+0.78	+0.175
-0.70	-0.49	+0.90	+0.203
-0.40	-0.23	+1.00	+0.226
-0.22	-0.04	+1.30	+0.298
-0.10	+0.12	+1.70	+0.395
0.00	+0.25	+2.00	+0.469
+0.08	+0.38	+3.00	+0.719

This table, giving  $\tau - \tau_r = f(\log y)$  solves the problem of the relation between temperature and pressure in a stellar atmosphere. For, as  $p = (g/k)y$ , we have

$$\tau - \tau_r = f(\log p - \log g/k).$$

This relation is represented in our diagram by a curve, which may be called the atmospheric curve. Its form is wholly given by theoretical deductions and is identical for all stars, for which our suppositions

\*) K. SCHWARZSCHILD, Ueber Diffusion and Absorption in der Sonnenatmosphäre (*Sitz. Berichte Berlin* 1914, 1183); E. A. MILNE, Radiative equilibrium in the outer layers of a star (*Monthly Notices R. A. S.* 81, 361, 1921).



on radiative equilibrium are valid. Its position for different stars depends horizontally on the effective temperature ( $\tau_x$ ) and vertically on the factor  $g/k$ . In the diagram the curve is drawn for  $\tau_x = 3.768$  (effective temperature of the sun  $T_x = 5860^\circ$ ) and  $g/k = 1$ ; the point  $T_1$  indicates  $\tau - \tau_x = 0$ .

The atmospheric curve at first runs nearly vertically for the boundary temperature  $T_0$  (which according to the table is  $0.816 T_x$ ); between  $y = -1$  and  $+1$  it makes a sharp bend, and then becomes nearly straight. In the diagram it intersects the ionization curves twice. As for the highest temperatures and pressures the atmospheric curve has a final direction  $d \log p / d \tau = 4$  while the ionization curve tends to a limiting value  $d \log y / d \tau = 2.5$  it would seem that there must be still a third intersection, where the atmospheric curve sinks definitely below the ionization curve. But for the interior of the star with increasing density and heat  $k$ , according to EDDINGTON, must increase very strongly. So the atmospheric curve is raised in its lower parts and probably its inclination will be smaller than for the ionization curve; the temperature and pressure computed by EDDINGTON for the interior will indeed (cf. EGGERT, *Physikalische Zeitschrift* 20.

570. 1919) cause a much higher degree of ionization than anywhere in the atmosphere. Thus we have only two intersections of the curves. While ionization is considerable in the outer atmosphere and is still greater in the interior there are layers between them, where the ionization is a minimum or absent. They are situated where the atmospheric curve has its sharpest bend, for  $\log y$  about  $-1$  to  $0$ .

#### *The gravitational distribution.*

If part of the atoms are ionized the free electrons are assumed to behave as a gas with atomic weight  $1/1847$ . They tend towards a distribution according to this weight, striving outward, but are hindered therein by their negative charge. In the state of equilibrium part of the electrons are occupying a higher level than their ions, and both electrons and ions are affected by gravitational and electrical forces at the same time. Putting  $p_1$  and  $p_2$  the partial pressure of the positive ions and the negative electrons,  $\mu_1$  and  $\mu_2$  their atomic weight,  $H$  the gas constant,  $e$  the charge of an electron,  $m$  the mass of a hydrogen atom, and assuming that outside of a sphere with radius  $r$

there are  $n$  electrons, whose ions are within, so that the sphere has a positive charge  $ne = E$ , we have the conditions of equilibrium for the ions:

$$\frac{d\dot{p}_1}{dz} = \frac{g\dot{p}_1\mu_1}{HT} - \frac{E}{r^2} \frac{e}{m} \frac{\dot{p}_1}{HT} = \frac{\dot{p}_1}{HT} (g\mu_1 - \gamma),$$

and for the electrons:

$$\frac{d\dot{p}_2}{dz} = \frac{g\dot{p}_2\mu_2}{HT} + \frac{E}{r^2} \frac{e}{m} \frac{\dot{p}_2}{HT} = \frac{\dot{p}_2}{HT} (g\mu_2 + \gamma),$$

where  $\gamma = E/r_2 (e/m)$ . From these formulae we find

$$\frac{d}{dz} (l\dot{p}_1 + l\dot{p}_2) = + \frac{g}{HT} (\mu_1 + \mu_2) = a$$

$$\frac{d}{dz} \left( \frac{1}{\mu_1} l\dot{p}_1 + \frac{1}{\mu_2} l\dot{p}_2 \right) = - \frac{\gamma}{HT} \left( \frac{1}{\mu_1} + \frac{1}{\mu_2} \right) = - \frac{b}{\mu_1\mu_2}$$

or

$$\dot{p}_1 = C_1 e^{\frac{a\mu_1 - b}{\mu_1 + \mu_2} z}; \quad \dot{p}_2 = C_2 e^{\frac{a\mu_2 + b}{\mu_1 + \mu_2} z}.$$

These values for  $\dot{p}_1$  and  $\dot{p}_2$ , giving the number of ions and electrons in the layer  $dz$ , determine the increase of  $E$  with  $r$ , which we will compare with the analogous increase of  $M$ , the mass of the sphere:

$$dE = d \left( \gamma r^2 \frac{m}{e} \right) = 4\pi r^2 dr \frac{\dot{p}_1 - \dot{p}_2}{HT} \frac{e}{m}$$

$$dM = d \left( gr^2 \frac{1}{f} \right) = 4\pi r^2 dr \frac{\mu_1 \dot{p}_1 + \mu_2 \dot{p}_2}{HT}$$

Thus

$$\frac{\dot{p}_1 - \dot{p}_2}{\mu_1 \dot{p}_1 + \mu_2 \dot{p}_2} = \frac{d(\gamma r^2)}{d(gr^2)} \frac{f}{(e/m)^2} = \frac{d(bTr^2)}{d(aTr^2)} \frac{f}{(e/m)^2}$$

