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1. THE LOCAL STARSYSTEM DEDUCED FROM THE DURCHMUSTERUNG CATALOGUES

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INTRODUCTION.

In investigating the structure of the universe our first aim is to find the distribution of the stars in space, i.e. the space density of the stars (the number of stars per unit volume) at every point of space. In this investigation the arrangement of the single stars, to be found only for a small number of stars in our neighbourhood whose parallaxes are measurable, will not concern us. The distribution of the stars in space manifests itself in the number of stars of different magnitude and their distribution over the sky. The irregularities of this distribution are shown also in the phenomenon of the Milky Way, whose brilliancy is caused by the accumulated light of densily clustered star masses. Both kinds of observational data have been used in theories on the structure of the universe.

The first investigations of WILLIAM HERSCHEL and STRUVE, based on limited data and proceeding from the simplifying assumption of equal luminosity of all the stars, established the fact of the predominating importance of the galactic plane as region of greatest density and widest extension of the star system. A basis for more exact researches was laid in the second half of the 19th century by the Durchmusterung Catalogues (Bonn, Cordoba, Cape-Groningen) and the photometric measures (Potsdam, and especially Cambridge). In several papers, published from 1884 till 1912, SEELIGER has done the pioneering work in deducing from these data the first reliable results on the variation of the mean density with distance and its dependance on galactic latitude. The chief source of difficulties in such researches lies in the fact that two unknown laws of distribution, the law of luminosity and the law of space density, are intermingled in the number of stars per unit surface of the sky (surface density) as a function of magnitude. SEELIGER overcame this difficulty by showing that, if space density as a function of distance and star number as a function of apparent brightness are simple exponential functions, their relation is independent from the luminosity law. Thus, assuming absorbtion of light in space to be absent, he found as a first approximation that the average density at distances 5, 50, 500, 5000 parsecs must be 1.0, 0.37, 0.14, 0.051. In treating separately the different galactic latitudes it was seen that the rate of decrease was smallest in the galactic plane and increased steadily with galactic latitude. In this second approximation the density is only a function of distance and galactic latitude, and the surfaces of equal density are flattened surfaces of revolution.

The next important step was made by KAPTEYN. From a discussion of parallaxes, proper motions and starnumbers he succeeded in 1901 in deriving the luminosity law as a quadratic-exponential function of the intrinsic brightness, thus following a Gaussian error curve. Soon afterwards from photometric measures and starcounts he was able to deduce the law of increase of starnumber with magnitude down to the 14th and 15th magnitude for each galactic latitude. The prosecution of these researches in Groningen with far more and better data has confirmed the luminosity law and ameliorated its coefficients; the agreement with the Gaussian error curve over a wide range is more exact here than in any other empirical law of nature. Furthermore the first results of the

„Selected Areas" afforded a more exact law of increase of starnumbers for faint photographic magnitudes. Thus the average distribution of stars in space, as a function of distance and galactic latitude, could be derived with great exactness. The result may be summarized in these few figures: for latitude 0° the densities 0.40, 0.063 and 0.010 are reached at distances 910, 3500, 8900 parsecs, for latitude 90° at distances 250, 660 and 1230 parsecs. KAPTEYN afterwards assumed the surfaccs of equal density to be similar ellipsoids of revolution and founded upon them his theory of the universe as a rotating system in statistical equilibrium.

These results have been obtained by neglecting all differences in surface density of the stars except the mean regular variation with magnitude and galactic latitude. In this regular universe the Milky Way is considered as a continuous belt of feeble light decreasing at both sides, the visual effect of increasing stardensity with decreasing latitude. An attentive study, however, shows the Milky Way as an extremely irregular series of bright patches and clouds, sometimes divided in two branches, interrupted by dark spaces and connected by long streams. The aspect of the Milky Way reveals the fundamental importance of the irregularities in the distribution of the stars, and it stands in direct opposition to the regular decrease of space density with distance given by the formulae of KAPTEYN. Galactic clouds and streams represent agglomerations of stars with a greater space density than in the surrounding parts of space beside, before and behind them; they are special objects with a character of their own, and this character is lost if by the process of computing mean values they are dissolved into the poor regions surrounding them. Space density as a function of distance as well as surface density as a function of magnitude will show an irregular course in the direction of such a cloud. Thus it is necessary, in studying the structure of the universe, to proceed to a third approximation, treating the different galactic features as special objects and determining the distribution functions separately for all these special regions of the sky.

In so far as may be presumed as yet, the main part of the features of the Milky Way occupies the galactic plane between distances of 1000 and 10.000 parsecs. EASTON has suggested a spiral arrangement around the Great Cygnus Cloud as a core. Definite knowledge regarding the distance of the different galactic objects — which is the necessary basis for a more exact theory — is almost entirely wanting. In the near future, however, considerable progress in this matter may be expected, from methods permitting us to separate the stars of different spectral class, the chief difficulty so far lying in the extreme faintness of the stars used in these researches.

The part of the universe lying within the distance of 1000 parsecs holds the special rank of a central core in the KAPTEYN universe. We do not know, of course, whether it really holds this dominant place of central system, or is equalled or surpassed by other distant agglomerations that appear to us as galactic clouds. But for us this part of the universe, which may be called the „local starsystem", holds a special place because it surrounds our sun and thus is the chief object of all studies on the physical constitution and the motions of the stars. Nearly all bright stars (down to the 12th magnitude) belong to this system. Nor can this local system have the regular ellipsoidal symmetry of the KAPTEYN universe, as is shown by the irregularities in the distribution of these brighter stars over the sky. Here the third approximation consists in taking into account the galactic longitude as well, thus representing the space density as a function of three coordinates. The knowledge of its irregularities is required, not only for a further study of the structure of this local system itself, but also for the study of the more remote galactic regions, in order to eliminate the influence of the foreground stars through which they are seen. The data for this research are afforded in the first place by the Durchmusterung Catalogues.

DERIVATION OF FORMULAE FOR STAR DENSITY.

Formulae for a regular distribution.

1. We put

m the magnitude of a star,

M its absolute magnitude, i.e. its magnitude at a distance of 1 parsec,

r its distance from the sun,

$\rho = 5 \log r$ its modulus of distance, thus $M = m - \rho$;

β, λ its galactic latitude and longitude,

$N(m)$ the number of stars per square degree from the brightest to magnitude m,

$A(m)$ the number per sq. d. between $m - \frac{1}{2}$ to $m + \frac{1}{2}$, for practical purposes $= \frac{d}{dm} N(m)$;

$D(r) = D(\rho)$ the space density, i.e. the number of stars pro cubic parsec, compared with the same number about the sun as unity,

$\Phi(M)$ the luminosity function, i.e. the number of stars having absolute magnitudes between $M - \frac{1}{2}$ and $M + \frac{1}{2}$, contained in one cubic parsec in the region about the sun.

For the numerical values ${}^{180}/_{\pi} = 57.296$ and $\log e = 0.4343$ we will write 57 and 0.43 in our formulae, though more exact values have been used.

The number of stars of magnitude m pro square degree is given by

$$A(m) = \frac{0.2}{57^2 \times 0.43} \int_{-\infty}^{+\infty} D(\rho)\, \Phi(m - \rho)\, 10^{0.6 \rho}\, d\rho. \qquad 1)$$

If $D(\rho)$ and $\Phi(M)$ are known the function $A(m)$ may be computed directly. In researches on the structure of the universe we have before us the inverse problem: to find $D(r)$, when $A(m)$ is given by observational data. SCHWARZSCHILD has deduced formulae to solve this problem in a general way. In order to solve

$$A(m) = \int D(\rho)\, \Phi(m - \rho)\, d\rho, \quad \text{he puts}$$

$$A(m) = \int a(q) e^{iqm}\, dq; \qquad D(\rho) = \int d(q) e^{iq\rho}\, dq; \qquad \Phi(M) = \int f(q) e^{iqM}\, dq,$$

(limits always $\pm \infty$). Then

$$a(q) = \frac{1}{2\pi} \int A(m) e^{-iqm}\, dm; \qquad f(q) = \frac{1}{2\pi} \int \Phi(M) e^{-iqM}\, dM; \qquad d(q) = a(q) : f(q).$$

From the known functions A and Φ the functions a and f may be found; from the resulting $d(q)$ the function $D(\rho)$ is found by means of an integral. Though this solution is a general one it is practicable only in the case of simple functions. SCHWARZSCHILD has used it for the case of

A, D and Φ all being quadratic-exponential functions (Gaussian error functions). As the luminosity function deduced by KAPTEYN follows this formula with the utmost exactness and the number of stars A according to VAN RHIJN may also be expressed by such a formula the special solution of SCHWARZSCHILD covers the practical case. Also, it may easily be derived directly. Thus putting:

$$\begin{aligned} \log A(m) &= a + bm - cm^2 \\ \log \Phi(M) &= p + qM - rM^2 \\ \log D(\rho) &= h + k\rho - l\rho^2 \end{aligned}$$

we have

$$10^{a + bm - cm^2} = \frac{0.2}{57^2 \times 0.43} \int_{-\infty}^{+\infty} 10^{(h + k\rho - l\rho^2) + \{p + q(m-\rho) - r(m-\rho)^2\} + 0.6\rho}\, d\rho$$

$$= \frac{0.2}{57^2 \times 0.43}\, 10^{h + p + qm - rm^2 + \frac{(0.6 + k - q + 2rm)^2}{4(l+r)}} \int_{-\infty}^{+\infty} 10^{-(l+r)\left(\rho - \frac{0.6 + k - q + 2rm}{2(l+r)}\right)^2} d\rho.$$

$$= \frac{0.2}{57^2} \sqrt{\frac{\pi}{0.43\,(l+r)}}\, 10^{h + p + qm - rm^2 + \frac{(0.6 + k - q + 2rm)^2}{4(l+r)}}$$

Thus

$$\left.\begin{aligned} a = h + p + \frac{(0.6 + k - q)^2}{4(l+r)} + \log \frac{0.2}{57^2} \sqrt{\frac{\pi}{0.43\,(l+r)}} = h + p - 3.786 + \frac{(0.6 + k - q)^2}{4(l+r)} - \tfrac{1}{2}\log.(l+r); \\ b = q + \frac{r\,(0.6 + k - q)}{l + r}; \quad c = \frac{rl}{r + l} \quad \text{or} \quad \frac{1}{c} = \frac{1}{r} + \frac{1}{l}; \end{aligned}\right\} \quad 3)$$

and by reversing these formulae we find:

$$\left.\begin{aligned} h &= a - p + 3.786 - \tfrac{1}{4}\,\frac{(b-q)^2}{r-c} - \tfrac{1}{2}\log\frac{r-c}{r^2} \\ k &= q - 0.6 + (b-q)\,\frac{rc}{r-c} \\ l &= \frac{rc}{r-c} \quad \text{or} \quad \frac{1}{l} = \frac{1}{c} - \frac{1}{r} \end{aligned}\right\} \quad 4)$$

These formulae[1]), by solving the problem of finding $D(\rho)$ from $A(m)$, are fundamental for all numerical researches on the structure of the stellar system. Introducing numerical values (after KAPTEYN and VAN RHIJN) we have

$$\begin{aligned} \log \Phi(M) &= -2.394 + 0.1858\,M - 0.03450\,M^2 && \\ \log A(m) &= -4.542 + 0.724\,m - 0.0141\,m^2 && (\text{for } \beta = 0^\circ) \\ \log A(m) &= -4.903 + 0.774\,m - 0.0221\,m^2 && (\text{for } \beta = 90^\circ);\ \text{thus} \\ \log D(\rho) &= -2.532 + 0.496\,\rho - 0.0237\,\rho^2 && (\text{for } \beta = 0^\circ) \\ \log D(\rho) &= -6.219 + 1.224\,\rho - 0.0615\,\rho^2 && (\text{for } \beta = 90^\circ). \end{aligned}$$

To the total number A of stars of magnitude m stars at the most various distances are contributing, each distance $\rho \pm \frac{1}{2}$ taking part in it according to the Gaussian error curve

$$10^{-(l+r)(\rho - \rho_m)^2}; \qquad \rho_m = \frac{0.6 + k - q + 2rm}{2(l+r)} = \frac{b-q}{2r} + \frac{r}{l+r}\,m \qquad 5)$$

The value ρ_m denotes the distance which contributes most to the stars of the magnitude m; then $m - \rho_m$ is the most numerous absolute magnitude of the stars m and at the same time their mean

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[1]) They coincide, though expressed in a different form, with the formulae used by KAPTEYN and VAN RHIJN, Astrophysical Journal **52**, 300 (Contr. Mt. Wilson 188).

absolute magnitude. If $l = 0$, thus $D(\rho)$ follows a simple exponential law, these most important distances increase at the same rate as m itself; if however $l > 0$ they increase more slowly. With the numerical data given we find for magnitudes 5, 10 and 15 the distance $\rho_m = 10.75$, 13.70 and 16.65 (i.e. 140, 550, 2200 parsecs) for the galactic plane and $\rho_m = 10.33$, 12.13 and 13.93 (i.e. 117, 270, 610 parsecs) for the galactic pole. The mean absolute magnitudes come out —5.7, —3.7, —1.6 and —5.3, —2.1, + 1.1 for these cases; for $m = 5$ they are too small, because for distances below 100 parsecs the formula for A does not hold.

While the formulae 3) and 4) are best adapted to numerical computations another form shows more clearly the relations existing between the functions A, D and Φ, viz:

$$A = A_0\, 10^{-1/\gamma^2 (m - m_0)^2}; \quad \Phi = P_0\, 10^{-1/\alpha^2 (M - M_0)^2}; \quad D = H_0\, 10^{-1/\beta^2 (\rho - \rho_0)^2} \tag{6}$$

$$\left.\begin{array}{lll} a = \log A_0 - m_0^2/\gamma^2; & b = 2m_0/\gamma^2; & c = 1/\gamma^2 \\ p = \log P_0 - M_0^2/\alpha^2; & q = 2M_0/\alpha^2; & r = 1/\alpha^2 \\ h = \log H_0 - \rho_0^2/\beta^2; & k = 2\rho_0/\beta^2; & l = 1/\beta^2 \end{array}\right\} \tag{7}$$

Then we have the relations

$$\left.\begin{array}{l} \gamma^2 = \alpha^2 + \beta^2; \quad m_0 = M_0 + \rho_0 + 0.3\beta^2 \\ \log A_0 = \log P_0 H_0 - 3.786 + 0.09\beta^2 + 0.6\rho_0 - \log \gamma/\alpha\beta. \end{array}\right\} \tag{8}$$

The values $\alpha\ \beta\ \gamma$ indicate the spreading of the three functions: for $m - m_0 = \pm\gamma$, for $M - M_0 = \pm\alpha$ and for $\rho - \rho_0 = \pm\beta$ the value of the function decreases to one tenth of the maximum values A_0, P_0, H_0. The constants for the luminosity curve and the density curves are given by

$$\begin{array}{lllll} M_0 = 2.693 & \alpha^2 = 29.0 & \alpha = 5.38 & \log P_0 = -2.144 & \\ \rho_0 = 10.4 & \beta^2 = 42 & \beta = 6.5 & \log H_0 = 0.05 & (\beta = 0^\circ) \\ \rho_0 = 9.8 & \beta^2 = 16 & \beta = 4.0 & \log H_0 = -0.22 & (\beta = 90^\circ) \end{array}$$

which means that from a maximum density of about 1 at a distance of 100 parsecs the density decreases with increasing distance, slower in the galactic plane, much faster in the direction of the poles, becoming 1/10 at distances 2000 and 600 parsecs. The curves for A (m) as they are given by observational data have the constants

$$\begin{array}{lllll} m_0 = 2.7 + 10.4 + 12.6 = 25.7 & \gamma^2 = 29 + 42 = 71 & \gamma = 8.4 & \log A_0 = 4.76 & (\beta = 0^\circ) \\ m_0 = 2.7 + 9.8 + 4.8 = 17.3 & \gamma^2 = 29 + 16 = 45 & \gamma = 6.7 & \log A_0 = 1.95 & (\beta = 90^\circ) \end{array}$$

The spreading of A is the square sum total of the spreadings of D and Φ; the central value of m, for which the function A is maximum, is the sum total of the central values of M and ρ, increased by $0.3\beta^2$; this last term is caused by the enormous increase of the volume elements with increasing distance.

It must be remarked that the Gaussian curve for D, used by SCHWARZSCHILD, cannot give the real density in the vicinity of the sun: for $r = 0$, $\rho = -\infty$ the formula gives $D = 0$ while in reality the density is nearly constant for these regions. This difference, which to an appreciable extent influences only the brighter magnitudes, must be taken into account in some other way. The formulae deduced will be used only for greater distances.

2. The foregoing formulae give the mutual interdependance of the functions $A(m)$ and $D(\rho)$, assuming them to prevail over an unlimited domain of m and ρ. In reality their validity is known only over a limited range covered by the observational data. Thus the question arises how the limits of ρ, between which $D(\rho)$ is known, depend on the limits m of $A(m)$, and in a more general way, whether and how the $A(m)$ for a certain m determines $D(\rho)$ for a certain ∂. We may answer it

in varying the coefficients a, b, c of the formula leaving $A(m)$ constant, or, in the inverse problem, in varying h, k, l leaving $D(\rho)$ constant. The variations of $A\ (m)$ caused by small variations in k and l (leaving h constant) are found from formulae 3:

$$\frac{dA}{dk} = \frac{d}{dk}(a + bm - cm^2) = \frac{0.6 + k - q}{2(l + r)} + \frac{r}{l + r}m = \frac{b - q}{2r} + \frac{r - c}{r}m.$$

$$\frac{dA}{dl} = \frac{d}{dl}(a + bm - cm^2) = -\left(\frac{0.6 + k - q + 2rm}{2(l + r)}\right)^2 - \frac{0.43}{2\,(l + r)} = -\left(\frac{b - q + 2(r - c)\,m}{2r}\right)^2 - \frac{0.43(r - c)}{2r^2}.$$

The first differential quotient dA/dk becomes zero for $m_1 = -(b - q)/2(r - c)$. The second, dA/dl, cannot become zero for any value of m; but it becomes minimum for the same value of m, that makes dA/dk zero, this minimum being $-0.43/2\ (l + r)$. Thus we may say that the adopted value of $h = D\ (0)$ determines the value $A(m)$ for this particular value $m = m_1$; for taking different k the resulting curves for A intersect in $A(m_1)$ [1]), and by taking different l the resulting curves for A approach nearest to one another at m_1. The same relations hold, if we count ρ from ρ_1 as zero point, thus writing

$$\log\ D(\rho) = h' + k'(\rho - \rho_1) - l(\rho - \rho_1)^2; \qquad k' = k - 2\,l\,\rho_1.$$

In order to leave p, q, r, i. e. M unchanged, we must also count m from this new zero point and introduce coefficients a', b' with $b' = b - 2c\rho_1$. Then the new $h' = D(\rho_1)$ determines $A(m_1)$ for the magnitude

$$m_1 = \rho_1 - \frac{b' - q}{2(r - c)} = \rho_1 \frac{r}{r - c} - \frac{b - q}{2(r - c)}. \tag{9}$$

It may easily be seen that a change in the rate of regular decrease of density does not influence the number of stars $A(m_1)$ because the influence of enlarged density on the one, and diminished density on the other side neutralize each other. A change in the curvature, however, e.g. an increase of l, diminishes the density at both sides of ρ, and causes a diminution of $A(m_1)$.

The inverse problem may be treated in the same way. From formulae 4) we find

$$\frac{dD}{db} = \frac{d}{db}(h + k\rho - l\rho^2) = -\frac{b - q}{2(r - c)} + \frac{r}{r - c}\rho.$$

$$\frac{dD}{dc} = \frac{d}{dc}(h + k\rho - l\rho^2) = -\left(\frac{2r\rho - b + q}{2\,(r - c)}\right)^2 + \frac{0.43}{2(r - c)}.$$

The first differential quotient dD/db becomes zero for $\rho_1 = (b - q)/2r$, the second becomes zero for $\rho = (b - q)/2r \pm \sqrt{0.43(r - c)/2r^2}$. In varying the coefficient c the resulting curves intersect in two points situated at both sides of ρ_1 while for ρ_1 itself their distance is $0.43\ dc/2\ (r - c)$. If now we express the function A in the vicinity of m_1 by

$$\log A(m) = a' + b'(m - m_1) - c\,(m - m_1)^2; \qquad b' = b - 2cm_1,$$

we find that a' or $A(m_1)$ determines the function $D(\rho_1)$ for the argument

$$\rho_1 = m_1 + \frac{b' - q}{2r} = \frac{r - c}{r}m_1 + \frac{b - q}{2r}, \tag{10}$$

the same distance, which after formula 5 contributes most to the stars of magnitude m_1. This is the same relation as expressed by formula 9. In a former publication [2]) it has been used to compute the limits of distance for which the densities deduced by KAPTEYN and VAN RHIJN hold good, if we assume them to be deduced from the number of stars A from the 4th to the 16th magnitude.

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[1]) Strictly speaking for finite variations $\triangle k$ the $A(m)$ for $h =$ Const. do not intersect in a point; $A(m_1)$ describes the envelope of the curves $a + bm - cm^2$ for h and l const., a and b varying with k.

[2]) The local starsystem. Proc. Amsterdam Academy, Vol XXIV, 1921.

For the galactic plane these limits are $\rho = 10$ and 17 ($r = 100 - 2500$ parsecs), for the polar regions $\rho = 9.5$ and 15 ($r = 80$ to 1000 parsecs). 1) For greater distances the densities, depending as they do on extrapolation, become uncertain in an increasing degree.

3. The formulae 9 and 10 show, that the values m_1 and ρ_1 are corresponding, conjugate values, belonging together, in this sense that the density D for distances about ρ_1 determines the number of stars A for magnitudes about m_1; and also inversely. Thus it will be more natural and appropriate to reduce the formulae for A and D to these conjugate values m_1 and ρ_1 as their zeropoints. Henceforward we will denote by a, b, c, h, k, l the coefficients of these new formulae, thus:

$$\log A(m) = a + b(m - m_1) - c(m - m_1)^2;$$
$$\log D(\rho) = h + k(\rho - \rho_1) - l(\rho - \rho_1)^2.$$

From the formulae 3, 4, 9, and 10 we find the relations

$$\left.\begin{aligned} a &= h + p - 3.786 + 0.6\rho_1 + \frac{q^2}{4r} - \frac{(k+0.6)^2}{4r} - \tfrac{1}{2}\log(l + r); \\ b &= k + 0.6; \quad c = \frac{rl}{r+l}; \quad \frac{1}{c} = \frac{1}{l} + \frac{1}{r}; \quad m_1 = \rho_1 - \frac{k + 0.6 - q}{2r}; \quad \triangle m = \frac{r}{l+r}\triangle\rho. \end{aligned}\right\} \quad 11)$$

$$\left.\begin{aligned} h &= a - p + 3.786 - 0.6m_1 - \frac{(q-0.6)^2}{4r} + \frac{(b-0.6)^2}{4r} + \tfrac{1}{2}\log\frac{r^2}{r-c}; \\ k &= b - 0.6; \quad l = \frac{rc}{r-c}; \quad \frac{1}{l} = \frac{1}{c} - \frac{1}{r}; \quad \rho_1 = m_1 + \frac{b-q}{2r}; \quad \triangle\rho = \frac{r-c}{r}\triangle m. \end{aligned}\right\} \quad 12)$$

The terms $+0{,}6\rho_1$ in a and $-0.6m_1$ in h are added, because in taking ρ_1 (or m_1) as the new zero point for the distances the volume elements are enlarged at the rate of $10^{0.6\rho_1}$. Introducing the numerical values for p, q, r we get

$$a = h - 5.929 + 0.6\rho_1 - \frac{(k+0.6)^2}{0.138} - \tfrac{1}{2}\log(l + 0.0345);$$
$$b = k + 0.6; \quad \frac{1}{c} = \frac{1}{l} + 29; \quad m_1 = \rho_1 - \frac{k + 0.414}{0.069}; \quad \triangle m = \frac{1}{29l+1}\triangle\rho.$$
$$h = a + 4.937 - 0.6m_1 + \frac{(b-0.6)^2}{0.138} + \tfrac{1}{2}\log(l + 0.0345);$$
$$k = b - 0.6; \quad \frac{1}{l} = \frac{1}{c} - 29; \quad \rho_1 = m_1 + \frac{b - 0.186}{0.069}; \quad \triangle\rho = (1 - 29c)\triangle m.$$

A few words may be said about the meaning of these formulae. The relation between b and k becomes extremely simple; according as the density at the distance ρ_1 will increase or decrease the coefficient b for the magnitudes about m_1 surpasses 0.6 or is smaller 2). The square spreading of the star number is the square spreading of the space density, increased by that of the luminosity function. The value of a still seems to depend on k, in contradiction to the meaning of ρ_1; but the term $(k + 0.6)^2/4r$ gives only the variation caused in a by a variation of m_1 (Cf. the note p. 6). The same holds good for the term $(b - 0.6)^2/4r$ in h. The term $\tfrac{1}{2}\log(l + r)$ expresses the influence of the curvature in D and A; an increase of l means a decrease of density at both sides of ρ_1 thus diminishing the number of stars for m_1.

These relations allow us to solve the problem of finding the space densities from the star numbers

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1) KAPTEYN and VAN RHIJN by another reasoning have come to analogous results (Astroph. Journal, 55, 268.)

2) In my paper of 1910 „Researches into the structure of the galaxy" (Proc. Amsterdam Acad. **1910**, 245) this relation was stated already in some indefinite way, without an exact demonstration.

in a more general way. The formulae of SCHWARZSCHILD suppose that these quantities, besides being regular and expressible by simple functions, are known for the whole range between $\pm \infty$, because we must integrate between these limits. In reality we have only to do with densities and starnumbers over a finite range. If the density along the visual radius varies irregularly it will be impossible to express it algebraically and compute the integral 1 (pag. 3). Now in this case we may divide the density curve in separate parts, each expressible by a quadratic-exponential function with different constants; each produces a partial A-curve, also standing for a limited range; by composing these different A-curves we get the whole curve for $A(m)$. Of course this procedure is not exact. The limits of the partial A-curves will not coincide, because they are founded on different coefficients. The ranges of the curves will overlap and they will not coincide in these common parts. The values of A themselves will not be correct, because outside the range we used, the D of the formulae do not coincide with the real densities, which have a small but perhaps appreciable influence. Indeed, for the problem to find A from D we had better make use of mechanical integration. But with the inverse problem it is otherwise. Here, to find the densities from the star numbers, we have no other means than using the quadratic-exponential function in an interpolatory manner, by dividing in just the same way the curve of A in separate parts, expressing them by a SCHWARZSCHILD function each, deducing from them the separate parts of the D curve and combining and linking these together as well as it may go.

In these partial curves the coefficients k and l may be negative as well as positive, denoting decreasing or increasing densities and concave or convex curves (representing thinner spaces or agglomerations of stars). The interdependence of the coefficients l and c, that determine the shape of the D and the A curve, may be seen from the following scheme of examples :

$l =$	∞	0.100	0.010	0	−0.010	−0.020	−0.030	−0.0345
$\beta^2 =$	0	10	100	∞	−100	−50	−33.3	−29
$\gamma^2 =$	29	39	129	∞	−71	−21	−4.3	0
$c =$	0.0345	0.0256	0.0070	0	−0.0141	−0.0473	−0.23	$-\infty$

For l and $c = 0$ we have the case, treated by SEELIGER, of a simple exponential increase of A and decrease of D; the luminosity curve is irrelevant in this case. For positive increasing values of l, c also increases at a slower rate; the spreading of the stars becomes smaller, they form a condensation widely spread at first, and contracting ever more, till at last, for $l \rightarrow \infty$ it represents a strongly condensed agglomeration at one definite distance. In the curve of A this condensation is much weakened by the dispersion of the luminosity curve, approaching for highly condensed masses to the spreading 5.4 of this curve. For negative values of l the density on account of this term increases at both sides of ρ_1, and only because the contribution of these increasing densities to $A(m_1)$ is decreasing (at least if the numerical value of l is not too great) with $\rho - \rho_1$ we get a finite value of A. As long as $r + l$ remains positive, the integration remains mathematically possible, though the contributions of distant ρ become ever more important, and therefore the use of a limited part of the curve becomes ever more risky; the negative curvature of A increases fast as $r + l$ approaches to zero. For $r + l = 0$ or negative the integration becomes impossible and the formulae lose their sense. The case of great negative l cannot be treated in this manner. It may actually occur with thinly populated parts of space of small extension between denser regions, and also when strong agglomerations rise rather suddenly amidst a region of mean density. These cases will be treated below. It must still be added that for positive l and c the range in m is wider than in ρ, while for negative l and c it is narrower; their proportion $1 + l/r$ decreases from ∞ to 0 for the limits of the above table.

For the inverse problem of the praxis, the finding of D from A, our table shows that c cannot have a greater positive value than 0.0345; but it may have any negative value. If, however, c does not deviate much from $r = 0.0345$ the corresponding great value for l is extremely uncertain. A real condensation of small extent presents itself so much effaced in the A curve that it will be wholly concealed in the errors of observational data; and it is highly hazardous to derive such a condensation from the small difference between r and the empirical c. In the other extreme case of great negative c the coefficient l may be computed with great accuracy; but in this case the influence of the neglected $D(\rho)$ for distant ρ is so great that the result will be illusory.

The formulae will be best applicable in the case of intermediate, moderate values of c and l, to compute a smoothly varying density along each visual radius from a smoothly irregular course of the number of stars of different magnitude. But also in this case they may sometimes give erroneous results. If the fluctuations of density are not very slow, so that only a portion of few units in ρ may be treated at once, the more distant values of ρ at both sides, which have a considerable influence on a (expressed in the term $\frac{1}{2}\log(r+l)$), in reality have a density which differs widely from what is supposed in the formula. This will be especially the case for small or negative l. The part of the integral for A contributed by the ρ outside $\rho_1 \pm 2$, being given by

$$2\int_2^\infty 10^{-(l+r)(\rho-\rho_1)^2}\,d(\rho-\rho_1),$$

amounts to 0.32, 0.43, 0.51 of the whole for the cases $l = +0.02$, 0 and -0.01. Thus in the last case a will be found considerably too great, and in the inverse problem h will be found too small from a, if the outlying realms, which in the formula are supposed to be of great density, in reality have a much smaller density. To avoid this error either a second approximation is necessary, as soon as from the discussion of adjacent intervals the real D has been found for them, or, if these are not available, for $\log(r+l)$ some value between the computed excessive and the normal value must be used. In this case the derivation of the constant h includes some rather uncertain estimates.

Condensations, voids and absorbing matter.

4. The regular or slow variations in density and starnumber, represented by the foregoing formulae, may be disturbed by more abrupt irregularities of density, such as local condensations or voids of small extension, and also in the case of absorbing nebulae. Also in this case we have the problem of finding the distribution of the stars in space from their distribution over the sky.

We suppose a star cloud of moderate extension superposed on the regular universe. If its dimensions tangentially and radially are of the same order of magnitude a depth expressed by $\rho \pm 1$ corresponds to an apparent breadth of 50°. Representing the density in this condensation by a quadratic-exponential function of ρ the value β in formula 6 will not exceed one unit. Then the coefficient γ expressing the spreading of $A(m)$ is given by $\gamma^2 = \alpha^2 + \beta^2 = 29 + 1$, not appreciably different from the case of $\beta = 0$. Thus we may as well assume the cloud to be condensed at one definite distance ρ_0 without any spreading. In this case a mass of stars dispersed over the different magnitudes around $m_0 = \rho_0 + M_0$ with the spreading 5.4 of the luminosity curve is added to the background of stars of the regular universe. As such condensations will chiefly occur in the galactic zone we have taken for this background the values of D and A for 5° latitude; the condensation has been assumed at distance $\rho = 11$, 12 .... 17. The total number of its stars for $\rho = 15$ was taken 6290 per square degree, giving 1000 for the coefficient A_0; for ρ increasing 1 this number was taken

$[0.5] = \sqrt{10}$ times greater. If we assume these stars to be distributed evenly between $\rho \pm 0.5$ their space density for $\rho = 15$ would be $[9.997] = 1.0$ (viz. 0.0449 stars per cubic parsec) and for ρ increasing 1 unit would be [0.1] or 1.26 times smaller. The surface brightness caused by such a condensation is given by

$$A_0 \int_{-\infty}^{+\infty} 10^{-r(m-m_0)^2 - 0.4m}\, dm = A_0 \sqrt{\frac{0.43\pi}{r}}\, 10^{0.04/r - 0.4m_0}$$

equaling 0.00756 stars of magnitude 0.0 per square degree for $\rho = 15$, and for ρ increasing 1 unit 1.26 times more. Only for greater ρ this approaches the brightness of the faintest galactic clouds. The following table gives in its first part the values $\log A_0(m)$ adopted for the background, and the differences $\triangle \log A = \log A - \log A_0$, all positive; the right hand columns contain the gradients of $\log A$, i.e. $\log A(m+1) - \log A(m)$, all expressed in units of the 3d decimal.

Table 1. Agglomerations of stars.

m	$\log A_0$	Increase $\triangle \log A$ for $\rho =$							Grad. A_0	Gradient of $\log A$ for $\rho =$						
		11	12	13	14	15	16	17		11	12	13	14	15	16	17
3	7.573	114	065	031	012	004	001	000	580	612	612	604	594	586	582	581
4	8.153	146	097	055	026	010	003	001	567	588	598	596	588	579	573	569
5	8.720	167	128	084	047	022	009	003	551	555	574	583	580	571	562	556
6	9.271	171	151	116	076	042	020	008	531	520	541	557	563	560	551	542
7	9.802	160	161	142	108	071	040	019	507	484	504	524	539	543	537	527
8	0.309	137	158	159	140	107	070	039	482	452	466	487	507	520	523	516
9	0.791	107	142	164	165	145	111	073	457	426	432	448	470	491	504	505
10	1.248	076	117	155	178	179	158	121	430	402	401	410	429	454	476	489
11	1.678	048	088	135	177	203	204	180	402	382	374	374	388	411	440	465
12	2.080	028	060	107	163	212	242	243	374	360	351	345	349	368	398	431
13	2.454	014	037	078	138	206	266	300	348	341	331	321	320	327	352	390
14	2.802	007	020	051	110	185	270	342	321	317	311	300	286	289	307	341
15	3.123	003	010	030	075	153	256	362	295	293	290	281	268	257	264	292
16	3.418	001	005	016	048	115	225	359								
max.		5.8	7.3	8.9	10.4	12.1	13.7	15.4		6.4	7.6	9.1	10.6	11.8	13.4	15.4

The increment of the number of stars is greatest for the magnitude at the bottom of the first part of the table, while the curvature of the curve for $\log A$ (appearing in the strongest decrease of the gradients) is greatest for the magnitudes at the bottom of the second part; they nearly coincide with one another. If the number of stars in the agglomeration becomes greater the gradients will show a reversal; after decreasing at first they increase over some magnitudes and then again decrease rapidly.

These values of A do not follow a SCHWARZSCHILD curve. Not knowing their origin, however, we will treat the limited portion given by observation as a part of such a regular curve and derive h,

k, l from the coefficients a, b, c satisfying this part of the curve. Then the result is a D curve, in which the stars of the condensation are dissipated over a large range of ρ (spreading $1/\sqrt{r+l}$) If the range in m used here covers the maximum of $\triangle \log A$ we get a maximum in D for nearly the same ρ where in reality the condensation is situated; if the maximum lies outside this range we get only an outer part of the dissipated curve. Thus the empirical data of $A(m)$ allow us to find the approximate distance at which there exists some accumulation of stars, but they do not represent its true character. A decision whether the wide dispersion found in D is real or is caused by a limited condensation must be obtained from the course of the gradient, which in the last case shows marked irregularities. The exact degree of condensation (e. g. whether β is < 1 or reaches 2) cannot be found at all from the number of stars as the distribution of $A(m)$ in these cases, owing to the large dispersion of the luminosity function, is practically identical.

5. As a second irregularity we suppose a void of stars in this way, that a part of space over an extent of one unit in ρ (from $\rho_0 - \frac{1}{2}$ to $\rho_0 + \frac{1}{2}$) is wholly empty. Subtracting the stars that in a regular universe would fill this space (D according to pag. 14) from the normal number (both for 5° latitude) the decrease of $\log A$ (in 0.001) for different assumptions as to ρ_0 is given in the following table.

Table 2. Void spaces.

ρ_0	$m = 3$	4	5	6	7	8	9	10	11	12	13	14	15	16
10	73	84	83	73	56	39	25	14	7	3	2	1	0	0
13	15	27	45	65	85	98	102	94	79	59	41	25	14	8
15	2	3	7	15	26	32	62	79	93	100	96	82	64	45
17	0	0	0	1	3	6	12	22	36	54	73	90	99	97

The influence of such a void upon the number of stars is greatest for the magnitude formerly called m_1 and decreases slowly for brighter and fainter classes. But also its maximum effect is small, diminishing $\log A$ only 0.1, thus reducing the number of stars at most to 0.8 of its normal value. Such a void, if globular, would have a lateral extension of 26°; thus empty spaces extending over a smaller surface would have an appreciable influence upon the number of stars only if they were of a long tubular shape, directed radially. Therefore the dark and starless regions often met with in the sky cannot be explained by assuming within the universe really empty parts of space.

6. A third cause of irregularities may be found in local clouds of absorbing matter in space. The influence of such absorbing matter on the distribution of the stars has been treated in my paper „The distance of the dark nebulae in Taurus". [1]) We will repeat the formulae here in the notation used in the foregoing paragraphs. The fraction of the total number of stars $A(m)$ contributed by stars at distance ρ is given by

$$\sqrt{\frac{r+l}{0.43\pi}}\,10^{-(r+l)\left(\rho-\frac{r}{r+l}m-\frac{0.6+k-q}{2(r+l)}\right)^2} d\rho.$$

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[1]) Proceedings Amsterdam Academy, XXIII, 708 (1920).

Integrating this expression between $\pm\infty$ we get 1. If at the distance ρ_2 there is an absorbing screen, diminishing the light of the more remote stars by ε magnitudes, only the part of this integral between $-\infty$ and ρ_2 will contribute to the stars m, the others having faded to $m+\varepsilon$. In their place the remoter stars that without a screen would have appeared ε magnitudes brighter will now add to the number of stars m. This second part is given by the same integral (with m in the exponent replaced by $m-\varepsilon$) between ρ_2 and $+\infty$. Putting

$$x_1 = \sqrt{r+l}\left(\rho_2 - \frac{0.6+k-q}{2(r+l)} - \frac{rm}{r+l}\right), \qquad x_2 = x_1 + \frac{r}{\sqrt{r+l}}\,\varepsilon,$$

$$\frac{1}{\sqrt{0.43\pi}}\int_{-\infty}^{x_1} 10^{-t^2}\,dt = \gamma_1, \qquad \frac{1}{\sqrt{0.43\pi}}\int_{x_2}^{\infty} 10^{-t^2}\,dt = \gamma_2,$$

we have

$$A'(m) = \gamma_1\,A(m) + \gamma_2\,A(m-\varepsilon).$$

Among the stars $A(m)$ a fraction γ_1 is situated before the screen and is seen unobscured. Among the stars $A(m-\varepsilon)$ a fraction γ_2 is situated behind the screen and appears as stars of magnitude m. The computation has been made for $\rho_2 = 9$, 12 and 15 (distance of the screen 63, 251 and 1000 parsecs) and for an absorbtion of 1, 2, 4 magnitudes and total obliteration of the remote stars; $A(m)$ has been taken from Table 4, pag. 15 for $\beta = 0°$.¹) The following table gives in its first part the diminution of log A for each magnitude ($\log A_0 - \log A$), and in its second part the gradient $\log A(m+1) - \log A(m)$ (unit always 0.001).

Table 3 Absorbing screen.

$\log A_0 - \log A$.

m	$\log A_0$	$\rho_2 = 9$				$\rho_2 = 12$				$\rho_2 = 15$			
		$\varepsilon = 1$	2	4	∞	$\varepsilon = 1$	2	4	∞	$\varepsilon = 1$	2	4	∞
3	7.575	295	395	422	423	042	049	050	050				
4	8.158	367	522	572	574	071	084	086	086				
5	8.728	429	659	752	756	110	134	138	138	006	007	007	007
6	9.281	472	791	962	972	160	201	210	210	013	015	015	015
7	9.814	493	895	1198	1223	217	287	305	306	024	027	028	028
8	0.324	491	947	1432	1495	273	385	422	423	041	049	050	050
9	0.810	480	958	1657	1818	328	498	570	574	067	083	086	086
10	1.270	459	934	1801	2177	367	609	746	756	102	131	138	138
11	1.703	433	891	1830	2574	386	698	946	972	145	194	209	210
12	2.111	408	841	1776	3009	388	748	1155	1223	191	273	304	306
13	2.492	381	789	1682	3462	374	752	1326	1495	231	357	419	423
14	2.847					354	726	1431	1818	266	444	562	574
15	3.175					329	683	1434	2177	284	516	725	756
16	3.478					304	633	1366	2574	287	556	891	972
17	3.758									277	560	1029	1223

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¹) There is an incongruity in this procedure as in the computation of γ a Schwarzschild function for D and A has been supposed; thus for $\rho_2 = 9$ the effect of the absorbing screen on the brighter classes is somewhat exaggerated. In my former paper, where also for $A(m)$ itself such a function was assumed this error must be still greater. For greater distances and faint magnitudes, however, it will have no appreciable influence.

Gradient of log A.

m	(A_0)	$\rho_2 = 9$				$\rho_2 = 12$				$\rho_2 = 15$			
		$\varepsilon = 1$	2	4	∞	$\varepsilon = 1$	2	4	∞	$\varepsilon = 1$	2	4	∞
3	583	511	456	433	432	554	548	547	547				
4	570	508	433	390	388	531	520	518	518				
5	553	510	421	343	337	503	486	481	481	546	545	545	545
6	533	512	429	297	282	476	447	438	437	522	521	520	520
7	510	512	458	276	238	454	412	393	393	492	488	488	488
8	486	497	475	261	163	431	373	338	335	460	452	450	450
9	460	481	484	316	101	421	349	284	278	425	412	408	408
10	433	459	476	404	036	414	344	233	217	390	370	362	361
11	408	433	458	462	neg.	406	358	199	157	362	329	313	312
12	381	408	433	475		395	377	210	109	341	297	266	264
13	355	381	408	459		375	381	250	032	320	268	212	204
14	328					353	371	325	neg.	310	256	165	146
15	303					328	353	371		300	263	137	087
16	280									290	276	142	029
17													

These tables show a considerable decrease in the number of stars already for moderate absorbtions. At first the curve for log A sinks in an increasing rate below the normal curve A_0. Then comes a region of transition, where the gradient shows a standstill or, for stronger absorbtions, a reversal, the A curve thus becoming concave. This takes place for $\rho_2 = 9$ between 6^m and 10^m (for stronger absorbtions lower), for $\rho_2 = 12$ between 10^m and 14^m, for $\rho_2 = 15$ below 14^m. For still smaller magnitudes the gradient becomes greater than normal, the stars are nearly all seen through the screen and show the same behaviour as normal stars ε magnitudes brighter, and log A again slowly approaches the normal curve.

If we treat such regions of the sky in the normal way, computing h, k, l and D from a, b, c, we find that at first the density decreases strongly, having nearly half the normal amount at the distance where the screen is situated. It reaches a (relative) minimum, with log D nearly 0.5, 1.0 and 2.0 below the normal value (in accordance with the number of stars 1^m, 2^m and 4^m brighter) for distances lying 3 to 5 units of ρ behind the screen. These great deviations from normal conditions usually give a sufficient indication that we have to do with absorbing nebulae. Moreover the irregularities of the gradient and especially the irregularities in the figure of the dark markings and starless regions of the sky will afford a sufficient proof, that in such cases really empty spaces, extending to the remotest distance, are out of the question.

Numerical data for a schematical universe.

7. In studying the distribution of the stars we deduce from observational data for smaller or greater parts of the sky the surface density A as a function of m, represent it by a quadratic-exponential formula and compute the space density D from its coefficients. Now some difficulties may arise from the rapid change of the regular coefficients with galactic latitude, which forbids the combining of large regions extending over greatly varying latitudes. As, however, we start from a regular universe as a basis for further approximations this difficulty is removed by computing the deviations of the real star numbers and densities from the schematical distribution of the

second approximation. In this schematical universe the density from 1 in the centre decreases regularly with increasing distance and the surfaces of equal density are flattened surfaces of revolution.

From the table of log $N(m)$ for different β, given by VAN RHIJN (Astron. Nachr. 5091, Bd. **213**, 47) we used the part between $m = 7$ and 14, derived quadratic formulae for log $A(m)$ and computed $D(\rho)$ for each latitude. The distances for which log $D = 9.8$, 9.5 and 9.2 were used to draw three meridian sections of equal density. As they showed many irregularities they must be smoothed; for these three densities ellipsoids of revolution were assumed and the axes were found

	for log $D = 9.8$	log $D = 9.5$	log $D = 9.2$
$\beta = 0°$	$\rho = 13.50$	15.15	16.40
$\beta = 90°$	10.60	12.15	13.21

The ellipsoids cannot be similar, as in this case the coefficients k and l, and also b and c, would be independent of latitude. These ellipsoids determine for other latitudes the values of ρ having log D 9.8, 9.5 and 9.2; from these values for each latitude quadratic formulae for log D were computed, which afforded the corresponding values of log $A(m)$. It must be remarked that the surfaces of equal density for other values of D thus derived will not be ellipsoids, though the deviations are only small.

The densities given by these formulae are adopted for greater distances only. For smaller distances where the quadratic formula would give a maximum and then a decrease to 0 for distance 0 the real density adopted in the schematical universe gradually increases from log $D = 9.8$ to the central value $D = 1$, which is reached over a range in ρ of 3.5. For these inner regions the surfaces of equal density are assumed to be similar, thus this range is the same for all latitudes. The corrections to be added to log $A(m)$ in consequence of this difference between the adopted density and the quadratic formula (or subtracted for the lower latitudes where $A(10)$ was found > 1) were computed for each magnitude by mechanical integration. Thus we get a distribution of density in the schematical universe and a distribution of stars over the sky exactly corresponding to each other and connected by the luminosity function of KAPTEYN. They are given in Table 4.

Table 4. Schematical universe.

log $D(\rho)$.

ρ	$\beta = 0°$	5°	10°	15°	20°	30°	40°	50°	60°	70°	80°	90°
7	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	9.998	9.996	9.994	9.992	9.992
9	0.000	0.000	0.000	0.000	0.000	9.996	9.989	979	970	962	958	956
10	0.000	0.000	9.999	9.995	9.989	971	948	926	906	889	880	877
11	9.990	9.987	979	967	948	907	861	819	783	759	743	738
12	951	944	926	898	863	787	714	651	602	565	543	536
13	867	854	819	770	716	607	504	418	352	303	275	266
14	721	703	655	588	513	363	228	117	034	8.973	8.937	8.926
15	532	507	442	352	252	057	8.887	8.749	8.648	574	531	518
16	302	269	181	060	8.931	8.686	479	313	194	107	057	042
17	034	8.988	8.871	8.715	553							
18	8.724	664	512									

log $A(m)$.

m	$\beta = 0°$	5°	10°	15°	20°	30°	40°	50°	60°	70°	80°	90°
2	6.984	6.982	6.981	6.976	6.971	6.964	6.955	6.946	6.940	6.936	6.934	6.933
3	7.575	7.573	7.568	7.561	7.555	7.539	7.520	7.507	7.496	7.485	7.482	7.481
4	8.158	8.153	8.146	8.135	8.123	8.097	8.070	8.052	8.037	8.024	8.017	8.015
5	8.728	8.720	8.709	8.694	8.675	8.637	8.603	8.576	8.554	8.537	8.528	8.525
6	9.281	9.271	9.255	9.232	9.207	9.156	9.110	9.072	9.044	9.023	9.011	9.008
7	9.814	9.802	9.779	9.750	9.716	9.648	9.589	9.542	9.507	9.482	9.466	9.462
8	0.324	0.309	0.280	0.241	0.197	0.112	0.039	9.983	9.941	9.909	9.891	9.886
9	0.810	0.791	0.755	0.705	0.651	0.547	0.459	0.392	0.342	0.305	0.284	0.278
10	1.270	1.248	1.202	1.141	1.076	0.951	0.848	0.770	0.710	0.668	0.644	0.637
11	1.703	1.678	1.621	1.547	1.470	1.323	1.204	1.114	1.046	0.998	0.970	0.962
12	2.111	2.080	2.012	1.924	1.833	1.663	1.526	1.425	1.348	1.293	1.263	1.254
13	2.492	2.454	2.372	2.270	2.164	1.971	1.817	1.702	1.616	1.554	1.521	1.511
14	2.847	2.802	2.706	2.587	2.465	2.245	2.072	1.943	1.850	1.783	1.746	1.735
15	3.175	3.123	3.010	2.872	2.734	2.488	2.295	2.154	2.051	1.976	1.936	1.924
16	3.478	3.418	3.284	3.128	2.971	2.696	2.484	2.329	2.216	2.135	2.092	2.079

log $N(m)$.

m	$\beta = 0°$	5°	10°	15°	20°	30°	40°	50°	60°	70°	80°	90°
4	7.997	7.994	7.989	7.981	7.973	7.955	7.935	7.921	7.909	7.900	7.895	7.894
5	8.578	8.572	8.564	8.552	8.539	8.511	8.484	8.464	8.447	8.434	8.427	8.424
6	9.144	9.136	9.124	9.107	9.088	9.048	9.013	8.984	8.961	8.944	8.935	8.932
7	9.694	9.683	9.667	9.644	9.616	9.564	9.517	9.478	9.449	9.428	9.416	9.413
8	0.224	0.211	0.188	0.158	0.123	0.054	9.994	9.947	9.912	9.886	9.871	9.866
9	0.732	0.717	0.686	0.646	0.602	0.517	0.443	0.387	0.345	0.314	0.296	0.291
10	1.216	1.197	1.160	1.109	1.055	0.952	0.864	0.798	0.749	0.712	0.692	0.686
11	1.675	1.653	1.607	1.545	1.481	1.357	1.256	1.180	1.122	1.080	1.056	1.050
12	2.109	2.084	2.028	1.954	1.878	1.734	1.618	1.530	1.464	1.417	1.390	1.383
13	2.519	2.488	2.421	2.335	2.245	2.081	1.949	1.850	1.776	1.723	1.694	1.686
14	2.904	2.866	2.787	2.687	2.584	2.398	2.250	2.140	2.058	2.000	1.968	1.959
15	3.264	3.220	3.127	3.012	2.895	2.685	2.521	2.400	2.311	2.247	2.212	2.202
16	3.599	3.548	3.440	3.309	3.176	2.944	2.764	2.631	2.534	2,464	2.427	2.415

The corrections applied to A of the formula are appreciable also for the faint magnitudes used in deriving the coefficients of the formula. Thus a second computation would be necessary, if we wished to make this schematical universe correspond to the average distribution of the stars. Indeed the log $N(m)$ of the Groningen tables forming the basis of our computations show systematic deviations from our final results, as is shown by the following values of log N (Groningen) — log N (table 4):

	$\beta = 0°$	20°	40°	90°
$m =$ 4	+ .180	+ .116	+ .031	— .093
8	+ .047	— .046	— .070	— .005
12	+ .034	— .050	— .049	+ .002
16	+ .011	— .002	— .000	— .097

For our purpose, however, these differences are not material, because the schematical distribution is only used as basis of reference, the deviations from which will be determined for every region; since a repetition of these computations would have cost much labour it was not deemed necessary. The deviations $\triangle A(m)$ or its coefficients $\triangle a$, $\triangle b$, $\triangle c$ applied to some mean value for this region affords the deviations $\triangle h$, $\triangle k$, $\triangle l$ and $\triangle D$ to be applied to the corresponding normal density. This procedure will give results near to the truth also for lower magnitudes where, owing to the influence of the central parts, the density and the starnumber themselves cannot be represented by corresponding quadratic formulae.

COMPARISON OF MAGNITUDE SCALES.

The Harvard photometric catalogues.

8. The magnitudes of the stars in the Durchmusterung Catalogues can be reduced to a photometric scale only by means of photometric catalogues. Among the two great photometric catalogue works, the Potsdam „*Photometrische Durchmusterung*" and the various catalogues of Harvard College Observatory, we have used only the latter.The Potsdam magnitudes doubtless are the most accurate existing; but as this catalogue contains only the stars down to 7.5 on the northern hemisphere, its use might cause a lack of homogeneity that would be especially detrimental to our purpose. Therefore we have based our investigation wholly upon the Harvard work, which extends over the entire sky and down to the faintest magnitude classes. SEELIGER has already used these measures in his first researches. Afterwards an extensive comparison of the Durchmusterung scales with the Harvard magnitudes has been made at Cambridge itself (Harvard Annals **72** and **84**). But, though the chief features are shown, a preliminary research proved its results to contain large systematic errors which rendered them inadequate for accurate investigations. The cause of these errors was found to lie in the incompleteness of some catalogues, cut off at some limit of magnitude, and in their compilation into one single catalogue. Thus a theoretical investigation of the influence of the incompleteness of the catalogues on the result of their comparison proved necessary.

The immense and invaluable labour bestowed on photometric measures of the stars at Harvard College Observatory, chiefly by the indefatigable zeal of EDWARD C. PICKERING, has been condensed in a number of photometric catalogues of different constitution and working plan. For the brightness of individual stars simple means may be taken; but if we wish to find mean corrections to other catalogues, corrections depending on the incompleteness of the catalogues must be applied. Thus a careful consideration of the choice of the stars included in the working list of each catalogue is necessary, to obtain reliable results. The data for each of these catalogues follow here.

9. *Harvard 14* (2 inch photometer, comparisons with α Ursae minoris, 1879—81) contains 4260 stars north of —30°, brighter than 6.0. It only concerns us in so far as it contributed to the general catalogue in H 50.

Harvard 24 (4 inch photometer, comparisons with λ Ursae minoris), „*Revision of the Durchmusterung*". In zones 20′ wide between declinations $5^\circ \pm 10'$ etc. according to the B.D. all stars 9.0 and brighter are measured; generally for 7.0—7.9 zones of 1°, for brighter stars zones of 2° were taken. The brightest stars are omitted; fainter classes are included only when a letter, indicating former meridian observations, is attached in B.D. The zones measured have mean parallels —1°, + 1°, + 5°, + 10°.... + 85°, —5°, —10°, —15°, —20°. The program comprised 20892 stars.

Harvard 34 (4 inch, Arequipa, 1889—91). The working list was composed of 1. all bright stars south of —30°, brighter than 6 (Uranometria Argentina > 6.3, BEHRMANN and HOUZEAU > 6); 2. zones of telescopic stars analogous to H 24: all stars of the *Argentine General Catalogue* and the *Zone Catalogue* (Resultados Cordoba, 7. 8. 14) in zones 20′ wide, and all these stars designated brighter than 8 in zones 40′ wide were observed; the middle parallels of the zones are —25°, —30° etc.; 3. miscellaneous stars from different sources (fundamental stars, GILL-Victoria stars c.a.). All these stars are collected into one catalogue arranged after AR. All stars occurring in H 14 were used as standards to determine the zeropoint.

Harvard 44 (4 inch, Cambridge, 1891—95) Remeasuring with the larger instrument of all stars of H 14 being 6^m and brighter, north of —30°.

Harvard 45 (4 inch, Cambridge, 1895—98) All stars 7.5 and brighter, selected from the Bonn and Cordoba Durchmusterung, north of —40° were measured (with the exception of the brighter stars that were taken from H 44 and H 14 and printed in italics). A small number of fainter stars (comparison stars for variables) was included (29587 stars).

Harvard 46 (4 inch, Arequipa) All stars south of —30°, designated 7.0 and brighter in the Argentine General Catalogue, the Zone Catalogue and the Cordoba Durchmusterung Vol. 16 and 17. It contains 5332 stars. The zero point was determined by stars between 0° and —30°, whose magnitudes were taken from H 14, H 24, H 44.

Harvard 50 „*Revised Harvard Photometry*", contains all stars down to 6.50, compiled from the former catalogues.

Harvard 54 contains all fainter stars, not in H 50, compiled in the same way from former catalogues.

Harvard 70 (12 inch photometer). In zones of 10′ width all stars of the B. D. down to the faintest class, have been measured. The northern border parallels are 0°, + 5°, + 10° .... + 85°; for the southern hemisphere the southern border parallels are —5°, —10°, —15°, —20°. In a zone —24° 50′ to —25° 0′ stars taken from the „*Cape Photographic Durchmusterung*" were observed. In order to limit the number of stars to be observed in the crowded regions a selection was made. „The first ten stars in each ten minutes were first selected. When, as often happened, less than ten stars were contained in these regions the number was completed from the zone 10′ south. The selection was then carefully revised, and compared with Harv. Ann. 34, so that at least two stars, contained in that work, should be included in each 10^m of Right Ascension" (H. A. **70**, 215). As the photometer is provided with a wedge, its scale of magnitudes is not independent. A discussion of its foundations and the tests afterwards applied is promised for a future volume (cf. H. A. **72**, 83).

Harvard 72 N⁰. 4 (*Rumford photometer*, Arequipa and Hannover, Cape Colony, 1902—09). The reduction to the scale of the Harvard Photometry was carried out by standard stars, that were also used to derive the reduction curve of the wedge; afterwards a correction $+ 0.24(m—9.0)$, deduced by means of 97 stars formerly observed with the 12 inch, was applied to all stars below 9.0. Durchmusterung zones 10′ wide, north of —30°, —40° and —50°, and a number of miscellaneous stars (comparison stars for variables, sequences in standard regions) were observed. On the construction of the working list for the D.M. zones nothing is said in the text. From the catalogues themselves we may infer that an analogous course is followed as for zone —24°: as a rule in every 10^m of AR 8 stars are taken, among them two of 10^m (the first met with), also two between 9.5 and 9.9, two of 9.0—9.5 and two brighter ones; if not available in this zone, the latter are taken from adjacent zones, preference being given (because they are used as standards) to stars occurring in former catalogues (H 34).

The influence of multitude and incompleteness on catalogue differences.

10. In deducing the reductions of some magnitude scale to photometric magnitudes we compare the magnitudes of the two scales and take the mean differences. Owing to the influence of the multitude of the stars and the incompleteness of the catalogues this mean difference will not be equal to the scale reduction. We will compute to what amount the mean magnitude of a number of specially chosen stars is modified by the modus of this choice.

We suppose that the errors of each catalogue follow a Gaussian law. The catalogue that is compared (by combining all stars of the same magnitude m_1 in this catalogue) will be called the first; the other, to which it is compared, will be called the second. The magnitudes are called m_1 and m_2 (all expressed already in the same scale unit); their mean error is μ_1 and μ_2, their combined mean error $\mu_{12} = \sqrt{\mu_1^2 + \mu_2^2}$. The true magnitude is always called m.

Case 1. Both catalogues are complete; the second gives the true magnitudes m.

We put the difference $m - m_1 = x$. The number of stars with magnitude m is taken $A(m) = A_0 e^{bm} = A_1 e^{bx}$, thus assuming a linear function for log $A(m)$, which is certainly permitted for these small ranges in m. The average difference $m - m_1$ is given by

$$\sqrt{\frac{h_1}{\pi}} \int_{-\infty}^{+\infty} A_1\, x\, e^{bx - h_1 x^2} dx \; : \; \sqrt{\frac{h_1}{\pi}} \int_{-\infty}^{+\infty} A_1 e^{bx - h_1 x^2} dx$$

$$= A_1 \frac{b}{2h_1} e^{b^2/4h_1} \; : \; A_1 e^{b^2/4h_1} = b/2h_1 = b\mu_1^2.$$

The divisor, being the number of stars counted m_1 in the first catalogue, is greater than the true number corresponding to m_1; put equal to $A_1 e^{b\triangle m}$ it is found to correspond to a magnitude $\triangle m = b/4h_1 = \frac{1}{2} b \mu_1^2$ greater than m_1. Because the number of stars increases geometrically with magnitude we will include in m_1 more faint stars estimated too bright than bright stars estimated too faint by the same amount; the average magnitude will be too faint by an amount $b\mu_1^2$. In the same way, if m_1 is derived statistically from the number of stars counted we find a value too faint by half that amount.

Case 2. Both catalogues are complete; the second also has accidental errors.

We put $m - m_1 = x$; $m_2 - m_1 = x_2$. The number of stars of magnitude m, measured m_2 in the second catalogue and estimated at the same time m_1 in the first catalogue is (omitting always the differential dm_1):

$$\frac{\sqrt{h_1 h_2}}{\pi} \int A_0 e^{bm} e^{-h_2(m_2 - m)^2} e^{-h_1(m_1 - m)^2} dm\, dm_2 = \frac{\sqrt{h_1 h_2}}{\pi} dx_2 \int A_1 e^{bx - h_1 x^2 - h_2(x_2 - x)^2} dx.$$

The average value of $m_2 - m_1$ is thus given by

$$\frac{\sqrt{h_1 h_2}}{\pi} \iint x_2 e^{bx - h_1 x^2 - h_2(x_2 - x)^2} dx_2\, dx \; : \; \text{same integral without factor } x_2$$

(both integrals between $\pm\infty$). Performing these integrations we find exactly the same results as in the preceding case; the quotient is $b/2h_1 = b\mu_1^2$ and the statistical magnitude $\triangle m = \frac{1}{2} b \mu_1^2$. The systematic deviation of the average magnitude is independent from the errors of the second catalogue.

Corollary 1. If the catalogues 1 and 2 are interchanged, i. e. the differences are taken once with argument m_p, another time with argument m_q, the difference found the first time is $q - p + b\mu_p^2$, the second time $q - p - b\mu_q^2$; the results will differ by the amount $b(\mu_p^2 + \mu_q^2)$. If the accuracy of the two catalogues is the same the true result is found by simply taking the mean.

Corollary 2. We suppose a table giving $A(m)$, constructed by counting catalogue p (e.g. Harvard 50—54); by means of this table we deduce statistical magnitudes from the number counted in catalogue q (e.g. a Durchmusterung Catalogue). The last values are $\frac{1}{2}\, b\, \mu_q^2$ too great; as, however, the first (argument of the table) are $\frac{1}{2}\, b\, \mu_p^2$ too great, the real error will be $\frac{1}{2}\, b\, (\mu_q^2 - \mu_p^2)$.

Case 3. The second catalogue is cut off at magnitude $m_2 = m_1 + l$.

The number of stars called m_2 in the second and at the same time called m_1 in the first catalogue is given by the same expression as in the preceding case. But now the integration over m_2 extends only to $m_1 + l$. Thus the average sought for is given by:

$$\frac{\sqrt{h_1 h_2}}{\pi} \int_{-\infty}^{l} A_1\, x_2\, dx_2 \int_{-\infty}^{+\infty} e^{bx - h_1 x^2 - h_2 (x_2 - x)^2}\, dx \ : \ \text{same integral without factor } x_2.$$

Performing the first integration we have

$$\sqrt{\frac{h_1 h_2}{\pi (h_1 + h_2)}}\, A_1\, e^{b^2/4h_1} \int_{-\infty}^{l} x_2\, e^{-\frac{h_1 h_2}{h_1 + h_2} (x_2 - b/2h_1)^2}\, dx_2 \ : \ \text{same integral without factor } x_2.$$

Putting

$$\frac{h_1 + h_2}{h_1 h_2} = \frac{1}{h_{12}} = 2\mu_{12}^2; \qquad \sqrt{h_{12}}\, (l - b/2h_1) = \frac{1}{\mu_{12}\sqrt{2}} (l - b\mu_1^2) = k; \qquad A_1 e^{b^2/4h_1} = A_1 e^{b \triangle m} = A'$$

$$\frac{1}{\sqrt{\pi}} \int_{-\infty}^{k} e^{-x^2}\, dx = \Psi(k)$$

we get for the average catalogue difference

$$A' \left(\frac{b}{2h_1} \Psi(k) - \frac{1}{2\sqrt{\pi h_{12}}}\, e^{-k^2} \right) \ : \ A' \Psi(k) = b\mu_1^2 - \frac{\mu_{12}}{\sqrt{2\pi}} \frac{e^{-k^2}}{\Psi(k)}. \tag{3A}$$

The differences $m_2 - m_1$ follow a Gaussian curve with maximum A' at the displaced zero magnitude $m_1 + b\,\mu_1^2$ and modulus h_{12}; this curve is cut off at argument k determined by this modulus and the difference of l with the displaced zero magnitude. The barycentrum of the remaining part gives the deviation caused by incompleteness, which is subtracted from the deviation caused by the increasing multitude of stars in the complete catalogue.

If the second catalogue consists of two parts, one part A cut off at l, the other part B complete to the faintest magnitude, we have for A the same expressions, while for B the function Ψ becomes 1 and $e^{-k^2} = 0$. Thus adding both parts we have.

$$\frac{A \left(b\,\mu_1^2 \Psi(k) + \mu_{12}/\sqrt{2\pi}\ .\ e^{-k^2} \right) + B\, b\mu_1^2}{A\, \Psi(k) + B} = b\mu_1^2 - \frac{\mu_{12}}{\sqrt{2\pi}} \frac{e^{-k^2}}{\Psi(k) + B/A}. \tag{3B}$$

The case also occurs that this part B is cut off at another fainter limit (completely or partly). In this case we must make use of a more general formula of which the foregoing is a special case

$$b\mu_1^2 - \frac{A\, \mu_{12}/\sqrt{2\pi}\ .\ e^{-k^2} + B\, \mu'_{12}/\sqrt{2\pi}\ .\ e^{-k'^2}}{A\, \Psi(k) + B\, \Psi(k') + C}. \tag{3C}$$

The denominator gives the number of stars as a function of m_1. If the limits l and l' are not too near, it appears that for practical computation we may substitute for this expression a series of partial corrections each computed for a single limit occurring alone:

$$b\mu_{12} - \left\{ \frac{\mu_{12}}{\sqrt{2\pi}} \frac{e^{-k^2}}{\Psi(k) + (B + C)/A} + \frac{\mu'_{12}}{\sqrt{2\pi}} \frac{e^{-k'^2}}{\Psi(k') + C/B} \right\}. \tag{3D}$$

Case 4. The first catalogue is cut off at $m_1 + l$.

In this case for every magnitude occurring in the first catalogue the matter is the same as if both catalogues were complete; the formulae for Case 2 are applicable.

Case 5. The second catalogue is incomplete; all stars are omitted that in a third catalogue fall below a certain limit.

For the 3d catalogue the quantities are m_3, h_3, μ_3; $m_3 - m_1 = x_3$; the limit l is counted from m_1. The number of stars having magnitude m_1 in the first, m_2 in the second, m_3 in the third catalogue, is

$$\sqrt{\frac{h_1 h_2 h_3}{\pi^3}}\, dm_2\, dm_3 \int A_m e^{-h_1(m_1 - m)^2 - h_2(m_2 - m)^2 - h_3(m_3 - m)^2} dm$$

between $\pm\infty$. The value sought for is given by the quotient

$$A_1 \sqrt{\frac{h_1 h_2 h_3}{\pi^3}} \int_{-\infty}^{l} dx_3 \int_{-\infty}^{+\infty} x_2\, dx_2 \int_{-\infty}^{+\infty} e^{bx - h_1 x^2 - h_2(x_2 - x)^2 - h_3(x_3 - x)^2} dx \text{ : the same without factor } x_2.$$

Performing the integrations between limits infinite we get

$$A_1 \sqrt{\frac{h_1 h_3}{\pi(h_1 + h_3)}} \int_{-\infty}^{l} \frac{b + 2h_3 x_3}{2(h_1 + h_3)} e^{-\frac{h_1 h_3}{h_1 + h_3}(x_3 - b/2h_1)^2 + b^2/4h_1} dx_3 \text{ : the same without factor before } e.$$

Putting, in the same way as in case 3

$$\frac{h_1 + h_3}{h_1 h_3} = \frac{1}{h_{13}} = 2\mu^2_{13} = 2(\mu_1^2 + \mu_3^2); \qquad (l - b/2h_1)\sqrt{h_{13}} = \frac{1}{\mu_{13}\sqrt{2}}(l - b\mu_1^2) = k,$$

we get for the deviation between the catalogues

$$b\mu_1^2 - \frac{\mu_1^2}{\mu_{13}\sqrt{2\pi}} \frac{e^{-k^2}}{\Psi(k)} \tag{5A}$$

In this formula the quantities belonging to the second catalogue have entirely disappeared; the errors of the second catalogue have no influence on the result, just as if — as in case 2 — it represented true magnitudes without error. The correction to be subtracted from the positive deviation for complete catalogues increases with μ_1; but it becomes smaller with increasing μ_3 since in this case the incompleteness of the second catalogue varies more slowly with magnitude. If the third catalogue is only partly cut off, the second part B being complete, we have, as in the third case

$$b\mu_1^2 - \frac{\mu_1^2}{\mu_{13}\sqrt{2\pi}} \frac{e^{-k^2}}{\Psi(k) + B/A}. \tag{5B}$$

In the statement of this case the second catalogue was assumed to be incomplete. If, however, we suppose the presence of a star in the first catalogue to be determined by the limit of the 3d catalogue, while the second is complete, our first integrals remain unchanged and the result is the same as found here. Whether the stars below the limit of the 3d catalogue are omitted in the first or in the second catalogue is of no importance; they do not exist for our comparisons. Thus we may interchange the catalogues 1 and 2. For a certain magnitude m_1 in the vicinity of the limit the chance for finding a brighter m_2 is greater than for finding a star with m_2 fainter by the same amount; but in the same way corresponding to a certain m_2 there are more stars with brighter m_1 than fainter.

11. It often occurs that in a catalogue stars below a certain magnitude become scarcer simply owing to the greater difficulties of observing them or to some other reason, without being determined by the precise limit of a third catalogue. This case has a great resemblance to the former and may be treated in the same way. Though the decrease will not follow exactly a simple mathematical law we will assume that with some approximation it may be represented by a Gaussian error law. Thus we have.

Case 6. The first catalogue is incomplete; about a certain magnitude the number of stars decreases according to the Gaussian law.

In this case it is the same as if the real number of stars at the sky should decrease according to this law; it may be represented by

$$A = A_0 \sqrt{\frac{h_3}{\pi}}\, e^{bm} \int_{-\infty}^{l} e^{-h_3(y-m)^2} dy = A_1 \sqrt{\frac{h_3}{\pi}}\, e^{bx} \int_{-\infty}^{l} e^{-h_3(z-x)^2} dz.$$

The integrals expressing the mean value of x_2 are wholly identical with the former case, if only z is substituted for x_3. The result is the same as in formula 5A. This case is realised by the Durchmusterung Catalogues containing the stars of the magnitudes 9 to 11 or 12 in a decreasing rate. The moduli l and h_3 determining this decrease can only be found by countings in a photometric catalogue, denoted here as second catalogue; the number of stars decreases here with m_2 at a rate determined by μ_{23}. Writing the correction in the form

$$\mu_1^2\left(b - \frac{1}{\mu_{13}\sqrt{2\pi}} \frac{e^{-k^2}}{\Psi(k)}\right) = b'\mu_1^2 \qquad 6)$$

it appears to come out as a change of the coefficient b, decreasing from its normal positive value to increasing negative values.

Case 7. The second catalogue is incomplete; about a certain magnitude the number of stars decreases according to the Gaussian law.

In this case the number of stars in the second catalogue must be multiplied by

$$\sqrt{\frac{h_3}{\pi}} \int_{-\infty}^{l} e^{-h_3(y-m_2)^2} dy.$$

Thus the number of stars having a magnitude m_1 in the first, m_2 in the secund catalogue is

$$A_1 \sqrt{\frac{h_1h_2h_3}{\pi^3}}\, dx_2 \int_{-\infty}^{l} e^{-h_3(z-x_2)^2} dz \int_{-\infty}^{+\infty} e^{bx - h_1x^2 - h_2(x_2-x)^2} dx.$$

The expressions for the deviation sought for become quite analogous to the preceding cases, and putting

$$\frac{h_1h_2 + h_1h_3 + h_2h_3}{h_1h_2h_3} = \frac{1}{h_{123}} = 2(\mu_1^2 + \mu_2^2 + \mu_3^2) = 2\mu^2_{123}$$

we get after two integrations

$$A_1 \sqrt{\frac{h_{123}}{\pi}} \int_{-\infty}^{l} \frac{h_2b + 2h_3(h_1+h_2)z}{h_1h_2 + h_1h_3 + h_2h_3}\, e^{-h_{123}(z - b/2h_1)^2 + b^2/4h_1} dz \quad : \text{ the same without factor before } e.$$

Calling $(l - b/2h_1)\sqrt{h_{123}} = k$ we get for the deviation

$$b\mu_1^2 - \frac{\mu^2_{12}}{\mu_{123}\sqrt{2\pi}} \frac{e^{-k_2}}{\Psi(k)}. \qquad 7)$$

The values μ_3 and l required for this computation must be found by counting the number of stars in the catalogue for every tenth magnitude, smoothing them and representing them by a formula of the kind used here. If the stars are counted in catalogue 1, the modulus determining the decrease is h_{123}; it is composed of h_3 and the modulus h_{12} determining the gradual decrease in catalogue 1 for the case 3 of catalogue 2 being cut off abruptly. Case 3 may be considered as a special case of 6 for $\mu_3 = 0$.

We may also proceed in another way, by using the numbers of stars A_2 actually counted in the second catalogue; then computing by mechanical integration

$$\int (m_2 - m_1)\, A_2 e^{-h_1 (m_2 - m_1)^2}\, dm_2 \quad : \quad \int A_2 e^{-h_1 (m_2 - m_1)^2}\, dm_2. \qquad 8)$$

between $\pm\,\infty$ we get the mean value of $m_2 - m_1$ for all stars m_1. Inserting for A_2 the value following from the formulae in case 7 after the first integration, it appears that we get just the same result by this method as by formula 7. If the variation of A_2 with magnitude deviates considerably from the error law assumed, this process of mechanical integration is likely to give results nearer to truth.

The question now arises whether in case 5 also mechanical integration may be substituted for the use of the formula. The number of stars A_2 found by counting catalogue 2 is given by

$$\frac{\sqrt{h_2 h_3}}{\pi} \int_{-\infty}^{l} dm_3 \int_{-\infty}^{+\infty} A_0 e^{bm - h_2 (m_2 - m)^2 - h_3 (m_3 - m)^2}\, dm = A_0\, e^{bm_2}\, e^{\frac{1}{2} b\mu_2^2}\, \Psi\left(\sqrt{h_{23}}(l - m_2 - b\mu_2^2)\right).$$

Treating these values as is done in the process of mechanical integration, we have

$$\int_{-\infty}^{+\infty} x_2 e^{bx_2 - h_1 x_2^2}\, dx_2 \int_{-\infty}^{l - b\mu_2^2} e^{-h_{23} (x_3 - x_2)^2}\, dx_3 \quad : \quad \text{the same without factor } x_2.$$

By changing the order of integrations and putting $\sqrt{h_{123}}(l - b\,\mu^2_{12}) = k'$ we get the result

$$b\mu_1^2 - \frac{\mu_1^2}{\mu_{123}\sqrt{2\pi}}\, \frac{e^{-k'^2}}{\Psi(k')}.$$

Comparing this formula with the result of the rigorous computation of case 5 we find that the second term, caused by incompleteness, differs somewhat in argument and in coefficient. In case 5, as we have seen, it is just as if catalogue 2 gives true magnitudes. In the mechanical computation, however, the real m_2 of this catalogue are used and so the displacement of its maximum as well as the greater spreading of its incompleteness by μ_2 are introduced into the result.

In the case that μ_2 is small, the difference of the results will be of no practical importance. If we wish to avoid this systematic difference we may subtract the values found by mechanical integration from $b\mu_1^2$, displace the argument by $b\mu_2^2$, multiply with μ_{123}/μ_{13} and subtract the result from $b\mu_1^2$ (method 9). If μ_2 is large, it is better to make use of the formulae of case 5.

Differences between the Harvard Catalogues.

12. The Harvard catalogues 24, 34, 44, 45, 46 have all been made with the same instrument. Systematic errors still occur between them, as has been shown by MüLLER and KEMPF in the Introduction of the Potsdam „*Photometrische Durchmusterung*". They may be attributed partly to a different treatment of the extinction and the standards used, partly to the influence of incompleteness.

Comparison of H 34 and H 45. The stars common to both catalogues were arranged once according to m_{45}, then according to m_{34} as argument; average differences were computed for each half magnitude (Table 7). The remaining deviations give $\mu_{12} = 0.22$; from a comparison of H 34 and H 72 0.20, nearly the same value, was found; as there is no reason to assume a different accuracy for these catalogues we will take $\mu_{12} = 0.21$ and $\mu_1 = \mu_2 = 0.15$, $\mu^2 = 0.022$. This value, as was to be expected, exceeds somewhat the mean error found from each catalogue separately by comparing its single results (0.08 to 0.13 according to FETLAAR, *Recherches astronomiques d' Utrecht*, IX, 9).

The number of common stars was counted for each tenth magnitude and added for both arrangements (N in table 5; 5.0 means 5.00 to 5.09). Dividing these numbers by the factor $10^{0.50(m - m_0)}$ (0.50 being the logarithmic increase of stardensity pro magnitude) we get relative values of completeness (R).