Now from  $f = 6.66 \cdot 10^{-8}$  gram $^{-1}$  cm $^3$  sec $^{-2}$  and  $e/m = 0.957 \cdot 10^4$  gram $^{-1/2}$  cm $^{3/2}$  sec $^{-1}$  we find  $f/(e/m)^2 = 7.3 \cdot 10^{-16}$ , while  $b/a$  has a moderate value. Thus  $\dot{p}_1 - \dot{p}_2$  must be extremely small; the number of electrons leaving the sphere of their ions is excessively small, as by their charges they soon cause a strong electric field preventing their going farther away. So we may put  $\dot{p}_1 = \dot{p}_2 = \sqrt{\dot{p}_1 \dot{p}_2} = Ce^{1/2 az}$ , and as  $\mu_2$  is small compared with  $\mu_1$  we have  $a\mu_1 - b = b$  or  $b = 1/2 a\mu_1$ , showing that indeed  $a$  and  $b$  do not differ much. For neutral atoms under the same conditions the distribution is given by  $p = Ce^{az}$ . Thus it appears, that both ions and electrons behave as atoms of half the normal atomic weight. It looks as if the electron striving upward carries by the mutual attraction half the weight of the ion; by their strong electrical forces the layers of electrons and ions are only slightly separated.

It is easily seen that for a mixture of several partly ionized gases the result is not changed. All ionized atoms behave as if their atomic weight is

halved. For a doubly-ionized atom, having lost two electrons, the electrical force is doubled and it is distributed as a gas with 1/3 of its atomic weight. Generally atoms that have lost  $n$  electrons will assume a gravitational distribution (as already adopted by EDDINGTON) corresponding to an atomic weight  $\mu/(\mu + 1)$ .

As the ionization is strongest in the interior of a star and least or nearly vanishing in the superficial layers where  $y = 0.1$  to 1, the active molecular weight in these layers will be much greater than in the interior. So there can be no equilibrium; just as in a mass where heavy matter lies above lighter one, the apparently light, strongly ionized atoms have a tendency to rise to cooler levels, where they combine with electrons, and the neutral atoms have a tendency to sink into the hotter interior where they are ionized. The amount of this circulation will depend on the viscosity of the matter; if it takes place in an appreciable degree its effect will be a mixing up of the atoms of different elements and a transfer of heat from the interior to the surface. Perhaps we have here an indication as to the cause of the occurrence of so many elements of high atomic weight in the reversing layer of the sun.

For the atmosphere above the level of minimum ionization the situation is just the reverse; as the ionization increases with height there will be no instability. SCHWARZSCHILD has deduced (*Gött. Nachr.* 1906) how in the case of radiative equilibrium pressure and temperature vary with height:

$$\frac{\mu g}{T_0 H} (z - z_0) = 4 \frac{T}{T_0} + l \frac{T - T_0}{T + T_0} - 2 t g^{-1} \frac{T}{T_0},$$

where

$$\left( \frac{T}{T_0} \right)^4 - 1 = \frac{k}{g} p.$$

For great height, where the second term is the most important, this tends to the limit

$$y = \frac{k}{g} p = e^{-\mu g (z - z_0) / T_0 H}$$

in accordance with an isothermal distribution. Inserting the data for a hydrogen atmosphere of the sun we have  $\log y = 4(z - z_0)'$ , indicating that the vertical scale of the diagram is at the same time a scale of height in the solar atmosphere, in which one unit corresponds to 0".25 or 180 km; it thus embraces only the lowest chromospheric level. For the higher levels, where hydrogen, calcium and iron vapours are abundant, these equilibrium formulae give imaginary low pressures. The only explanation seems to lie in selective radiation pressure, as shown by Dr. SAHA (*Journal Dept. of Science Calcutta, Febr. 1920*) and Dr. GROOT (*Physica, I. 7; 49. 1921*). While the

general radiation pressure in the superficial layers with small  $k$  and low density of radiation is wholly insignificant, the selective pressure on some special elements (*Ca, H, Fe, Mg*) may be appreciable and cause them to ascend high above the level of the other elements. For these high level atoms we should insert a small value of  $g$  in our formulae and the scale of heights would be strongly compressed for them.

*The reversing layer.*

The question arises what part of the atmospheric curve corresponds to the layers where the Fraunhofer spectrum takes its origin. We suppose that the volume-elements of the atmosphere radiate and absorb according to KIRCHHOFF'S law (having a large coefficient of absorption for some wave lengths, a small one for others) and are scattering the light too. As each is in thermal equilibrium with the radiation traversing it, both by absorbing and radiating and by scattering, it emits and distributes over all directions just as much radiant energy as it receives in total from all the traversing rays. The coefficients of absorption and scattering (at least if we neglect the anisotropism of scattering) therefore enter in the same way into the formulae and are both included in the factor  $k$  (which will still be called simply absorption).

If for a certain wave length  $\lambda$  the coefficient of absorption is  $k_1$  (including the effect of concentration of the absorbing atoms in the mixture) the light  $k_1 \rho E d\lambda dv$  emitted radially by a volume element  $dv$  will reach the boundary with an intensity diminished as  $e^{-y_1}$ , if  $dy_1 = k_1 \rho dz$ . Thus the total intensity of this wave length will be

$$I_1 = d\lambda \int_0^\infty E e^{-y_1} dy_1.$$

The temperature upon which the black body emission  $E$  depends is determined by the mean absorption  $k$ . Putting  $k_1 = \kappa k$  and taking for  $E$  its mean value  $cT^4$  we find

$$I_1 = d\lambda \int_0^\infty cT^4 e^{-\kappa y} \kappa dy = d\lambda \int_0^\infty \kappa c T_0^4 (1+y) e^{-\kappa y} dy = \left(1 + \frac{1}{\kappa}\right) c T_0^4 d\lambda.$$

Though here the well known fact is shown, that within a strong absorption line the intensity corresponds to the black body radiation of boundary temperature, it will be necessary, for precise results, to make use of the exact emission function. In this case however, the integral

$$\int_0^\infty \kappa E e^{-\kappa y} dy = \int_0^\infty \frac{\kappa e^{-\kappa y}}{e^{c/\lambda T} - 1} dy$$

cannot be found by direct methods. We have therefore computed its value for some values of  $k$  by mechanical integration, taking  $\lambda = 4000 \text{ \AA}$  and  $T_1 = 5860^\circ$ , using MILNE'S table of  $T^4$  as function of  $y$  and neglecting the term 1 in the denominator of PLANCK'S function. So the following results were got for the total intensity of the light (the boundary emission taken as unity), for the value of  $y$  for which the integrand is maximum, and for the „mean”  $y$ , which separates the layers each contributing half the total amount.

$\kappa$	$I$	$y$ (max)	mean $y$	$I$ comp.	mean $y$ comp.
$\infty$	1.—	0	0	1	0
11.51	1.46	0	0.08	1.46	0.14
1	6.27	0.8	1.57	6.30	1.6
0.230	25.1	4.5	6.9	24.0	7.0

It appears that within these limits  $I$  may be represented rather exactly by the formula  $I = 1 + 5.3/\kappa$ , while the  $y$  for maximum does not differ very much from  $1/\kappa$  and the mean depth is nearly  $y_m = 1.6/\kappa$ .

The mean absorption  $k$  determining the gradient of temperature may be considered to consist of two parts: a general absorption  $k_0$  affecting all wave lengths (though it may be continuously variable with  $\lambda$ ), such as scattering, and the mean of all selective absorptions, each affecting a small range of  $\lambda$ . For the background of the luminous spectrum we must use  $k_0 = \kappa_0 k$ , which is smaller than  $k$ , while in the dark line the coefficient of absorption is  $k_1 = \kappa k$ , usually very large. The contrast between the line and the adjacent wave lengths, which determines the blackness of the line, is given by

$$\frac{I_1}{I_0} = \frac{1 + 5.3/\kappa}{1 + 5.3/\kappa_0}$$

According to ABBOT (*The Sun*, p. 251) the brightness in the Fraunhofer lines is estimated from 1/5 to 1/10 of the continuous background. SCHWARZSCHILD (*Sitz. Ber. Berlin 1914*. 1198) finds the centre of the lines *H* and *K* 2.5 mag. fainter than the adjacent spectrum, i. e.  $I_1/I_0 = 0.10$ . Using this value and supposing  $\kappa$  so large that  $5.3/\kappa$  may be neglected, we find  $\kappa_0 = 0.6$ . This means, that the mean absorption  $k$  is 1.7 times the general absorption, or that the sum total of all selective absorptions amounts to 40%, the general absorption to 60% of the whole absorption. It may be of interest to test this result by direct measures of the total absorption by Fraunhofer lines. If in other lines smaller values for  $I_1/I_0$  are found, these must be used and  $\kappa_0$  will become smaller; e. g. for  $I_1/I_0 = 0.05$ ,  $1/\kappa_0 = 3.6$ .

The question may now be answered in what depth the Fraunhofer spectrum originates. The mean  $y$  for the continuous background is  $1.6/\kappa_0 = 2.7$ ; i. e. half

of the light emitted at the boundary comes from higher, the other half from deeper layers. From the layers between  $y = 0.8$  and  $y = 7$  comes 80% of the light. Atoms in these depths will manifest themselves as absorption lines in the spectrum; the fainter light within these lines comes from more superficial layers. This mean  $y$  is indicated on the atmospheric curve in the diagram by  $S$  and corresponds to a temperature of  $7400^\circ$ . Of course by making our computations for other values of  $\lambda$  the numerical results would have been somewhat different; but those given here may be sufficient to fix the character of the solar spectrum. It is determined chiefly by just the layers for which the ionization is minimum and the layers immediately below them, where the ionization begins to increase.

#### The Fraunhofer spectrum.

In comparing the actual conditions of the solar spectrum with theoretical results we can only make use of elements, whose ionization potentials are known, viz: the metals of the 1<sup>st</sup> and 2<sup>d</sup> group. SAHA and RUSSELL have already given a detailed comparison of these lines; from their results we will take here only as chief features the behaviour of the resonance lines  $1s-2p$ : for  $Ba$ ,  $Sr$ ,  $Ca$  from the single line spectrum, for  $K$ ,  $Na$  and for the ionized earths the well known doublets.

$K$	{ 7699	Int. 5	$K +$	some lines 1 - 3 *)
	{ 7964	" 6		
$Na$	{ 5895	" 20	$Na +$	?
	{ 5889	" 30		
$Ba$	5535	absent	$Ba +$	{ 4934 Int. 7
				{ 4554 " 8
$Sr$	4607	Int. 1	$Sr +$	{ 4216 " 5
				{ 4078 " 8
$Ca$	4226	" 20	$Ca +$	{ 3968 " 700
				{ 3933 " 1000

By these data it might appear as if the alkaline earths are more strongly ionized than the alkali metals, while their ionization potential, on the contrary,

\*) The lines 3312.63, 3530.85, 3955.27, 3972.61, 3995.08, 4001.20, 4012.07, 4114.96, measured by FOOTE, MEGGERS and MOHLER (*Aph. J.* 55, 158) in the enhanced spectrum of Potassium coincide with lines in ROWLAND's Preliminary Table of the solar spectrum with intensities 1, 3, 1, 1, 2, 3, 00, 2, to which no other origin is ascribed. Several other equally strong lines are missing or very faint in the solar spectrum. By comparison with the table of enhanced  $K$  lines by ZEEMAN and DIK (*Proc. Ac. Amsterd.* Apr. 1922, of which advance sheets were kindly communicated by Prof. ZEEMAN) it appears that nearly all the lines conspicuous in the solar spectrum belong to the series  $S$ , while most  $P$ ,  $Q$  and  $R$  lines are faint or absent. [Exceptions are 3972.61(1), a  $P$ -line and 3955.27(1), a  $Q$ -line, while the strong  $S$ -line 3363.25 is faint in the sun]. The solar lines 3927.43(1) and 3650.54(2) may also correspond to the lines  $S$  12 and  $S$  18, which are not observed by FOOTE, MEGGERS and MOHLER.

is higher. This contradiction can only partly be explained by what is known on the behaviour of such lines. The enhanced spectrum of the alkali metals is very difficult to produce, as the absorbing electrons, just as in the Argon atom, form part of a very stable shell or ring. It seems to follow from this solar spectrum, that the doublets, produced by atoms with one single electron in the outer ring are more easily incited than the single lines of atoms with two electrons in the outer ring; the resonance potentials, however, required to incite them do not show such a difference (For  $Ba$  and  $Na$  they are 2.23 and 2.09 Volts). Still there is left a good deal of contradiction which is most marked for the  $Ba$  line, which is wholly absent while the  $Na$  doublet is very strong, notwithstanding the fact that their ionization potentials are nearly equal. This fact, already mentioned by RUSSELL (*Aph. J.* 55 140) proves, that the difference between both groups of metals cannot be wholly explained by different conditions of excitation. Perhaps the explanation may be sought for in the occurrence of metastable orbits in the case of the earths, by which the ionization effect is increased.