Table 5. Multitude of stars.

m	H 34 · H 45		H 34 · H 46		m	H 34 · H 45		H 34 · H 46		m	H 34 · H 45		H 34 · H 46	
	N	R	N	R		N	R	N	R		N	R	N	R
5.0	44	44	103	103	6.0	47	15	115	36	7.0	52	5.2	46	4.6
5.1	43	38	119	105	6.1	43	12	111	32	7.1	62	5.5	49	4.3
5.2	45	36	136	108	6.2	36	9.0	98	25	7.2	70	5.6	32	2.5
5.3	74	52	168	119	6.3	30	6.7	86	19.3	7.3	45	3.2	24	1.7
5.4	67	42	164	104	6.4	41	8.2	98	19.6	7.4	52	3.3	19	1.2
5.5	73	41	192	108	6.5	37	6.6	107	19.1	7.5	48	2.7	13	0.73
5.6	75	37	181	91	6.6	46	7.3	88	13.9	7.6	45	2.2	11	0.55
5.7	74	33	160	71	6.7	49	6.9	84	11.9	7.7	33	1.5	8	0.35
5.8	68	27	158	62	6.8	68	8.6	89	11.2	7.8	34	1.4	8	0.32
5.9	46	16	141	49	6.9	63	7.1	52	5.9	7.9	19	0.7	5	0.18

The first decrease of completeness between 5.5 and 6.3 is due to H 34 for bright stars being complete and below 6^m being confined to zones of 40′ width. The second decrease between 7.0 and 8.0 is due to H 45 nearly vanishing at 7.5 Cordoba D. M. In order to find μ_{13} determining the modus of decrease the values R were smoothed; the tangens drawn in the midst between the constant values 42 (for 5.0—5.5) and 7.3 (for 6.3—7.0) intersects the horizontal lines 42 and 7.3 at a distance $\mu_{13}\sqrt{2\pi}$ to both sides of l. On account of the irregularities in the course of the values R this cannot be a very accurate procedure; but we have no other and for the purpose it suffices. In this way $l = 5.95$ and $\mu_{13}\sqrt{2} = 0.33$ was found; the same method applied to the second decrease afforded $l = 7.45$ and $\mu_{13}\sqrt{2} = 0.51$. In table 6 the corrections computed with formula 5 are given in the first line; the second line contains the results found by mechanical integration according to formula 8, while they are strictly reduced by method 9 to the correct values in the third line ($\mu_1^2 = 0.024$, $b\mu_1^2 = 0.028$ has been used here).

Table 6. Corrections for multitude (in 0.001_m).

m	5.35	5.65	5.95	6.25	6.55	6.85	7.15	7.45	7.75
Formula 5. .	— 027	— 011	+ 030	+ 031	— 028	— 021	— 004	+ 003	+ 067
Formula 8. .	— 022	+ 004	+ 031	+ 019	— 023	— 018	+ 004	+ 022	+ 042
Method 9 . .	— 018	+ 015	+ 045	+ 023	— 024	— 015	+ 009	+ 028	+ 050

The differences caused by the lack of exact correspondence of the numbers practically used in mechanical integration and the formula underlying case 5, are not great, some few hundredths of a magnitude at most.

In table 7 the results for H 45—H 34 with argument m_{34}, and H 34—H 45 with argument m_{45} are given. As the same μ is assumed for both catalogues the correction for multitude is the same in both cases; thus by combining the results after the sign of the second has been reversed the true difference of the catalogues is found free from systematic error. The last columns contain the correction for multitude and incompleteness deduced from the difference of the observed results and computed from the last line of table 6.

Table 7. Comparison of H 34 and H 45.

m	H 45 — H 34	H 34 — H 45	True H 45 — H 34	Correction for multitude	
				obs.	comp.
5.0 — 5.5	+ .030 (134)	+ .027 (139)	+ .00	+ .03	+ .03
5.5 — 6.0	— .036 (175)	+ .012 (162)	— .02	— .01	.00
6.0 — 6.5	— .056 (94)	+ .050 (103)	— .05	— .00	— .02
6.5 — 7.0	— .002 (130)	+ .041 (143)	— .02	+ .02	+ .02
7.0 — 7.5	— .089 (136)	+ .037 (145)	— .06	— .03	.00
7.5 — 8.0	— .132 (95)	+ .098 (85)	— .11	— .02	— .06

The true catalogue difference seems to increase with magnitude. For the magnitudes 6.0—7.0, the only ones where H 45 will be used, the mean difference H 45—H 34 = 0.04 may be adopted.

13. *Comparison of H 34 and H 46.* The southern stars below — 30°, which are common to these catalogues, have been treated in the same way; the numbers counted are contained in table 5. The first decrease in completeness due to H 34 is the same as in the preceding case; the second, due to H 46 being limited to stars > 7.0, is confluent with the first in their outer parts. We get $l = 5.87$, $\mu_{13}\sqrt{2} = 0.38$ for the first, $l = 6.95$, $\mu_{13}\sqrt{2} = 0.51$ for the second decrease. The corrections have been computed with formula 5. Table 8 gives the results just in the same way as Table 7 for the preceding case.

Table 8. Comparison of H 34 and H 46.

m	H 46 — H 34	H 34 — H 46	True H 46 — H 34	Correction for multitude	
				obs.	comp.
5.0 — 5.5	+ .050 (345)	— .038 (345)	+ .04	+ .01	+ .02
5.5 — 6.0	+ .045 (415)	— .043 (415)	+ .04	.00	— .01
6.0 — 6.5	+ .045 (238)	— .040 (270)	+ .04	.00	— .02
6.5 — 7.0	— .009 (204)	— .017 (216)	+ .00	— .01	— .02
7.0 — 7.5	— .022 (76)	— .053 (94)	+ .02	— .04	— .05
7.5 — 8.0	— .07 (25)	— .07 (21)	00	— .07	— .07

Here the true differences H 46—H 34 show the same course as H 45—H 34, decreasing with magnitude, but they are on the average + 0.06 greater. The magnitudes of H 46 are independent of H 45, because as its standards the former magnitudes of H 14, H 24 and H 44 were adopted; thus a constant difference may occur. Yet it is not certain that this value represents a real constant difference between the catalogues H 45 and H 46, that should be applied to the whole of one of them. Different zones of H 34, north and south of — 30°, are used in deriving these reductions and a different treatment of the correction for extinction as well as irregularities in the atmospheric extinction itself may cause differences of this kind. If this is the real origin the difference between various parts of the same catalogue may be of the same order as that between the different catalogues; and there is no reason to apply constant reductions. The variation of H 45—H 34 and H 46—H 34 with magnitude will be more difficult to explain.

14. *Comparison of H 24 and H 45.* This comparison is made by Müller and Kempf in the Introduction of their Potsdam Catalogue. Besides a dependence on AR, which they think may be caused by a different treatment of the extinctions, they find a marked variation of the difference with magnitude (argument m_{45}). The number of stars common to both catalogues increases strongly to 6.5 and then decreases as rapidly; the corrections for multitude computed by formula 8 from the numbers (third column of Table 9) make the variation found by M. and K. disappear. The mean difference becomes H 24 — H 45 = + 0.02.

Table 9. Comparison of H 24 and H 45.

m	H 24 — H 45	Correction for multitude	H 24 — H 45 corrected
4.5 — 5.0	+ .17 (21)	— .05	+ .12
5.0 — 5.5	+ .02 (66)	— .04	— .02
5.5 — 6.0	+ .07 (182)	— .05	+ .02
6.0 — 6.5	+ .06 (539)	— .06	00
6.5 — 7.0	+ .01 (896)	+ .02	+ .03
7.0 — 7.5	— .05 (214)	+ .08	+ .03

A comparison of H 54 and H 70 has been made by Dr. H. Nort (*Hemel en Dampkring* 1921). He gives the differences H 54—H 70 for each half magnitude

7.0	7.5	8.0	8.5	9.0	9.5	10.0
+ .02	+ .03	+ .03	+ .04	+ .03	— .02	— .16

It is clear from these figures, though nothing is said about it, that m_{70} is the argument. The strong negative value indicates the increasing incompleteness of H 54 for magnitudes below 9. Then for the complete brighter stars a negative correction $b\mu_1^2 = 0.03$ should be applied, making the true systematic difference between these catalogues nearly zero.

15. *Comparison of H 72 and H 34.*

This comparison has been made for the zones —39° and —49°. In order to eliminate the error caused by the increasing incompleteness of H 34 (cut off at 9.0 Cordoba G. C. and Z. C.) the differences were arranged twice, according to both arguments, and combined; supposing μ equal for both catalogues the average is free from error. The abrupt-irregular correction + 0.24 (m — 9) applied below 9.0 in H 72 was subtracted before in order to use the measured magnitudes uncorrected. In this way the true differences H 34—H 72 are found (Table 10, first part).

Table 10. Comparison of H 72.

m_{72}	H 34 — H 72	Harvard Annals 72, pag. 85			
		m	H 70 — H 72	m	H 70 — H 72
7.26	— .06 (26)	7.37	+ .07	9.84	+ .44
7.76	— .10 (46)	7.77	.00	10.12	+ .43
8.04	— .07 (75)	8.14	+ .08	10.54	+ .34
8.40	+ .00 (84)	8.52	+ .03	10.85	+ .27
8.79	+ .04 (97)	8.76	— .01	11.09	+ .36
9.17	+ .06 (125)	9.00	— .04	11.36	+ .67
9.57	+ .07 (136)	9.40	+ .18		
9.94	+ .07 (34)	9.60	+ .03		

The second part of this table gives the averages of sets of 5 stars, found in Harv. Annals 72 by comparing with the measures of H 70. For the brighter stars they have far smaller weight than the averages in the first part. We may represent the different results by a smooth curve that for the fainter classes corresponds with PICKERINGS straight line. It is given by the upper values of Table 11.

Table 11. Corrections for H 72 (in 0.01^m).

7.0	7.5	8.0	8.5	9.0	9.5	10.0
— 08	— 08	— 06	— 02	+ 04	+ 12	+ 22
— 08	— 08	— 06	— 02	+ 04	00	— 02

The correction to the catalogue values (as they are given in Harv. Annals 72, the linear correction having been applied already) is given in the last line; it must show an abrupt edge at 9.0.

THE MAGNITUDES OF THE „BONNER DURCHMUSTERUNG".

The decimal error.

16. Though the magnitudes in the Bonner Durchmusterung are given down to tenths, this decimal is chiefly a result of computation. Already the predominance of the decimals 0 and 5 and the scarcity of the figures 1, 4, 6 and 9 shows that they are obtained from much coarser direct estimates. In the Introduction to the B. D. volumes hardly anything is said about the method of making the estimates and deriving catalogue magnitudes from them. Afterwards SCHOENFELD has given further information in a letter to Prof. S. PEIRCE, printed in Vol. 9 of the Harvard Annals (p. 26—28). From this letter we learn the following details.

At first only half magnitudes were estimated (in all the zones observed by THORMANN till May 1853, and the first part of the zones of SCHOENFELD and KRÜGER). After one fifth of the work had been finished (since the end of 1854) the practice was adopted of noting striking deviations from these magnitudes (7 bright and $7\frac{1}{2}$ faint stand for 7.2 and 7.3). Smaller deviations corresponding to other decimals could also be perceived, but there was no time to write them down, except in poor zones. Since 1857, when 70 % of the work had been finished, and high declinations were reached, also the other tenths were estimated; but still the decimals 4, 9 and especially 1 and 6 were used much less than the others. In the zones of revision, observed by ARGELANDER with the meridiancircle, the accurate estimates form a somewhat greater part of the whole. In the deduction of catalogue magnitudes, mostly founded on two observations, as a rule in the case of an odd difference the fainter decimal is adopted (thus generally for the mean of estimates 0 and 5 the decimals 3 or 8 are put down).

The relative scantiness of some decimals means that they extend over a range smaller than 0.1^m in a homogeneous scale. This range may be found for each decimal from the number of stars having this magnitude. The number of stars for each tenth magnitude in the Northern B.D. has been counted by VON LITTROW.[1] Comparing the sum total of all stars from the brightest downward including e. g. 7.3 with a table giving log $N(m)$ as a function of m we get the ,,statistical magnitude'' corresponding to the limit between 7.3 and 7.4 (that will be called 7.35). As a basis for these statistical magnitudes we will use the log $N_0^{90}(m)$ given by VAN RHYN (Astron. Nachr. **213**), reduced to half the hemisphere. The difference between two consecutive limits is the range of the included tenth magnitude. The following table 12 gives the total number of stars for each limit, the corresponding statistical magnitude and the range of each tenth magnitude.

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[1] K. VON LITTROW, Zählung der nördlichen Sterne im Bonner Sternverzeichnisse nach Grössen. (Sitzungsberichte der . Ak. d. Wiss., II$^{\text{er}}$ Abt., Bd. 59, 1869.)

Table 12. Statistical magnitudes of B. D.

m	Number	Stat. m.	Range	Corr.	m	Number	Stat. m.	Range	Corr.
5.9					7.7			.09	
	1490	5.60				16224	7.69		— .04
6.0			.30		7.8			.13	
	2108	5.90				18708	7.82		— .04
6.1			.05		7.9			.05	
	2214	5.95		— .22		19699	7.87		— .02
6.2			.10		8.0			.22	
	2507	6.05		— .21		25321	8.09		— .01
6.3			.09		8.1			.07	
	2782	6.14		— .19		27099	8.16		+ .01
6.4			.03		8.2			.11	
	2883	6.17		— .18		30749	8.27		+ .03
6.5			.31		8.3			.13	
	4122	6.48		— .17		35358	8.40		+ .05
6.6			.04		8.4			.08	
	4281	6.52		— .14		38459	8.48		+ .07
6.7			.09		8.5			.21	
	4738	6.61		— .12		48247	8.69		+ .09
6.8			.15		8.6			.07	
	5639	6.76		— .10		52436	8.76		+ .12
6.9			.03		8.7			.12	
	5876	6.79		— .08		59235	8.88		+ .15
7.0			.28		8.8			.16	
	8017	7.07		— .07		70198	9.04		+ .19
7.1			.03		8.9			.09	
	8362	7.10		— .05		77794	9.13		+ .24
7.2			.10		9.0			.25	
	9346	7.20		— .04		101071	9.38		+ .30
7.3			.12		9.1			.14	
	10702	7.32		— .04		116686	9.52		+ .37
7.4			.04		9.2			.16	
	11218	7.36		— .04		137420	9.68		+ .45
7.5			.21		9.3			.20	
	14078	7.57		— .04		168698	9.88		
7.6			.03		9.4			.18	
	14687	7.60		— .04		203649	10.06		
7.7			.09		9.5			.44	
						314925	10.50		

The irregularities in the range have a period of 0.5^m. Eliminating them by taking the mean of every five consecutive values of the statistical magnitude we get the smoothed differences statistical minus observed magnitude, which under the head „corrections" have been put in the last column. They clearly show some fluctuations, an observed magnitude being at first (6^m to 7^m) too great, then becoming normal, and from 8^m again increasing. As the ranges of each decimal take part in this general fluctuation of scale, we will reduce them to „subjective ranges" by dividing them by 2 × the range of the half magnitude in which they are included. Thus we get

Table 13. Range of decimals.

	Obs. range	Decimals				
		3;8	4;9	0;5	1;6	2;7
6.25 — 6.75	0.56	08	03	28	04	08
6.75 — 7.25	0.59	13	03	24	03	08
7.25 — 7.75	0.49	12	04	22	03	09
7.75 — 8.25	0.58	11	04	19	06	09
8.25 — 8.75	0.61	11	07	17	06	10
8.75 — 9.25	0.80	10	06	16	09	10

The increasing subjective certainty of the estimates with decreasing brightness becomes manifest in the decreasing range of the decimals 0 and 5 and the increasing range of 1, 4, 6 and 9. On the average these ranges are 11, 04, 21, 05, 09, thus dividing a half magnitude .25 to .75 by the limits .36, .40, .61, .66. The average magnitude of each decimal .3 to .7 (or .8 to .2) is .305, .38, .505, .635, .705, thus the greatest systematic error made in adopting the observed magnitudes is only 0.035. It is different, however, with counts of stars including a certain decimal. SEELIGER has counted the Bonner Durchmusterung [1]), dividing the stars in groups 6.6 to 7.0, 7.1 to 7.5 etc.; these counts will also form the basis of our research. The limits of these groups (7.05, 7.55 etc.) are appreciably displaced by the decimal error and are lying 0,10^m, sometimes more, below the mean brightness of the stars 7.0 7.5 etc.

After the communication of SCHOENFELD the decimal error may be expected to vary with declination. Therefore in the same way as for the whole northern hemisphere statistical magnitudes have been derived for separate zones of 5°. As such a zone cannot be assumed to have the same mean density as the whole sky, reductions were computed by ascertaining which portions of it are situated in the three Kapteyn zones with $\beta \pm 20°$, between 20°—40°, between 40°—90°; these reductions vary a little with magnitude. The decimal ranges, deduced from these results, show hardly any trace of the expected variation with declination; the scarcity of the decimals 1, 4, 6 and 9 and the preference of 0 and 5 is just as great in the higher as in the lowest zones of declination.

The density error.

17. By using the photometric measures in Harvard Annals, SEELIGER and PICKERING have shown that the Durchmusterung magnitudes depend on the position relative to the Milky Way. In the Milky Way stars of the same brightness are estimated fainter than in higher galactic latitudes. In the densely filled galactic regions the stars follow one another so rapidly that the observer has no time to record them all, and stars that in poor regions are observed, are omitted here. Thus the lowest magnitude 9.5 is here applied to brighter stars and this displaces the scale also for brighter classes. Another cause tending in the same direction is the stronger illumination of the field of view by a great multitude of stars; then the retina becomes less sensitive and by the contraction of the pupil the actual aperture (as only a power of 9 with an aperture of 7 c.M. was used) is diminished.

In deriving the numerical influence of density the data of SEELIGER and PICKERING are used. SEELIGER divides the northern sky into 8 zones of galactic latitude called I—VIII and determines in each zone for magnitude 6.3—6.7, 6.8—7.2 etc. the mean difference H 24 — Bonn. He does not give the density for each zone itself but only the density relative to the galactic zone V:

Latitude	+ 90°	+ 70°	+ 50°	+ 30°	+ 10°	— 10°	— 30°	— 50°	— 70°
Zone	I	II	III	IV	V	VI	VII	VIII	
Density	0.35	0.37	0.45	0.68	1.00	0.77	0.47	0.41	

After some trials I have assumed that this unit represents 25 stars per square degree. Combining the zones with nearly equal density (with the weights put after their name) we get the results of the first half of the following table.

In Harvard Annals 23 the same observations are grouped by dividing the northern sky into 24 equal areas (denoted *A* 1—3, *B* 1—9, *C* 1—12). We have computed the mean density *D*

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[1]) Neue Annalen, München, Bd. II.

for each area, arranged them and combined them four to four in the second part of the same table. The differences H—B are given in hundredths of a magnitude.

Table 14. Influence of density.

Regions	*D*	6.5	7.0	7.5	8.0	8.5	9.0
S. I (2) II (5) VIII (1)	9.2	— 03	+ 09	+ 09	+ 10	+ 18	+ 33
III (8) VII (4)	11.4	+ 08	+ 09	+ 09	+ 07	+ 16	+ 23
IV (12) VI (8)	17.9	+ 05	+ 08	+ 10	+ 09	+ 12	+ 19
V (12)	25.0	00	+ 04	+ 03	+ 05	+ 07	+ 12
P. B 5, 6, C 6, 7,	8.5	— 05	00	+ 10	+ 15	+ 22	+ 44
A 2, B 4, C 2, 8.	10.0	— 11	+ 03	+ 04	+ 06	+ 19	+ 33
A 1, C 1, 5, 12	12.0	+ 02	+ 01	— 02	— 02	+ 17	+ 30
A 3, B 3, 7, C 9	15.0	+ 07	+ 12	+ 14	+ 13	+ 18	+ 29
B 1, 2, C 3, 11	18.2	00	+ 05	+ 04	+ 03	+ 15	+ 25
B 8, 9, C 4, 10	25.5	— 01	— 03	— 03	— 01	+ 08	+ 20

If we express the differences Harvard—Bonn by linear functions of the density *D* we find the coefficients of *D* from both parts of the table (in 0.0001^m).

S	— 3	— 32	— 34	— 23	— 71	— 110
P	+ 35	— 18	— 49	— 60	— 73	— 113
Mean	+ 16	— 25	— 42	— 42	— 72	— 112

For the brightest classes the influence of density is zero (the positive value cannot be real), and it increases fairly regularly for the fainter classes.

18. For the faintest classes below 9.0 the density error may be found from the data in Harvard Annals 72, N⁰. 6. („Scale of the Bonn Durchmusterung”). Here in Tables IV—VI the differences Harvard—Bonn are given for each hour in AR for each of the three Volumes of the B. D. Computing *D* for each hour and grouping them according to *D* for each volume we get (in 0.01^m).

Table 15. Influence of density.

Vol. Hours	*D*	9.0	9.1	9.2	9.3	9.4	9.5
III 10 — 15	9.0	+ 24	+ 33	+ 39	+ 79	+ 86	+ 111
V 7 — 16	9.2	27	52	43	62	70	93
IV 4, 9 — 16	9.7	45	57	67	78	103	142
III 0 — 3, 9, 23	10.3	41	48	58	69	82	122
III 4, 8, 16, 21, 22	13.3	25	28	45	62	74	107
IV 0 — 3, 8, 23	13.5	28	36	52	62	77	115
V 3 — 6, 17, 18	15.0	16	37	33	50	66	74
IV 7, 17, 18, 21, 22	19.3	29	38	50	59	70	109
III 5, 7, 17, 20	19.6	03	16	14	37	60	92
V 0, 1, 2, 19	21.1	24	25	47	46	59	79
V 20 — 23	23.8	24	41	31	45	52	73
III 6, 18, 19	26.2	01	15	17	31	50	82
IV 5, 6, 19, 20	28.0	19	23	55	39	62	89

The coefficients of D are found from these figures (in 0.0001):

—114 —131 —93 —203 —177 —216.

The increase of the coefficient for fainter stars is much stronger here than for magnitudes 6—9. This was to be expected, and is still increased by the widening of the scale of magnitudes below 9.0^m. The coefficients found from tables 14 and 15 have been smoothed by a quadratic function of the photometric magnitude (vide the following chapters). In this way the following values (rounded to 3 decimals) have been found.

Table 16. Adopted coefficient of density (in 0.001^m).

6.5	7.0	7.5	8.0	8.5	9.0	9.1	9.2	9.3	9.4	9.5
0	—1	—3	—5	—7	—12	—13	—14	—16	—18	—22

They have been used always to reduce the Northern Bonner Durchmusterung magnitudes to a mean density $D = 15$.

Corrections depending on declination.

19. The scale used in observing the Durchmusterung magnitudes exists only in the mind of the observer; thus it may be subject to variations with the state of his mind as well as the state of the sky. Properly speaking each zone should have its own reductions to photometric scale. Since, however, the zones have not been published and since only for a small part of them stars have been measured photometrically we must proceed in another way. By the arrangement of the work, slowly proceeding during many years from lower to higher declinations, variations of the scale corrections with time will appear as variations with place at the sky. Temporary fluctuations will affect the different hours in the same zone of declination, while slow „secular" variations of scale will make it vary with declination. We are able to find only these more or less regular average variations, while the irregular deviations from them must be treated as accidental errors. The zones of declination, each treated as a whole, will show the more important changes in the estimates, while minor irregularities appear as variations with Rectascension within each zone. We will make use of the zones 0°—5°, 5°—10° etc. because these are the zones counted by SEELIGER; the zone —2° to 0° will be treated separately.

The results of SEELIGER have shown already that the corrections for the zones 0°—10° deviate considerably from the higher ones; between 10° and 15° declination there is a sudden and strong variation. Perhaps it is due to the first observer, THORMANN, ceasing to observe May 1853 after having finished 110 zones, while SCHOENFELD and KRÜGER, who executed the chief part (909 and 810 zones) commenced to observe February and August 1853. Afterwards the variations of the scale are smaller and may, as PICKERING has shown, be attributed partly to atmospheric extinction. Thus for some investigations we have combined the zones of declination into sections reaching from —2° to + 10°, + 10° to + 35° and + 35° to + 90°.

20. The differences Harvard—Bonn were derived from the catalogues H 45, H 24 and H 70; for the first two the stars were grouped in half-magnitudes (6.3—6,7 called 6.5, 6.8—7.2 called 7.0 etc.). For each hour of AR the density correction was applied, and the mean difference H—B was computed for the whole zone. The remaining deviations from this mean value were considered as accidental errors and used to derive the mean error of a difference. The mean error was in each case found by the quartil method, arranging the single errors according to their amount and dividing their number in four equal parts; the probable error thus found was divided by 0.674.[1]) The results are contained in the next table.

Table 17. Mean errors μ_{12} (H — B) (in 0.01^m).

Zones	H 45		H 24							H 70					
	6.5	7.0	6.5	7.0	7.5	8.0	8.5	9.0	Mean	9.0	9.1	9.2	9.3	9.4	9.5
— 2° to + 10°	36	—	40	36	43	39	36	33	38	—	—	—	—	—	45
+ 10° „ + 35°	35	—	36	40	38	31	28	24	33	—	—	—	—	—	51
+ 35° „ + 90°	34^5	35	31	36	35	32	29	26	32	—	—	—	—	—	46
All............	35	35	36	37	39	34	31	28	34	35	29	31	35	40	47
Stars..........	2257	2454	892	1268	1573	1800	2469	1897	—	—	—	—	—	—	—

The mean error clearly diminishes from the bright stars down to 9.0; below this magnitude it again increases rapidly. As it seems to be greater for the first zone, in smoothing these values we have treated this first zone separately from the rest. Subtracting from μ^2_{12} the amount μ^2 (Harv.) $= 0.022$ we get μ_1^2 and μ_1; using for the coefficient b values regularly decreasing from 1.13 (for 6.0) to 0.97 (for 10.0) we find the data of the following table.

Table 18. Adopted values of μ and $b\mu^2$.

m		6.5	7.0	7.5	8.0	8.5	9.0	9.1	9.2	9.3	9.4	9.5
μ	— 2° to + 10°	0.37	.35	.34	.32	.30	.29	.28	.29	.32	.37	.44
	+ 10° to + 90°	0.35	.33	.30	.27	.25	.24	.25	.28	.32	.37	.44
$b\mu^2$	— 2° to + 10°	0.15	.14	.12	.11	.09	.08	.08	.09	.10	.13	.18
	+ 10° to + 90°	0.14	.12	.10	.08	.07	.06	.06	.08	.10	.13	.18

Taking into account that the width of the Bonn scale increases strongly for stars below 9.0 it appears that the subjective mean error (expressed in estimated magnitudes) decreases steadily from 0.31 (for 6.5) to 0.16 (for 9.0) and 0.12 (for 9.5).

In these mean errors the variations with AR are included as well as the accidental errors proper; for the systematic errors depending on multitude this has no sensible influence. According as the first or the second kind of error is predominant the general mean for each zone must be computed by giving each hour, or by giving each star the same weight. If the number of stars is very

---

[1]) The distribution of the errors has been investigated for some magnitude groups (H 45: 6.5 7.0; H 24: 8.0 8.5 9.0). The fraction of the whole number included between 0, $\frac{1}{2}\rho$, ρ, 2ρ, 3ρ, ∞ was found for H 45: .259 .241 .328 .131 .042 (3675 stars), for H 24: .237 .236 .312 .126 .058 (6178 stars), while the theoretical fractions are .264 .236 .323 .134 .043. Only a slight excess of great errors is found in the second group. Hence there is no reason to exclude all differences beyond a certain limit.

great, the first case may be assumed, if small, the second. As during the course of the work the relation between the two kinds of errors was not always clear, both methods of computing means have often been followed; they are distinguished by the figures *h* and *s*.

21. *Comparison with H 45.* As all stars of the B. D. down to 7.5 are contained in **H 45**, the magnitude groups 6.3—6.7 and 6.8—7.2 are complete in this catalogue. Thus the differences H—B must be corrected for multitude by $b\mu_1^2$. In order to test these corrections a second arrangement was made (only for the three sections as a whole) with the Harvard magnitudes as argument, and the mean catalogue differences for H 6.00—6.59 and H 6.50—6.99 were determined.

Table 19. Comparison of corrected and uncorrected differences.

	Uncorrected differences H—B				Corrected differences H—B		
	— 2° to + 10°	+ 10° to + 35°	+ 35° to + 90°		— 2° to + 10°	+ 10° to + 35°	+ 35° to + 90°
B 6.5	— 31	— 06	+ 04	B 6.5	— 44	— 19	— 09
B 7.0	— 30	— 03	+ 14	H 6.25	— 50	— 23	— 13
H 6.25	— 52	— 25	— 15	B 7.0	— 43	— 16	+ 01
H 6.75	— 43	— 16	— 05	H 6.75	— 41	— 14	— 03

The uncorrected differences clearly show a systematic difference, the amount of which is 0.17; the theoretical value $b\mu_{12}^2 = 0.15$ (from $\mu_{12} = 0.35$). Applying the corrections — 0.13 and + 0.02 we get the corrected differences, where the systematic deviation has disappeared.

The corrections applied to the separate zones of 5° are the values $b\mu^2$ of Table 14 for 6.5 and 7.0, corrected for the systematic difference + 0.02 between H 45 and H 24, thus reduced to the scale of H 24. Table 20 contains the resulting reductions of B.D. to Harvard 45.

Table 20. Reductions of B. D. to Harvard 45.

		— 2°–0°	0°–5°	5°–10°	10°–15°	15°–20°	20°–25°	25°–30°	30°–35°	35°–40°	40°–45°	45°–50°	50°–55°	55°–60°	60°–65°	65°–70°	70°–75°	75°–80°	80°–90°
6.5	*h*	— 47	— 43	— 45	— 21	— 21	— 16	— 17	— 17	— 07	— 08	— 07	— 10	— 15	— 11	— 04	+ 01	— 02	— 06
	s	— 50	— 40	— 43	— 26	— 19	— 14	— 16	— 16	— 09	— 08	— 06	— 06	— 13	— 08	— 05	+ 02	— 06	— 14
7.0	*h*	— 44	— 45	— 44	— 25	— 12	— 07	— 12	— 10	+ 04	+ 04	+ 07	+ 01	— 03	+ 03	+ 09	+ 13	+ 05	— 01
	s	— 43	— 45	— 38	— 22	— 12	— 11	— 14	— 07	+ 02	+ 03	+ 06	+ 03	— 04	+ 02	+ 10	+ 13	+ 05	+ 02
Number of stars		57	90	92	142	183	166	136	171	197	187	178	145	162	121	81	70	54	29
		76	239	234	313	345	317	322	370	419	411	367	293	248	235	173	139	104	58

22. *Comparison with H 24.* In the zones of H 24, expressly observed as a „Revision of the Durchmusterung", each magnitude class of B.D., except the stars below 9.0, is complete within the assigned limits (20′ for 8.0—9.0, 1° for 7.0—7.9, 2° for > 7.0). Therefore only a correction $-b\mu_1^2$ must be applied (from table 17). The stars below 9.0 are excluded and the faintest group consists only of the classes 8.8, 8.9 and 9.0 (mean 8.92). The hourly means are corrected for density error. The resulting mean reduction of B.D. to Harvard 24 for each zone is contained in Table 21.

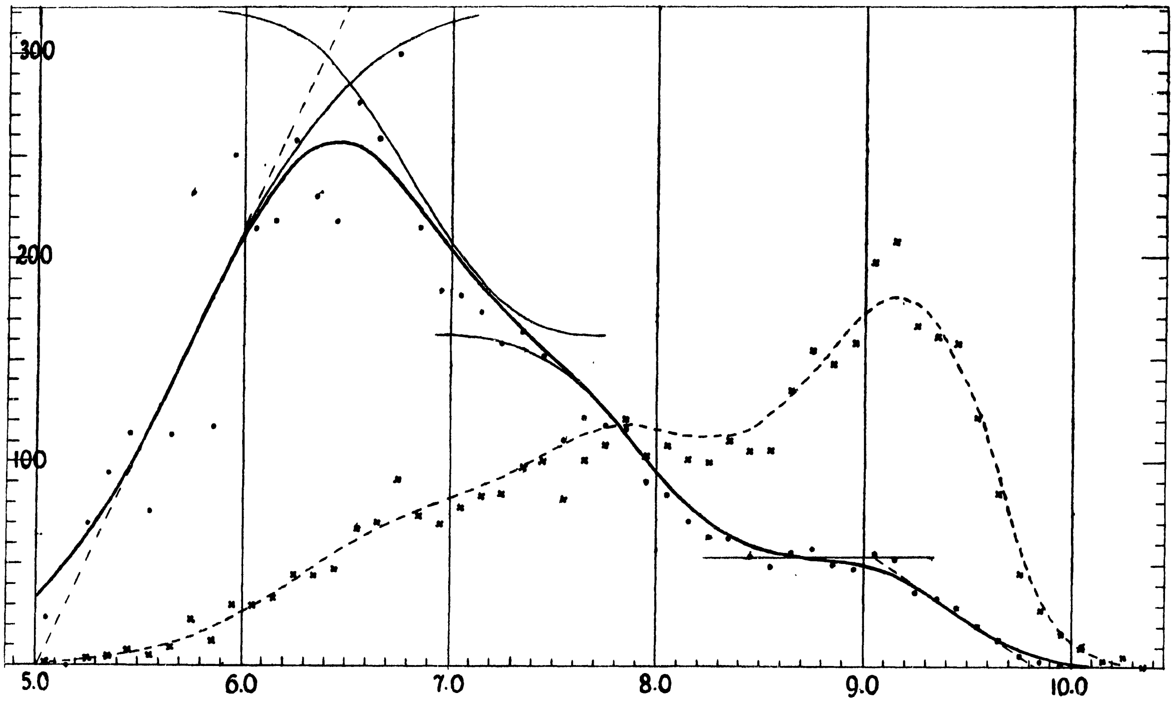
Table 21. Reduction of B. D. to Harvard 24.

	6.5	7.0	7.5	8.0		8.5		9.0 (8.92)		Number of stars
	s	*s*	*s*	*s*	*h*	*s*	*h*	*s*	*h*	6.5, 7.0, 7.5, 8.0, 8.5, 9.0
— 1°, — 0°	— 38	— 33	— 40	— 24	— 26	— 05	— 01	+ 13	+ 14	31, 21, 45, 91, 151, 267
+ 0°, + 1°	— 47	— 42	— 34	— 33	— 34	— 03	— 02	+ 12	+ 14	19, 43, 72, 103, 213, 354
+ 4°, 5°	— 36	— 37	— 34	— 34	— 31	— 06	— 07	+ 16	+ 15	21, 41, 61, 99, 226, 315
+ 9°, 10°	— 43	— 32	— 32	— 18	— 20	— 07	— 08	+ 09	+ 08	34, 73, 86, 108, 197, 263
+ 14°, 15°	— 15	— 11	— 13	— 05	— 09	+ 01	— 01	+ 12	+ 10	79, 92, 87, 117, 132, 202
+ 19°, 20°	— 11	— 08	— 11	+ 03	+ 05	+ 12	+ 12	+ 23	+ 23	74, 90, 108, 93, 157, 179
+ 24°, 25°	— 11	— 01	+ 02	— 05	+ 02	+ 17	+ 18	+ 29	+ 29	64, 76, 113, 102, 150, 200
+ 29°, 30°	— 09	— 05	— 10	+ 01	+ 06	+ 09	+ 14	+ 23	+ 23	45, 96, 109, 139, 148, 203
+ 34°, 35°	— 14	— 03	+ 05	+ 09	+ 10	+ 20	+ 21	+ 28	+ 28	77, 89, 93, 114, 173, 204
+ 39°, 40°	— 06	+ 04	+ 09	+ 19	+ 20	+ 32	+ 33	+ 43	+ 40	69, 133, 163, 142, 162, 240
+ 44°, 45°	— 06	— 02	+ 16	+ 15	+ 16	+ 18	+ 21	+ 40	+ 41	67, 92, 104, 112, 139, 196
+ 49°, 50°	— 07	+ 10	+ 11	+ 15	+ 18	+ 18	+ 17	+ 32	+ 31	54, 80, 104, 95, 130, 199
+ 54°, 55°	— 13	— 02	+ 07	+ 07	+ 03	+ 10	+ 11	+ 21	+ 16	59, 62, 86, 93, 84, 139
+ 59°, 60°	— 06	+ 01	+ 04	+ 05	+ 07	+ 20	+ 19	+ 24	+ 23	62, 76, 76, 89, 105, 111
+ 64°, 65°	— 12	+ 11	+ 02	+ 15	+ 08	+ 14	+ 11	+ 18	+ 14	34, 73, 61, 63, 72, 98
+ 69°, 70°	— 01	+ 07	+ 11	00	— 01	+ 01	— 04	+ 01	— 01	29, 37, 40, 51, 57, 85
+ 74°, 75°, 76°	— 09	+ 01	+ 03	— 02	00	— 05	— 05	+ 03	+ 03	30, 43, 53, 73, 131, 187
+ 79°, 80°, 81°	— 11	+ 05	— 07	— 03		— 09		+ 06		30, 30, 50, 61, 32, 39
+ 82° to 90°	— 26	00	+ 01	+ 02		— 01		+ 05		14, 35, 45, 55, 16, 25

In order to diminish the influence of accidental grouping, the differences between the catalogues were also deduced with argument m(Harvard) for groups 6.00—6.49, 6.50—6.99 .... 9.50—9.49. As here the ,,second'' catalogue, the Bonn catalogue, is cut off for part of the zones at certain limits, the finding of the corrections is a complicated matter. The multitude of the stars and the law of their incompleteness was deduced by counting in zones 15°—35° (where the proportion of the surfaces is the normal one 6 : 3 : 1) the number of stars for every 0.10^m H. In the figure on pag. 36 they are indicated by crosses. Bij dividing these numbers by $A(m)$, the normal number pro square degree, the regular increase with magnitude is eliminated and we get the relative number of stars. The irregular course of these numbers (represented by dots in the figure) shows the influence of different causes of incompleteness. The normal values (for 5 zones of 2°, 1°, 20′ at mean declination 25°) for the bright, middle and faint stars should be 325, 162, 54. These data determine two limits l beforehand : 6.95 B = 6.8 H where the number of stars decreases from 325 to 162, and 7.95 B = 7.9 H where it decreases again from 162 to 54; the coefficients $\mu_{12}\sqrt{2}$ determining k are 0.52 and 0.48. The numbers actually counted indicate a somewhat smaller mean star density. Moreover for bright stars above 5^m and for faint stars below 10^m the completeness decreases to zero; the moduli that determine this vanishing must be taken from the curve and they are found by means of the tangents drawn in the figure: $l = 5.75$, $\mu_{123}\sqrt{2} = 0.85$ and $l = 9.45$ (0.08 beyond the value corresponding to 9.05 B), $\mu_{123}\sqrt{2} = 0.45$. As the domains of these different curtailments overlap, the total expression for the relative number of stars becomes :

$$\Psi\left(\frac{m-5.75}{0.85}\right)\left\{162\,\Psi\left(\frac{6.8-m}{0.52}\right)+108\,\Psi\left(\frac{7.9-m}{0.48}\right)+54\right\}\Psi\left(\frac{9.45-m}{0.45}\right).$$

The figure shows this curve, as well as its separate constituents. With these moduli the corrections for incompleteness were computed for every tenth magnitude, by means of formulae 3C and 7. It appeared, however, that without sensible difference, they could be obtained as well by adding



the separate corrections for each limit according to formula 3D (where the coefficient for the first and the last term, according to case 7, must be taken $\mu^2_{12}/\mu_{123}\sqrt{2\pi}$).

Table 22. Separate corrections for incompleteness (in 0.001^m).

m	6.0	6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	7.0	7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9
I	+ 123	103	85	69	56	43	33	25	19	13	9	6	4	3	2	1	1			
II	— 6	10	18	27	40	55	72	84	96	99	94	83	68	50	34	21	11	6	3	2
III	—									1	2	3	7	15	22	38	58	80	102	123
m	8.0	8.1	8.2	8.3	8.4	8.5	8.6	8.7	8.8	8.9	9.0	9.1	9.2	9.3	9.4	9.5	9.6	9.7		
III	— 133	131	116	92	66	40	20	8	4	2	1									
IV	—					1	3	6	12	23	38	60	91	127	168	216	269	310		

Since the limits 6.95, 7.95, 9.05 Bonn for different zones correspond to different m_H, and also μ_{12} is somewhat different for lower and higher zones, this method of computation has the advantage that, by simply displacing the separate error curves and changing their amplitude and width, the corrections for each zone may be had without much labour. The abnormal proportions of the surfaces used for bright, middle and faint stars in the zones —1—0 (7 : 4 : 2), + 0 + 1 (7 : 4 : 3), 74—75—76 (27 : 21 : 17) and 79—90 (184 : 167 : 15) could be taken into account in the same way by varying the amplitude of the curves. For the correction for e.g. 6.25 H the mean of the six values 6.0—6.5 (after adding $-b\mu_1^2$) was taken, and likewise for others.

The results of these comparisons are given in table 23 in condensed form. For the first group was found : H 6.25 mean difference B—H = + 0.28, thus mean $m_B = 6.53$; correction for multitude + 0.05 to be subtracted from B—H, makes the corrected difference H—B = — 0.23; from the figures in Table 23 : 6.53 and — 0.23 the different data may be found. The first and the last result (6¼ and 9¼) may have smaller weight, small changes in the moduli having a great influence on the correction.

Table 23. Comparison of B. D. with H 24 (argument H).

	6.25	6.75	7.25	7.75	8.25	8.75	9.25
— 1°, — 0°	6.53 — 23	7.10 — 38	7.64 — 42	8.00 — 31	8.34 — 10	8.70 + 04	8.95 + 14
+ 0,° + 1°	6.66 — 36	7.16 — 43	7.55 — 31	8.11 — 37	8.41 — 15	8.68 + 06	8.97 + 12
+ 4°, 5°	6.68 — 39	7.04 — 34	7.65 — 46	7.97 — 33	8.39 — 17	8.70 + 04	8.98 + 09
+ 9°, 10°	6.70 — 41	6.95 — 25	7.41 — 22	7.92 — 28	8.33 — 11	8.68 + 06	9.00 + 07
+ 14°, 15°	6.53 — 20	6.84 — 13	7.23 — 03	7.71 — 05	8.20 — 01	8.66 + 08	8.99 + 13
+ 19°, 20°	6.43 — 10	6.87 — 16	7.20 00	7.67 + 01	8.07 + 11	8.59 + 16	8.91 + 25
+ 24°, 25°	6.47 — 14	6.78 — 07	7.28 — 08	7.61 + 07	8.05 + 13	8.57 + 18	8.89 + 27
+ 29°, 30°	6.43 — 10	6.89 — 18	7.30 — 10	7.66 + 02	8.11 + 07	8.59 + 16	8.89 + 27
+ 34°, 35°	6.50 — 17	6.82 — 11	7.17 + 04	7.60 + 10	8.10 + 04	8.51 + 23	8.95 + 22
+ 39°, 40°	6.41 — 07	6.79 — 07	7.14 + 06	7.56 + 15	7.87 + 28	8.43 + 30	8.79 + 41
+ 44°, 45°	6.49 — 15	6.71 + 01	7.11 + 09	7.55 + 16	8.05 + 10	8.46 + 27	8.87 + 33
+ 49°, 50°	6.32 + 02	6.75 — 03	7.17 + 03	7.57 + 12	8.02 + 14	8.48 + 27	8.85 + 34
+ 54°, 55°	6.52 — 18	6.76 — 04	7.17 + 02	7.59 + 08	8.02 + 15	8.59 + 16	8.95 + 21
+ 59°, 60°	6.38 — 04	6.76 — 04	7.17 + 02	7.57 + 10	8.06 + 11	8.52 + 23	8.90 + 26
+ 64°, 65°	6.53 — 19	6.74 — 02	7.13 + 06	7.54 + 13	8.11 + 06	8.52 + 22	8.94 + 20
+ 69°, 70°	6.23 + 11	6.68 + 04	7.09 + 10	7.62 + 05	8.10 + 07	8.67 + 06	9.11 — 03
+ 74°,75°,76°	6.41 — 06	6.69 + 08	7.28 — 03	7.64 + 10	8.23 + 02	8.76 — 03	9.07 + 01
+ 79° to 90°	6.42 — 15	6,83 — 07	7.33 — 09	7.76 — 14	8.06 — 05	8.72 — 05	9.00 + 08

23. *Comparison with H 70.* In Harvard Annals 72, N⁰. 6, Table II the differences H 70—Bonn for each tenth magnitude 9.0—9.5 are given in hourly means for each zone. They were first corrected for density error; then means were taken, giving equal weight to each star for 9.0—9.2, to each hour for the more numerous 9.3—9.5. The faint B. D. stars in each zone of 10′ breadth are observed completely in H 70; but the B. D. stars themselves are incomplete, so that Case 6 with formula 6 must be applied. The moduli of decrease were determined by counting the stars in zones 0° to 34° for each tenth magnitude m_H and making them comparable by dividing through $A(m)$. The results R, expressing the relative completeness, are contained in Table 24. They fit very well into a Gaussian curve with R = 55 for the complete classes and the constants, determined graphically, $l = 10.35$ $\mu_{23} = 0.61$; the computed values are given in the third column. The corrections and the values b' were computed with $\mu_3 = 0.60$, $\mu_{13} = 0.68$ and 0.75 ($\mu_1^2 = 0.10$ and 0.20), using the first values for $m = 9.0$, the latter for $m = 11.0$ with

Table 24. Relative completeness of B. D.

m	R	C	m	R	C	m	R	C	m	R	C
8.0	58	55	9.0	59	54	10.0	38	38	11.0	6.5	7.0
1	53	55	1	53	54	1	33	35	1	4.4	5.3
2	43	55	2	56	53	2	30	31	2	2.5	3.9
3	54	55	3	55	52	3	28	27	3	2.3	2.9
4	54	55	4	49	51	4	23	24	4	1.9	2.0
5	51	55	5	48	50	5	21	20	5	1.0	1.4
6	62	55	6	53	48	6	16	17	6	0.8	0.9
7	55	55	7	47	46	7	14	14	7	0.6	0.6
8	58	55	8	44	44	8	10.5	11.4	8	0.4	0.4
9	65	54	9	42	41	9	8.2	9.0	9	0.2	0.2

gradual transition; thus for $m = 9.0$, 9.5, 10.0 10.5, 11.0 the values of b' used are: 0.93, 0.67, 0.22, —0.39, —1.05. In Table 25 the differences H 70—Bonn are given for each zone, at first uncorrected, then corrected for $-b'\mu_1^2$ (all positive except one number printed in italics).

Table 25. Reduction of B. D. to Harvard 70.

Zone	—1°	+1°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	80°—85°
m	Uncorrected																		
9.0	25	17	40	15	16	31	29	35	29	49	38	43	32	31	22	10	02	17	21
9.1	36	26	40	22	17	38	38	51	41	50	49	55	42	42	38	20	12	24	34
9.2	44	24	35	32	28	44	50	48	53	68	51	56	40	38	41	13	21	37	60
9.3	59	63	56	56	58	57	57	70	60	77	70	56	61	59	52	41	27	30	32
9.4	69	75	67	76	68	72	79	80	86	91	80	83	73	62	62	39	35	55	63
9.5	97	109	100	105	102	112	112	133	125	117	104	106	85	82	79	62	52	68	78
	Corrected																		
9.0	18	09	33	07	11	26	24	30	24	45	33	39	27	26	17	05	*03*	12	16
9.1	30	20	35	16	12	33	33	47	37	46	45	51	38	38	34	15	07	19	29
9.2	39	18	30	26	23	40	46	44	49	65	47	52	36	34	37	07	16	33	57
9.3	56	60	53	53	55	54	54	68	57	75	68	52	58	56	48	36	21	24	26
9.4	67	74	65	75	66	71	79	80	87	93	80	83	72	60	60	34	29	52	61
9.5	104	118	108	114	110	123	123	149	138	128	113	115	89	85	81	60	48	68	80

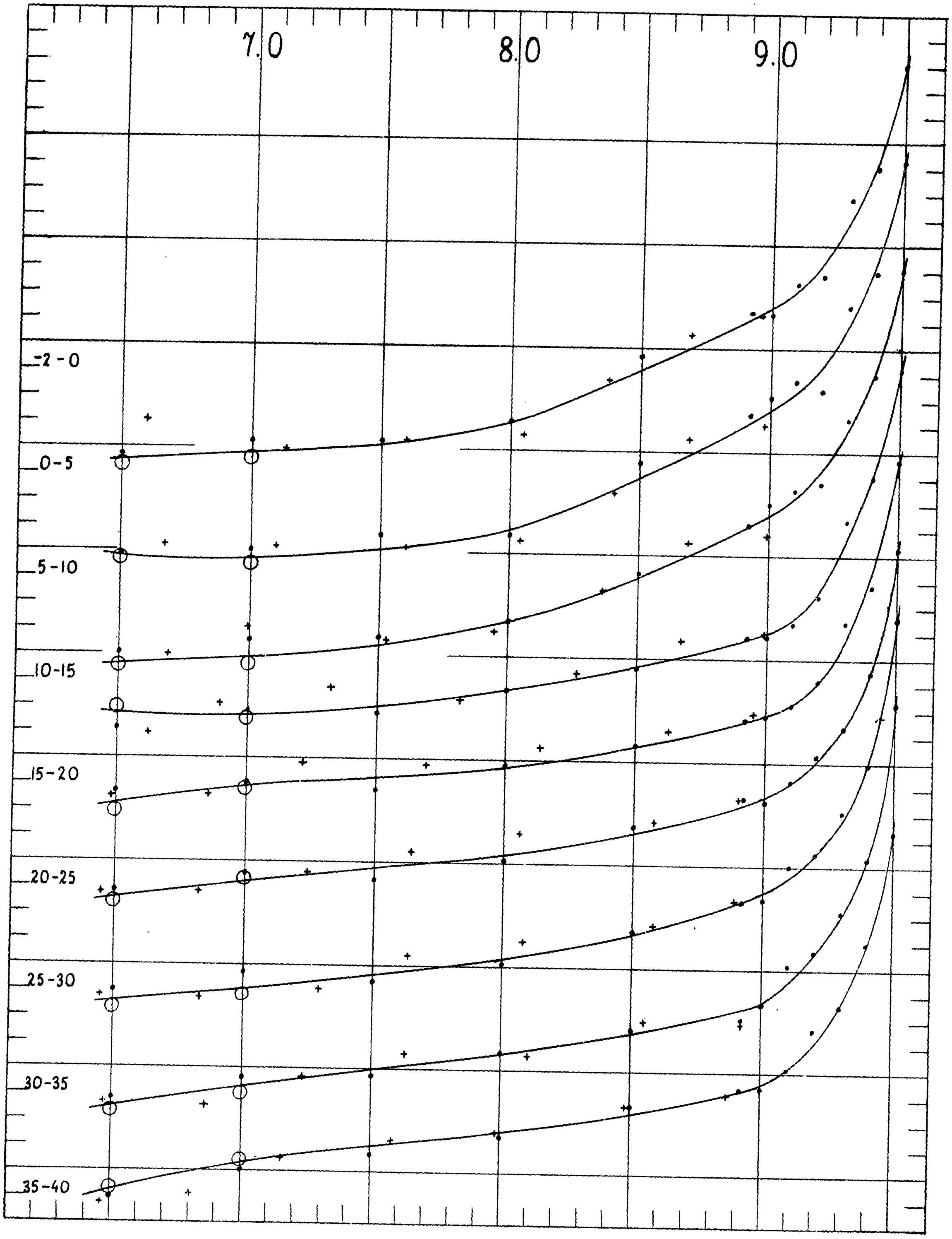
24. The reductions to photometric scale are needed for the zones of declination —2° to 0°, 0° to + 5°, etc. Thus we may use the results from H 45 directly; for the catalogues H 24 and H 70 we must take for each zone of 5° the mean of the results for the limiting narrow zones observed at parallels 0°, 5°, 10° etc. For zones 10°—20° where there is a sudden or rapid change in the scale, the mean of the narrow limiting zones may deviate appreciably from the whole zone; for the other zones such deviations are hardly to be feared. For H 45 the results h were used, for H 24 for the brighter stars *s*, for 8.0 and 8.5 the mean of *s* and *h*, for 9.0 the results *h*. For the brighter classes the results from H 24 have smaller weight than those from H 45.

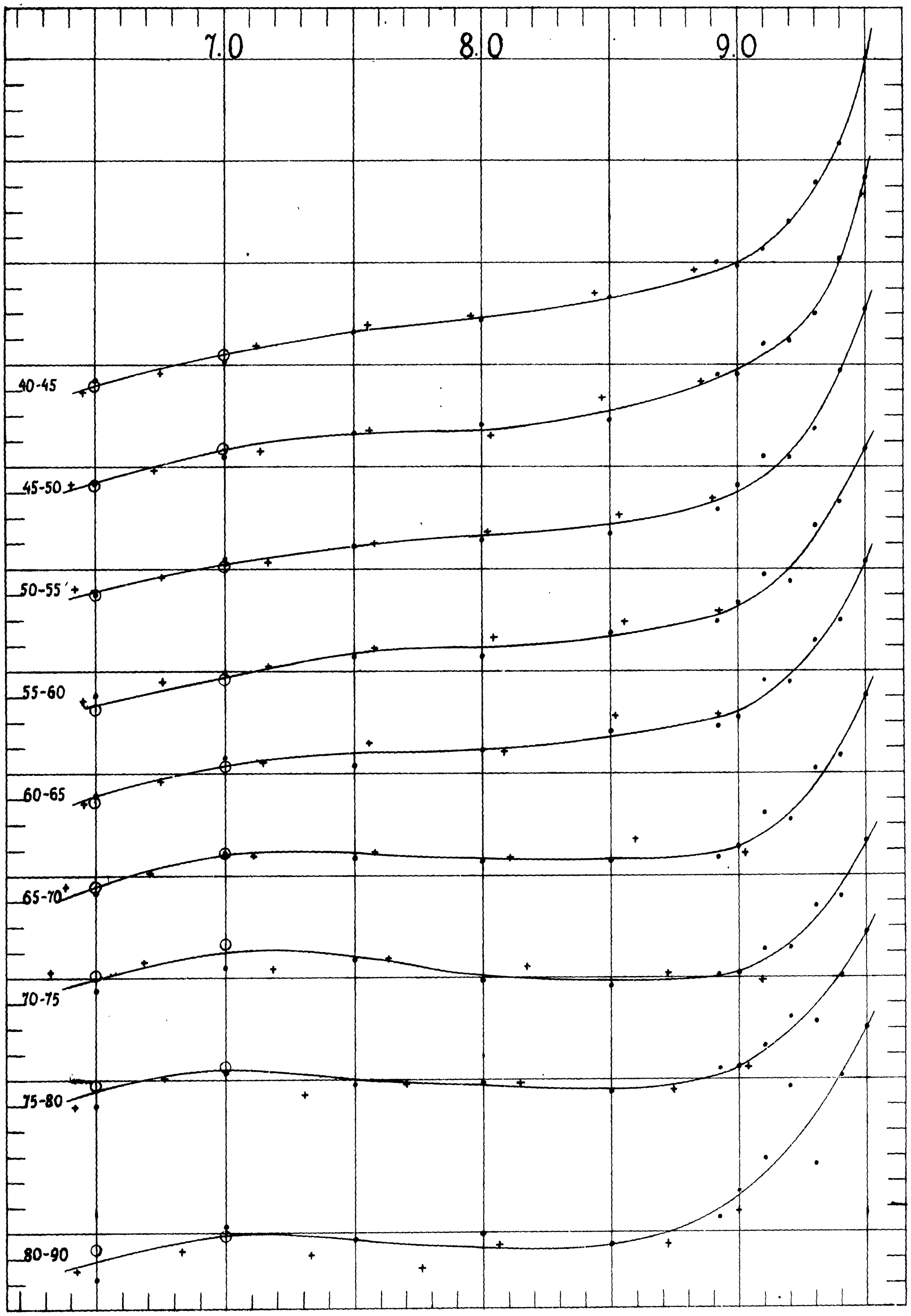
The definite reductions to photometric scale were deduced in a graphical way. On the diagrams pag. 40 and 41 the results from H 45 are represented by circles, those from H 24 (table 21) by dots, from H 24 (table 23) by crosses, and from H 70 (9.0—9.5) again by dots. Curves were drawn representing them as well as possible. It appears that there are still some systematic errors left; now and then the outstanding deviations are greater than could be expected beforehand (m. e. of a mean of 100 stars 0.03). Especially some series of crosses deviate systematically from the curve of the dots; so those from class 8.75 H, all lying too high. If a greater spreading of the error curves μ_{12}, used in deducing the correction for incompleteness, had been assumed, this correction itself would have been greater and the discrepancy would have diminished; but we could not find any reason for it in the data available.

From these curves the reduction to photometric scale for each half magnitude was read. Table 26 contains the photometric magnitude corresponding to the estimated magnitudes 6.5, 7.0, ... 9.0 for each zone.

Table 26. Photometric magnitudes of the B. D. scale.

	6.5	7.0	7.5	8.0	8.5	9.0
— 2°	6.05	6.58	7.12	7.72	8.42	9.16
0°	6.08	6.57	7.11	7.69	8.40	9.17
+ 5°	6.06	6.59	7.14	7.74	8.42	9.16
10°	6.27	6.76	7.30	7.87	8.47	9.11
15°	6.32	6.89	7.43	7.97	8.56	9.18
20°	6.36	6.92	7.48	8.03	8.62	9.27
25°	6.36	6.91	7.47	8.03	8.63	9.30
30°	6.35	6.93	7.50	8.06	8.65	9.27
35°	6.42	7.03	7.60	8.16	8.74	9.36
40°	6.42	7.04	7.63	8.18	8.76	9.40
45°	6.44	7.07	7.63	8.14	8.72	9.38
50°	6.41	7.02	7.59	8.13	8.67	9.30
55°	6.37	6.98	7.57	8.09	8.64	9.25
60°	6.41	7.03	7.58	8.09	8.64	9.24
65°	6.46	7.08	7.59	8.06	8.56	9.11
70°	6.50	7.10	7.58	8.01	8.49	9.02
75°	6.46	7.04	7.50	7.98	8.46	9.05
80°	6.39	6.99	7.47	7.94	8.45	9.14
90°						


7.0
8.0
9.0
-2 - 0
0-5
5-10
10-15
15-20
20-25
25-30
30-35
35-40


7.0
8.0
9.0
40-45
45-50
50-55
55-60
60-65
65-70
70-75
75-80
80-90

25. It may be of some interest to compare the statistical magnitudes for the separate zones, mentioned in § 16, with the photometric magnitudes just derived. At first the decimal error must be eliminated by taking means of every five consecutive values. (In the vicinity of 9.0 these means are nearly 0.02 too high owing to the strong increase of the scale-width below 9^m). According to § 10 Case 2, Corollary 2, they must be corrected by $\frac{1}{2}\,b(\mu_1^2-\mu_2^2)$. The Groningen table of log $N(m)$, used in deriving statistical magnitudes, is founded on the number of stars in Harvard 50 for 6.5 and brighter ($\frac{1}{2}\,b\,\mu^2 = 0.01$), on the number in B.D. itself for 7.5—9.25 ($\frac{1}{2}\,b\,\mu^2$ decreasing from 0.05 to 0.03). Thus in the smoothed values of log N $\frac{1}{2}\,b\,\mu_2^2$ may be assumed to increase from 0.02 for 6.5 to 0.04 for 9.5, and the correction $\frac{1}{2}\,b(\mu^2_1-\mu_2^2)$ becomes for $m = 6\frac{3}{4}, 7\frac{1}{4} \ldots . 9\frac{1}{4}$

— .04 — .03 — .02 — .01 — .00 — .00

The statistical magnitudes, corrected in this way, (and for 9.05 diminished by 0.02) have been combined for the four large zones between 0°, 10°, 35°, 60°, 80°; the results for $m_B = 6\frac{3}{4}, 7\frac{1}{4} \ldots . 8\frac{3}{4}$, 9.05 are contained in the next table, with the photometric results from the curves and the differences between them.

Table 27. Comparison of statistical and photometric magnitudes.

m	Statistical reductions				Photometric reductions				Differences Stat.—Phot.			
	0°—10°	10°—35°	35°—60°	60°—80°	0°—10°	10°—35°	35°—60°	60°—80°	0°—10°	10°—35°	35°—60°	60°—80°
6.75	— 54	— 17	— 03	— 09	— 43	— 14	— 03	+ 02	— 11	— 03	00	— 11
7.25	— 46	— 10	+ 08	00	— 40	— 09	+ 07	+ 07	— 06	— 01	+ 01	— 07
7.75	— 42	— 10	+ 09	— 02	— 34	— 04	+ 12	+ 05	— 08	— 06	— 03	— 07
8.25	— 24	— 02	+ 16	00	— 20	+ 04	+ 17	+ 03	— 04	— 06	— 01	— 03
8.75	+ 01	+ 12	+ 27	+ 03	+ 02	+ 15	+ 26	+ 05	— 01	— 03	+ 01	— 02
9.05	+ 22	+ 26	+ 40	+ 10	+ 19	+ 25	+ 36	+ 13	+ 03	+ 01	+ 04	— 03

The differences found here may be caused either by real deviations of the number of stars in these zones from the mean regular number over the sky, or by a deviation of the scale underlying the Groningen table of $N(m)$ from the photometric Harvard scale used here. This may be tested by computing in the same way statistical magnitudes from the number of stars counted in Harvard 50 down to 6.5 (0—6.49). Then we find for the same four zones :

— .09 — .01 — .01 — .14 (whole northern hemisphere —.03).

For the corresponding B.D. magnitudes (6.93, 6.63, 6.53, 6.48) we have

corrected stat. reduction	— .51	— .19	— .10	— .17
photometric reduction . .	— .42	— .15	— .08	— .03
difference	— .09	— .04	— .02	— .14.

The discrepancy for the brighter classes has nearly disappeared, only some few hundredths of a magnitude being left. The scales are hardly different, as the southern hemisphere gives a statistical result 6.52; the chief cause lies in local deviations from the average number of stars. In nearly all zones the statistical magnitudes for 8^m are found to be too bright; and the number of stars for these magnitudes is smaller than corresponds to a quite regular increase with m.

26. As the counts of SEELIGER have been made for the classes 0 to 6.5, 6.6 to 7.0, etc. the photometric magnitude for the limits 6.55, 7.05.... B.D. is required. Thus half the extension of the decimals 6.5, 7.0.... must be added to the values of Table 26. The extension of these decimal classes is found from the statistical magnitudes; as the scales are not wholly identical the slope of the photometric and the statistical curves was read and used to reduce the width of these decimals in statistical scale to photometric scale. On the whole the fluctuations are small. The averages for broad zones are contained in Table 28, where the polar cap has been taken separately, because the modus of observing was different from the other zones.

Table 28. Width of decimals 5 and 0 in photometric scale (in 0.01^m).

Zone	6.5	7.0	7.5	8.0	8.5	9.0
0°—15°	30	25	22	23	25	30
15°—35°	33	26	24	22	20	23
35°—60°	31	27	23	23	20	22
60°—80°	34	25	22	21	18	22
80°—90°	37	31	20	18	20	26

For the greater middle part the width may be taken constant. Thus for the half-width of these decimals we have adopted:

Table 29. Half-width adopted (in 0.01^m).

Zone	6.5	7.0	7.5	8.0	8.5	9.0
— 2° to + 15°	15	13	11	11	12	15
+ 15° to 60°	16	13	12	11	10	11
60° to 80°	17	13	11	11	09	11
80° to 90°	18	15	10	09	10	13

If these quantities are added to the values of Table 26, we get the photometric aequivalent for the limits 6.55 D.M. etc., used in the starcounts.

27. The magnitudes of the B.D. below 9.0 bear another character than those for brighter stars. The scale becomes much wider, nearly twice the normal amount. The decimal error becomes less marked, the decimals 1 and 4 becoming more numerous, in accordance with the small subjective mean error. At the same time the magnitude classes become incomplete. This incompleteness is shown by a comparison of photometric and statistical magnitudes. The mean photometric magnitudes (found by adding the corrections read from the curves to the B.D. magnitudes, and taking the means for all zones) for 9.0 to 9.5 are:

9.23, 9.42, 9.60, 9.85, 10.14, 10.59,

while the middle values between the statistical magnitudes of the limits, given in table 9, are

9.25, 9.45, 9.60, 9.78, 9.97, 10.28.

The value 9.42 must be too low, because by the decimal error 9.1 denotes a somewhat fainter magnitude. Then the difference is nearly zero for 9.0—9.2, and from 9.3 downward it becomes decidedly negative. Thus the incompleteness becomes appreciable for 9.3 and fainter classes, while for 9.2 the number of stars counted seems still to be normal. The stars below 9.2 therefore must be excluded in statistical investigations.

Variation with Rectascension.

28. For each hour of AR in each zone the deviation of the hourly means (already corrected for density error) from the average of the whole zone was computed. These deviations were taken together into the groups H 45: 6.5 and 7.0; H 24: 6.5 and 7.0; H 24: 7.5, 8.0, 8.5; H 24: 8.8—9.0 (all with weight = number of stars) and H 70: 9.0—9.5 (each magnitude weight 1). In these five groups the average number of stars per hour was 19, 5, 16, 10, 30. In a graphical representation these hourly means (especially after having been smoothed into averages of 3) presented conspicuous fluctuations, that for the different magnitudes were nearly alike in character. Also for adjacent zones of declination they showed an analogous course, continually changing with declination, but sometimes also varying considerably and abruptly with the zone. For H 24 and H 70 the mean of every two consecutive narrow zones was taken to get values standing for the 5° zones (e.g. H 24 9°—10° and 14°—15° are combined for zone 10°—15°). As the weight of H 24 6^m—7^m is small (some hours having only 1 or 2 stars), the final result was obtained by taking the mean of the four other groups, giving the same weight to each. Considering their differences as errors the m.e. of a group is found to be 0.116 for the lower, 0.088 for the higher declinations (the differences between H 45: 6.5 and 7.0 afforded 0.09 for the first of these values). Then the mean error a priori of a final result is 0.058 or 0.044. In a graphical representation of these results curves were drawn from which were read the corrections dependent on Rectascension. Table 30 gives for each zone and each

Table 30. Corrections depending on AR.

Hour	— 2° to 0°	0° to 5°	5° to 10°	10° to 15°	15° to 20°	20° to 25°	25° to 30°
0	+ 28 + 22	+ 20 + 12	+ 07 + 09	+ 08 + 10	+ 02 + 03	+ 08 + 04	— 01 + 01
1	+ 32 + 26	+ 18 + 16	+ 14 + 09	+ 15 + 09	+ 01 00	+ 01 — 01	+ 04 — 07
2	+ 13 + 21	+ 06 + 14	+ 07 + 08	+ 03 + 05	— 04 — 03	— 10 — 08	— 24 — 14
3	+ 09 + 11	+ 15 + 09	+ 13 + 03	+ 04 — 03	+ 02 — 06	— 16 — 12	— 05 — 10
4	+ 08 — 03	+ 02 — 02	— 12 — 05	— 16 — 13	— 11 — 10	— 13 — 13	+ 06 — 03
5	— 36 — 22	— 15 — 14	— 14 — 14	— 14 — 18	— 15 — 14	— 06 — 13	— 03 00
6	— 25 — 29	— 27 — 21	— 23 — 18	— 19 — 20	— 12 — 17	— 13 — 12	00 — 02
7	— 26 — 20	— 08 — 14	— 06 — 14	— 14 — 14	— 16 — 14	— 08 — 10	— 11 — 03
8	— 01 — 11	00 — 04	— 09 — 09	— 07 — 10	— 12 — 08	— 16 — 07	— 04 — 01
9	00 — 02	+ 01 + 03	00 — 04	00 — 06	00 — 02	+ 07 00	+ 09 + 05
10	+ 02 + 01	+ 04 + 06	— 04 — 02	+ 01 — 04	+ 06 + 03	+ 02 + 08	+ 08 + 13
11	+ 03 + 03	+ 02 + 05	— 14 — 01	— 13 — 03	— 02 + 07	+ 19 + 16	+ 24 + 18
12	+ 08 + 01	+ 04 + 02	— 04 00	— 01 — 03	+ 16 + 06	+ 23 + 18	+ 16 + 17
13	— 14 — 02	00 — 01	+ 06 — 02	— 04 — 02	— 12 — 02	— 02 + 10	+ 09 + 13
14	— 07 — 06	— 06 — 05	— 01 — 05	+ 05 — 01	— 04 — 06	+ 08 + 03	+ 10 + 08
15	— 05 — 06	— 08 — 08	— 19 — 12	— 06 00	— 04 00	+ 13 + 01	+ 04 + 03
16	— 03 — 03	— 10 — 08	— 05 — 08	+ 03 + 01	+ 12 + 08	— 04 — 01	+ 02 — 03
17	+ 04 00	+ 02 — 05	+ 03 + 02	+ 08 + 05	+ 14 + 10	+ 02 — 03	— 16 — 09
18	— 01 + 02	+ 04 00	+ 13 + 08	+ 10 + 08	+ 10 + 11	+ 02 — 05	— 13 — 13
19	+ 10 + 03	+ 07 + 02	+ 12 + 10	+ 09 + 10	+ 03 + 11	— 08 — 05	— 11 — 13
20	— 04 + 01	— 05 + 05	+ 02 + 11	+ 10 + 11	+ 06 + 10	— 08 — 03	— 13 — 10
21	— 05 — 01	— 05 + 06	+ 14 + 11	+ 11 + 11	+ 07 + 09	+ 04 + 02	— 01 — 03
22	+ 01 + 01	+ 17 + 03	+ 12 + 10	+ 10 + 11	+ 10 + 08	+ 14 + 08	+ 06 00
23	+ 06 + 09	— 13 — 10	+ 05 + 10	+ 04 + 10	+ 03 + 07	+ 02 + 08	— 03 + 02

Hour	30° to 35°		35° to 40°		40° to 45°		45° to 50°		50° to 55°		55° to 60°		60°—65° 65°—70°		
0	+ 02	— 03	— 03	00	— 04	— 02	+ 08	+ 08	+ 03	+ 10	00	— 01	+ 06	+ 06	+ 04
1	— 10	— 04	+ 01	+ 02	— 13	— 06	+ 02	+ 03	+ 21	+ 11	00	— 04	— 15	— 12	— 12
2	+ 01	00	00	+ 05	— 03	— 09	— 05	— 02	+ 08	+ 08	+ 03	— 07	— 18	— 08	— 12
3	+ 05	+ 08	+ 10	+ 08	— 16	— 10	— 14	— 07	+ 04	+ 03	— 15	— 09	+ 02	— 03	00
4	+ 17	+ 09	+ 12	+ 09	— 02	— 09	+ 01	— 10	— 05	— 04	— 10	— 10	+ 11	+ 03	+ 06
5	— 06	— 04	— 01	+ 04	— 12	— 08	— 14	— 11	— 14	— 10	— 09	— 11	— 02	+ 02	00
6	00	— 03	+ 01	— 04	+ 02	— 06	— 14	— 12	— 14	— 15	— 08	— 12	+ 06	+ 02	+ 04
7	— 03	+ 01	— 14	— 09	— 08	— 04	— 06	— 11	— 13	— 18	— 11	— 10	+ 11	+ 13	+ 10
8	+ 08	+ 06	00	— 06	— 02	— 03	— 02	— 10	— 18	— 18	— 11	— 07			
9	+ 10	+ 09	00	— 02	+ 01	— 01	— 13	— 09	— 18	— 16	— 02	— 02			
10	+ 14	+ 11	+ 04	+ 02	— 02	+ 01	— 05	— 07	— 09	— 09	+ 02	+ 03			
11	+ 06	+ 10	— 06	+ 06	00	+ 06	— 08	— 03	— 03	+ 01	+ 06	+ 09			
12	+ 05	+ 08	+ 15	+ 08	+ 09	+ 10	+ 08	+ 01	+ 12	+ 07	+ 14	+ 10	70°—75° 75°—80°		
13	+ 04	+ 05	+ 08	+ 05	+ 16	+ 10	+ 04	+ 05	— 08	— 03	+ 02	+ 03			
14	+ 02	— 00	— 04	— 02	— 02	+ 05	+ 04	+ 08	— 12	— 09	+ 02	— 04	00	— 05	— 05
15	— 07	— 06	— 10	— 09	+ 02	00	+ 12	+ 10	+ 01	— 05	— 14	— 09	— 01	— 06	— 05
16	— 16	— 12	— 10	— 08	+ 02	+ 03	+ 08	+ 11	+ 02	+ 01	— 07	— 09	+ 02	+ 03	— 01
17	— 14	— 16	+ 03	+ 02	+ 14	+ 10	+ 15	+ 12	+ 13	+ 08	00	— 01	— 06	— 02	+ 02
18	— 12	— 13	+ 09	+ 04	+ 08	+ 06	+ 09	+ 09	+ 08	+ 11	+ 07	+ 12	+ 02	+ 03	+ 05
19	— 10	— 09	— 08	— 03	— 03	00	— 04	00	+ 07	+ 13	+ 25	+ 18	+ 04	+ 09	+ 07
20	— 09	— 02	— 14	— 05	— 06	— 01	— 05	— 06	+ 14	+ 10	+ 10	+ 13	+ 03	+ 02	+ 03
21	+ 10	+ 05	+ 02	— 05	+ 08	+ 02	+ 02	— 03	+ 06	+ 05	+ 05	+ 07	— 03	— 04	— 03
22	+ 15	+ 07	+ 02	— 04	+ 01	+ 03	+ 01	+ 03	— 04	+ 02	00	+ 04			
23	— 09	+ 02	— 03	— 02	+ 05	+ 01	+ 16	+ 10	+ 10	+ 05	+ 01	+ 01			

hour the final results from the observations and the values smoothed by the curves; for the zones 60°—70° and 70°—80° for every three consecutive hours 0—2, 3—5, etc. the results for 5° zones and the adopted corrections for 10° zones are given. From the remaining differences a mean error a posteriori of 0.065 and 0.055 is computed; thus it seems that the curves might have been adjusted somewhat closer to the observed deviations. Sudden leaps in the correction may actually have occurred (especially where by the vicinity of the Milky Way the stardensity changes rapidly), that are not accounted for in the smooth course of the curves.

The question arises, whether these corrections may be assumed to be the same for all magnitudes. It is an important question, because a magnitude coefficient in this correction implies a local change of scale value, difficult to determine exactly and having a great influence upon the results for the distribution of the stars. A priori there is no reason to be sure that, while the deviation from a constant photometric scale is continually changing, the width of the scale should remain constant. There are indeed many instances where the values for 6¾ and 9¼ deviate to different sides of the curve. The number of times that they are the extreme of the four groups (av. 7 from 24) is greater than may be expected by mere chance; but the chief cause of this excess was found in the value for 6¾ coming from H 45 and standing for other regions than the results from H 24; the yoften were contradicted by H 24 6.5—7.0. After a careful comparison of all the data I have abstained from introducing any variation with magnitude; though doubtless such variations occur they cannot be found from the data available with sufficient certainty.

The „Südliche Durchmusterung".

29. In his extension of the Bonner Durchmusterung with a 4th section from — 2° to — 23° SCHOENFELD (1876—1881) used a somewhat greater instrument (159 *mm* aperture) and observed in a field faintly illuminated by red light, whereby stars were still visible whose magnitudes lie a whole class below the limit of the catalogue. The lowest magnitude included was called 10^m; in the introduction the observer states his belief that in poor regions he has gone below this limit. Completeness was only aimed at for stars as faint as 9.3 or 9.4.

As the S. D. M. has not been counted for all tenth magnitudes, we have tried to deduce statistical magnitudes and decimal error by counting a part of the work, viz three degrees (— 2°, — 12°, — 22°) for the numerous stars 9.0—10, and four degrees more (— 5°, — 9°, — 15°, — 19°) for the brighter stars. The sevenfold or the threefold of these numbers may be taken to represent the whole zone of 21° breadth. That this cannot be far from the truth is shown by the total number of stars thus computed, 133163, deviating only 0.4 % from the real number 133659. The results are contained in Table 31.

Table 31. Statistical magnitudes of the S. D. M.

m	Number	Stat. m.	Range	Corr.	m	Number	Stat. m.	Range	Corr.
5.9					8.0			25	
	534	5.66				7971	8.05		— 07
6.0			22		8.1			03	
	690	5.88				8265	8.08		— 05
6.1			05		8.2			11	
	714	5.91				9441	8.19		— 03
6.2			12		8.3			17	
	819	6.03		— 20		11274	8.36		— 01
6.3			17		8.4			05	
	1005	6.20		— 19		11913	8.41		+ 02
6.4			04		8.5			24	
	1044	6.24		— 18		15381	8.65		+ 05
6.5			18		8.6			08	
	1287	6.42		— 18		16620	8.73		+ 08
6.6			03		8.7			12	
	1332	6.45		— 18		18951	8.85		+ 12
6.7			10		8.8			16	
	1500	6.55		— 18		22314	9.01		+ 17
6.8			14		8.9			09	
	1758	6.69		— 16		24705	9.10		+ 24
6.9			03		9.0			29	
	1815	6.72		— 14		33616	9.39		+ 32
7.0			32		9.1			20	
	2610	7.04		— 11		41344	9.59		
7.1			03		9.2			15	
	2682	7.07		— 09		48071	9.74		
7.2			09		9.3			16	
	2979	7.16		— 06		56751	9.90		
7.3			16		9.4			18	
	3576	7.32		— 07		67895	10.08		
7.4			03		9.5			27	
	3675	7.35		— 08		89133	10.35		
7.5			16		9.6			08	
	4431	7.51		— 09		95706	10.43		
7.6			02		9.7			07	
	4536	7.53		— 10		103273	10.50		
7.7			08		9.8			14	
	4959	7.61		— 11		118309	10.64		
7.8			16		9.9			03	
	5913	7.77		— 10		121501	10.67		
7.9			03		10.			09	
	6081	7.80		— 09		133163	10.76		

Table 32. Range of decimals.

	3;8	4;9	5;0	6;1	7;2
6.25—6.75	16	04	17	03	10
6.75—7.25	11	02	26	02	07
7.25—7.75	17	03	19	02	09
7.75—8.25	14	03	21	03	10
8.25—8.75	13	04	18	06	09
8.75—9.25	09	05	17	11	08

The decimals show the same general behaviour as in the Northern D. M.; in table 32 they are reduced to a subjective scale.

The ranges of 0 and 5 do not show the regular decrease of the N. D. M, but they show a clear difference between 0 and 5 themselves. Moreover here the decimal error shows dissymmetry to a stronger degree than before, the decimals 3 and 8 being used more often to round the quarters than 2 and 7. By this dissymmetry the limits 6¾, 7¼.... are displaced relative to the mean magnitudes 7.0, 7.5...., which must be considered in comparing statistical and photometric results. If, however, we wish to find photometric equivalents for the limits 6.55...., it appears that we have only to add half the range of 6.5 to the mean corrections photometer—estimate for all the stars 6.3—6.7.

30. *The density error* was derived from the differences Harvard 24—Bonn given in Table XLI of Harvard Annals **23** for sections D 1—12, as they practically coincide with the zones of the S. D. M. (only —0°, —1° of the N. D. M. are included). The mean density for every two hours denotes the number of stars down to 9.5 (not down to 10) per square degree; in Table 33 these double hours are arranged according to D. At the bottom of each column the coefficient of D deduced for this magnitude is put down (in 0.001^m).