Turning now to the diagram, where the atmospheric curve has been drawn for the effective temperature of the sun and the arbitrary value  $\log g/k = 0$ , we must look for the position of the point  $S$  (indicating the reversing layer) relative to the ionization curves of these elements. The vertical distances of  $S$  from these curves, i. e. the values of  $\xi$  are

for	$K$	$Na$	$Ba$	$Sr$	$Ca$	$Mg$ .
	-0.3	-0.8	-0.9	-1.2	-1.4	-2.4.

From these  $\xi$  we cannot find immediately the amount of ionization, as  $\xi = \log \frac{x}{1-x} \frac{x_0}{1+x_0}$  and the mean ionization of all the elements is unknown. As, however,  $Ca$  is one of the chief elements of these layers, and only few other elements may be compared with it in importance, we may estimate its partial pressure as a moderate fraction of the total pressure. If we suppose  $x_0 = 1/5 x$  we may solve the equation for  $Ca$  and find  $x_0 = 0.08$ . Using this value we find the amount of ionization for the six elements named

$$87\% \quad 69\% \quad 63\% \quad 46\% \quad 35\% \quad 5\%.$$

For  $Ba$ ,  $Sr$  and  $Ca$  these figures are clearly too low; their ionization must be stronger to account for the strength of the enhanced lines and the faintness or absence of the arc lines. An increase of +0.7 for  $\xi$  would give the values 84%, 74%, 64%, which do not appear too high. This means that the atmospheric curve should be laid 0.7 higher, or

$$\log p - \log y = \log g/k = -0.7.$$



The certainty of this result, however, is diminished by the discrepancy with the alkali metals; the ionization found for  $K$  and  $Na$  scarcely allows an increase. Until the cause of this difference has been cleared up we must content ourselves with the somewhat vague result, that according to the state of ionization in the solar spectrum the quantity  $g/k$  must lie in the vicinity of 1 or 0.2 atm.

Still it may lead to some important conclusions as to the coefficient of absorption  $k$ . Expressing  $p$  in CGS units the two results found become  $p/y = 0.98 \cdot 10^6$  and  $0.20 \cdot 10^6$  gram.  $\text{cm}^{-1} \text{sec}^{-2}$ . With  $g = 26.7 \cdot 10^3$   $\text{cm sec}^{-2}$  we find  $k = 0.027$  and  $0.13$  gram  $^{-1} \text{cm}^2$ .

There is a wide-spread opinion that in the absorption of the solar atmosphere scattering plays the main part. Computing the coefficient of scattering by Lord RAYLEIGH's well known formula, adopting an atmosphere of hydrogen, we find for  $\lambda = 4000$  and  $5500 \text{ \AA}$ .  $k_0 = 0.0012$  and  $k_0 = 0.00032$ , values much (at least a twenty times) smaller; with this coefficient of absorption we would have  $p = 23 y$  and  $83 y$ . *The absorption in the solar atmosphere is much greater than can be accounted for by scattering.*

This conclusion is confirmed by what is known on the pressure in the reversing layer. Its mean depth was found to be  $y = 1.6/x_0$ ; the pressure in this depth  $p = (g/k)y = 1.6g/k_0$ . Taking for  $k_0$  the two values computed for scattering this pressure would be 40 and 130 atmospheres. Now the pressure in the reversing layer is always given as being some few (1–5) atmospheres; so again we reach the conclusion, that in the case of scattering alone the transparency of the solar atmosphere would be much greater and we would get light from much deeper, more compressed layers than is actually the case.

Both evidences, however, relate to somewhat different quantities. The temperature gradient and the ionization are determined by the mean absorption  $k$ , the pressure in the reversing layer is determined by the general absorption  $k_0$ . We found from the lines  $H$  and  $K$ , that  $k = 1.7 k_0$  as a minimum, and roughly speaking we may say that general and selective absorption account for equal parts in the total absorption. The suppositions that  $k/k_0$  amounts to some few units, and the results from pressure  $1.6g/k_0 = 1-5$  atmospheres, and from ionization  $g/k = 0.2-1$  atm. harmonize well with another. This means that the greater absorption required by the ionization cannot be found in the sum total of selective absorptions in the Fraunhofer lines. *The general absorption  $k_0$ , affecting the background of the spectrum must be much greater than the effect of scattering.* Besides the selective absorptions playing also an important part there must be some cause working in the same way as

scattering, diminishing the light of all wave lengths to an amount much greater (about ten times) than scattering. Perhaps this cause may be found in the irregular refractions by small irregular variations of density in the solar atmosphere, suggested by W. H. JULIUS and treated mathematically by ORNSTEIN and ZERNIKE. \*)

#### *The chromospheric spectrum.*

In the chromospheric spectrum we see the conditions in the solar atmosphere at a higher level, with smaller  $p$  and somewhat lower temperature. Here the ionization is stronger than in the reversing layer. If we try to deduce more quantitative results, however, we meet with considerable difficulties.

In the first place the physical conditions for emission and for absorption of the same lines are somewhat different; the higher series emanating from the  $2p$  or  $3d$  orbits appear less easily as absorption lines, because the orbits as a normal state of the atom occur only with strong excitation, at high temperatures and perhaps high pressure (The behaviour of helium in the Fraunhofer and the chromospheric spectrum is the best known instance). But far more important is the practical difficulty of rightly comparing intensities. Each observer expresses them by arbitrary numbers in some subjective scale of his own. In flash spectra the matter is still complicated by the influence of the height of the emission on the breadth of the line. The relation of the estimated numbers to an exactly defined intensity is not known. We may say that as long as we have the intensities of lines in celestial spectra expressed only in such arbitrary scale numbers we cannot hope to derive exact results from them.

From the list of flash spectrum lines by MITCHELL (*Aph. J.* **38** 407) we give here for the alkali metals and alkaline earths (for which SAHA has already given a detailed comparison) the chief features:

		Int. Sun.	Int. Chrom.	Height Chrom.
$Na$	{ 5895	20	10	1000
	{ 5889	30	10	1000
$Sr$	4607	1	2	350
$Ca$	4226	20	25	5000
$Ba +$	{ 4934	7	12	750
	{ 4554	8	20	1200
$Sr +$	{ 4216	5	40	6000
	{ 4078	8	40	6000
$Ca +$	{ 3933	700	80	14000
	{ 3933	1000	100	14000

\*) W. H. JULIUS, Anomalous dispersion and Fraunhofer lines (*Aph. J.* **43** 1916. vide p. 64). L. S. ORNSTEIN and F. ZERNIKE, The scattering of light by irregular refraction in the sun (*Proc. Amsterdam Ac.* **21** 115, 1917).

The increase of ionization is especially noticeable in the *Sr* lines; but as by the strong ionization of these elements in the Fraunhofer spectrum the intensities for the same element ionized and neutral are extremely different, it will be almost impossible to derive numerical relations between the ionization in the chromosphere and in the reversing layer. More appropriate to this purpose are the elements *Sc*, *Ti*, *V*, *Cr*, *Mn*, *Fe*, *Co*, *Ni* following *Ca* in the same row of the periodic system; they play a most important part in stellar spectra and their intensities are expressed by moderate, readily comparable numbers. They have, however, the drawback, that their ionization potential has not yet been measured by laboratory experiments.

For some of them we have collected the enhanced lines (according to LOCKYER) and the strongest arc lines, excluding all that were blended with foreign lines in the flash spectrum. The average intensities (*o* was taken  $\frac{1}{2}$ ,  $\infty$   $\frac{1}{4}$  etc.) are:

Neutral Arc lines.				Enhanced lines.					
Arc Spark Sun Chrom.				Arc Spark Sun Chrom.					
<i>Sc</i> (14)	12	3	0,9	1,0	<i>Sc</i> +(14)	14	17	2,4	8,0
<i>Ti</i> (18)	12	4	2,6	2,4	<i>Ti</i> +(16)	3	12	3,8	14,0
<i>Fe</i> (22)	20	7	9,2	6,1	<i>Fe</i> +(8)	1	2	3,3	10,6

Supposing tentatively that for such moderate values the numbers expressing the intensity are proportional to the concentration of the atoms we will have  $\log \frac{x}{1-x} = \log \frac{E}{N} + i$ , where *E* and *N* are the mean intensities of the enhanced and the arc lines and *i* is some constant. For  $\log E/N$  we find:

	Sun	Chrom.	Diff.
<i>Sc</i>	+0.43	+0.90	+0.47
<i>Ti</i>	+0.17	+0.77	+0.60
<i>Fe</i>	-0.44	+0.23	+0.67

According to RUSSELL for two elements of the same atmosphere

$$\frac{x_1}{1-x_1} : \frac{x_2}{1-x_2} = \frac{K_1}{K_2} = f(T, V_1, V_2)$$

( $\log f = \xi_1 - \xi_2$  = the vertical distance of the ionization curves). Here we find for  $\log K_2/K_1 = \xi_2 - \xi_1$  for

$$Ti - Sc: -0.26 + i_2 - i_1(\text{Sun}) - 0.13 + i_2 - i_1(\text{Chrom});$$

$$Fe - Ti: -0.61 + i_3 - i_2(\text{Sun}) - 0.54 + i_3 - i_2(\text{Chrom}).$$

The accordance of the results from the Fraunhofer and the flash spectrum indicates that the scales used are fairly parallel. If we knew the constants *i*, we would be able, by inclusion of similar data for *Ca*, to find the ionization potentials for these elements. Now, by assuming that these *i* are not very different, we may get the qualitative result, already probable

from other data, that the ionization potential is increasing from Scandium to Titanium and still more so from Titanium to Iron.

Assuming that in the difference  $\log E/N(\text{chr}) - \log E/N(\text{sun})$  the differences of subjective scale are eliminated we may take it as the value of

$$\log \frac{x}{1-x}(\text{chr}) - \log \frac{x}{1-x}(\text{sun}) = \xi_c - \xi_s - \left\{ \log \frac{x_o}{1+x_o}(\text{chr}) - \log \frac{x_o}{1+x_o}(\text{sun}) \right\}.$$

For the three elements used the mean is +0.58. Remedying our lack of knowledge on the mean ionization by the supposition, that it varies about 5 times slower than the ionization of each element, we get  $\xi_c - \xi_s = +0.7$ . Thus the point indicating the mean chromospheric level on the atmospheric curve is lying 0.7 higher within the region of ionization; if *S* denotes the reversing layer *C* may indicate the mean origin of the chromospheric spectrum, corresponding to a pressure of  $10^{-2.5}$  atmospheres. But the highly hypothetical character of these deductions — given chiefly as a specimen, how with really exact data important results may be reached — needs not to be stated expressly. If the estimated scale-values do not correspond to the intensities themselves but to some higher power or perhaps — in accordance with the practice of star magnitudes — to their logarithms, much larger values of  $\xi_c - \xi_s$  would be obtained. If we will apply the theory of ionization to these elements, that play a most important role in all celestial spectra, it is necessary to have their ionization potential determined by laboratory work, to investigate the behaviour of their lines under different known conditions of temperature and pressure, and to measure accurately their real intensities in the laboratory as well as in celestial spectra.