Table 33. Influence of density.

Hours	D	6.5	7.0	7.5	8.0	8.5	9.0
0, 1	8.2	+ 10	+ 15	+ 20.	+ 27	+ 45	+ 56
12, 13	8.5	— 12	— 15	— 03	+ 04	+ 16	+ 28
22, 23	8.7	+ 01	— 07	+ 05	+ 03	+ 31	+ 47
10, 11	8.8	— 10	— 03	+ 08	+ 00	+ 19	+ 25
2, 3	8.9	+ 07	+ 16	+ 12	+ 17	+ 41	+ 48
14, 15	9.2	— 17	00	— 60	00	+ 08	+ 30
16, 17	10.5	— 08	+ 09	+ 18	+ 21	+ 21	+ 46
20, 21	11.8	— 03	+ 02	— 06	— 05	+ 14	+ 19
4, 5	13.7	— 14	— 18	— 03	— 12	+ 02	+ 19
8, 9	14.3	— 06	00	+ 08	+ 08	+ 18	+ 22
18, 19	16.5	— 11	— 04	+ 13	+ 06	+ 22	+ 35
6, 7	21,6	— 29	— 17	— 29	— 13	— 08	+ 06
Coefficient		— 017	— 014	— 020	— 016	— 024	— 024
Adopted		— 015	— 016	— 018	— 019	— 021	— 026

For the fainter classes the differences given in Table VII of Harvard Annals **72** for each hour of AR are condensed into five groups of nearly equal weight. They are contained in Table 34, the coefficients deduced from them being added at the foot (in 0.001^m).

Table 34. Influence of density (in 0.01^m).

Hours	D	9.0	9.1	9.2	9.3	9.4	9.5	9.6	9.7	9.8	9.9	10.
0, 1, 11—13, 23	8.3	+ 39	+ 56	+ 74	+ 78	+ 92	+ 115	+ 122	+ 112	+ 140	+ 106	+ 125
2, 3, 10, 14, 16, 22	9.1	42	58	66	80	92	112	106	118	122	126	134
4, 9, 15, 17, 21	10.9	31	51	61	76	78	109	112	111	118	105	112
5, 8, 19, 20	15.5	23	34	55	54	60	88	83	81	90	96	91
6, 7, 18	20.2	14	14	36	42	56	72	61	67	70	79	69
Total nr. of stars.		247	236	221	330	351	685	193	216	488	84	404
Coefficient.		— 23	— 37	— 28	— 34	— 33	— 38	— 49	— 45	— 56	— 31	— 53
Adopted.		— 26	— 28	— 30	— 33	— 36	— 39	— 42	— 46	— 49	— 52	— 56

Whereas one would expect that by the use of the illuminated field the influence of the multitude or the scantiness of the stars on the estimates would vanish or at least be diminished strongly, just the contrary is shown in these figures. The coefficient of D is not only much greater than in the N. D. M., but it does not vanish, either, for the bright stars 6.5, where already it reaches as great a value as for stars 9.0 in the northern sections.

The coefficients used in the further reductions are obtained by smoothing the computed results and have been given in the last line of each table.

31. *The corrections depending on declination* have been deduced in the same way and from the same catalogues as for the northern sections. The region covered by this Durchmusterung has been divided for the star counts into four zones, between 2°, 7°, 12°, 17°, 22° of southern declination. Thus the same zones are used in the comparison with H 45, and the narrow zones of H 24 and H 70 along the parallels of —5°, —10°, —15°, —20° fall in the midst of each zone. The catalogue differences are all corrected for density error (reduced to $D = 15$). In computing the corrections for incompleteness for H 24, argument m_H, the first of the four separate error curves could be omitted since for these zones all stars up to the brightest were observed completely (as expressly stated for —5°, —15°, —20°). In the following tables the comparisons are given in the same manner as in Table 17, 18, 20, 21, 23, 25 for the northern part.

Table 35. Mean error μ_{12} Harvard—Bonn IV (in 0.01^m).

	6.5	7.0	7.5	8.0	8.5	9.0	9.1	9.2	9.3	9.4	9.5	9.6	9.7	9.8	9.9	10.
H 45	39	38														
H 24	36	32	37	32	34	33										
H 70						36			34		44	53			54	
μ_1 adopted	35	33	32	31	30	29	30	32	35	38	41	44	46	48	50	52
$b\mu_1^2$	.14	.12	.11	.10	.09	.09	.09	10	.12	.14	.16	.18	.20	.22	.24	.25

Table 36. Reduction of S. D. M. to Harvard 45.

m	—2° —7°			—7° —12°			—12° —17°			—17° —22°		
	h	s	n	h	s	n	h	s	n	h	s	n
6.5	— 28	— 27	(162)	— 23	— 23	(151)	— 23	— 22	(134)	— 22	— 24	(152)
7.0	— 23	— 23	(328)	— 19	— 18	(367)	— 16	— 16	(394)	— 11	— 11	(365)

Table 37. Reduction of S. D. M. to Harvard 24.

Zone	6.5	7.0	7.5	8.0		8.5		9.0 (8.92)		Number of stars
				s	*h*	*s*	*h*	*s*	*h*	
— 5°	— 23	— 20	— 08	— 07	— 09	+ 07	+ 06	+ 14	+ 12	66, 85, 99, 113, 180, 218
— 10°	— 25	— 14	— 10	— 03	— 07	+ 10	+ 06	+ 17	+ 13	66, 87, 68, 102, 137, 227
— 15°	— 29	— 15	— 07	— 01	— 08	+ 07	+ 08	+ 18	+ 15	62, 112, 104, 125, 141, 224
— 20°	— 22	— 16	— 10	— 09	— 09	+ 02	+ 03	+ 13	+ 12	72, 75, 113, 104, 169, 225

Table 38. Comparison of S. D. M. with H 24 (argument H).

Zone	6.25	6.75	7.25	7.75	8.25	8.75	9.25
— 5°	6.45 — 22	6.89 — 21	7.36 — 16	7.74 — 09	8.17 + 01	8.66 + 07	8.99 + 12
— 10°	6.39 — 16	6.82 — 14	7.33 — 13	7.70 — 05	8.19 — 01	8.63 + 10	8.97 + 14
— 15°	6.44 — 21	6.91 — 23	7.24 — 04	7.72 — 07	8.17 + 01	8.68 + 05	8.98 + 13
— 20°	6.41 — 18	6.86 — 18	7.30 — 10	7.63 + 02	8.25 — 07	8.66 + 07	9.03 + 08

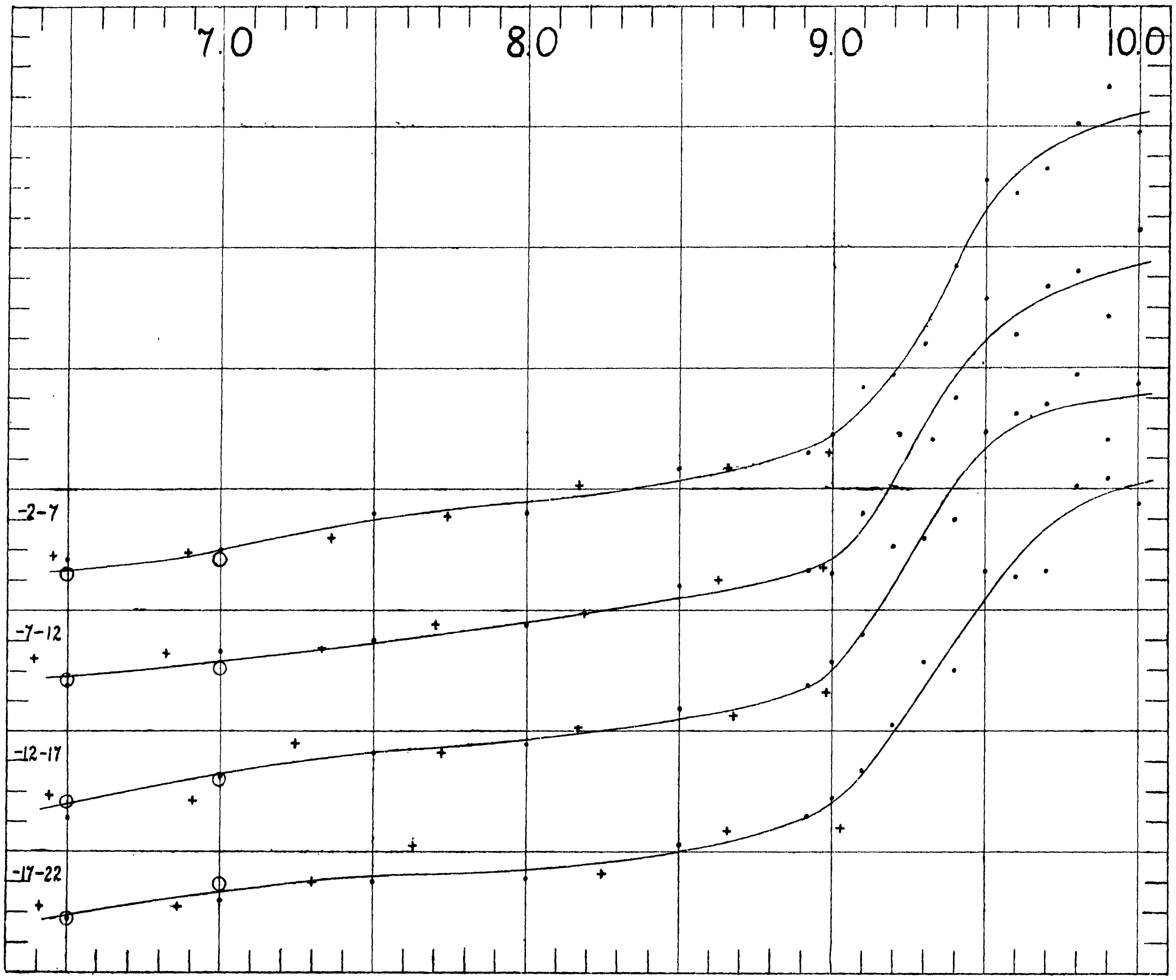
Table 39. Reduction of S. D. M. to Harvard 70.

m	9.0	9.1	9.2	9.3	9.4	9.5	9.6	9.7	9.8	9.9	10.0
Zone						Uncorrected					
— 5°	25	41	44	53	75	96	91	94	103	110	93
— 10°	20	39	58	58	71	97	85	95	96	81	99
— 15°	30	39	65	68	71	93	97	96	101	80	91
— 20°	25	34	48	67	62	87	85	83	103	102	91
						Corrected					
— 5°	18	34	38	48	74	102	98	106	121	134	118
— 10°	12	32	53	53	70	103	91	107	112	97	126
— 15°	23	32	61	64	70	99	105	108	118	96	115
— 20°	18	27	42	63	60	93	91	93	121	124	115

Table 40. Photometric magnitudes of the S. D. M. scale.

	6.5	7.0	7.5	8.0	8.5	9.0
— 2° — 7°	6.23	6,80	7.40	7.96	8.53	9.18
— 7° — 12°	6.28	6.83	7.39	7.96	8.54	9.17
— 12° — 17°	6.26	6.86	7.43	7.97	8.54	9.20
— 17° — 22°	6.29	6.87	7.42	7.94	8.50	9.16

The graphical representation of these results and the curves drawn through them (pag. 50) show some systematic deviations; so the observed values for 8.5 all lie higher, those for 8.0 lower than the curve. We have no right to claim that in a subjective mental scale irregularities of some hundredths of a magnitude will not occur. Still it did not seem advisable to adjust the curves more closely by admitting such minor fluctuations into them. The corrections read from these curves for 6.5, 7.0.... afford the photometric magnitudes contained in Table 40.



It may be emphasized as a remarkable fact that the scale shows no continuous variation with declination. While by the atmospheric extinction the stars in zone —20° must appear 0.30^m fainter than in zone —5° the observer manifestly has been able to counterbalance this difference by varying his scale with the varying aspect of the stars.

The width of the tenth magnitudes 6.5, 7.0 .... in photometric scale was deduced in the same way as for the N.D.M. The values adopted for half width, to be added to Table 40, are

0.10, 0.15, 0.10, 0.11, 0.11, 0.14.

32. *The corrections depending on Rectascension* are derived quite in the same way as described for the northern sections. The final results from the four magnitude groups and the values adopted after smoothing are given in Table 41.

33. Below 9.0 the scale for the southern zones at first shows the same behaviour as for the northern section; the width of the scale increases suddenly to nearly double the amount it has

Table 41. Corrections depending on AR.

	— 2° to — 7°		— 7° to — 12°		— 12° to — 17°		— 17° to — 22°	
0h	— 20	— 11	+ 07	+ 05	+ 19	+ 15	+ 11	+ 11
1	+ 09	— 06	+ 10	+ 10	+ 14	+ 22	+ 23	+ 19
2	— 09	— 01	+ 16	+ 12	+ 29	+ 22	+ 14	+ 22
3	+ 04	+ 02	+ 05	+ 10	+ 12	+ 10	+ 15	+ 20
4	+ 13	+ 01	+ 07	+ 02	— 12	— 08	+ 26	+ 12
5	— 16	— 04	— 08	— 06	— 19	— 22	— 12	— 06
6	— 16	— 07	+ 05	+ 03	— 29	— 23	— 18	— 15
7	+ 03	— 10	+ 15	+ 21	+ 02	— 01	— 04	— 07
8	— 11	— 12	+ 38	+ 32	+ 08	+ 03	+ 07	+ 02
9	— 07	— 13	+ 16	+ 16	— 24	— 14	+ 03	+ 04
10	— 19	— 09	— 05	00	— 12	— 17	+ 02	00
11	+ 04	— 02	— 09	— 10	— 10	— 12	— 08	— 07
12	— 06	— 02	— 09	— 14	00	— 07	— 09	— 14
13	— 02	— 05	— 11	— 16	— 06	— 02	— 22	— 20
14	— 15	— 06	— 17	— 15	+ 08	+ 02	— 26	— 22
15	— 02	+ 02	— 05	— 08	— 10	+ 07	— 09	— 10
16	+ 21	+ 17	— 05	00	+ 21	+ 11	+ 04	+ 05
17	+ 33	+ 28	+ 12	+ 12	+ 21	+ 13	+ 18	+ 15
18	+ 29	+ 33	+ 25	+ 19	+ 02	+ 06	— 02	+ 10
19	+ 34	+ 26	+ 13	+ 05	+ 03	— 02	+ 09	00
20	+ 01	+ 09	— 23	— 19	— 01	— 07	— 12	— 10
21	— 16	— 06	— 32	— 28	— 06	— 07	— 20	— 17
22	— 04	— 13	— 06	— 18	— 10	— 02	— 10	— 13
23	— 15	— 15	— 07	— 07	+ 04	+ 06	+ 03	— 01

for brighter stars. For stars below 9.5, however, the curves become less steep and the scale contracts again. Comparing the photometric aequivalents of 9.0—10 S. D. M. (mean of the readings from the four curves) with the corresponding values for the N.D.M.:

9.18	9.41	9.62	9.89	10.16	10.40	10.60	10.77	10.92	11.06	11.18	(S.D.M.)
9.23	9.42	9.60	9.85	10.14	10.59	—	—	—	—	—	(N.D.M.),

we see that down to 9.4 the scales run nearly parallel; then 9.5 N. D. M. covers the decimals 9.5, 9.6, 9.7 of the S. D. M., while the remaining decimals represent fainter stars. The statistical magnitudes for these decimals (middle values between consecutive limits) and their differences with the photometric magnitudes are

9.25	9.49	9.66	9.82	9.99	10.21	10.39	10.47	10.57	10.66	10.72
+ 07	+ 08	+ 04	— 07	— 17	— 19	— 21	— 30	— 35	— 40	— 46.

Thus it appears that perhaps class 9.2 may be considered complete, but 9.3 already shows a lack of stars, which increases for each lower class. Therefore, also in the southern D. M. stars below 9.2 must not be used for statistical purposes.

THE MAGNITUDES OF THE CORDOBA DURCHMUSTERUNG.

The decimal error.

34. The extension of the Durchmusterung over the southern sky at Cordoba was begun by Juan Thomé, 1885—1891, by observing a zone of 20°, from 22° to 42° southern declination, published in Vol. XVI and XVII of the „*Resultados del Obs. Nac. Argentina*". The instrument (aperture 125*mm*) was used with dark field; thus a great number of faint stars could be observed (the AR was registered), especially in poor fields where the limit lies much lower than in the galaxy. Nevertheless the faintest class observed was always noted 10^m. A comparison of the common zone —22° with Schoenfeld showed that on the average in Cordoba the number of stars observed was three times that of Bonn. The following part was begun in 1894, but owing to the lack of means it could proceed but slowly. The section —42° to —52° was finished December 1897 and published by Thomé in Vol XVIII; but at the death of this zealous observer the observations had proceeded no farther than —62°; this section was published by Prof. C. H. Perrine, his successor, in Vol. XXI. The completion of this enormous work therefore (containing between —22° and —52° nearly 500.000 stars) is still to be awaited from the Cordoba astronomers.

The long duration of the work, which occupied the chief observer for nearly twenty years, was not favorable to its homogeneity. Especially the gap between the first and the second half has caused a strong discontinuity. This is manifest at once by the comparison with photometric magnitudes given in Harvard Annals **72**, N^0. 7, and **80**, N^0. 7. (*Scale of the Cordoba Durchmusterung*). The mean correction to Cordoba 9.0 for zones 22°—27°, 27°—32° .... 57°—62° is given there + 0.46, + 0.50, + 0.44, + 0.41, —0.01, 0.00, + 0.03 + 0.03. In our investigation we will make use of the same 8 zones of 5° breadth, as a rule combining them to two halves of the work, or sometimes to four sections of 10°.

35. The Cordoba Durchmusterung has been counted here for each tenth magnitude down to 9.2 (some parts still farther). The statistical magnitudes derived from these counts in the same way as described in § 16, are contained for the separate zones in Table 42. The ranges for each tenth magnitude found from them show the following peculiarities. The decimals 5 and 0 occupy too great a space at the cost of the adjacent decimals 9, 1, 4, 6. The range for the decimals 3 and 7 is greater than for 2 and 8, especially in zone III and IV, where they extend thrice as much. These irregularities increase steadily during the four first zones; then at 42° they disappear, a sudden return to nearly the normal values occurs and this state is maintained during the second half, though in the last zones the same irregularities again become perceptible. These decimal errors are hardly perceptible for the stars brighter than 7^m, at least in the first six zones; zone VII and VIII, on the contrary, show a strong decimal error of the same character as the fainter stars.

Table 42. Statistical magnitudes and decimal ranges.

m	22°--27°		27°—32°		32°—37°		37°—42°		42°—47°		47°—52°		52°—57°		57°—62°	
5.9																
	5.72		5.53		5.69		5.75		5.84		5.74		5.68		5.84	
6.0		10		12		10		08		08		07		18		14
	82		65		79		83		92		81		86		98	
1		10		07		07		10		04		07		02		03
	92		72		86		93		96		88		88		6.01	
2		09		08		09		09		06		09		09		05
	6.01		80		95		6.02		6.02		97		97		06	
3		13		09		08		06		09		09		12		08
	14		89		6.03		08		11		6.06		6.09		14	
4		06		13		14		10		09		13		05		06
	20		6.02		17		18		20		19		14		20	
5		13		15		12		15		12		14		20		25
	33		17		29		33		32		33		34		45	
6		13		08		10		13		13		10		05		07
	46		25		39		46		45		43		39		52	
7		12		12		16		11		13		13		12		10
	58		37		55		57		58		56		51		62	
8		10		15		11		11		10		12		07		07
	68		52		66		68		68		68		58		69	
9		12		13		15		16		10		12		05		04
	80		65		81		84		78		80		63		73	
7.0		12		20		18		15		17		14		18		22
	92		85		99		99		95		94		81		95	
1		09		11		13		17		09		08		15		11
	7.01		96		7.12		7.16		7.04		7.02		96		7.06	
2		13		13		07		09		10		09		13		11
	14		7.09		19		25		14		11		7.09		17	
3		16		20		19		19		13		15		15		13
	30		29		38		44		27		26		24		30	
4		08		09		05		04		09		07		11		11
	38		38		43		48		36		33		35		41	
5		15		25		29		27		15		16		19		23
	53		63		72		75		51		49		54		64	
6		12		11		05		05		08		11		11		10
	65		74		77		80		59		60		65		74	
7		08		12		15		17		11		14		13		12
	73		86		92		97		70		74		78		86	
8		10		11		07		04		11		10		12		11
	83		97		99		8.01		81		84		90		97	
9		07		07		06		06		06		05		09		07
	90		8.04		8.05		07		87		89		99		8.04	
8.0		18		25		27		27		13		16		14		18
	8.08		29		32		34		8.00		8.05		8.13		22	
1		12		10		06		04		10		10		11		11
	20		39		38		38		10		15		24		33	
2		12		10		05		04		09		08		12		12
	32		49		43		42		19		23		36		45	
3		15		16		20		20		12		13		15		12
	47		65		63		62		31		36		51		57	
4		12		11		07		05		08		09		11		11
	59		76		70		67		39		45		62		68	
5		17		19		28		28		10		11		14		17
	76		95		98		95		49		56		76		85	
6		15		12		07		06		12		11		13		14
	91		9.07		9.05		9.01		61		67		89		99	
7		12		16		18		19		09		09		11		10
	9.03		23		23		20		70		76		9.00		9.09	
8		14		12		06		05		09		08		08		11
	17		35		29		25		79		84		08		20	
9		11		11		09		08		09		09		07		08
	28		46		38		33		88		93		15		28	
9.0		20		24		25		25		13		12		13		18
	48		70		63		58		9.01		9.05		28		46	
1		17		16		10		10		14		11		10		10
	65		86		73		68		15		16		38		56	
2		15		11		06		05		13		14		12		15
	80		97		79		73		28		30		50		71	

In searching for the cause of these variations we must keep in mind that the direct estimates were made in quarters of a magnitude. To explain the relative scarcity of the decimals 1, 4, 6, 9 we must then admit that 0 and 0.5 were estimated oftener than ¼ and ¾. The difference between 2 and 3, 7 and 8 must have its origin in the practice of the computer, who, in the case of the mean of two observations coming out ¼ or ¾, has given preference to the odd decimal. But it is not clear whether and why this practice has not been followed in the same way in the other zones, where this difference is smaller. The different behaviour of the bright stars is easily understood by the fact that for these stars the magnitudes of the „*Uranometria Argentina*" have been given in the catalogue instead of the estimates of the Durchmusterung zones themselves, as has been expressly stated pag. XXXI of the Introduction to Vol. XVI. For the last section, however, where the results of the observations of THOMÉ have been published by his successor without further discussion, the Durchmusterung results are also given for the bright stars.

The sudden change at —42° coincides with a change of the same character in the reduction to photometric scale. Obviously the observer after the completion of the first half, by various investigations, has become aware of the existence and the character of the systematic errors of the Durchmusterung magnitudes; and in his second series he has tried to avoid them and better to adapt his scale to the photometric scale. The gradual restoration of the former peculiarities may be an indication that such a subjective scale is not a fortuitously erroneous scale but obeys to some causality in the psychical processes underlying this kind of observations. As in all personal errors a subjective scale may become fixed and constant for an experienced observer; and then the relation to a theoretical scale may be found accurately. If he tries, however, to correct these personal „errors", the resulting unsteadiness of his scale will make its use much more difficult.

36. Reducing the ranges of each decimal (between 7.0 and 9.0) to „subjective ranges" (with the sum total 1.0) we may use these to find the real limits (half-tenth magnitudes) of each class in a homogeneous subjective scale. They are contained, separately for each zone, in Table 43, where the mean deviation is made zero for each line.

Table 43. Limits of tenth magnitudes.

Zones	0		1		2		3		4		5		6		7		8		9		0
I		06		14		24		36		44		57		67^5		76		85		92	
II		08		14^5		23		36		42		58		66		76^5		85		91	
III		08		15		20		35		40		62		66^5		79		84		92^5	
IV		07		15		20^5		36		40		62		66		80		84		89	
V		06		15		24		36		44		56		66		75^5		85		92	
VI		06		14		22		36		43		56		66^5		77		86		93	
VII		03		14		24		36		45		58		67		77		85		91	
VIII		05		14		23		33		42		58^5		68		77		86		92	

The limits between 0 or 5 and the adjacent decimals are strongly variable; this is also the case with the limits 2—3 and 7—8; in both cases a consequence of the variable amount of the decimal errors pointed out above. The limits 1—2, 3—4, 6—7 and 8—9, on the other hand, are not influenced by them, and their variations appear to be much smaller. The number of stars to be used for investigations on stardensity, should therefore be counted between these limits. Thus in counting the stars of the Cordoba Durchmusterung over small fields the decimal classes 9, 0, 1 were combined (called .0), also 2 and 3 (group ¼), also 4, 5 and 6 (group .5) and also 7 and 8 (group ¾). If the correction to photometric scale is deduced for 7.0 7.5 etc. from all stars 6.8—7.2, 7.3—7.5.... the corrections for the limits 7.15, 7.35...., taken from these results, do not require any systematic correction for decimal error surpassing 0.01^m.

The density error.

37. For the differences Harvard—Cordoba, contained in „*Scale of the Cordoba Durchmusterung*" (Harv. Ann. **72**,7, Tables I—X and **80**,7, Tables I and II) for each hour of AR in each zone, the deviations of the hourly means from the average difference for this magnitude (at the bottom of each column) were computed. They were combined into groups according to galactic latitude, separately for the first half 22°—42° (containing for the faint stars below 9.0 the zones 24°, 29°, 39°) and the second

Table 44. Influence of density.

Zones 22°—42°.

m	β 70°—90°	60°—70°	40°—60°	30°—40°	20°—30°	10°—20°	0°—10°	coeff. of D
6.5	+ 05 (55)	00 (25)	+ 03 (20)	— 07 (28)	— 03 (33)	+ 01 (23)	— 04 (7)	— .003
7.0	+ 20 ,(112)	+ 07 (75)	+ 02 (125)	+ 01 (194)	+ 02 (304)	— 05 (222)	— 16 (192)	— .019
7.5	+ 18 (148)	+ 05 (131)	+ 02 (210)	+ 04 (256)	+ 06 (466)	— 10 (395)	— 20 (427)	— .022
8.0	+ 11 (45)	— 05 (32)	+ 03 (50)	+ 09 (67)	+ 06 (107)	— 02 (113)	— 24 (122)	— .017
8.5	+ 05 (63)	— 01 (36)	— 01 (62)	+ 13 (101)	+ 09 (144)	— 04 (183)	— 22 (205)	— .015
9 0	+ 07 (40)	+ 03 (40)	— 05 (44)	+ 09 (121)	+ 03 (167)	— 04 (143)	— 18 (233)	— .014
8.6—9.2	+ 15 (126)	+ 03 (88)	+ 04 (105	+ 09 (195)	+ 05 (184)	— 07 (159)	— 20 (134)	— .021
9.3—9.6	+ 26 (115)	+ 03 (77)	— 07 (103)	+ 07 (184)	— 01 (172)	— 07 (135)	— 23 (122)	— .023
9.7—10	+ 29 (148)	+ 18 (100)	+ 08 (143)	+ 13 (208)	— 03 (232)	— 11 (243)	— 21 (229)	— .032
D_1	12.3	12.4	14.3	16.5	20.3	23.1	25.7	
D_2	12.2	12.0	14.2	16.3	19.3	23.7	26.2	

Zones 42°—62°.

m	β 60°—90°	50°—60°	40°—50°	30°—40°	20°—30°	10°—20°	0°—10°	coeff. of D
6.5	+ 11 (26)	+ 16 (30)	+ 04 (31)	00 (16)	+ 03 (32)	— 07 (40)	— 14 (111)	— .016
7.0	+ 18 (71)	+ 16 (46)	+ 06 (49)	+ 04 (31)	+ 01 (65)	— 08 (148)	— 13 (184)	— .018
7.5	+ 16 (31)	+ 07 (42)	+ 13 (30)	+ 06 (38)	— 09 (40)	— 05 (105)	— 09 (183)	— .014
8.0	+ 08 (31)	+ 06 (21)	+ 07 (29)	+ 06 (19)	— 02 (38)	— 03 (59)	— 07 (122)	— .010
8.5	+ 01 (31)	+ 05 (30)	+ 07 (31)	— 03 (28)	— 04 (55)	+ 01 (72)	— 04 (125)	— .005
9.0	+ 12 (29)	+ 05 (32)	+ 02 (34)	+ 01 (23)	+ 01 (32)	— 01 (111)	— 02 (235)	— .005
8.6—9.2	+ 14 (26)	+ 07 (6)	+ 06 (14)	+ 05 (12)	— 09 (20)	— 03 (59)	— 05 (41)	— .017
9.3—9.6	+ 21 (26)	+ 17 (23)	+ 12 (22)	— 11 (23)	— 23 (25)	— 05 (84)	— 01 (65)	— .018
9.7—10	+ 33 (118)	+ 18 (53)	+ 15 (52)	— 06 (51)	— 33 (48)	— 07 (169)	— 19 (116)	— .040
D_1	7.8	9.4	9.2	9.1	12.7	14.0	20.8	
D_2	7.8	7.7	8.8	8.3	10.5	14.6	16.5	

half 42°—62° (for the faint stars only zone 49°). The faint classes in tables VII—X were combined into three groups 8.6—9.2, 9.3—9.6, 9.7—10. The results are contained in Table 44 (in 0.01^m). When afterwards the densities became available the mean density was computed for each zone of galactic latitude. For D always 3 × the number of stars from the brightest down to 9.05 was taken. Somewhat different values D_1 and D_2, had to be taken for the bright and for the faint stars.

Computing linear formulae we find the coefficients of D contained in the last column. In the two parts of the Durchmusterung they show the same behaviour : at first they decrease somewhat with magnitude, but then below 9.0 they increase rapidly.1) Since the bright stars are taken from the Uranometria Argentina, they must show the same density error; from PICKERINGS result (Harvard Annals **23**, 159) for the difference between galactic and extragalactic regions in U. A. 0.16^m we find the coefficient of D 0.018. But also the Durchmusterung magnitudes themselves show the decrease from 7^m to 9^m; obviously its scale for the brighter classes has been adapted to the U. A. and thus borrows its systematic dependence on galactic latitude in a rate diminishing for the fainter stars. For the stars below 9.0, however, just as in Bonn, the influence of stardensity on the scale increases strongly for decreasing brightness.

As the magnitude 9.0 C. in the first half corresponds to 9,4, in the second half to 9.0 H, the values D in the two parts do not mean the same real density; for the second part they are smaller (as is also shown by D_1 and D_2) and should be multiplied by 1.56. But by this change the harmony of the two parts is considerably vitiated. While apparently their density coefficients come out nearly equal, in reality they are considerably smaller in the second half of the Durchmusterung. Perhaps this is caused by the occurrence of regions with very high densities (in Carina) not showing corresponding strong deviations in magnitude scale. It is also possible that the fall of the density coefficient is due to the observer trying to avoid it after having become acquainted with its existence. For the reductions of the magnitudes it seemed the best way to take the error as being simply dependent on the apparent density D with coefficients valid for the whole work and adopted as in Table 45.

Table 45. Coefficients of density error adopted (in 0.001^m).

m	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
Coefficient.....	— 20	— 19	— 17	— 16	— 15	— 16	— 24	— 37

The mean density, to which the magnitudes were reduced, was taken $D_0 = 20$ for 22°—42°, $D_0 = 10$ for the 3d section 42°—52°, and $D_0 = 15$ for the 4th section 52°—62°.

Corrections depending on declination.

38. The comparisons of Cordoba with the Harvard catalogues were first made with H 50 and H 54. When the systematic influence of the incompleteness of these catalogues was perceived, the original data of H 34 were used for the bulk of the Cordoba magnitudes, from 6.3 to 9.2. For the brighter magnitudes, however, H 50 and H 54 may be assumed to be complete, for the northern part, resting on H 45, down to 7¼, for the southern part, below — 35°, resting on H 46, down to 6¾; thus for these classes, for which H 34 contains only few stars, comparisons with H 50 and

---

1) The result for 6.5 is wholly untrustworthy; all the values for 6.5 given in Harvard Annals 72 being rendered incorrect by the omission of the stars of H 50, it must be found too small. As it had got a small weight already, its influence on the results is very slight.

H 54 are also used. For the faintest classes below 9^m we have used the zones observed with the Rumford photometer in H 72.

The general method of treating them consisted in computing for each zone hourly means of the differences H—C for each class, correcting them for density error, and deriving general means, either by giving equal weight to each hour (*h*) or to each star (*s*). The deviations of the single stars from the mean of the zone (corrected for density) were used to find the mean error (combined mean error μ_{12} for the two catalogues). These deviations doubtlessly include some gross errors, caused by misidentifications, as well as irregularities of the scale; thus they will give a somewhat exagerated value of the mean error. But it is just this exagerated value that determines the dispersion of the m_2 belonging to the same m_1 and must therefore be used in deriving the corrections for multitude. The combined mean errors μ_{12}, derived, just as in other cases, by the quartil method, are contained in the upper part of Table 46.

Table 46. Mean errors μ_{12} and μ_1 (Cordoba) (in 0.01^m).

<table>
<tr><th></th><th>6.5</th><th>7.0</th><th>7.5</th><th>8.0</th><th>8.5</th><th>9.0</th><th></th><th>9.0</th><th>9.1</th><th>9.2</th><th>9.3</th><th>9.4</th><th>9.5</th><th>9.6</th><th>9.7</th><th>9.8</th><th>9.9</th><th>10.0</th></tr>
<tr><td>22°—42° . . . .</td><td>29</td><td>25</td><td>35</td><td>39</td><td>40</td><td>31</td><td>29°</td><td colspan="2">45</td><td colspan="3">49</td><td colspan="2">42</td><td colspan="2">45</td><td colspan="2">40</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>39°.</td><td colspan="2">45</td><td colspan="3">40</td><td colspan="2">48</td><td colspan="2">47</td><td colspan="2"></td></tr>
<tr><td>42°—62° . . . .</td><td>32</td><td>20</td><td>17</td><td>11</td><td>14</td><td></td><td>49°</td><td colspan="2">30</td><td colspan="3">29</td><td colspan="2">40</td><td colspan="2">49</td><td colspan="2"></td></tr>
<tr><td>μ_1 . . . . . .</td><td>21</td><td>25</td><td>29</td><td>34</td><td>38</td><td>41</td><td></td><td>41</td><td colspan="3">42</td><td colspan="3">42</td><td colspan="4">41</td></tr>
<tr><td>μ_1^2 . . . . . .</td><td>04</td><td>06</td><td>08</td><td>12</td><td>15</td><td>17</td><td></td><td>17</td><td colspan="3">18</td><td colspan="3">18</td><td colspan="4">17</td></tr>
<tr><td>μ_1 } app.. . .</td><td>21</td><td>16</td><td>10</td><td>10</td><td>10</td><td>10</td><td></td><td>25</td><td colspan="3">32</td><td colspan="3">40</td><td colspan="4">41</td></tr>
<tr><td>μ_1^2 } 42°—62°.</td><td>04</td><td>03</td><td>01</td><td>01</td><td>01</td><td>01</td><td></td><td>06</td><td colspan="3">10</td><td colspan="3">16</td><td colspan="4">17</td></tr>
</table>

For the first half of the C. D. M. the mean error increases regularly for fainter stars; the low value for 9.0 (H 34) is caused by the cutting off of faint stars, and it is found to be consistent with a real mean error 0.40. For the second half, however, we get the curious result that the combined mean error of the Harvard and the Cordoba magnitudes about 8^m should be only 0.11^m—0.14^m, smaller than the mean error of a Harvard magnitude alone. It is clear that this is impossible; it shows that the Cordoba estimates cannot be independent from the Harvard magnitudes. This is confirmed by comparing the same Cordoba magnitudes between 8^m and 9^m with the later measures in H 72; here a combined mean error 0.21 is found, corresponding with the mean value of a difference H 72—H 34, 0.20, found from the same stars. The origin of this dependence may perhaps be elucidated by the following sentences. In the Introduction to Vol. 18 of the „*Resultados*" (written 1900) THOMÉ, opposing the idea that photographic magnitudes must necessarily surpass visual estimates and make them superfluous, says: „I have, however, experienced no greater difficulty in conforming to PICKERINGS photometric scale in the milky way than in any other region of the heavens". Thus it is manifest that he has tried to adapt his scale as much as possible to the photometric magnitudes. Now in the first section of the work the adaptation to the scale of the Zone Catalogue is described in this way: „I have been able to diminish the laps in declination to 5′, by the aid of a programm containing the position, magnitude and place upon the scale of 5 or 6 stars in each hour, usually selected from among the fainter ones given in the Z. C. As the transits of these stars approach, the assistant calls out the scale reading and the magnitude, and the observer.... can adhere more closely to a consistent scale in his estimates of magnitude". (*Resultados,* Vol XVI, page XIII.). If the same method has been followed for the zones in the second half, the dependence of his estimates on the Harvard magnitudes becomes intelligible. But then we have no control a posteriori that for the bulk of the other stars the same relation between the two catalogues holds that is introduced a priori for the common narrow 20′ zones.

For the first half a regular increase of μ_{12} with magnitude is adopted, till some maximum value 0.45 is reached for 9.5, and a small decrease afterwards. With $\mu(H) = 0.15$ we find the values μ (Cordoba) contained in the lower part of Table 46. The mean errors of a catalogue magnitude derived by THOMÉ from the concordance of the separate observations, viz 0.18, 0.18, 0.15, 0.11, for $m = 7.5$, 8.5, 9.2, 9.7, which reduced to photometric scale become 0.24, 0.25, 0.28 and 0.29, are appreciably smaller; they overrate the accuracy really attained while our results underrate it. In computing the systematic errors depending on multitude for the second half (where the dependence on the magnitudes of H 34 is perceptible down to 9.5) the small apparent values of μ_{12}, indicating the actual small dispersion of the differences, must be used. Thus the values of the two last lines of Table 46 have been adopted here.

39. *Comparison with H* 34. As the working list of H 34 was composed of G. C. and Z. C. stars we have for the systematic corrections for multitude case 5, the limits depending on a third catalogue. The basis of the computation is formed by the number of stars for each tenth magnitude (6.50—6.59, etc.) counted in H 34, and given in Table 44 (N); the second column gives smoothed values (S), the third column (R) gives the numbers reduced (by dividing by $A(m)$), indicating relative completeness.

Table 47. Number of stars in H 34.

m	N	S	R	m	N	S	R	m	N	S	R	m	N	S	R
6.5	41	37	164	7.5	101	94	135	8.5	140	145	64	9.5	300	285	50
6	47	42	174	6	81	97	97	6	157	153	65	6	255	256	38
7	45	47	145	7	96	100	103	7	162	161	61	7	211	204	29
8	71	53	208	8	121	103	117	8	154	171	52	8	145	148	18
9	47	59	124	9	102	107	89	9	169	187	51	9	89	100	10
7.0	53	65	124	8.0	114	112	89	9.0	222	215	61	10.0	66	66	7
1	77	71	161	1	108	117	76	1	259	240	64	1	43	40	4
2	84	78	156	2	118	124	75	2	260	256	58	2	12	20	2
3	76	85	127	3	132	131	75	3	246	267	50				
4	91	90	135	4	148	138	76	4	295	281	54				

From the curve of R, completed by the analogous numbers from our former Table 5, the following moduli of decrease were derived:

$l = 5.9$ $\mu_{13}\sqrt{2} = 0.33$ $B/A = 0.21$ decrease from total number to zones of 40′,
$l = 7.8$ $\mu_{13}\sqrt{2} = 0.56$ $B/A = 1$ decrease of zones 40′ tot 20′ for 7¾ C. G. C.,
$l = 9.6$ $\mu_{13}\sqrt{2} = 0.68$ $B/A = 0$ gradual disappearing of the stars of C. G. C. and Z. C.