A few remarks may still be added. MITCHELL gives among his flash lines a line 4686.0 which is identified with the chief line of ionized helium, well known in Wolf-Rayet stars and nebulae; it has also been found in earlier chromospheric spectra by LOCKYER, EVERSHED a. o. (cf. LOCKYER and BAXANDALL, *Proc. R. S.* 74 546). SAHA considers the presence of ionized helium in the highest layers of the solar atmosphere as possible, as he finds for  $T = 7000^\circ$  and  $p = 10^{-6}$  a degree of ionization  $x = 0.1\%$ . According to our diagram, however, this must be considered as highly improbable. Though the temperature of the reversing layer may amount to  $7000^\circ$ , for the highest atmospheric layers it must be much lower,  $1000^\circ$  below the effective temperature. For  $\log T = 3.7$  we find  $\xi \log p = -16.9$ ; by SAHA's



formula for a single gas this would give  $x = 10^{-5}$ , but taking account the fact that the average ionization of all elements is much greater,  $x$  must become still smaller ( $10^{-9}$  for  $x_0 = 1\%$ ). If thus the presence of this line in the chromospheric spectrum is real, another explanation must be sought for than by regular ionization according to the equilibrium-theory.

Much easier will be the double ionization of the alkaline earths. The energy needed to drive away the second electron of the outer ring has been computed from the doublet series of these elements; for  $Ca+$  the ionization curve has been drawn in the diagram, while for  $Ba+$  and  $Sr+$  they lie somewhat lower. Regarding the abundance of  $Ca+$  in the highest levels of the solar atmosphere the supposition does not seem too hazardous, that doubly-ionized calcium may occur in still higher levels, and that its spectrum may be found among the coronal lines. On the spectra of doubly-ionized elements hardly anything is known by laboratory experiments; as, however, in stellar atmospheres extremely high temperatures and small pressures are found by which ionized atoms must lose a second electron (SAHA ascribes the disappearance of the lines  $H$  and  $K$  beyond the helium stars to the ionization of  $Ca+$ ) we may look here perhaps for the source of the many lines of unknown origin playing an important role in celestial spectra.

#### *The spectroscopic parallaxes.*

The importance of the enhanced lines in stellar spectra has been first remarked and often emphasized by Sir NORMAN LOCKYER, who made these lines, ascribed by him to hypothetical proto-elements, the basis for the division of the stars in an ascending and a descending branch of evolution.

In Potsdam they had already been found unusually strong in the spectra of  $\alpha$  and  $\gamma$  Cygni,  $\alpha$  Persei and  $\alpha$  Ursae minoris (*Publ. Potsdam* 7. II). The  $c$ -division which Miss MAURY separated from the main mass of stellar spectra (*Annals Harvard* 28. I) was characterised by a greater intensity of the enhanced lines. HERTZSPRUNG recognised these stars as stars with high luminosity; the spectral differences between giant and dwarf stars consist in the relative strength of enhanced and arc lines. The spectroscopic determination of luminosity and parallax at Mount Wilson is founded chiefly on the relative strength of some enhanced and some arc lines, viz the  $Sr+$  lines 4077 and 4215, the  $Ti+$  line 4290, the  $Zr+$  lines 4461 and 4494 compared with the  $Fe$  lines 4072, 4250, 4271 and the  $Ca$  line 4454. (*Contrib. Mt Wilson*. 199. 13).

Now the relative intensities of arc lines and enhanced

lines of an element in stellar spectra, as they depend on the degree of ionization of this element, is wholly determined by the relative position in the pressure-temperature-diagram of the ionization curve and the atmospheric curve, defined by

$$\log p = -10^{\tau-6.49} + 2.5\tau - 6.49$$

and  $\tau - \tau_1 = f(\log p - \log g/k)$ .

The position of the atmospheric curve depends on two physical properties of the stellar atmosphere: the effective temperature  $T_1$  and the factor  $g/k$ . Stars of the same spectral class, thus having the same effective temperature\*) will show differences depending solely on this factor  $g/k$ .

As we have as yet no data on differences of  $k$ , the coefficient of mass-absorption, we will assume it to be constant as a first approximation. So we find, that the physical quantity, directly given by the spectra used for the determination of spectroscopic parallaxes is the gravitation at the surface of the star.

As, however,  $g = fM/R^2$  and  $L = 4\pi\sigma R^2$ , if  $M$  is the mass,  $L$  the luminosity,  $R$  the radius and  $\sigma$  the surface brightness of the star we will have

$$g \sim \sigma M/L.$$

The reduction curves used at Mount Wilson are adjusted by empirical data (of trigonometrical parallaxes) for each spectral class separately. As within each class  $\sigma$  is constant\*\*) and the masses  $M$  are not very different, it is the great differences of  $L$  that causes the great differences in  $g$ , and practically the luminosity of the star may be derived from the curves. But it must be borne in mind, that this is not exact. The real quantity determined by the Mount Wilson curves is  $LM_0/M$ , if  $M_0$  denotes the mean mass for this class.

\*) Spectral class may be defined by the relative brightness of different wavelengths, determined solely by the temperature and indicated by colour index and effective wave length, or by the characteristics of important lines (hydrogen,  $H-K$ ). Though the second way is used in practice, the first definition should be preferred, as the aspect of such lines also depends on other causes. Another question is, how far the observed colour index, effective wave length and distribution of energy are modified by absorption and how they should be corrected to theoretically regular values.

\*\*) The general decrease of luminosity with advancing type for the same value of relative line intensity, which is shown (with many irregularities) by most reduction curves (Table I, *Contr. Mt Wilson* 199 13), corresponds to the decrease of  $\sigma$ , as for the same  $g$  and  $R$  smaller surface brightness means smaller luminosity. If we take account, however, of the direct influence of temperature on ionization, which acts much more strongly in the opposite direction, we must expect equal ionization in the more advanced types for much smaller  $g$  and higher luminosities, contrary to the empirical reduction curves. It looks as if this effect is compensated by some other direct influence of temperature on the spectrum.