With these data and $\mu_1 = 0.20$ the following corrections for multitude were found (in 0.001^m).

$m = 6.3$	6.7	7.1	7.5	7.9	8.3	8.7	9.1	9.5
+018	—043	—040	—027	—016	—025	—033	—018	+027.

A table of corrections was also computed by mechanical integration (formula 8 § 11) for regularly increasing μ_1 (0.10 to 0.40), using the smoothed numbers S, and corrected by method 9, § 11. The resulting values for $\mu_1 = 0.20$ are, for the magnitudes just used,

$$+007 \quad -040 \quad -031 \quad -019 \quad -017 \quad -021 \quad -030 \quad -028 \quad +035,$$

showing that both methods give nearly concordant results. From this table the corrections for multitude were taken for the different magnitude groups, using μ from Table 46, to be applied to the observed mean differences H 34—C. Table 48 gives in its upper part the mean differences H 34—C for the groups 6.3—6.7, 6.8—7.2…., with equal weight to each star (s) as well as to each hour (h); the mean of the two results, corrected, as well as the correction for multitude itself are contained in the lower part of the table. In zone III the narrow strip measured photometrically in 17^h AR runs right through a coarse cluster. By the agglomeration of bright stars the Cordoba estimates are much too faint here; thus we get here a great number of large negative differences, which are not neutralised by the density correction belonging to the whole zone. In order to diminish their influence on the results, for 17^h only one third of their number, with the same mean deviation as the whole, has been included in the sums total.

Table 48. Reductions of Cordoba D. M. to Harvard 34 (in 0.01^m.)

Zone	6.5 C		7.0 C		7.5 C		8.0 C		8.5 C		9.0 C	
	s	*h*	*s*	*h*	*s*	*h*	*s*	*h*	*s*	*h*	*s*	*h*
22°–27°	— 29 (22)	— 29	— 09 (38)	— 12	— 01 (65)	— 03	+ 18 (85)	+ 18	+ 36 (168)	+ 34	+ 41 (240)	+ 43
27°–32°	— 17 (28)	— 18	00 (49)	— 01	+ 17 (78)	+ 15	+ 31 (106)	+ 30	+ 53 (178)	+ 50	+ 55 (232)	+ 52
32°–37°	— 23 (32)	— 26	— 15 (57)	— 17	+ 08 (85)	+ 14	+ 26 (84)	+ 22	+ 45 (174)	+ 46	+ 46 (133)	+ 45
37°–42°	— 31 (21)	— 32	— 10 (38)	— 11	+ 16 (86)	+ 16	+ 20 (69)	+ 21	+ 41 (155)	+ 37	+ 44 (146)	+ 40
42°–47°	— 24 (31)	— 25	— 10 (48)	— 09	+ 01 (59)	+ 02	+ 02 (69)	+ 02	+ 02 (82)	+ 01	+ 03 (153)	+ 03
47°–52°	— 28 (26)	— 31	— 13 (39)	— 13	+ 02 (57)	+ 02	+ 04 (68)	+ 02	+ 04 (68)	+ 03	+ 07 (98)	+ 04
52°–57°	— 37 (38)	— 39	— 08 (36)	— 06	— 02 (49)	— 05	— 04 (55)	— 06	+ 08 (93)	+ 07	+ 07 (98)	+ 04
57°–62°	— 23 (34)	— 27	— 02 (31)	— 06	— 01 (67)	— 01	+ 04 (72)	+ 02	+ 05 (104)	+ 03	+ 17 (120)	+ 06
	Corrections and Corrected Values											
22°–27°	+ 03	— 26	— 06	— 17	— 05	— 07	— 06	+ 12	— 09	+ 26	+ 09	+ 51
27°–32°	+ 01	— 16	— 06	— 06	— 04	+ 12	— 06	+ 25	— 08	+ 44	+ 15	+ 69
32°–37°	+ 02	— 22	— 05	— 21	— 04	+ 07	— 06	+ 18	— 09	+ 36	+ 11	+ 57
37°–42°	+ 03	— 28	— 06	— 16	— 04	+ 12	— 06	+ 15	— 09	+ 30	+ 09	+ 51
42°–47°	+ 02	— 22	— 03	— 13	00	+ 01	00	+ 02	— 01	+ 01	— 01	+ 02
47°–52°	+ 03	— 26	— 03	— 16	00	+ 02	00	+ 03	— 01	+ 03	— 01	+ 05
52°–57°	+ 04	— 34	— 03	— 10	00	— 03	00	— 05	— 01	+ 07	— 01	+ 05
57°–62°	+ 04	— 28	— 03	— 07	00	— 01	00	+ 03	— 01	+ 03	— 01	+ 11

For the second arrangement of the differences H 34—C according to m_H (6.00—6.49, 6.50—6.99.... called 6.25, 6.75.... 9.25) the same formulae of Case 5 must be used. The corrections for multitude for the 7 magnitude groups are

zone I—IV.	— 01	+ 03	+ 01	+ 01	+ 01	+ 02	+ 01
zone V—VIII	— 01	+ 01	00	00	00	+ 01	+ 01

For 6.25 H a special correction was necessary, because the lists of stars common to H 34 and Cordoba, constructed for these researches, contained no stars brighter than 6.3. Thus the mean Cordoba magnitude corresponding to 6.25 H is found too faint and a somewhat cursory computation showed that an additional correction —.08 must be applied to C—H. For 6.75 H this error proved insignificant. In Table 49 the results for each zone are given in condensed form; for the first group 6.25 H = 6.67 C, difference — 42, after correction — 01 + 08 becomes — 35. The number of stars is added in parentheses.

Table 49. Reductions of Cordoba D. M. to Harvard 34 (in 0.01^m).

Zone	6.25 H	6.75 H	7.25 H	7.75 H	8.25 H	8.75 H	9.25 H
22°							
	6.67 — 35 (17)	7.07 — 29 (24)	7.41 — 15 (47)	7.79 — 03 (45)	8.19 + 07 (71)	8.53 + 24 (120)	8.89 + 37 (174)
27°							
	6.55 — 23 (19)	6.88 — 10 (39)	7.30 — 04 (45)	7.61 + 15 (60)	8.05 + 21 (81)	8.35 + 42 (102)	8.75 + 51 (187)
32°							
	6.68 — 36 (17)	6.96 — 18 (48)	7.44 — 18 (47)	7.69 + 07 (56)	8.07 + 19 (89)	8.46 + 31 (107)	8.77 + 49 (137)
37°							
	6.62 — 30 (24)	6.96 — 18 (25)	7.30 — 04 (46)	7.70 + 06 (67)	8.04 + 22 (69)	8.42 + 35 (81)	8.82 + 44 (152)
42°							
	6.57 — 25 (25)	6.85 — 09 (31)	7.26 — 01 (44)	7.76 — 01 (62)	8.28 — 03 (81)	8.76 00 (103)	9.22 + 04 (186)
47°							
	6.61 — 29 (21)	6.81 — 05 (27)	7.22 + 03 (41)	7.74 + 01 (62)	8.23 + 02 (66)	8.75 + 01 (68)	9.17 + 09 (135)
52°							
	6.39 — 07 (11)	6.79 — 03 (23)	7.30 — 05 (43)	7.75 00 (52)	8.28 — 03 (61)	8.69 + 07 (88)	9.21 + 05 (134)
57°							
	6.53 — 21 (18)	6.79 — 03 (21)	7.25 00 (51)	7.76 — 01 (71)	8.25 00 (78)	8.64 + 12 (93)	9.12 + 14 (150)
62°							

40. *Comparison with H 50 and H 54.* For zones I and II (22°—32°) the Cordoba stars 6.3—6.7 and 6.8—7.2, for the remaining zones only the stars 6.3—6.7 have been used. As the catalogues may be assumed to be complete, the mean differences H—C (equal weight to each hour) must be corrected — $b\mu_1^2$, which has been taken — 0.06. In the same way the differences for the argument *m*(Harvard) (for groups 5.50—5.99, 6.00—6.49 and for zone I—II also 6.50—6.99, equal weight to each star) have been corrected by + 0.025. The question arises whether a systematic reduction to H 34 should be applied. We have found the systematic difference H 45—H 34 = — 0.04, H 46—H 34 =

Table 50. Reductions of Cordoba D. M. to Harvard 50 and 54 (in 0.01^m).

	22°—27°	27°—32°	32°—37°	37°—42°	42°—47°	47°—52°	52°—57°	57°—62°
6.5 C (*h*)	— 37 (193)	— 25 (159)	— 25 (201)	— 31 (156)	— 29 (149)	— 22 (144)	— 38 (101)	— 25 (104)
7.0 C (*h*)	— 23 (332)	— 09 (372)						
5.75 H (*s*)	— 40 (90)	— 24 (74)	— 34 (81)	— 36 (74)	— 35 (70)	— 29 (52)	— 32 (68)	— 31 (57)
6.25 H (*s*)	— 35 (148)	— 26 (118)	— 29 (130)	— 30 (164)	— 31 (113)	— 26 (121)	— 31 (102)	— 31 (97)
6.75 H (*s*)	— 30 (236)	— 15 (241)						

+ 0.02 for 6^m—7^m. In the catalogues H 50 and H 54 these differences will be diminished to perhaps half the amount; as it is doubtful, besides, whether they represent real constant differences, we have abstained from applying them. Table 50 contains the corrected mean differences H—C.

41. *Comparison with H* 72. The Cordoba stars below 9.0 have been measured in four zones of 10′ width, at 24°, 29°, 39°, 49°. In Harvard Annals **72**, N⁰. 7, the mean of the differences is given for each tenth magnitude and each hour of AR. We have corrected them for density error and computed mean values for 9,0, 9.2 (9.1—9.3), 9.5 (9.4—9.6), 9.8 (9.7—9.9) and 10.

For zone — 24° the working list was made by means of the Cape P.D.; thus § 11 case 5 determines the correction for multitude. The numbers of stars counted for each tenth magnitude were smoothed and used to compute corrections by mechanical integration. The smoothed number of stars and the corrections (in 0.001^m) were found for

8.5	9.0	9.5	10.0	10.5	11.0	11.5	12.0
15	25	34	42	45	37	21	7
	— 014	— 078	— 036	+ 022	+ 098	+ 210	

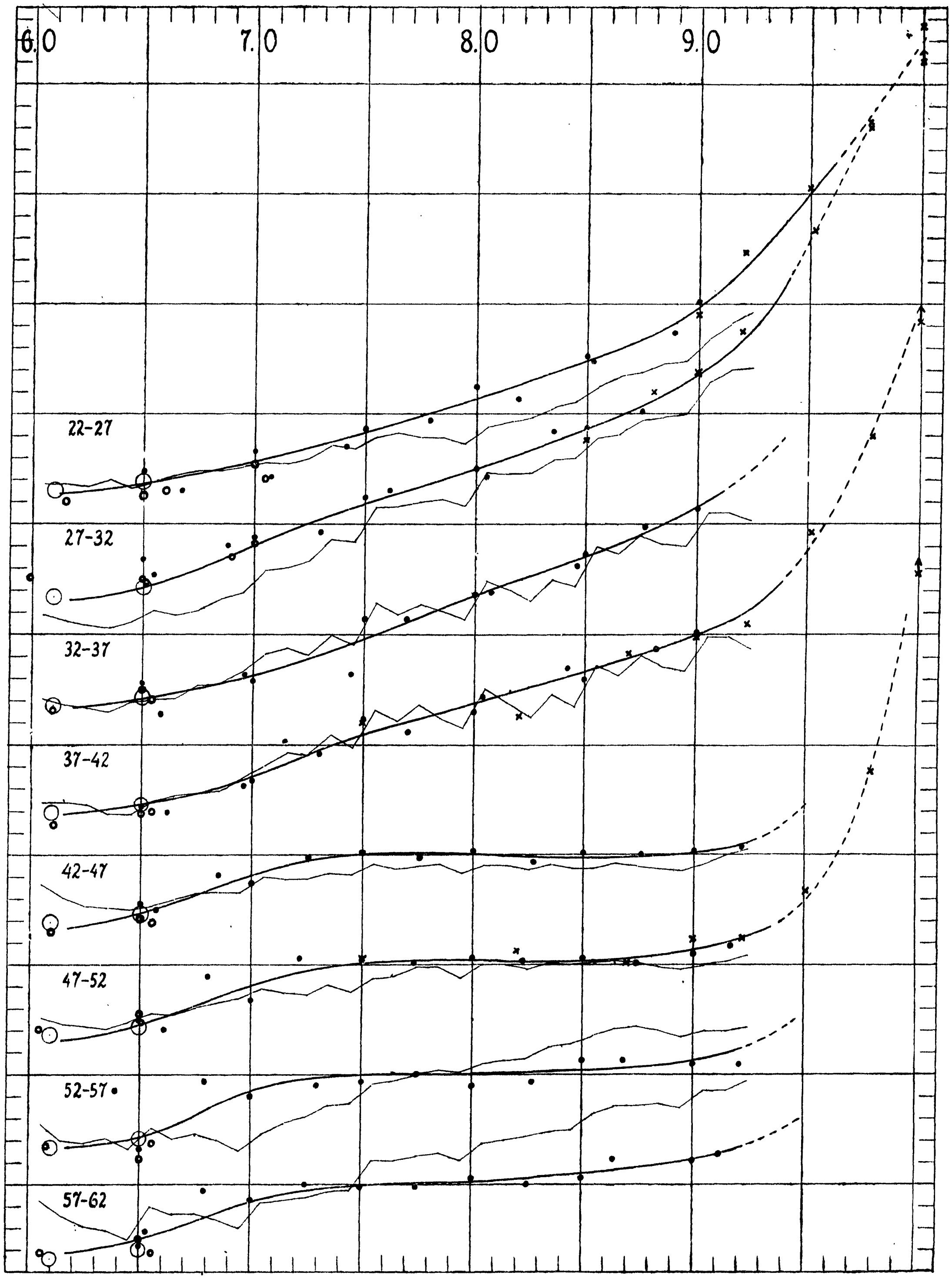
For the magnitudes needed they could be interpolated from this table.

For the zones — 29°, — 39°, — 49°, observed with the Rumford photometer, the matter was much more difficult, because it was not possible to ascertain in a clear way the principles guiding the choice of the stars. If for each 10 minutes of AR only two 10^m stars are observed, the first encountered or taken wholly at random — the modus followed also for other faint classes below 9.5 — their average is determined quite in the same way as if all stars had been observed. Thus the correction for multitude will be — $b' \mu^2$, where b' denotes the apparent rate of increase, smaller than b or negative on account of the incompleteness of the C.D.M. for fainter stars. I have not found it possible to derive exact values for this incompleteness as a function of the Harvard magnitude and to compute a formula for it; we may only say with some probability that for 9.2 C its influence is not yet perceptible, while between 11^m and 12^m b' becomes zero. Thus for 9.8 C we will assume $b' = 0$, for 9.5 C we will take $b' = \frac{1}{2}b$, while for 10 C it is negative of an unknown amount.

For the classes brighter than 9.5 numerous stars outside the zone were observed, and because they should be used as standards, preference was given to stars formerly observed, i. e. occurring in the H 34 zones. The stars brighter than 9.0 within the zone are almost exclusively H 34 stars, and stars missing in H 34 are also missing here. Thus for these classes the same corrections for multitude will be necessary as for the comparisons with H 34 itself. For 9.0 the matter is less clear; within the zone they seem to be taken at random while among the smaller number outside the zone

Table 51. Reductions of Cordoba D. M. to Harvard 72 (in 0.01^m).

Zone — 24°					Zone — 29°					Zone — 39°					Zone — 49°				
										7.5	+ 23	— 04 — 08	+ 11	75	7.5	+ 10	00 — 08	+ 02	52
					8.5	+ 44	— 08 + 02	+ 38	78	8.2	+ 23	— 07 — 03	+ 13	64	8.2	+ 10	00 — 04	+ 06	48
					8.8	+ 59	— 01 + 02	+ 60	70	8.7	+ 43	— 06 + 04	+ 41	81	8.7	00	+ 01 00	+ 01	64
9.0	+ 53	— 08	+ 45	107	9.0	+ 75	— 06	+ 69	64	9.0	+ 58	— 09	+ 49	74	9.0	+ 14	— 05 + 03	+ 12	30
9.21	+ 77	— 04	+ 73	221	9.20	+ 105	— 18	+ 87	160	9.23	+ 73	— 18	+ 55	131	9.22	+ 21	— 10 + 01	+ 12	107
9.50	+ 100	+ 03	+ 103	292	9.52	+ 142	— 09	+ 133	204	9.52	+ 105	— 09	+ 96	185	9.51	+ 42	— 08	+ 34	227
9.77	+ 119	+ 11	+ 130	199	9.77	+ 182	00	+ 182	204	9.79	+ 140	00	+ 140	239	9.80	+ 88	00	+ 88	294
10	+ 149	+ 27	+ 176	82	10	+ 210	+ ..	......	283	10	+ 192	+ ..	......	237	10	+ 178	+ ..	.....	313


6.0
7.0
8.0
9.0
22-27
27-32
32-37
37-42
42-47
47-52
52-57
57-62

80 % are H 34 stars. Thus in this case we have adopted a correction composed for $^2/_3$ of the correction for random choice and for $^1/_3$ of the H 34 correction. For the magnitudes 9.1—9.3 also some preferently chosen H 34 stars outside the zone occur besides the numerous zone stars; as, however, the H 34 correction for these faint classes cannot be found very well, we have neglected it and adopted the random correction — $b' \mu^2$ (μ from Table 46) also for these magnitudes. For zone — 49° the apparent μ^2 from the last line of Table 46 was used.

To reduce the differences H 72—C.D.M. to the scale of H 34 the corrections of Table 11 have been applied. Table 51 contains for each zone the mean magnitude m_C, the mean difference H 72—Cordoba, the correction for multitude and (sometimes) the reduction to H 34, the corrected difference and the number of stars.

42. The data afforded by the foregoing tables have been plotted in the figure pag. 62. (Table 48 and 49 by dots, Table 50 by small circles, Table 51 by crosses). Before we draw curves through them, they must be modified somewhat for the bright stars. By their small number strong differences between adjacent zones are caused, which cannot be real, because these U. A. magnitudes have been estimated with the naked eye and the fieldglass, without regard to the 5° zones. Thus they must be smoothed. This was done while at the same time the irregular corrections depending on Rect-ascension (cf. § 45) were derived. The mean corrections adopted for the eight zones for 6.5 C are

— 31 — 29 — 28 — 27 — 27 — 28 — 29 — 30,

while for 6.1 C they are — 0.04 greater. They are represented by large circles in the figure.

The curves at their beginning at 6.0 are nearly horizontal; the corrections for the Uranometria Argentina, derived by Pickering (Harvard Annals **23**, 161) show that there must be a minimum about 5.5. Then the curves, for the first half of the work, rise gradually with some minor fluctuations, and the rise becomes steeper below 9.0. For the faintest classes they become somewhat hypothetical by the uncertainty of the corrections in Table 51. For the second half of the work the curves are much more irregular, a consequence of the adaptation to different other sources: for the bright stars they show the corrections of U. A., for the magnitudes 7—9 the corrections are nearly zero, and then a steep rise of the same character as in the former zones takes place. As for the zones III, V, VII and VIII there are no data below 9.0 we must suppose the corrections to have the same character as in the adjacent zones; this includes some uncertain extrapolation, especially for the most southern zones.

From the curves the reductions to photometric scale are read for the limits 6.35, 6.65, 6.85.... used in the starcounts. The resulting photometric magnitudes corresponding to these limits are contained in the following table.

Table 52. Photometric magnitudes of the Cordoba D. M. scale.

Zone	6.35	6.65	6.85	7.15	7.35	7.65	7.85	8.15	8.35	8.65	8.85	9.15	9.35
22°													
	6.01	6.36	6.60	6.97	7.22	7.61	7.87	8.27	8.54	8.95	9.24	9.77	10,18
27°													
	6.03	6.41	6.69	7.11	7.39	7.79	8.05	8.45	8.73	9.14	9.44	9.94	10.36
32°													
	6.04	6.39	6.63	7.01	7.28	7.69	7.97	8.38	8.65	9.06	9.35	9.82	10.20
37°													
	6.06	6.41	6.66	7.07	7.35	7.74	8.00	8.38	8.64	9.03	9.30	9.71	10.04
42°													
	6.05	6.43	6.71	7.10	7.34	7.65	7.86	8.15	8.34	8.64	8.85	9.18	9.45
47°													
	6.04	6.43	6.70	7.10	7.34	7.66	7.87	8.17	8.36	8.67	8.90	9.25	9.52
52°													
	6.04	6.41	6.72	7.12	7.34	7.65	7.85	8.16	8.37	8.68	8.00	9.25	9.52
57°													
	6.07	6.44	6.72	7.11	7.33	7.65	7.86	8.17	8.39	8.72	8.94	9.30	9.58
26°													

43. For the first half of the C.D.M. the curves are well determined as far as they are needed. For the second half, however, some uncertainty remains because the estimated magnitudes, used in deriving them, are influenced by the Harvard magnitudes. Thus some control a posteriori is desirable. A first control may be found in the statistical magnitudes of Table 42. They must deviate from the photometric magnitudes firstly because the latter have not been reduced to the mean density of the zone but to some other D_0; to stand for the mean D the photometric results should get the reductions

$$+07 \quad -01 \quad +01 \quad +02 \quad -02 \quad -02 \quad -01 \quad +02$$

for the eight zones. Secondly according to § 10 the statistical magnitudes are $\frac{1}{2}b(\mu_1^2-\mu_0^2)$ too great; this difference amounts to 0.02 on the average. Thus the eight values given above, increased by + 0.02 are subtracted from the statistical magnitudes, to make them comparable with the photometric values. They are inserted in the figure by a thin broken line.

For the northern half the course of the statistical values, though for the fainter classes in zone I and for zone II they indicate a shortness of stars, does not deviate much from the curves. For the southern zones their course is more gradual than the photometric curves, though in the third section the deviations remain small. In the fourth section 52°—62°, however, they show a regular rise, wholly different from the curves. This may be an indication that the close concordance between the Cordoba and the Harvard magnitudes 7—9 holds only for the narrow zone for which the estimates are influenced by the photometric catalogue, while for the bulk of the stars the estimates follow a more regular scale. But it is also possible that the strong increase of the number of stars with magnitude in these zones indicates a real divergence from the average of the sky.

A second control may be found in the „miscellaneous stars" contained in Table VI of Harvard Annals **72**, 5., observed at the same time as the Durchmusterung zones and consisting chiefly of comparison stars for variables. They are not numerous and are distributed irregularly; moreover it is difficult to find out the systematic corrections they require; so the information they may give must needs be rather crude. After correction for D they are collected into zones of 10° wide; the average differences H—C are contained in Table 53.

Table 53. Miscellaneous stars, Harvard—Cordoba.

	22°—32°	Dev.	32°—42°	Dev.	42°—52°	Dev.	52°—62°	Dev.
7.5	+ 37 (9)	+ 37	+ 07 (15)	+ 05	+ 15 (17)	+ 15	+ 18 (9)	+ 18
8.2	+ 44 (15)	+ 22	+ 33 (18)	+ 09	+ 09 (24)	+ 08	— 09 (14)	— 11
8.7	+ 44 (21)	+ 02	+ 57 (25)	+ 16	— 05 (19)	— 06	+ 05 (22)	00
9.1	+ 80 (21)	+ 14	+ 56 (23)	— 04	+ 13 (29)	+ 08	+ 34 (13)	+ 23
9.4	+ 108 (19)	— 16	+ 78 (26)	— 05	+ 19 (38)	+ 02	+ 33 (13)	+ 10
9.7	+ 157 (23)		+ 116 (28)		+ 82 (36)		+ 36 (14)	
9.97	+ 179 (26)		+ 181 (45)		+ 145 (74)		+ 101 (19)	

For zone 52°—62° the last values are certainly too low; as the positions are given only to whole minutes in AR the identification of the stars is difficult and especially many of the faintest stars must be omitted. Though the averages are not accurate and the deviations from the curves, contained in the last columns, are rather great, they indicate for magnitudes 8—9 in the southern zones a difference H—C near to zero, thus confirming the curves by wholly independent data.

For the zones 52°—62° also they correspond better to the curves than to the regular rise of the statistical magnitudes, thus making the second explanation more probable.

44. In order to test the scale of the Cordoba D.M. by independent data we have also compared in the common zone —22° the Cordoba with the Bonn magnitudes of SCHOENFELD. By computing the differences once with argument m_C, once with argument m_B, the correction for multitude was eliminated. The results, contained in the second column of Table 54, show a nearly constant small difference between the two catalogues. This contradicts the corrections derived from the curves for the whole 5° zones, which show for Cordoba a much stronger slope than for Bonn. Adding the differences H—B from the curve pag. 50 we get values for H—C in the fourth column, quite different from the direct values H—C taken from the curve pag. 62.

Table 54. Comparison Cordoba—Bonn for zone —22°.

m (Cordoba)	B—C	H—B	H—C computed	H—C curve
7.53	+ 05 (74)	— 08	— 03	— 08
7.87	+ 06 (102)	— 06	00	+ 03
8.20	+ 05 (197)	— 03	+ 02	+ 14
8.53	+ 06 (342)	+ 02	+ 08	+ 26
8.87	+ 06 (726)	+ 12	+ 18	+ 41
9.15	+ 02 (650)	+ 31	+ 33	+ 62
9.40	00 (1462)	+ 58	+ 58	+ 91

Thus it appears that our aim to compare the Cordoba and the Bonn scale by their common zone is frustrated by a new discrepancy: for this 1° zone the Cordoba scale deviates from the mean scale of the 5° zone. Evidently THOMÉ has at first adapted his scale to SCHOENFELDS by adhering to the Bonn magnitudes in the zone —22°; but then gradually he has formed his own scale, based on the former meridian zones of GOULD. If this change has taken place regularly, as is probable also judging from the difference between the zones I and II, we may assume that the curve derived from stars in the middle of the zone will represent the average reductions for this zone without considerable error.

Corrections depending on Rectascension.

45. In computing for each zone and magnitude class the mean difference H—C for the whole zone, used in Tables 48—51, the deviations of each hour from the general means were derived at the same time. In plotting them it became at once visible that they bear a wholly different character for the magnitudes estimated in the D.M. work and for the bright stars, whose magnitudes had been taken from the Uranometria; also for the faint stars there seemed to be some differences. Thus separate results were at first derived for the classes 7.5, 8.0 and 8.5 C. Then the residuals of the hourly means 7.0—9.0 in Table I—VI Harvard Annals 72.7 and Table I—II Harvard Annals 80.7, found by subtracting the general means and corrected for density, were combined with them. In Table 55 in each of the eight divisions the first column contains the results for every hour. (The second column for the two last zones will be explained in the next paragraph). They were smoothed, — taking account also of the adjacent zones — by curves, from which the values in the last column of each division were read.

In the first four zones the variations are considerable, showing sudden jumps especially where the Milky Way crosses the zone. In the second half, as could be expected, the variations are much smaller, again increasing, however, in the last zones.

46. For the magnitudes of the U. A. the differences H—C were formed for the classes 6.5 C, 5,75 H and 6.25 H and corrected — 06, + 07, + 03, to make the mean differences coincide. They must be smoothed in two directions, for declination as well as for Rectascension, because these magnitudes have not been observed in zones of declination. In this process for the mean differences for each zone the values given in § 42 were found. The deviations of the smoothed hourly values from these means are contained in the right hand part of Table 56. In the first, left hand part the deviations of the observed differences from the general means — 0.23 are given.

Table 55. Corrections depending on AR (7.5^m—9^m).

Hour	22°—27°		27°—32°		32°—37°		37°—42°		42°—47°		47°—52°		52°—57°			57°—62°		
0	+ 03	00	+ 10	— 09	00	+ 02	— 04	00	+ 01	+ 03	00	— 01	— 01	+ 01	— 03	— 03	— 16	— 02
1	— 10	— 04	— 07	— 10	— 12	— 04	— 10	— 04	+ 12	+ 05	00	— 02	— 14	+ 07	— 07	— 08	+ 03	— 01
2	— 12	— 07	— 13	— 10	— 16	— 09	00	— 08	— 01	+ 04	— 06	— 03	— 11	— 28	— 09	+ 02	—	00
3	— 02	— 07	— 16	— 09	— 04	— 12	— 17	— 10	— 06	+ 03	— 08	— 03	— 10	+ 12	— 10	+ 06	— 11	— 01
4	+ 08	00	— 02	— 04	— 21	— 13	— 01	— 11	+ 04	+ 01	+ 05	— 02	— 07	— 52	— 10	+ 08	— 21	— 05
5	+ 06	+ 10	+ 02	+ 04	— 08	— 12	— 23	— 10	+ 09	00	— 04	— 01	— 06	— 12	— 09	— 19	— 14	— 08
6	+ 22	+ 14	+ 20	+ 11	— 31	— 10	+ 04	— 08	— 01	— 01	— 07	+ 01	— 14	— 09	— 06	— 06	+ 07	— 09
7	— 22	— 10	+ 04	+ 13	— 07	— 08	— 06	— 06	— 12	— 02	+ 06	+ 03	— 07	— 04	— 01	— 14	+ 14	— 07
8	+ 08	+ 12	+ 16	+ 14	— 02	— 03	— 10	— 02	— 03	— 03	+ 12	+ 05	+ 04	+ 07	+ 03	00	— 17	— 02
9	+ 44	+ 30	+ 18	+ 15	00	+ 05	+ 08	+ 02	00	— 02	+ 04	+ 06	+ 12	+ 15	+ 06	+ 06	+ 19	+ 03
10	+ 34	+ 25	+ 12	+ 14	+ 20	+ 10	— 02	+ 08	+ 04	— 01	+ 05	+ 06	+ 04	+ 31	+ 07	+ 02	— 04	+ 07
11	+ 10	+ 19	— 01	+ 13	+ 07	+ 14	+ 25	+ 14	+ 05	00	+ 02	+ 05	+ 06	— 18	+ 07	+ 16	— 02	+ 08
12	— 01	+ 09	+ 14	+ 12	+ 26	+ 18	+ 08	+ 19	— 05	+ 01	+ 02	+ 04	+ 06	+ 05	+ 06	+ 14	+ 12	+ 08
13	— 08	00	+ 15	+ 10	+ 16	+ 21	+ 24	+ 22	— 01	+ 02	— 07	+ 03	+ 08	— 10	+ 06	+ 03	— 11	+ 07
14	+ 14	— 07	+ 12	+ 07	+ 28	+ 20	+ 22	+ 23	+ 04	+ 03	+ 02	+ 03	+ 03	00	+ 07	+ 03	(— 29)	+ 07
15	— 06	— 12	— 14	00	+ 21	+ 17	+ 25	+ 22	+ 08	+ 02	— 02	+ 02	+ 18	+ 06	+ 07	+ 04	— 07	+ 06
16	— 21	— 15	+ 02	— 17	+ 08	+ 05	+ 19	+ 18	— 02	00	— 04	+ 01	+ 16	— 06	+ 06	+ 04	+ 01	+ 05
17	— 24	— 17	— 34	— 28	— 36	— 13	+ 16	+ 09	00	— 02	+ 02	+ 01	00	— 05	+ 03	+ 14	— 01	+ 03
18	— 20	— 16	— 14	— 19	— 23	— 20	— 21	— 14	— 13	— 05	+ 06	00	+ 06	— 10	+ 02	+ 02	+ 03	00
19	— 16	— 13	+ 02	— 02	— 20	— 14	— 10	— 19	— 08	— 08	— 02	— 03	00	+ 10	+ 01	— 20	+ 02	— 06
20	— 10	— 10	+ 06	+ 03	— 01	— 03	— 10	— 16	— 04	— 07	— 09	— 07	— 02	+ 02	+ 01	— 12	(+ 57)	— 07
21	— 22	— 06	+ 06	+ 02	+ 22	+ 06	— 02	— 11	00	— 04	— 04	— 05	00	+ 10	00	— 10	— 04	— 06
22	+ 13	— 01	— 22	— 04	00	+ 08	— 17	— 06	+ 01	— 01	— 03	— 01	— 01	+ 12	00	+ 01	+ 08	— 04
23	+ 04	+ 02	— 13	— 08	+ 32	+ 07	— 04	— 02	+ 03	+ 01	+ 05	00	— 02	+ 03	— 01	+ 03	— 14	— 03

The zones 52°—62° are included in this table because they have taken part in the process of smoothing. The results, however, have **not** been used, because in this section the estimated magnitudes of the D.M. zones are given also for these bright stars. Computing here the hourly deviations for the classes 5.75 H, 6.25 H, 6.5 C (H 50) and 6.5—7.0 C (H 34) we get the values given in the second column of the two last divisions of Table 55. (Figures in parentheses, depending on one star, have smaller weight). The general course of these values is quite different from the last columns of Table 56 and corresponds on the whole with the first columns in Table 55. Thus they were combined with half weight with the latter to derive the smoothed curve.

The classes 7.0 C and 6.75 H consist partly of U.A. stars, partly of stars below 7.0; thus we may expect variations with AR in which the curves found for the bright and for the faint stars both take part. Deriving the residuals (H 50—C) for the zones 22°—32° they were found to show actually a course that could be represented by the mean of the two curves.

47. For the faintest classes the hourly residuals in the last column of Tables VII—X (zones —24°, —29°, —39°, —49°) in Harvard Annals **72**, 7, were taken and corrected for density error. They were combined with analogous residuals taken from the classes 8.6—9.5 only. After plotting them it was seen that, though their accuracy was less, they showed for 29° and 39° the same characteristics as the curves derived from 7^m—9^m in the 5° zones in which they lie. For zone 49° they do not themselves indicate the small fluctuation of the curve of the brighter stars, but they do not deviate considerably from it. For zone 24° the residuals :

Table 56. Corrections depending on AR (6.5^m).

Hour	Observed								Adopted							
0	+ 03	+ 18	+ 12	+ 19	— 06	+ 12	+ 04	+ 04	+ 08	+ 07	+ 07	+ 06	+ 07	+ 09	+ 11	+ 13
1	+ 04	+ 02	+ 02	— 01	+ 17	+ 14	+ 01	+ 16	+ 09	+ 07	+ 06	+ 04	+ 03	+ 03	+ 04	+ 05
2	+ 04	+ 13	+ 01	+ 05	+ 11	— 08	— 09	—	+ 09	+ 05	+ 03	00	— 02	— 03	— 04	— 05
3	+ 10	+ 22	+ 08	— 11	+ 10	— 06	— 05	— 20	+ 06	+ 01	— 02	— 05	— 07	— 08	— 09	— 09
4	+ 04	— 07	— 12	— 17	— 02	— 07	— 06	— 28	+ 01	— 03	— 06	— 09	— 10	— 11	— 12	— 13
5	— 14	+ 09	— 08	— 09	— 32	— 05	— 27	— 25	— 02	— 05	— 07	— 10	— 11	— 12	— 13	— 14
6	00	+ 11	— 17	— 06	— 06	+ 01	— 24	+ 02	— 03	— 06	— 08	— 10	— 12	— 13	— 14	— 15
7	— 22	— 09	— 05	— 13	— 10	— 13	— 15	— 02	— 02	— 05	— 06	— 08	— 10	— 11	— 11	— 12
8	— 05	00	— 05	+ 03	— 03	— 01	— 11	— 23	— 01	— 03	— 03	— 04	— 05	— 05	— 06	— 07
9	00	+ 08	+ 01	— 20	— 06	+ 15	— 10	— 01	00	— 01	— 01	— 01	— 01	— 01	— 01	— 01
10	— 09	+ 10	+ 18	— 06	+ 11	+ 12	+ 20	— 01	+ 01	+ 01	+ 01	+ 01	+ 02	+ 03	+ 03	+ 03
11	— 02	+ 09	+ 12	— 05	+ 05	+ 10	— 11	— 03	+ 03	+ 02	+ 02	+ 02	+ 01	+ 02	+ 02	+ 02
12	— 01	+ 01	+ 14	— 16	— 18	+ 08	00	+ 04	+ 04	+ 02	+ 02	+ 01	+ 01	+ 01	00	00
13	— 08	— 02	+ 06	+ 01	— 15	+ 02	— 13	— 11	00	00	+ 01	+ 01	00	00	— 01	— 02
14	— 17	+ 04	+ 15	— 04	— 08	— 01	— 14	— 04	— 06	— 03	00	00	00	— 01	— 02	— 03
15	— 23	— 12	— 13	+ 07	— 12	+ 03	+ 06	+ 11	— 07	— 04	— 01	— 01	00	00	— 01	— 03
16	— 13	+ 04	— 07	— 04	— 05	+ 26	— 05	+ 13	— 08	— 04	— 01	00	+ 01	+ 01	00	— 02
17	— 15	+ 05	00	+ 15	+ 13	— 10	— 11	— 03	— 07	— 03	— 01	00	+ 01	+ 02	+ 01	00
18	— 11	+ 01	+ 08	+ 11	+ 05	00	— 10	+ 07	— 06	— 02	00	+ 01	+ 02	+ 03	+ 03	+ 02
19	— 01	+ 19	+ 05	+ 03	— 06	+ 09	+ 09	+ 16	— 05	— 01	+ 01	+ 02	+ 04	+ 06	+ 07	+ 06
20	— 06	+ 11	+ 14	+ 03	+ 29	+ 13	— 23	(+ 61)	— 03	00	+ 02	+ 04	+ 07	+ 09	+ 11	+ 10
21	— 09	+ 12	+ 04	— 05	+ 21	+ 23	+ 07	+ 04	+ 01	+ 02	+ 03	+ 05	+ 08	+ 11	+ 13	+ 14
22	— 07	— 05	+ 03	— 08	+ 13	+ 14	(+ 22)	+ 11	+ 05	+ 05	+ 05	+ 06	+ 09	+ 12	+ 14	+ 17
23	+ 11	+ 10	+ 07	— 04	+ 09	+ 29	+ 14	+ 22	+ 07	+ 06	+ 06	+ 06	+ 08	+ 11	+ 13	+ 16

+44 +18 +19 +12 + 24+13 +43 —06 +13 +65 +17 —03 +08 —08 +06 —27 —35 —26 —38 —23 —08 —34 —29 —28

show a decided deviation from the curve, though the minor fluctuations (e.g. the steep valley in 7^h) are represented in them. After subtracting the curve of Table 55 we get

+44 +22 +26 +19 +24 +03 +29 +04 +01 +35 —08 —22 —01 —08 +13 —15 —20 —09 —22 —10 +02 —28 —29 —30,

which numbers can be represented by a simple harmonic oscillation with maxima + 0.18 and — 0.18 for 6^h and 18^h.

Collecting in the same way the residuals for class 9.0 C (H 34) the same result was arrived at; on the whole there was a sufficient accordance with the curves, but in the first zone an additional oscillation with amplitude 0.10 should be added. Thus it seems, that in the first zone the variation with AR changes with magnitude, in this way that in the first 12 hours the scale for the fainter classes is wider, for the last 12 hours the scale is narrower than for the average.