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In order to test this theoretical result we have made use of the list of hypothetical parallaxes for double stars by JACKSON and FURNER (*Monthly Notices R. A. S.* 81 2) computed with the adopted mass  $2 \odot$ . Putting  $h$  the hypothetical,  $s$  the spectroscopic,  $t$  the real trigonometric parallax, we have the relations

$$M = 2 (h/t)^3; \bar{M} = (s/t)^2 M_0.$$

The stars from Table III were taken, for which spectroscopic parallaxes occur in the Mount Wilson Catalogue of 1646 stars (*Contr. M. W.* 199; *Aph. J.* 53 13), and the trigonometric parallaxes were also mostly taken from this source, only in some few cases completed by later determinations. Excluded

Bu. G. C.	$h$	$s$	$t$	$n$	Sp.	$\log h/t$	$\log s/t \sqrt{M_0}$
13 Ceti	53	42	52	1	G0	00	-10
335	61	66	104	2	K0	-23	-28
$\eta$ Cassiop.	152	132; 151	187	5	F8;K5	-09	-10; -19
$\gamma_1$ Androm.	19	33	10	4	K0	+28	+44
1144	24	25	73	1	F0	-48	-26
20 Pers. A	12	22	13	2	F0	-03	+43
2381	23	24	24	3	G0	-02	00
2383	25	28	7	1	G0	+55	+60
2535	29	17	22	1	F0	+12	+09
3474	22	29	40	2	G0	-26	-14
4310	67	79	79	3	G0	-07	00
4414	32	8	88	1	G5	-43	-110
$\zeta$ Cancri	44	38; 48	38	3	G0;G0	-06	00; +10
$\epsilon$ Hydrae	30	52; 16	25	Spec.	G0;F5	+08	+32; -10
5005	51	52	67	3	K0	-12	-19
$\gamma$ Leonis	29	44; 16	2	4	K0;G5	+116	+126; +84
$\xi$ Ursae	130	126; 120	158	3	G0;G0	-09	-10; -12
5805	15	12	42	1	F5	-44	-44
5811	36	44	55	1	F5	-18	00
$\gamma$ Virginis	92	50; 55	68	3	F0;F0	+13	+07; +11
42 Comae	61	55	70	3	F5	-07	-01
6578	22	11	15	5	F0	+16	+06
6842 BC.	16	9	18	2	F0	-06	-11
$\xi$ Bootis	134	158; 105	178	2	G5;K5	-12	-11; -33
$\eta$ Coron. bor.	59	55	83	2	G0	-15	-18
$\mu_2$ Boötis	28	32	46	3	G0	-21	-15
7332	47	55	46	1	K0	+01	00
$\xi$ Scorpii	45	44; 36	46	2	F5;G5	-01	+08; -16
$\sigma$ Coron. bor.	59	32; 38	44	2	G0;F8	+13	-13; -01
7642	45	58	69	2	G5	-19	-14
$\zeta$ Herc. Br.	101	76	114	5	G0	-06	-18
7783	20	22	24	3	F5	-08	+06
8038	66	63	65	2	G5	+01	-07
26 Draconis	57	55	84	4	G0	-16	-18
70 Ophiuchi	183	200; 174	187	5	K0;K5	-01	-05; -13
99 Herculis	62	69	45	5	F5	+14	+29
8679	26	32	35	1	G0	-13	-03
8933	64	83	95	4	G0	-17	-06
9114 BC.	21	26	15	3	K0	+14	+13
9650	18	17	21	1	F5	-06	+01
9979	20	30	49	1	G5	-39	-27
$\beta$ Delphini	43	46	27	4	F5	+20	+33
$\epsilon$ Equulei AB.	23	19	19	2	F0	+08	+20
$\delta$ » AB.	67	60	67	Spec.	F5	00	+05
$\tau$ Cygni	55	42	42	5	F0	+12	+20
$\alpha$ Pegasi	46	30	38	5	F5	+08	00
Kr. 60 A	157	200	261	4	M	-22	-24
12290	30	30	11	4	K0	+44	+36
85 Pegasi A	74	91	96	5	G0	-11	-02
12755	51	60; 36	36	1	G5;G5	+15	+16; -06

were Sirius, Procyon,  $\alpha$  Centauri, 61 Cygni, because by the largeness of their parallaxes the reduction curves and the spectroscopic parallaxes must have been brought too near to the real parallax; furthermore 40 Eridani, Castor and  $\mu$  Herculis, because the hypothetical and the spectroscopic parallaxes belong to different objects. The table on the left hand side of this page contains the data used: the three parallaxes in units of 0."001, the weight of  $t$ , the spectral class (usually rounded to the next half), the quantities  $\log h - \log t$  and  $\log s - \log t + \frac{1}{2} \log M_0$  in units of 0.01. For  $\frac{1}{2} \log M_0$  from SEARES, Table IV (The masses and densities of the stars. *Aph. J.* 55 179) the values  $F0: +20$ ;  $F5: +10$ ;  $F8: +05$ ;  $G0: 00$ ;  $G5: -06$ ;  $K0: -08$ ;  $K5: -10$ ;  $M: -12$  were adopted. Half the mass of the system is then given by  $\log M = 3 (\log h - \log t)$ , the mean mass of each star used is given by  $\log M = 2 (\log s - \log t + \frac{1}{2} \log M_0)$ .

Already a superficial glance at this table shows, that  $h$  and  $s$  usually deviate on the same side from  $t$  (in 31 cases, while in 9 cases only they are lying on different sides of  $t$ ). The correlation becomes still more manifest by grouping the values  $\log h/t$  and  $\log s/t \sqrt{M_0}$  according to the first named quantity and taking averages (where both components are given, their mean is taken).

Stars	$\log \frac{h}{t}$	$\log \frac{s}{t} \sqrt{M_0}$	$\log M_h$	$\log M_s$
5	+0.53	+0.53	+1.59	+1.06
6	+0.14	+0.16	+0.42	+0.32
9	+0.06	+0.13	+0.18	+0.26
8	-0.04	+0.06	-0.12	+0.12
7	-0.09	-0.05	-0.27	-0.10
7	-0.17	-0.12	-0.51	-0.24
4	-0.23	-0.25	-0.69	-0.50
4	-0.44	-0.47	-1.29	-0.94

This table confirms very clearly our theoretical result, that the quantity determined at Mount Wilson is not the luminosity itself but the luminosity modified by mass. Of course the greatest positive values have not the meaning of real determinations of mass, as they are computed from very small, uncertain trigonometrical parallaxes. But they are included, because they too afford evidence of the influence of mass; for in the case of very remote great masses both the orbital motion and the spectrum, but not the direct measures, show an appreciable parallax. If their results were sufficiently accurate they could even afford an independant determination of mass (and thus of the real parallax) because  $M = (s/h)^6$ , which formula for the case, that the spectrum of both components is observed, becomes  $M = (s_1^2 + s_2^2)^3 / h^6$ .

That the variation of  $\log M_s$  in our table comes out smaller than the variation of  $M_h$  may be readily accounted for by the fact, that the mean mass increases with the luminosity.

Some investigations which are based on the Mount Wilson spectroscopic parallaxes will now have to be considered anew from this point of view. Several results will of course still hold; so the values found by SEARES for the mean mass of each spectral class probably are still right. The small dispersion in mass found by him (*Aph. J.* 55, 184), however, is a consequence of the hypothetical and the spectroscopic parallax being both affected by mass in the same direction.\*

If now for a star independent data are known determining the luminosity itself (e. g. the trigonometrical parallax) *the spectral differences used at Mount Wilson afford a means for finding the mass of the star.* So we are now able to find the mass for all stars in the vicinity of the sun, for which reliable trigonometric parallaxes are known. If the reduction curves are first readjusted by stars of known mass, this determination may be made independent of the knowledge of the mean mass for each class. But also the published data of Mount Wilson already allow to note some cases of great and small masses. The values for  $\alpha$  Tauri  $(96/55)^2 = 3.1$ ,  $\gamma$  Serpentis  $(120/63)^2 = 3.6$ ,  $\alpha$  Bootis  $(158/75)^2 = 4.4$ ,  $\epsilon$  Hydrae AB  $(52/25)^2 = 4.3$ , Boss 5180  $(110/42)^2 = 7$ , Groombr. 2875  $(100/30)^2 = 11$ , all expressed in the mean mass of their class as unity, are samples of rather reliable great masses, the last named showing a great mass in a class *K* dwarf star. Greater masses are uncertain, as they would depend on too small trigonometrical parallaxes. On the other hand small masses can be ascertained with great accuracy, as they depend on large trigonometrical parallaxes; we note Groombr. 3689  $(24/51)^2 = 0.22$ ,  $\alpha$  Serpentis  $(25/56)^2 = 0.20$ , Ll 30271  $(23/54)^2 = 0.18$ , Ll 14146  $(17/51)^2 = 0.11$ ,  $\rho$  Persei  $(13/48)^2 = 0.07$ ,  $\epsilon$  Andromedae  $(14/63)^2 = 0.05$ ,  $\alpha$  Ursae minoris  $(10/41)^2 = 0.06$ , RT Aurigae  $(3/31)^2 = 0.01$ ,  $\rho$  Cassiopeiae  $(2/32)^2 = 0.004$ . The three last named are Cepheids or Pseudocephids, that are reduced with special curves computed from the parallactic motions; whether there is a large diversity of mass among them (regarding their mean mass we have no empirical data) or whether the cause of this divergence lies in some peculiarities of the spectrum cannot be decided without knowing more details on the lines used in these spectra.

For double stars the relative mass of their com-

ponents may be found from spectroscopic observations alone, as it is independent of the trigonometrical parallax. Among those of special interest are such giant stars of advanced type, who have companions of younger type. Three instances of them are contained in the Mount Wilson list; the following table contains the spectroscopic parallaxes of both components,  $\log s_1/s_2$  corrected for mean mass of their class, the concluded relative mass  $\log \frac{M_1}{M_2} = \log \left( \frac{s_1}{s_2} \right)^2 \frac{M_{01}}{M_{02}}$ ;  $\log R_1/R_2 = 0.2 (m_1 - m_2) - \frac{1}{2} \log \sigma_1/\sigma_2$ , where  $\sigma$  has been taken from SEARES, *Aph. J.* 55 Table XII;  $\log \rho_1/\rho_2 = 3 \log R_2/R_1 + \log M_1/M_2$ .

Star	$m_1$	$m_2$	Spectr.	$s_1$	$s_2$	$\frac{s_1}{s_2} \sqrt{\frac{M_{01}}{M_{02}}}$	$\log \frac{M_1}{M_2}$	$\log \frac{R_1}{R_2}$	$\log \frac{\rho_1}{\rho_2}$
$\alpha$ Herculis	3.5	5.4	<i>Mb F9</i>	7	18	-0.55	-1.10	1.24	-4.82
<i>Bu</i> 8114	6.5	8.6	<i>K2 F0</i>	7	6	-0.22	-0.44	1.18	-4.78
$\gamma$ Delphini	4.5	5.5	<i>K1 F6</i>	21	32	-0.35	-0.70	0.76	-2.98

These stars thus show the curious fact *that the bluer component has a greater mass (13, 3 and 5 times greater) than the much bigger brighter star of redder color*; its density is 1000—60000 times greater. For the other double stars with both components in the Mt. Wilson list, where the smaller star has the same or a more advanced type, it has at the same time a smaller mass. Grouping them according to the difference of magnitude we find:

Stars	$m_1 - m_2$	$\log \frac{M_1}{M_2}$
10	0.3	0.09
9	0.8	0.18
10	1.3	0.26
9	2.9	0.36

The masses are the more different, as are also the spectral classes, as the difference in magnitude is larger.

It does not appear safe to make here a more extensive use of the published Mt. Wilson data. As the empirical reduction tables have been deduced in the supposition, that the luminosity alone determines the relative intensities of spectral lines, it will be at first necessary to readjust them by means of stars, for which luminosity and mass are known; only in this way we may free ourselves from the statistical element of mean mass. The data for these curves then become rather meagre, as only double stars with measured parallaxes can be used; but this will partly be made good by the greater mutual concordance of the single data.

While we gain in this way the determination of

\*) From this apparently small dispersion in mass RUSSELL already suspected that the mass must in some way have falsified the real luminosities (*Aph. J.* 55, 238).

a new element, the mass, instead of having a double determination of the element luminosity, it may seem that we have a decided loss for the more remote stars, that are only accessible by their spectra, as by these spectra we cannot find luminosity and parallax, but only a combination of two unknown

elements, luminosity (or parallax) and mass. It may be expected, however, that a more minute and quantitative investigation of stellar spectra will reveal other characteristics, which in some other way than the state of ionization are connected with diameter, density, temperature, mass and luminosity.