Thus the following corrections depending on AR have been adopted: for zones I—VI for the magnitudes down to 6.65 the values of Table 56, from magnitude 7.35 downward the values of Table 55 (standing for the middle of each hour, and interpolated for other AR), for 6.85 and 7.15 intermediate values computed with weights 1 : 2 and 2 : 1. For zones VII and VIII the curves of Table 55 have been adopted for all magnitudes, interpolated for the middle AR of each field. For zone I a second correction is added for the fainter classes, varying with the sine of AR, having amplitudes 0.03 0.07, 0.12 and 0.15 for magnitudes 8.65, 8.85, 915 and 9.35.

THE DISTRIBUTION OF SURFACE DENSITY.

Data for the separate areas.

48. The Bonn Durchmusterung has been counted by SEELIGER for the magnitude groups *B*—6.5, 6.6—7.0,.... 9.1—9.5, for areas extending one degree in declination and 20^m in Rectascension (Neue Annalen München II). STRATONOFF has used these data to compute for the same magnitude groups for areas extending 5° in declination and 20^m in AR. (for 50°—60° 40^m, for 60°—80° 1^h, 80°—85° 3^h, 85°—90° 6^h) the number of stars pro square degree. (Annales Tachkent I). For the brightest group, for which SEELIGER had taken fields of 40^m, STRATONOFF gives the results for 20^m. For the northern hemisphere we have taken the results of STRATONOFF as the basis of our work. For the southern hemisphere we have taken the limits of the areas at 2°, 7°, 12°....62° of declination, thus beginning with a narrow zone of 2^0 width (52°—62° 40^m in AR). The numbers pro square degree have been computed by us from SEELIGERS counts for the zones 0°—22°; for the remaining zones we have used our own counts of the Cordoba Durchmusterung, made for the magnitude groups *B*—6.3, 6.4—6.6, 6.7—6.8, 6.9—7.1.....

As the photometric aequivalent for each limit is known, we are able to deduce a formula $a + bm - cm^2$ for each area separately. We have, however, followed another way. In order to judge the distribution of a certain class of stars over the sky it is necessary to have the surface density in each area for this class with constant limits in a photometric scale. The photometric values of the limits used in the counts, however, are different for each area. Moreover it is desirable to smooth the accidental differences of adjacent areas; this can be done only if their numbers relate to exactly the same magnitude classes. Therefore we have first deduced from the data of STRATONOFF and its aequivalents for the southern sky other data for some definite normal magnitudes identical for the whole sky.

These photometric magnitudes were chosen in such a way, that the brightest limits should not come above 6.5 D.M. and the faintest should include the last group assumed to be complete, going down as far as possible, but not so far that extrapolation from this last class should be necessary for too many areas. Between them three other limits were inserted in order that the number of results should have some concordance with the number of independent data that may be assumed to underlie the observations. Thus as normal limits 6.7, 7.5, 8.2, 8.8, 9.3 on the photometric scale were chosen, taking decreasing intervals because the number of stars increases for them. Adding the limit 5.5 for which the number may be taken from the Harvard catalogues, we will deduce the number of stars per square degree for the normal magnitude intervals:

B to 6.7; 5.5 to 7.5; 6.7 to 8.2; 7.5 to 8.8; 8.2 to 9.3.

In order to strengthen by independent data the first result, which rests always on very few stars, we have still added the number of stars B to 6.50, counted in Harvard 50.

49. Since a normal limit, as a rule, falls between two limits of the counted groups, we must divide the stars of a counted group over two normal groups. Owing to the increase of the number of stars with magnitude this division cannot be proportional to the differences of magnitude.

We may take the number of stars in a group proportional to 10^{bm}, if m is the mean magnitude of the group, and b for the magnitudes considered here varies from 0.50 to 0.38. If m_1 and m_2 are the limits of the counted group and m is the normal limit, the fraction to be assigned to the lower part will be

$$\frac{(m-m_1)\,10^{\frac{1}{2}b\,(m+m_1)}}{(m_2-m_1)\,10^{\frac{1}{2}b(m_2+m_1)}} = \frac{m-m_1}{m_2-m_1}\,10^{-\frac{1}{2}b\,(m_2-m)} = \frac{q_1}{Q}\,10^{-\frac{1}{2}bq_2}\,,$$

putting $m_2 - m_1 = Q$, $m - m_1 = q_1$, $m_2 - m = q_2$. A table with double entry was constructed with the average value $b = 0.44$, giving this fraction for different values of Q and q_1. As a change of b to 0.38 or 0.50 did not change the fraction more than 0.01 this table could be used for all magnitudes. As an exemple we give here the computation for one case (15°—20°, AR $4^h\ 30^m$):

Limits B.D.		6.55		7.05		7.55		8.05		8.55		9.05	
Numbers per sq.d.	0.71		0.20		0.25		0.50		0.70		1.80		
Photometric m		6.38		6.92		7.46		7.99		8.57		9.21	
q_1/Q			32/54				4/53		21/58		23/64		73/64
First fraction			0.53				0.06		0.30		0.29		1.19
Division of numbers			0.11+0.09				0.03+0.47		0.21+0.49		0.52+1.28		2.14
Results		0.82			1.19			1.05		1.69		2.63	

where the second value, standing for B—7.5, must be diminished by 0.42 for the stars B—5.5.

50. The logarithm of the number of stars per square degree between definite normal limits of magnitude, obtained in this way, must be compared with the logarithm of the number between the same limits in the schematical universe, that is found from Table 4. Now the counted numbers are, according to § 10, too great by the ratio $e^{\frac{1}{2}b^2\mu^2}$. If henceforward we denote by b the coefficient of m in the common logarithm of the stardensity (as is done in § 1—4), this means that the logarithm of the counted numbers must be diminished by $\frac{1}{2} \times 2.30\, b^2\mu^2$. These corrections vary with m and with β; the values adopted are (in 0.001)

	B—6.7	5.5—7.5	6.7—8.2	7.5—8.8	8.2—9.3
Bonn 0°	042	038	027	021	018
„ 90°	032	021	019	015	012
Cordoba 0°	020	023	030	037	048
„ 90°	014	016	022	026	029

and 0.008 for B—6.5 Harvard. Instead of correcting the results of each area the corrections were applied with opposite sign to the tables for the schematical universe. The differences log. number from the counts minus log. number for the schematical universe are the differences $\log A - \log A_0$,

giving for some medium magnitude of each magnitude interval the deviations of log A in each area from the normal value in the schematical universe. These differences are the basis of the further researches. They are called $\triangle_1$ (for B—Harvard 6.5), $\triangle_2$ (B—6.7), $\triangle_3$, $\triangle_4$, $\triangle_5$, $\triangle_6$; table 57 contains $\triangle_3$, $\triangle_4$, $\triangle_5$, $\triangle_6$, and for the brighter classes (where the deviations are relatively great) the differences $A—A_0$, called $\triangle'_1$ and $\triangle'_2$.

51. The medium magnitude for which $\log A - \log A_0$ may be assumed to stand, is found by the condition that in varying the coefficients in $A(m)$ the value $A(m_0)$ remains constant together with the total number for the interval $N(m_2) - N(m_1)$. Practically only the variations in the coefficient b have to be considered. Representing, as is allowed for moderate intervals, $\log A$ by $\log A_0 + b\,(m - m_0)$ this condition is expressed by

$$\frac{d}{db}\int_{m_1}^{m_2} A_0 10^{b\,(m\,-\,m_0)}\,dm = 0.$$

From this formula we find that m_0 is simply the mean magnitude of the group:

$$m_0 = \int_{m_1}^{m_2} A_0\, m\, 10^{b\,(m\,-\,m_0)}\,dm \;\; : \;\; \int_{m_1}^{m_2} A_0 10^{b\,(m\,-\,m_0)}\,dm = m_1 + \frac{A_2}{A_2 - A_1}(m_2 - m_1) - \frac{1}{2.3b}\;.$$

Putting $(A_1 - A_2)/(A_1 + A_2) = \alpha$ we may represent it also by an infinite series

$$m_0 = {}^1/_2\,(m_1 + m_2) + {}^1/_{2.3b}\;({}^1/_3\,\alpha^2 + {}^1/_5\,\alpha^4 + \ldots.),$$

which, however, is less fit for practical computation.

For $\triangle_1$ and $\triangle_2$ representing the differences for $N(6.5)$ and $N(6.7)$ we have in an analogous manner, that the condition

$$\frac{d}{db}\int_{-\infty}^{m_1} 10^{\,a\,+\,b(m\,-\,m_0)\,-\,c\,(m\,-\,m_0)^2}\,dm = 0$$

is fulfilled, if m_0 is the mean magnitude of all these stars. This mean magnitude is found to be

$$m_0 = \frac{b}{2c} - \frac{10^{\,a\,+\,bm_1\,-\,cm_1{}^2}}{2.3 \times 2c\displaystyle\int_{-\infty}^{m_1} 10^{\,a\,+\,bm\,-\,cm^2}\,dm} = m_1 - \frac{1}{2.3(b - cm_1)} = m_1 - \frac{1}{2.3\,b_1}$$

where b_1 denotes the variation of $\log A(m)$ with m for $m = m_1$.

Strictly speaking the medium magnitudes are different for different galactic latitudes, because b varies with β. As these differences have no practical importance we will adopt the same mean values for the whole sky, viz.:

5.6 5.8 6.9 7.7 8.3 8.9.

52. The results for the deviations of the real surface densities in each area from the schematical universe are given in Table 57. The areas in each zone of declination are arranged after Rectascension, the middle AR of each area being designated in the first column by the hour and the number of ten minutes. The 2d and 3d column give the density and the galactic latitude used in the computation of the photometric magnitude and the normal number of stars. Then follow the six differences $A - A_0$ or $\log A - \log A_0$. The last column will be explained afterwards.

Table 57. Deviations of surface density.

AR	D	β	Δ'₁	Δ'₂	Δ₃	Δ₄	Δ₅	Δ₆	Region
+ 85° + 90°									
3.0	13	25	+ 09	+ 03	— 24	— 34	— 23	— 10	+3.9
9.0	15	29	— 11	— 16	— 21	— 10	— 01	— 01	+3.9
15.0	14	29	— 11	— 08	— 05	— 15	— 06	00	+3.9
21.0	17	26	— 01	+ 03	— 16	+ 02	+ 10	— 01	+3.9
+ 80° + 85°									
1.3	15	20	— 06	— 08	— 10	— 21	— 19	— 11	+1.9
4.3	15	23	— 02	— 12	— 37	— 11	— 15	— 12	+3.9
7.3	14	29	+ 03	00	— 03	— 15	— 15	— 18	+3.9
10.3	15	33	00	+ 01	— 12	— 03	+ 06	+ 05	+3.9
13.3	15	34	+ 07	+ 03	+ 12	— 05	— 04	— 01	+3.9
16.3	16	31	— 10	— 13	— 07	— 02	+ 01	— 02	+3.9
19.3	16	26	+ 02	+ 06	+ 08	00	— 02	— 10	+3.9
22.3	18	21	+ 01	— 07	— 10	+ 01	— 03	— 16	+3.9
+ 75° + 80°									
0.3	12	15	— 06	+ 08	— 04	— 19	— 18	— 22	+1.9
1.3	11	15	+ 32	+ 26	+ 11	— 09	— 16	— 20	+1.9
2.3	12	16	— 18	— 21	— 03	— 01	— 05	— 07	+1.9
3.3	12	18	— 11	— 18	— 14	— 12	+ 05	+ 13	+1.9
4.3	11	20	— 17	— 10	— 11	— 06	— 07	— 15	+1.11
5 3	11	23	— 10	— 21	— 18	— 02	00	— 02	+3.11
6.3	11	26	— 03	— 01	+ 08	+ 10	— 01	— 10	+3.11
7.3	12	29	— 15	—12	— 07	+ 07	+ 04	+ 02	+3.11
8.3	9	32	— 08	+ 01	+ 14	— 18	— 29	— 17	+3.11
9.3	11	35	— 01	— 13	00	— 07	— 04	+ 04	+3.11
10.3	10	37	— 07	— 06	— 04	+ 03	— 05	— 12	+3.9
11.3	10	39	— 19	— 14	— 23	— 12	— 12	— 12	+3.9
12.3	10	40	+ 12	— 06	— 02	— 19	— 12	+ 04	+3.9
13.3	8	39	— 07	— 01	+ 06	+ 08	— 01	— 14	+3.9
14.3	9	38	00	— 08	— 19	— 12	— 22	— 25	+3.9
15.3	10	36	— 07	— 08	— 69	— 09	+ 06	— 02	+3.9
16.3	11	33	+ 23	+ 16	+ 14	— 05	— 01	+ 05	+3.7
17.3	13	30	+ 04	+ 02	+ 18	+ 08	00	— 01	+3.7
18.3	12	27	+ 10	+ 14	+ 04	— 08	— 04	— 05	+3.7
19.3	12	24	— 03	— 06	— 17	— 09	— 08	— 15	+3.7
20.3	14	21	— 04	— 05	— 06	— 04	— 07	— 07	+3.7
21.3	12	19	— 11	+ 09	+ 11	— 13	— 20	— 21	+1.9
22.3	14	17	— 05	— 11	+ 10	+ 07	— 04	— 06	+1.9
23.3	14	16	— 12	— 24	— 26	— 04	00	— 04	+1.9
+ 70° + 75°									
0.3	14	10	— 03	— 08	— 07	— 22	— 20	— 12	+1.9
1.3	14	10	+ 01	+ 09	+ 01	— 10	— 11	— 10	+1.9
2.3	12	12	— 16	— 09	— 04	+ 04	+ 03	— 10	+1.9
3.3	11	14	— 02	+ 10	— 03	— 10	— 14	— 13	+1.11
4.3	12	17	+ 03	— 06	— 36	— 10	+ 06	+ 05	+1.11
5.3	10	21	— 14	— 14	— 02	— 09	— 18	— 14	+3.11
6.3	10	25	— 04	+ 02	— 13	— 18	— 13	— 10	+3.11
7.3	10	30	— 12	— 19	— 15	— 05	— 05	+ 04	+3.11
8.3	10	34	— 11	— 04	+ 06	— 06	— 04	— 06	+3.11
9.3	9	38	+ 16	+ 06	+ 02	+ 05	— 07	— 14	+3.11
10.3	8	41	— 05	— 07	— 16	— 20	— 13	— 04	+5.10
11.3	8	44	— 18	— 06	— 07	— 04	— 10	— 06	+5.10
12.3	9	45	00	+ 03	— 12	— 12	— 02	+ 02	+5.10
13.3	8	44	— 01	— 11	— 08	— 01	— 12	- 17	+5.7
14.3	8	42	— 14	— 16	— 34	— 19	— 15	— 10	—5.7
15.3	10	40	— 10	— 13	— 3	— 01	+ 07	+ 15	+3.7
16.3	11	36	— 06	— 05	— 06	+ 05	+ 08	+ 04	+3.7
17.3	12	31	— 07	— 11	+ 02	+ 05	+ 04	+ 01	+3.7
18.3	12	27	— 04	— 02	— 29	— 13	— 04	00	+3.7
19.3	12	22	— 05	— 23	— 34	— 10	— 05	00	+3.7
20.3	10	18	— 15	— 23	— 27	— 08	— 04	— 18	+1.7
21.3	11	15	— 02	— 05	— 16	— 14	— 20	— 25	+1.7
22.3	15	12	+ 02	— 11	— 16	+ 04	+ 02	— 02	+1.9
23.3	14	11	+ 06	+ 04	— 02	— 02	— 10	— 11	+1.9
+ 65° + 70°									
0,3	16	5	— 05	— 16	— 09	— 14	— 08	— 03	+1.9
1.3	15	5	— 05	— 11	— 27	— 27	— 20	— 15	+1.9
2.3	12	7	— 08	— 17	— 31	— 19	— 17	— 16	+1.11
3.3	11	10	— 01	— 04	— 11	— 15	— 22	— 25	+1.11
4.3	11	14	— 14	— 15	— 09	— 10	— 07	— 07	+1.11
5.3	9	18	— 13	— 14	— 15	— 10	— 12	— 19	+1.11
6.3	8	24	— 08	— 15	— 39	— 12	— 11	— 20	+3.11
7.3	9	30	— 10	— 06	+ 02	00	— 03	— 05	+3.11
8.3	10	36	+ 08	+ 12	— 08	— 11	+ 02	+ 02	+3.11
9.3	9	41	— 01	— 13	— 13	+ 02	+ 04	— 01	+5 10
10 3	8	45	+ 14	+ 05	— 13	— 06	— 03	— 05	+5 10
11.3	8	48	+ 03	+ 03	+ 11	+ 04	— 04	— 01	+5.10
12.3	7	50	— 03	— 03	— 14	— 15	— 14	— 07	+5.10
13.3	8	49	00	— 02	00	+ 03	— 03	+ 02	+5.7
14.3	9	46	— 11	— 12	— 08	+ 09	00	— 06	+5.7
15.3	9	42	— 04	+ 05	+ 07	— 06	— 05	+ 07	+5.7
16.3	10	37	+ 05	+ 03	— 17	— 12	— 01	+ 06	+3.7
17.3	12	32	— 03	— 05	— 20	— 07	+ 05	+ 07	+3.7
18 3	12	26	00	— 03	— 03	— 16	— 11	— 01	+3.7
19 3	16	21	— 02	— 06	— 20	— 05	+ 03	+ 03	+3.7
20 3	14	16	— 10	— 09	— 10	— 24	— 22	— 16	+1.7
21 3	20	12	— 11	— 16	+ 04	+ 02	+ 01	+ 07	+1.7
22.3	18	8	— 05	— 13	— 15	— 03	— 01	— 03	+1.7
23.2	18	6	— 05	— 13	— 22	00	+ 03	— 07	+1.9
+ 60° + 65°									
0.3	25	0	+ 02	— 07	— 21	— 14	— 06	— 03	—1.9
1.3	27	1	+ 14	+ 02	— 14	— 07	+ 01	+ 05	—1.9
2.3	20	3	— 18	— 09	+ 10	+ 03	— 04	— 09	+1.11
3.3	17	6	— 06	— 07	— 18	— 20	— 14	— 07	+1.11
4.3	11	11	— 02	— 04	— 14	— 22	— 26	— 27	+1.11
5.3	11	16	+ 02	— 09	— 16	— 18	— 19	— 19	+1.11
6.3	11	23	— 14	— 16	— 13	— 13	— 11	— 04	+3.11
7.3	10	29	— 09	— 10	— 21	— 17	— 13	— 09	+3.13
8.3	9	36	+ 04	— 06	— 22	— 09	— 03	— 08	+3.11
9.3	9	43	— 01	— 03	— 11	— 05	— 07	— 05	+5.10
10.3	9	49	— 06	— 03	— 12	— 21	— 14	00	+5.10
11.3	8	53	— 02	00	+ 06	+ 03	— 04	— 13	+5.10
12.3	8	55	+ 06	+ 03	+ 05	— 08	— 07	— 13	+5.10
13.3	9	54	— 02	— 04	— 07	— 02	— 01	— 03	+5.7
14.3	9	50	— 03	— 04	— 02	— 08	— 02	— 04	+5.7
15.3	10	45	+ 02	+ 04	+ 02	— 03	— 04	— 08	+5.7

AR	D	β	Δ′1	Δ′2	Δ3	Δ4	Δ5	Δ6	Region
16.3	11	39	— 02	+ 04	+ 01	+ 05	00	— 10	+3.7
17.3	11	33	— 03	— 02	+ 02	— 03	— 06	— 12	+3.5
18.3	12	26	— 04	— 08	— 17	— 18	— 10	— 03	+3.7
19.3	19	19	— 12	— 06	+ 04	+ 03	+ 12	+ 15	+1.7
20.3	15	13	+ 10	— 01	— 22	— 17	— 06	— 04	+1.7
21.3	20	8	+ 18	+ 18	+ 11	+ 12	+ 07	— 01	+1.7
22.3	20	4	+ 14	+ 07	00	— 05	— 10	— 12	+1.7
23.3	27	1	+ 05	+ 21	+ 24	+ 09	+ 02	+ 02	+1.9

+ 55° + 60°

AR	D	β	Δ′1	Δ′2	Δ3	Δ4	Δ5	Δ6	Region
0.2	25	5	— 04	— 04	+ 06	+ 15	+ 12	+ 11	—1.9
1.0	28	6	+ 12	+ 05	— 03	+ 05	+ 10	+ 13	—1.9
1.4	32	4	00	00	+ 07	+ 10	+ 19	+ 24	—1.9
2.2	33	2	00	+ 24	+ 27	+ 24	+ 27	+ 26	—1.11
3.0	17	0	— 12	— 04	— 15	— 04	— 04	— 08	—1.11
3.4	14	3	+ 27	+ 03	— 10	— 19	— 18	— 19	+1.11
4.2	14	6	+ 04	— 08	— 19	— 26	— 16	— 08	+1.11
5.0	10	10	— 03	— 06	— 22	— 24	— 36	— 38	+1.11
5.4	11	15	+ 06	+ 05	— 08	— 14	— 18	— 15	+1.13
6.2	13	20	+ 15	+ 18	— 02	+ 08	+ 06	00	+1.13
7.0	12	25	— 03	— 08	— 36	— 10	+ 02	+ 03	+3.13
7.4	10	30	+ 02	+ 05	+ 06	— 05	— 10	— 12	+3.13
8.2	9	36	— 04	— 13	— 18	— 06	— 10	— 19	+3.13
9.0	8	41	— 03	00	— 02	— 04	— 19	— 18	+5.13
9.4	8	46	+ 01	— 03	— 12	— 14	— 21	— 15	+5.13
10.2	8	51	— 06	— 03	— 23	— 24	— 18	— 12	+5.10
11.0	8	55	+ 10	— 01	— 04	— 03	— 11	— 08	+5.10
11.4	7	58	+ 10	+ 01	— 14	— 16	— 07	— 09	+5.10
12.2	8	59	+ 06	+ 05	— 14	— 18	— 05	— 01	+5.10
13.0	8	60	— 05	— 05	— 32	— 18	— 03	+ 04	+5.7
13.4	9	58	+ 06	— 01	— 26	— 24	— 12	— 09	+5.7
14.2	9	55	— 09	+ 03	+ 01	+ 06	+ 34	+ 01	+5.7
15.0	8	52	— 13	— 06	— 04	— 07	+ 15	— 13	+5.7
15.4	9	47	+ 13	+ 15	+ 02	— 13	— 12	— 08	+5.4
16.2	11	41	00	— 11	— 31	— 10	+ 03	+ 06	+5.4
17.0	11	36	— 08	— 03	— 09	— 08	— 13	— 09	+3.5
17.4	13	31	— 02	00	— 20	— 15	+ 07	+ 01	+3.5
18.2	12	25	— 06	— 11	— 17	— 13	— 27	— 16	+3.5
19.0	17	20	+ 11	— 06	— 11	— 03	— 04	— 01	+3.5
19.4	24	15	+ 10	+ 26	+ 04	— 18	— 04	+ 10	+1.5
20.2	23	11	+ 02	— 01	— 05	— 02	+ 04	+ 01	+1.7
21.0	18	7	+ 12	+ 12	+ 08	+ 04	— 07	— 10	+1.7
21.4	21	3	+ 04	— 11	— 10	— 02	— 04	+ 02	+1.7
22.2	23	0	+ 23	+ 05	— 12	— 05	+ 03	+ 03	—1.7
23.0	27	3	+ 11	+ 16	+ 05	00	+ 06	+ 09	—1.7
23.4	23	4	+ 19	+ 01	— 03	— 02	+ 02	+ 04	—1.9

+ 50° + 55°

AR	D	β	Δ′1	Δ′2	Δ3	Δ4	Δ5	Δ6	Region
0.2	25	9	+ 04	— 02	— 10	+ 01	+ 05	+ 06	—1.9
1.0	26	10	+ 01	— 11	— 03	— 05	— 09	— 01	—1.9
1.4	28	9	— 06	— 21	— 14	— 13	— 11	+ 04	—1.9
2.2	22	7	— 06	+ 02	— 04	— 24	— 09	— 05	—1.11
3.0	16	4	— 03	— 07	— 17	— 21	— 25	— 24	—1.11
3.4	16	1	— 10	— 01	— 17	— 29	— 29	— 24	—1.11
4.2	17	3	— 04	— 13	— 27	— 16	— 16	— 15	+1.11
5.0	15	8	— 03	— 15	— 37	— 21	— 15	— 16	+1.13
5.4	14	13	— 02	— 02	— 18	— 27	— 07	— 05	+1.13
6.2	14	18	— 10	— 17	— 15	— 02	00	— 02	+1.13
7.0	13	24	— 09	— 10	— 11	— 04	+ 03	+ 05	+3.13
7.4	11	30	— 14	— 08	— 05	— 06	— 04	— 02	+3.13
8.2	10	36	— 03	— 04	— 11	— 08	— 01	— 06	+3.13
9.0	9	42	— 02	+ 01	— 01	— 03	+ 01	— 02	+5.13
9.4	9	48	+ 06	+ 05	+ 14	+ 06	— 03	— 04	+5.13
10.2	8	53	— 04	+ 02	+ 05	— 11	— 14	— 16	+5.13
11.0	8	58	— 07	+ 06	+ 11	+ 01	— 07	— 07	+5.10
11.4	9	62	— 06	— 14	— 32	— 12	— 02	— 04	+7.9
12.2	8	64	00	— 03	— 19	— 21	— 15	— 13	+7.9
13.0	9	65	— 09	— 06	— 16	— 16	— 03	+ 01	+7.9
13.4	9	62	+ 10	+ 11	+ 17	+ 03	00	00	+7.9
14.2	9	58	+ 07	+ 11	+ 19	+ 14	+ 10	+ 06	+5.7
15.0	8	54	00	— 05	+ 11	+ 01	— 01	+ 04	+5.4
15.4	10	48	+ 12	+ 15	+ 08	— 26	+ 01	+ 03	+5.4
16.2	10	42	— 09	— 04	— 04	— 10	— 13	— 12	+5.4
17.0	10	36	— 06	— 07	— 10	— 12	— 11	— 10	+3.5
17.4	14	30	— 04	— 04	— 10	— 09	— 14	— 10	+3.5
18.2	19	24	+ 01	— 06	— 03	— 03	+ 03	+ 04	+3.5
19.0	21	18	+ 16	+ 07	— 13	— 19	— 09	+ 03	+1.5
19.4	27	13	+ 15	+ 05	— 16	— 13	+ 04	+ 11	+1.5
20.2	27	8	— 03	+ 01	+ 06	+ 03	+ 05	+ 10	+1.5
21.0	22	3	+ 03	+ 09	+ 10	+ 02	— 06	— 08	+1.5
21.4	29	1	+ 03	+ 05	+ 05	+ 04	+ 07	+ 08	—1.7
22.2	34	4	— 07	— 06	— 04	+ 09	+ 21	+ 24	—1.7
23.0	27	7	— 03	— 16	— 19	— 08	00	+ 08	—1.7
23.4	26	9	— 22	— 13	— 07	— 05	+ 07	+ 16	—1.9

+ 45° + 50°

AR	D	β	Δ′1	Δ′2	Δ3	Δ4	Δ5	Δ6	Region
0.1	31	15	— 01	— 03	+ 01	+ 08	+ 11	+ 13	—1.9
0.3	29	16	+ 06	— 02	— 04	+ 02	+ 01	+ 08	—1.9
0.5	30	16	— 12	+ 10	+ 22	+ 25	+ 15	+ 11	—1.9
1.1	27	15	— 06	— 12	00	+ 16	+ 07	+ 08	—1.9
1.3	26	15	— 01	— 05	— 05	— 11	— 03	+ 04	—1.9
1.5	24	14	— 07	— 19	— 14	— 16	— 04	+ 03	—1.11
2.1	21	13	+ 28	+ 17	— 04	— 09	— 02	00	—1.11
2.3	22	11	— 19	— 06	— 08	+ 02	00	— 03	—1.11
2.5	21	9	— 08	00	— 01	— 14	— 12	— 13	—1.11
3.1	21	8	+ 33	+ 35	+ 11	+ 12	+ 09	+ 01	—1.11
3.3	18	6	+ 62	+ 26	— 04	— 07	— 11	— 14	—1.11
3.5	17	4	— 15	— 10	— 26	— 35	— 18	— 13	—1.11
4.1	17	1	— 03	— 08	— 28	— 15	— 17	— 26	—1.13
4.3	15	1	— 15	— 09	— 05	— 31	— 36	— 31	—1.13
4.5	18	3	— 21	— 05	— 11	— 23	— 24	— 21	+1.13
5.1	16	6	— 08	— 11	— 30	— 18	— 16	— 13	+1.13
5.3	12	9	— 14	— 09	+ 03	— 06	— 29	— 31	+1.13
5.5	16	12	+ 10	+ 11	+ 04	— 06	— 03	— 06	+1.13
6.1	14	15	— 01	+ 06	— 03	— 07	— 17	— 27	+1.13
6.3	14	19	— 11	— 22	— 28	— 15	— 19	— 18	+1.13
6.5	13	22	+ 13	— 04	— 10	— 12	— 14	— 13	+3.13
7.1	13	25	+ 02	+ 23	+ 08	+ 16	+ 04	— 10	+3.13
7.3	13	28	+ 08	— 08	— 01	+ 02	+ 02	— 03	+3.13
7.5	12	31	— 09	— 11	— 06	+ 07	— 04	— 23	+3.13
8.1	12	35	— 20	— 10	— 02	+ 07	+ 13	+ 08	+3.13
8.3	11	38	+ 04	+ 03	+ 06	00	— 04	— 02	+3.13
8.5	11	41	+ 11	+ 05	— 26	— 08	— 06	— 09	+5.13
9.1	11	45	— 12	— 10	— 15	— 01	+ 01	— 01	+5.13
9.3	11	48	00	— 04	+ 05	00	— 04	+ 02	+5.13
9.5	11	51	— 11	— 20	— 33	— 05	00	+ 05	+5.13
10.1	11	55	— 05	+ 03	+ 04	— 08	+ 01	+ 04	+5.13

AR	D	β	Δ'_1	Δ'_2	Δ_3	Δ_4	Δ_5	Δ_6	Region	AR	D	β	Δ'_1	Δ'_2	Δ_3	Δ_4	Δ_5	Δ_6	Region
10.3	9	58	− 05	− 11	− 14	+ 10	+ 03	− 08	+5.13	5.1	29	3	+ 06	− 06	− 14	+ 06	+ 08	+ 11	+1.13
10.5	9	60	− 10	− 13	− 08	+ 01	− 11	− 07	+7.15	5.3	24	6	− 05	− 24	+ 08	− 04	− 07	+ 03	+1.13
11.1	9	63	− 10	− 12	− 09	− 05	− 06	− 07	+7.15	5.5	24	9	+ 01	− 11	− 07	− 07	+ 02	+ 02	+1.13
11.3	9	65	+ 14	+ 19	+ 15	− 02	− 09	− 15	+ 7.9	6.1	22	13	− 14	− 19	+ 02	+ 12	+ 02	− 06	+1.13
11.5	8	67	− 04	− 18	− 17	+ 12	+ 04	+ 01	+ 7.9	6.3	20	16	− 08	+ 01	− 04	− 06	− 05	+ 02	+1.13
12.1	8	69	− 10	− 08	− 11	00	− 08	− 04	+ 7.9	6.5	16	20	− 01	− 09	− 10	− 02	+ 03	− 01	+1.15
12.3	9	70	− 16	− 12	− 04	− 09	− 21	− 15	+ 7.9	7.1	15	23	− 11	− 10	− 26	− 17	− 17	− 15	+3.15
12.5	8	70	+ 08	+ 10	− 05	− 15	− 14	− 13	+ 7.9	7.3	15	27	− 16	− 17	− 04	− 04	− 09	− 09	+3.15
13.1	8	69	− 10	− 15	− 36	− 14	− 06	− 06	+ 7.9	7.5	13	30	− 15	− 11	00	− 04	− 13	− 10	+3.15
13.3	9	67	− 04	− 03	− 22	− 08	00	− 08	+ 7.9	8.1	14	34	+ 02	− 15	− 38	− 30	− 11	− 06	+3.15
13.5	8	65	− 04	− 14	− 27	− 11	− 08	− 09	+ 7.9	8.3	11	38	− 19	− 17	− 10	− 03	− 16	− 09	+3.15
14.1	8	63	+ 02	− 02	− 15	− 16	− 14	− 06	+ 7.3	8.5	11	41	+ 08	+ 04	− 22	− 49	− 18	+ 01	+5.13
14.3	9	60	− 10	− 10	− 13	− 07	− 05	− 10	+ 7.3	9.1	11	45	− 13	− 15	− 19	− 04	+ 04	+ 01	+5.13
14.5	10	57	+ 07	+ 14	+ 21	00	− 02	+ 04	+ 5.4	9.3	11	49	− 01	+ 02	00	− 03	− 05	− 01	+5.13
15.1	9	54	− 05	+ 01	− 18	− 21	− 07	− 06	+ 5.4	9.5	10	52	− 12	− 13	− 16	− 07	− 03	+ 03	+5.13
15.3	11	51	00	− 11	+ 03	− 21	− 05	+ 02	+ 5.4	10.1	10	56	00	+ 09	+ 05	− 02	− 02	00	+5.13
15.5	11	48	− 12	− 12	− 12	− 15	− 10	+ 01	+ 5.4	10.3	8	59	− 06	− 14	− 10	+ 01	00	− 11	+5.13
16.1	11	45	00	− 06	− 06	00	− 05	− 03	+ 5.4	10.5	10	63	+ 06	+ 02	− 15	− 11	− 02	− 02	+7.15
16.3	11	42	+ 05	+ 05	− 09	− 18	− 09	− 03	+ 5.4	11.1	8	66	00	+ 02	− 12	− 16	− 08	− 14	+7.15
16.5	13	38	− 07	− 15	+ 02	+ 08	+ 04	+ 04	+ 3.5	11.3	10	69	− 05	+ 07	+ 08	− 04	− 04	− 04	+7.15
17.1	13	35	+ 04	+ 15	00	− 12	00	+ 02	+ 3.5	11.5	9	71	− 10	− 03	+ 02	− 04	− 02	+ 03	+7.15
17.3	16	31	− 03	− 04	− 07	− 01	+ 04	+ 04	+ 3.5	12.1	9	73	− 05	− 03	+ 11	+ 19	+ 14	+ 02	+7.9
17.5	16	28	+ 08	+ 24	+ 12	− 08	+ 08	+ 08	+ 3.5	12.3	9	75	+ 01	+ 02	+ 10	+ 04	− 06	− 04	+7.9
18.1	17	25	+ 02	+ 06	+ 07	+ 10	+ 02	− 06	+ 3.5	12.5	10	75	− 15	− 08	− 02	+ 06	+ 10	+ 06	+7.9
18.3	18	22	− 22	− 07	+ 02	+ 07	+ 01	− 05	+ 3.5	13.1	9	74	+ 12	+ 17	− 04	− 33	− 16	− 08	+7.9
18.5	20	19	+ 06	− 01	− 09	− 25	− 07	+ 02	+ 1.5	13.3	8	72	− 05	+ 01	− 20	− 31	− 22	− 20	+7.9
19.1	21	15	− 06	+ 06	+ 08	+ 04	+ 08	+ 08	+ 1.5	13.5	8	69	− 10	− 03	+ 10	+ 04	− 05	− 07	+7.3
19.3	23	12	− 01	+ 09	+ 14	+ 10	+ 03	+ 04	+ 1.5	14.1	10	66	00	− 02	+ 08	+ 04	− 12	− 07	+7.3
19.5	24	10	+ 16	+ 10	− 04	− 16	− 10	+ 06	+ 1.5	14.3	8	63	+ 06	+ 13	− 04	− 41	− 14	+ 01	+7.3
20.1	30	7	+ 27	+ 21	− 09	+ 02	+ 03	+ 16	+ 1.5	14.5	8	60	− 11	− 08	− 16	− 09	− 16	− 15	+7.3
20.3	26	4	+ 09	+ 12	+ 08	+ 09	+ 09	+ 09	+ 1.5	15.1	10	56	− 06	− 03	− 13	− 25	− 01	+ 04	+5.4
20.5	32	1	+ 26	+ 35	+ 17	+ 16	+ 13	+ 14	+ 1.5	15.3	10	53	− 01	− 02	00	+ 04	− 02	− 07	+5.4
21.1	32	1	+ 15	+ 13	+ 14	+ 17	+ 17	+ 19	− 1.5	15.5	10	49	− 01	+ 05	− 04	− 09	− 02	− 02	+5.4
21.3	36	3	+ 21	+ 38	+ 16	+ 13	+ 23	+ 27	− 1.5	16.1	11	45	− 13	− 03	+ 10	+ 03	− 09	− 08	+5.4
21.5	39	6	− 14	− 04	− 07	+ 07	+ 14	+ 21	− 1.7	16.3	11	42	− 08	− 07	− 27	− 16	− 06	− 08	+5.4
22.1	35	7	− 08	− 07	− 01	+ 11	+ 10	+ 13	− 1.7	16.5	12	38	+ 08	− 06	+ 03	− 07	− 15	− 10	+3.3
22.3	33	9	+ 04	− 10	− 25	+ 05	+ 17	+ 21	− 1.7	17.1	12	34	+ 02	+ 01	− 11	− 20	− 17	− 11	+3.3
22.5	32	11	− 02	− 03	− 10	+ 15	+ 25	+ 20	− 1.7	17.3	18	31	− 15	− 05	− 04	− 11	− 05	00	+3.3
23.1	31	12	+ 10	+ 14	00	+ 02	− 02	+ 15	− 1.7	17.5	20	27	− 10	− 06	− 06	+ 02	+ 05	+ 05	+3.3
23.3	32	13	− 01	− 10	+ 08	+ 08	+ 12	+ 24	− 1.7	18.1	22	24	+ 10	+ 05	+ 11	− 02	− 06	+ 03	+3.3
23.5	30	14	+ 05	+ 02	+ 06	+ 10	+ 13	+ 15	− 1.9	18.3	24	20	− 12	− 11	+ 06	00	− 01	+ 12	+3.3
				+ 40°	+ 45°					18.5	25	17	+ 03	+ 01	+ 37	+ 14	+ 01	+ 01	+1.3
0.1	22	20	− 01	+ 08	+ 07	− 10	− 10	+ 01	−3.9	19.1	26	13	− 14	00	+ 06	− 08	− 05	+ 13	+1.5
0.3	20	21	− 06	− 06	− 21	− 16	− 05	+ 04	−3.9	19.3	34	10	+ 07	+ 06	+ 02	− 05	+ 07	+ 20	+1.5
0.5	20	21	+ 04	+ 04	+ 02	− 07	− 08	− 04	−3.9	19.5	35	7	+ 06	− 13	− 13	+ 03	+ 14	+ 19	+1.5
1.1	18	20	− 07	+ 36	+ 24	+ 05	− 03	− 02	−1.9	20.1	36	4	+ 17	+ 11	+ 11	+ 25	+ 22	+ 20	+1.5
1.3	19	19	+ 36	− 02	− 16	− 13	− 03	+ 06	−1.9	20.3	33	1	+ 06	+ 18	+ 21	+ 10	+ 10	+ 20	+1.5
1.5	17	18	− 13	− 11	− 05	+ 03	− 04	− 05	−1.11	20.5	33	2	+ 27	+ 25	+ 23	+ 07	+ 11	+ 15	−1.5
2.1	20	17	− 13	− 08	− 23	− 12	− 03	− 06	−1.11	21.1	33	5	+ 01	− 01	− 10	− 06	+ 02	+ 11	−1.5
2.3	23	16	− 19	− 16	− 01	+ 09	+ 10	+ 11	−1.11	21.3	33	7	+ 06	+ 08	+ 04	+ 03	+ 10	+ 19	−1.5
2.5	18	14	− 14	− 06	− 12	− 21	− 15	− 10	−1.11	21.5	27	9	− 09	− 14	+ 05	+ 13	+ 11	+ 09	−1.5
3.1	21	12	+ 24	− 06	− 26	− 14	− 08	− 04	−1.11	22.1	26	11	+ 07	− 05	− 06	− 04	− 04	+ 03	−1.7
3.3	20	10	− 09	− 04	− 02	+ 01	− 01	− 06	−1.11	22.3	27	14	+ 03	00	+ 08	00	− 06	+ 08	−1.7
3.5	19	8	− 15	− 19	− 18	− 18	− 22	− 12	−1.13	22.5	23	16	+ 25	+ 20	− 03	− 02	+ 02	− 01	−1.7
4.1	20	5	− 05	− 05	+ 02	− 04	− 14	− 16	−1.13	23.1	24	17	+ 20	+ 13	+ 09	+ 02	+ 02	+ 03	−1.7
4.3	24	2	00	+ 10	− 01	− 10	− 12	− 06	−1.13	23.3	21	18	+ 04	+ 10	+ 14	+ 02	− 01	00	−1.7
4.5	30	0	+ 11	− 05	− 19	− 03	00	+ 07	−1.13	23.5	22	19	− 02	+ 01	+ 05	+ 06	+ 02	− 02	−1.7

+ 35° + 40°

AR	D	β	Δ'₁	Δ'₂	Δ₃	Δ₄	Δ₅	Δ₆	Region
0.1	17	25	— 01	— 05	+ 02	— 01	— 02	— 03	—3.9
0.3	18	25	— 12	— 12	— 19	— 12	— 04	+ 02	—3.9
0.5	19	25	— 06	— 06	— 03	— 04	— 16	— 07	—3.9
1.1	18	24	— 02	— 08	— 27	— 09	— 04	— 06	—3.9
1.3	18	24	— 12	— 05	+ 04	— 08	— 03	00	—3.9
1.5	21	23	— 02	— 01	00	00	+ 04	+ 13	—3.11
2.1	19	22	— 12	— 19	— 15	— 05	— 12	+ 03	—3.11
2.3	20	20	+ 18	+ 07	+ 10	00	— 10	— 03	—3.11
2.5	19	18	+ 22	+ 02	— 11	— 10	— 02	— 03	—1.11
3.1	17	16	— 14	— 20	— 04	00	— 09	— 10	—1.11
3.3	16	14	— 04	— 12	— 01	— 16	— 31	— 24	—1.13
3.5	15	11	00	— 03	00	— 08	— 15	— 16	—1.13
4.1	15	9	— 15	— 15	— 09	— 12	— 24	— 36	—1.13
4.3	12	6	— 16	— 15	— 15	— 25	— 28	— 33	—1.13
4.5	23	3	+ 04	— 01	— 26	— 22	— 17	— 17	—1.13
5.1	35	0	— 11	— 10	— 14	— 12	+ 02	+ 17	—1.13
5.3	29	4	— 21	— 14	— 12	+ 04	+ 07	+ 03	+1.13
5.5	25	7	+ 15	— 06	— 06	+ 02	— 05	— 03	+1.15
6.1	23	11	— 20	— 21	— 04	— 05	— 05	00	+1.15
6.3	22	15	+ 06	+ 12	+ 13	— 02	— 08	+ 06	+1.15
6.5	20	18	+ 02	+ 06	00	— 19	— 09	+ 02	+1.15
7.1	17	22	— 02	— 17	— 22	— 17	— 20	— 08	+3.15
7.3	14	26	— 06	— 03	— 10	— 15	— 10	— 12	+3.15
7.5	14	29	— 06	00	00	— 04	— 16	— 11	+3.15
8.1	13	33	— 10	— 11	— 03	— 03	+ 05	+ 01	+3.15
8.3	12	37	+ 06	— 01	+ 01	+ 01	— 01	— 06	+3.15
8.5	12	41	— 08	— 03	+ 07	+ 16	+ 09	+ 01	+5.16
9.1	11	45	+ 13	— 05	— 23	— 07	— 01	+ 09	+5.16
9.3	9	49	+ 08	+ 02	— 01	— 16	— 21	— 11	+5.16
9.5	10	53	— 17	+ 06	+ 06	— 04	— 02	— 06	+5.16
10.1	9	57	— 06	— 05	— 20	— 35	— 20	— 09	+5.16
10.3	10	61	+ 09	— 01	— 23	— 23	— 06	+ 01	+5.16
10.5	9	65	— 01	— 04	— 01	— 06	— 12	— 06	+7.15
11.1	8	68	00	— 09	— 04	— 06	— 08	— 06	+7.15
11.3	7	72	— 05	— 10	— 31	— 26	— 14	— 16	+7.15
11.5	7	76	00	+ 10	— 04	— 29	— 17	— 15	+7.15
12.1	7	78	— 15	— 09	— 10	— 17	— 22	— 18	+7.9
12.3	9	80	— 05	— 14	— 19	— 05	— 02	+ 02	+7.9
12.5	9	80	00	— 05	— 28	— 10	+ 02	— 02	+7.9
13.1	11	78	+ 15	+ 09	+ 33	+ 16	+ 01	+ 11	+7.9
13.3	9	75	+ 05	+ 06	— 09	— 10	— 02	+ 04	+7.3
13.5	10	72	+ 05	+ 12	+ 10	— 25	— 18	+ 03	+7.3
14.1	10	69	00	— 02	+ 11	+ 14	+ 08	00	+7.3
14.3	10	65	+ 10	+ 02	+ 02	— 11	— 21	— 02	+7.3
14.5	9	61	+ 04	— 01	00	— 04	— 06	— 06	+7.3
15.1	9	57	— 01	+ 02	+ 03	— 04	— 06	— 09	+5.1
15.3	10	53	+ 08	+ 10	— 04	— 23	— 23	+ 01	+5.1
15.5	10	49	+ 08	+ 05	— 20	— 40	— 31	— 10	+5.1
16.1	11	45	— 03	— 03	+ 04	— 02	— 11	+ 02	+5.1
16.3	12	42	— 03	— 11	— 17	— 13	00	00	+5.1
16.5	12	38	— 19	— 19	— 32	— 13	+ 04	+ 03	+3.3
17.1	12	34	00	+ 01	— 22	— 10	— 14	— 10	+3.3
17.3	18	30	+ 05	— 17	— 28	— 13	— 13	+ 05	+3.3
17.5	21	26	— 11	+ 11	+ 17	+ 08	+ 07	+ 17	+3.3
18.1	24	22	+ 03	+ 01	— 03	00	00	+ 13	+3.3
18.3	29	19	+ 02	+ 11	+ 07	+ 11	+ 13	+ 16	+1.3
18.5	32	15	+ 27	— 02	+ 05	+ 13	+ 11	+ 15	+1.3
19.1	35	11	+ 06	+ 14	+ 14	+ 20	+ 23	+ 24	+1.3
19.3	39	8	+ 10	+ 13	+ 02	+ 01	+ 16	+ 28	+1.3
19.5	40	4	+ 40	+ 17	+ 12	+ 05	+ 20	+ 35	+1.3
20.1	43	1	+ 29	+ 29	+ 30	+ 29	+ 36	+ 45	+1.5
20.3	35	2	+ 04	— 06	— 08	+ 09	+ 21	+ 28	—1.5
20.5	35	5	— 11	— 03	— 01	+ 08	+ 10	+ 16	—1.5
21.1	35	8	+ 15	+ 28	+ 14	+ 04	+ 15	+ 24	—1.5
21.3	28	11	+ 06	— 04	— 21	+ 03	+ 06	+ 05	—1.5
21.5	20	13	— 14	— 17	— 23	— 09	— 01	00	—1.5
22.1	21	16	— 04	— 03	— 16	— 13	— 15	— 02	—1.5
22.3	24	18	+ 28	+ 33	+ 09	+ 03	+ 05	+ 10	—1.7
22.5	19	20	+ 13	+ 01	— 07	— 11	— 08	— 04	—3.7
23.1	18	21	— 18	— 14	00	— 06	— 18	— 12	—3.7
23.3	16	23	+ 08	+ 01	— 18	— 35	— 21	— 12	—3.7
23.5	14	24	— 22	+ 01	— 06	— 29	— 21	— 10	—3.7

+ 30° + 35°

AR	D	β	Δ'₁	Δ'₂	Δ₃	Δ₄	Δ₅	Δ₆	Region
0.1	14	30	— 02	+ 04	+ 11	— 08	— 09	— 05	—3.9
0.3	14	30	— 02	00	— 10	— 10	— 10	— 07	—3.9
0.5	15	30	— 11	— 15	— 06	+ 14	+ 08	+ 05	—3.9
1.1	15	29	— 02	+ 09	+ 02	+ 03	+ 08	+ 06	—3.9
1.3	14	29	+ 03	— 12	— 29	— 16	— 11	— 08	—3.11
1.5	14	28	— 12	— 18	— 54	— 08	00	— 02	—3.11
2.1	15	27	+ 03	+ 03	— 10	00	00	— 07	—3.11
2.3	14	25	+ 07	+ 04	+ 19	+ 14	— 01	— 13	—3.11
2.5	15	22	— 08	— 03	— 12	— 11	— 08	— 08	—3.11
3.1	15	20	+ 06	+ 09	— 02	— 04	+ 03	— 07	—3.13
3.3	12	18	— 09	00	— 10	— 24	— 36	— 26	—1.13
3.5	15	15	+ 14	+ 07	— 09	— 26	— 28	— 14	—1.13
4.1	15	12	+ 04	+ 05	— 11	— 24	— 26	— 20	—1.13
4.3	9	9	— 11	— 17	— 29	— 47	— 42	— 46	—1.13
4.5	14	6	— 02	— 22	— 30	— 31	— 34	— 37	—1.13
5.1	25	3	+ 17	+ 19	+ 03	— 08	— 03	+ 07	—1.15
5.3	26	1	+ 21	— 09	— 09	+ 01	+ 01	+ 03	+1.15
5.5	23	5	+ 02	— 02	— 07	— 06	— 10	— 04	+1.15
6.1	23	8	— 21	— 16	— 11	— 03	— 01	— 02	+1.15
6.3	24	12	— 01	— 14	— 17	00	+ 08	+ 10	+1.15
6.5	18	16	00	— 02	— 12	— 11	— 08	— 06	+1.15
7.1	18	20	— 04	— 12	— 06	— 14	— 10	+ 01	+1.15
7.3	15	24	+ 07	— 03	— 22	— 16	— 13	— 07	+3.15
7.5	16	28	— 12	— 22	— 31	— 14	+ 02	+ 03	+3.15
8.1	15	32	— 20	— 19	— 14	— 07	+ 01	+ 05	+3.15
8.3	14	36	— 05	— 07	+ 07	— 06	— 08	— 01	+3.15
8.5	12	40	+ 20	+ 21	+ 11	— 15	— 20	— 08	+3.15
9.1	11	44	+ 11	— 19	— 58	— 34	— 11	— 04	+5.16
9.3	11	49	+ 02	— 04	— 02	— 13	— 25	— 13	+5.16
9.5	11	53	— 07	+ 02	00	— 16	— 04	— 01	+5.16
10.1	11	57	+ 03	+ 07	+ 07	— 05	+ 08	+ 06	+5.16
10.3	10	62	+ 03	+ 01	— 28	— 40	— 15	— 01	+7.15
10.5	10	66	+ 03	— 05	— 12	— 02	— 15	— 14	+7.15
11.1	9	70	— 06	— 06	— 26	— 22	— 18	— 12	+7.15
11.3	8	74	+ 04	— 11	— 10	— 09	— 24	— 19	+7.15
11.5	7	78	+ 04	— 10	— 10	— 07	— 14	— 19	+7.15
12.1	8	81	— 06	— 11	— 28	— 07	— 11	— 11	+9
12.3	9	84	— 06	— 13	— 07	— 02	— 02	— 02	+9

AR	D	β	Δ'_1	Δ'_2	Δ_3	Δ_4	Δ_5	Δ_6	Region
12.5	9	84	— 06	— 09	— 08	— 16	— 21	— 16	+9
13.1	8	82	— 10	— 13	— 12	— 11	— 18	— 12	+9
13.3	9	78	— 10	— 01	— 14	— 35	— 15	— 01	+7.3
13.5	9	74	— 01	— 12	— 35	— 13	— 10	— 11	+7.3
14.1	10	70	— 06	— 06	— 07	— 24	— 25	+ 05	+7.3
14.3	9	66	+ 03	— 05	— 25	— 05	— 03	— 06	+7.3
14.5	9	62	— 02	— 12	— 09	— 12	— 10	+ 03	+7.3
15.1	10	58	+ 08	+ 06	— 05	— 06	— 09	— 04	+5.1
15.3	10	54	— 02	— 03	— 13	— 30	— 21	— 05	+5.1
15.5	11	49	— 03	— 10	— 14	— 08	— 04	+ 03	+5.1
16.1	12	45	+ 11	+ 03	— 31	— 17	— 07	— 01	+5.1
16.3	13	41	— 14	— 09	00	— 08	— 06	+ 01	+5.1
16.5	14	37	+ 05	+ 05	— 05	— 03	+ 03	+ 12	+3.3
17.1	15	32	— 06	— 05	— 06	— 02	+ 04	+ 08	+3.3
17.3	16	28	+ 08	+ 13	+ 03	— 03	+ 02	+ 05	+3.3
17.5	17	24	— 03	+ 16	00	— 08	— 04	— 03	+3.3
18.1	20	20	— 04	— 14	— 09	+ 02	+ 02	+ 06	+3.3
18.3	22	16	+ 10	+ 05	— 03	+ 06	+ 11	+ 09	+1.3
18.5	27	12	+ 23	+ 10	+ 06	+ 06	+ 16	+ 29	+1.3
19.1	30	9	+ 13	+ 21	+ 14	+ 10	+ 16	+ 21	+1.3
19.3	36	5	— 07	+ 13	+ 10	+ 22	+ 26	+ 28	+1.3
19.5	38	1	+ 12	+ 16	+ 08	+ 15	+ 28	+ 34	+1.3
20.1	34	2	+ 02	+ 01	— 03	+ 15	+ 25	+ 28	—1.3
20.3	30	6	+ 03	— 05	00	— 03	+ 06	+ 14	—1.5
20.5	32	9	— 02	— 10	— 06	+ 06	+ 13	+ 17	—1.5
21.1	27	12	— 06	— 12	+ 05	+ 08	+ 06	+ 11	—1.5
21.3	22	15	— 19	— 12	— 06	— 03	+ 05	+ 07	—1.5
21.5	20	17	— 19	— 19	+ 04	+ 01	— 11	— 05	—1.5
22.1	20	20	+ 01	— 15	— 11	— 02	— 04	00	—3.5
22.3	20	22	— 08	00	+ 05	— 06	+ 02	+ 07	—3.5
22.5	17	24	— 17	— 07	+ 02	+ 01	+ 01	— 04	—3.7
23.1	16	26	— 02	00	+ 01	13	— 12	— 03	—3.7
23.3	16	28	— 07	— 06	— 04	— 01	— 07	+ 02	—3.7
23.5	16	29	— 11	— 07	00	— 09	— 02	+ 10	—3.7

+ 25° + 30°

AR	D	β	Δ'_1	Δ'_2	Δ_3	Δ_4	Δ_5	Δ_6	Region
0.1	15	35	— 02	— 12	— 18	— 10	— 08	— 02	—3.9
0.3	14	35	— 02	— 05	— 36	— 31	+ 01	+ 10	—3.9
0.5	12	35	— 02	— 01	— 04	— 08	— 10	— 08	—3.9
1.1	14	34	— 02	— 06	— 08	— 12	— 02	+ 01	—3.9
1.3	13	33	— 11	— 14	— 04	— 08	— 05	— 02	—3.11
1.5	13	32	— 07	+ 09	+ 07	+ 03	+ 06	— 03	—3.11
2.1	12	31	+ 06	+ 16	+ 02	— 06	— 02	— 02	—3.11
2.3	12	29	+ 06	— 05	— 18	— 10	— 14	— 13	—3.11
2.5	10	26	— 12	— 14	— 27	— 15	— 18	— 24	—3.13
3.1	10	24	+ 14	+ 13	— 14	— 35	— 32	— 39	—3.13
3.3	13	22	— 13	— 14	— 21	— 11	— 04	— 12	—3.13
3.5	12	19	— 19	— 12	— 19	— 22	— 17	— 24	—1.13
4.1	8	16	— 02	— 16	— 99	— 42	— 49	— 63	—1.13
4.3	7	13	— 07	00	— 20	— 74	— 74	— 62	—1.13
4.5	12	9	— 15	— 18	— 18	— 26	— 35	— 37	—1.15
5.1	12	6	— 08	00	— 14	— 43	— 44	— 32	—1.15
5.3	27	2	+ 18	— 12	— 10	00	— 02	— 04	—1.15
5.5	29	2	— 04	— 04	— 03	00	— 01	+ 01	+1.15
6.1	29	6	— 13	— 16	— 18	00	+ 04	+ 03	+1.15
6.3	28	10	+ 02	— 03	— 21	— 14	— 04	+ 03	+1.15
6.5	24	14	— 06	— 10	+ 04	— 05	— 03	+ 03	+1.15
7.1	19	18	+ 12	+ 02	— 08	— 11	— 09	— 04	+1.15
7.3	19	22	+ 05	— 04	— 71	— 27	— 14	— 04	+3.15
7.5	17	26	+ 01	— 11	— 13	— 14	— 10	— 04	+3.17
8.1	14	30	— 07	00	— 09	— 09	— 01	— 06	+3.17
8.3	13	34	— 11	— 15	— 13	— 19	— 05	— 08	+3.17
8.5	11	39	+ 08	+ 06	+ 03	— 09	— 07	— 05	+3.17
9.1	11	43	+ 05	— 06	— 11	— 12	— 08	— 08	+5.16
9.3	11	48	— 09	— 11	— 15	— 05	— 12	— 30	+5.16
9.5	10	52	— 08	— 09	— 11	+ 01	— 05	— 03	+5.16
10.1	10	56	+ 01	— 10	— 06	— 04	— 06	— 09	+5.16
10.3	10	61	— 12	— 13	— 11	— 13	— 09	— 06	+7.15
10.5	9	66	+ 07	— 07	— 19	— 25	— 26	— 15	+7.15
11.1	9	70	— 11	— 13	— 14	— 13	— 20	— 17	+7.15
11.3	9	75	— 06	— 12	— 04	+ 04	— 06	— 18	+7.15
11.5	9	79	— 15	— 20	— 11	— 02	— 16	— 19	+7.15
12.1	9	83	+ 43	+ 43	+ 36	+ 02	— 04	— 03	+9
12.3	9	87	+ 12	+ 03	— 67	— 06	+ 08	+ 04	+9
12.5	9	88	— 02	— 07	— 12	— 03	— 12	— 09	+9
13.1	9	84	+ 03	— 06	— 13	— 07	— 13	— 10	+9
13.3	9	79	— 06	— 01	— 12	— 20	+ 03	— 02	+9
13.5	8	75	+ 03	— 05	+ 08	+ 03	— 19	— 20	+7.3
14.1	9	70	— 07	— 01	— 05	— 07	— 14	— 08	+7.3
14.3	9	66	— 07	— 10	— 14	— 06	+ 02	+ 01	+7.3
14.5	9	61	— 03	— 05	— 13	— 06	— 12	— 19	+7.3
15.1	10	57	+ 15	+ 07	— 11	— 06	— 07	— 12	+5.1
15.3	10	53	+ 01	— 05	— 43	— 24	— 01	+ 03	+5.1
15.5	12	48	— 09	— 14	— 56	— 19	+ 04	+ 04	+5.1
16.1	13	44	— 05	— 03	+ 04	— 12	— 11	— 12	+5.1
16.3	10	40	— 10	— 06	— 14	— 07	— 12	— 14	+3.1
16.5	14	36	— 02	— 03	+ 01	— 06	00	— 02	+3.1
17.1	15	31	— 07	— 02	— 07	— 06	— 01	+ 05	+3.1
17.3	18	27	— 03	— 02	— 11	— 08	— 01	+ 03	+3.1
17.5	19	23	00	— 01	— 17	00	+ 09	+ 13	+3.1
18.1	18	19	+ 04	+ 06	— 09	— 05	+ 01	— 05	+1.3
18.3	20	15	— 11	— 09	— 03	— 04	00	+ 06	+1.3
18.5	26	11	— 03	+ 01	— 03	+ 10	+ 13	+ 08	+1.3
19.1	28	7	— 04	+ 05	+ 13	+ 06	+ 12	+ 21	+1.3
19.3	28	3	+ 14	+ 15	+ 04	+ 06	+ 13	+ 16	+1.3
19.5	31	1	— 04	00	+ 11	+ 02	+ 23	+ 26	—1.3
20.1	29	5	+ 05	+ 03	— 01	+ 12	+ 21	+ 21	—1.3
20.3	28	8	— 03	— 11	— 08	+ 01	+ 09	+ 13	—1.3
20.5	24	12	— 02	— 08	— 07	+ 09	+ 07	+ 04	—1.3
21.1	21	15	— 02	— 03	— 14	— 12	— 02	00	—1.5
21.3	21	18	+ 04	— 11	— 22	— 10	+ 01	+ 08	—1.5
21.5	19	21	00	— 06	— 25	— 13	+ 01	+ 06	—3.5
22.1	17	24	+ 05	— 05	+ 03	— 08	— 06	+ 04	—3.5
22.3	19	26	— 04	— 07	— 16	— 01	+ 08	+ 13	—3.5
22.5	18	29	— 16	— 27	— 58	— 16	— 03	+ 06	—3.7
23.1	14	31	— 02	+ 04	+ 14	+ 09	+ 01	— 06	—3.7
23.3	14	32	— 11	— 07	— 12	— 29	— 23	— 06	—3.7
23.5	15	34	— 06	00	+ 09	— 04	— 08	— 02	—3.7

+ 20° + 25°

AR	D	β	Δ'_1	Δ'_2	Δ_3	Δ_4	Δ_5	Δ_6	Region
0.1	12	40	— 10	— 01	— 02	— 07	— 01	+ 02	—3.9
0.3	12	40	— 06	+ 04	+ 03	— 07	+ 03	+ 04	—5.7
0.5	11	40	+ 03	+ 10	+ 02	— 12	— 14	— 04	—5.10
1.1	11	39	+ 02	— 03	— 06	— 17	— 16	00	—3.9

AR	D	β	Δ'_1	Δ'_2	Δ_3	Δ_4	Δ_5	Δ_6	Region
1.3	10	38	— 15	— 10	— 18	— 27	— 22	— 11	—3.11
1.5	9	37	+ 02	+ 08	— 11	— 24	— 22	— 13	—3.11
2.1	11	35	+ 02	— 05	— 27	— 14	— 06	— 07	—3.11
2.3	10	33	— 07	— 05	— 08	— 06	— 10	— 14	—3.11
2.5	9	31	— 03	— 05	— 22	— 30	— 21	— 23	—3.13
3.1	12	28	00	+ 09	— 10	— 18	— 17	— 07	—3.13
3.3	16	25	+ 17	+ 33	+ 16	+ 14	+ 10	+ 02	—3.13
3.5	14	22	+ 42	+ 06	+ 03	+ 05	00	— 02	—3.13
4.1	12	19	+ 20	+ 34	+ 11	— 26	— 46	— 24	—1.13
4.3	11	15	+ 10	— 08	— 30	— 38	— 38	— 34	—1.15
4.5	17	12	— 03	+ 04	— 18	— 27	— 16	— 15	—1.15
5.1	16	8	00	— 22	— 79	— 51	— 32	— 19	—1.15
5.3	30	4	+ 12	— 05	— 09	00	+ 08	+ 12	—1.15
5.5	32	0	+ 03	— 10	— 18	— 02	+ 08	+ 12	—1.15
6.1	32	4	+ 12	+ 04	+ 01	— 02	+ 07	+ 13	+1.15
6.3	29	8	— 04	— 23	— 14	— 02	+ 09	+ 19	+1.15
6.5	27	12	— 03	— 16	— 20	— 04	+ 05	+ 13	+1.15
7.1	23	16	— 02	— 01	+ 01	— 06	— 01	+ 06	+1.17
7.3	20	21	+ 20	+ 05	+ 08	+ 01	+ 01	+ 03	+1.17
7.5	19	25	— 04	+ 02	+ 06	— 02	+ 01	+ 02	+3.17
8.1	16	29	+ 01	— 09	— 18	— 18	— 06	— 06	+3.17
8.3	15	34	+ 10	+ 10	+ 10	— 05	— 02	+ 05	+3.17
8.5	12	38	— 15	— 19	— 14	— 09	— 06	+ 03	+3.17
9.1	10	42	— 01	— 01	— 13	— 08	— 18	— 17	+5.16
9.3	11	47	— 09	+ 04	+ 09	— 05	— 17	— 15	+5.16
9.5	10	51	00	— 17	— 49	— 16	— 25	— 08	+5.16
10.1	10	55	+ 05	+ 11	— 06	— 23	— 30	— 18	+5.16
10.3	9	60	— 08	— 15	— 32	— 19	— 17	— 14	+5.19
10.5	10	64	— 07	— 07	— 07	— 04	— 04	— 07	+7.21
11.1	9	69	— 03	— 11	— 24	— 03	— 14	— 16	+7.21
11.3	8	73	— 07	— 07	— 01	+ 02	— 07	— 10	+7.21
11.5	7	77	— 07	— 11	— 29	— 38	— 30	— 12	+7.21
12.1	8	81	— 02	— 03	— 08	— 08	— 16	— 19	+9
12.3	8	84	+ 19	+ 06	+ 01	— 01	— 07	— 14	+9
12.5	8	85	— 07	+ 11	+ 20	— 07	— 23	— 13	+9
13.1	8	82	+ 02	— 15	— 26	— 18	— 09	— 05	+9
13.3	9	78	+ 06	+ 02	00	+ 01	— 02	— 07	+7.33
13.5	9	74	— 07	— 02	— 08	00	+ 01	+ 02	+7.33
14.1	11	69	— 07	— 11	— 16	— 02	+ 14	+ 15	+7.33
14.3	8	65	+ 01	— 02	— 31	— 46	— 30	— 18	+7.33
14.5	9	60	+ 01	00	+ 10	— 08	— 16	— 13	+7.33
15.1	10	56	— 08	— 09	— 19	— 32	— 14	— 02	+5.1
15.3	10	52	— 13	— 11	— 10	+ 04	+ 07	— 01	+5.1
15.5	11	47	— 05	— 02	— 13	— 11	— 02	+ 04	+5.1
16.1	10	43	— 05	— 08	— 21	— 26	— 05	— 06	+5.1
16.3	12	38	— 06	— 09	— 19	— 10	00	— 03	+3.1
16.5	14	34	— 03	— 09	— 36	— 17	— 07	+ 01	+3.1
17.1	15	30	+ 05	+ 10	— 09	— 22	— 11	— 06	+3.1
17.3	16	26	+ 04	— 03	— 22	— 14	+ 04	+ 07	+3.1
17.5	18	21	+ 07	+ 01	+ 01	+ 02	+ 04	+ 08	+3.1
18.1	21	17	+ 19	— 01	— 08	00	— 02	+ 02	+1.1
18.3	26	13	— 08	+ 01	— 11	— 05	+ 03	+ 06	+1.1
18.5	24	8	+ 09	+ 13	— 11	— 15	— 01	+ 04	+1.3
19.1	23	4	+ 08	+ 03	— 06	00	— 02	— 13	+1.3
19.3	26	0	+ 12	— 09	+ 04	+ 10	+ 10	+ 09	+1.3
19.5	30	4	+ 08	+ 13	+ 01	00	+ 06	+ 11	—1.3
20.1	29	7	+ 17	+ 16	— 03	+ 02	+ 09	+ 13	—1.3
20.3	28	11	+ 05	+ 03	— 10	— 11	+ 08	+ 22	—1.3
20.5	23	15	— 16	— 16	— 13	+ 04	+ 03	+ 04	—1.3
21.1	18	18	+ 02	+ 02	— 06	— 12	— 13	— 11	—1.3
21.3	17	22	— 01	+ 13	+ 18	— 03	— 04	+ 02	—3.5
21.5	15	25	— 13	— 12	— 12	+ 02	+ 02	— 06	—3.5
22.1	16	28	00	— 04	+ 01	— 10	— 20	— 14	—3.5
22.3	15	31	— 20	— 13	— 06	— 14	— 12	— 04	—3.5
22.5	13	33	— 07	— 14	— 16	— 19	— 14	— 05	—3.5
23.1	13	35	+ 02	+ 05	— 11	— 13	— 05	+ 01	—3.7
23.3	12	37	+ 02	— 01	— 01	— 20	— 12	— 07	—3.7
23.5	11	39	— 06	— 03	— 03	— 07	— 06	— 02	—3.7

+ 15° + 20°

AR	D	β	Δ'_1	Δ'_2	Δ_3	Δ_4	Δ_5	Δ_6	Region
0.1	11	44	— 10	— 06	— 11	— 15	— 12	— 05	—5.7
0.3	10	45	— 05	— 03	— 04	— 18	— 28	— 07	—5.7
0.5	10	45	— 05	— 12	— 23	— 14	— 08	+ 01	—5.10
1.1	10	44	— 01	+ 03	— 10	— 11	— 13	— 08	—5.10
1.3	10	43	+ 15	+ 03	— 24	— 44	— 23	— 04	—5.10
1.5	8	42	+ 02	— 12	— 48	— 30	— 13	— 06	—5.10
2.1	9	40	— 06	— 11	— 12	— 22	— 26	— 18	—5.10
2.3	12	38	— 02	— 05	— 31	— 32	— 22	— 20	—3.13
2.5	10	35	+ 01	+ 05	— 06	— 38	— 38	— 23	—3.13
3.1	13	32	— 08	— 08	— 07	— 05	— 07	— 10	—3.13
3.3	12	29	+ 04	— 07	— 16	— 13	— 10	— 20	—3.13
3.5	11	26	— 09	— 05	— 26	— 30	— 20	— 22	—3.15
4.1	10	22	+ 24	+ 28	+ 06	— 10	— 27	— 38	—3.15
4.3	13	19	+ 40	+ 50	+ 01	— 15	— 20	— 19	—1.15
4.5	12	15	+ 05	+ 02	— 07	— 30	— 39	— 29	—1.15
5.1	16	11	00	+ 19	— 07	— 18	— 19	— 18	—1.15
5.3	23	7	+ 20	+ 02	— 14	— 10	— 03	+ 02	—1.15
5.5	28	3	— 06	— 17	— 08	+ 03	+ 12	+ 16	—1.15
6.1	28	1	+ 20	+ 19	+ 04	— 03	+ 10	+ 18	+1.17
6.3	26	6	— 01	— 04	+ 11	+ 12	+ 11	+ 15	+1.17
6.5	27	10	00	— 07	+ 07	+ 04	+ 13	+ 22	+1.17
7.1	21	14	— 03	— 06	— 04	+ 03	+ 11	+ 15	+1.17
7.3	20	19	— 06	— 09	— 36	— 18	00	+ 09	+1.17
7.5	20	23	+ 07	+ 01	— 01	— 08	— 04	+ 03	+3.17
8.1	17	27	— 04	— 03	— 03	— 22	— 14	— 04	+3.17
8.3	18	32	+ 01	— 13	— 18	00	+ 07	+ 11	+3.17
8.5	16	36	+ 02	+ 03	— 01	— 08	— 02	+ 01	+3.17
9.1	14	40	— 06	— 03	+ 01	— 02	— 05	— 06	+3.17
9.3	11	45	— 10	— 11	— 29	— 22	— 16	— 14	+5.19
9.5	10	49	— 18	— 18	— 38	+ 04	+ 07	+ 01	+5.19
10.1	9	54	00	— 18	— 32	— 23	— 08	— 01	+5.19
10.3	8	58	— 12	— 09	— 08	— 15	— 12	— 10	+5.19
10.5	9	62	— 12	— 16	— 34	— 09	— 06	— 03	+7.21
11.1	9	67	— 12	— 09	— 16	— 15	— 06	— 16	+7.21
11.3	8	71	+ 01	— 04	— 11	— 34	— 26	— 08	+7.21
11.5	9	75	— 03	— 01	— 14	— 14	— 16	— 14	+7.21
12.1	9	78	— 03	+ 06	+ 05	— 06	— 10	— 08	+7.21
12.3	9	80	— 07	— 07	— 21	— 08	— 14	— 10	+9
12.5	8	80	+ 06	00	— 11	— 40	— 35	— 07	+9
13.1	8	79	— 03	+ 01	+ 11	+ 10	+ 03	— 03	+7.33
13.3	9	75	— 11	— 15	— 29	— 10	— 05	— 05	+7.33
13.5	9	71	+ 05	+ 06	00	00	— 06	— 04	+7.33
14.1	9	67	+ 01	— 01	— 09	— 02	+ 03	+ 09	+7.33
14.3	10	63	— 04	+ 04	— 01	— 14	— 02	+ 08	+7.33

AR	D	β	Δ′₁	Δ′₂	Δ₃	Δ₄	Δ₅	Δ₆	Region
14.5	10	59	+ 13	+ 02	+ 01	+ 01	+ 03	— 06	+5.34
15.1	9	54	— 04	— 08	— 10	— 08	— 08	— 02	+5.34
15.3	10	50	+ 16	+ 20	+ 16	— 11	— 16	— 06	+5.34
15.5	10	46	+ 16	— 03	— 51	— 42	— 18	+ 04	+5.34
16.1	11	41	+ 02	— 02	— 12	— 04	+ 02	00	+5.34
16.3	12	37	— 11	— 09	— 08	— 01	— 04	— 08	+3.1
16.5	15	32	— 04	+ 08	+ 14	— 02	— 16	— 11	+3.1
17.1	16	28	— 08	+ 05	+ 04	— 06	— 09	— 11	+3.1
17.3	19	24	+ 12	— 05	— 04	— 02	— 04	— 06	+3.1
17.5	20	19	+ 02	+ 09	— 05	— 12	— 10	00	+1.1
18.1	27	15	— 07	— 08	— 31	— 11	00	+ 07	+1.1
18.3	25	11	— 08	+ 01	— 02	— 02	+ 01	+ 01	+1.1
18.5	28	6	+ 07	+ 06	+ 02	— 07	— 02	+ 07	+1.1
19.1	24	2	— 10	+ 03	+ 11	00	— 08	— 06	+1.1
19.3	28	2	+ 03	+ 02	+ 04	— 08	— 09	+ 03	—1.3
19.5	29	6	+ 16	+ 13	— 02	+ 02	+ 10	+ 11	—1.3
20.1	24	10	— 17	— 11	— 06	00	00	+ 04	—1.3
20.3	20	14	+ 09	+ 09	+ 08	— 11	— 11	— 08	—1.3
20.5	19	18	— 11	— 12	— 13	— 08	— 07	+ 03	—1.3
21.1	18	22	— 14	— 08	+ 03	— 12	— 25	— 04	—3.3
21.3	14	25	— 01	— 04	— 36	— 30	— 21	— 13	—3.3
21.5	14	29	00	— 03	+ 04	— 10	— 19	— 04	—3.5
22.1	14	32	— 04	— 11	— 16	— 31	— 17	— 04	—3.5
22.3	13	35	+ 01	— 06	— 30	— 34	— 23	— 05	—3.5
22.5	12	37	— 07	— 06	— 04	— 10	+ 06	+ 09	—3.5
23.1	11	39	— 02	— 13	— 16	— 11	— 08	— 06	—5.7
23.3	12	41	— 10	— 02	— 08	— 12	— 20	— 11	—5.7
23.5	12	43	— 14	— 11	— 24	— 04	— 12	— 18	—5.7

+ 10° + 15°

AR	D	β	Δ′₁	Δ′₂	Δ₃	Δ₄	Δ₅	Δ₆	Region
0.1	10	49	+ 03	— 04	— 16	— 16	— 17	— 24	—5.7
0.3	10	50	— 05	— 03	— 03	— 24	— 19	— 08	—5.7
0.5	10	50	— 01	— 04	— 21	— 27	— 18	— 10	—5.10
1.1	10	49	— 13	— 09	— 20	— 44	— 14	+ 02	—5.10
1.3	11	48	— 05	— 12	— 26	— 33	— 19	+ 04	—5.10
1.5	11	46	— 06	— 09	— 16	— 16	— 09	— 07	—5.10
2.1	11	44	— 10	— 14	— 28	— 15	— 13	— 06	—5.13
2.3	11	42	+ 02	— 06	— 03	— 03	— 07	— 08	—5.13
2.5	10	39	— 11	— 14	— 27	— 10	— 07	00	—3.13
3.1	8	36	— 03	— 06	— 09	— 20	— 26	— 22	—3.13
3.3	9	33	— 08	— 08	— 25	— 36	— 26	— 13	—3.15
3.5	10	29	— 08	— 12	— 32	— 10	— 09	— 14	—3.15
4.1	8	26	+ 11	+ 24	+ 10	— 19	— 24	— 26	—3.15
4.3	10	22	+ 14	+ 14	— 26	— 30	— 30	— 32	—3.15
4.5	16	17	+ 05	— 06	— 04	— 01	— 10	— 13	—1.15
5.1	16	13	— 21	— 15	— 20	— 10	— 03	— 05	—1.15
5.3	21	9	— 09	+ 04	+ 02	— 05	+ 01	+ 03	—1.15
5.5	25	5	+ 15	+ 08	+ 04	+ 09	+ 17	+ 18	—1.17
6.1	29	1	+ 22	+ 25	+ 15	+ 09	+ 21	+ 26	—1.17
6.3	27	3	+ 18	+ 13	— 02	+ 03	+ 19	+ 02	+1.17
6.5	29	8	+ 07	00	— 02	+ 09	+ 20	+ 22	+1.17
7.1	24	12	— 17	— 12	— 07	— 01	+ 11	+ 18	+1.17
7.3	19	17	+ 01	+ 02	— 08	— 12	+ 08	+ 17	+1.17
7.5	18	21	— 06	— 16	— 01	+ 15	+ 13	+ 12	+3.17
8.1	15	25	+ 03	— 11	— 14	— 14	— 06	+ 01	+3.17
8.3	15	30	+ 08	— 02	+ 02	— 02	00	+ 07	+3.19
8.5	13	34	— 04	— 12	— 22	— 11	— 03	— 01	+3.19

AR	D	β	Δ′₁	Δ′₂	Δ₃	Δ₄	Δ₅	Δ₆	Region
9.1	12	39	— 07	— 18	— 35	— 17	— 06	— 02	+3.19
9.3	12	43	+ 02	+ 13	+ 03	00	00	+ 02	+5.19
9.5	11	47	— 05	— 13	— 21	— 14	— 09	— 05	+5.19
10.1	11	52	— 01	— 06	— 17	— 11	— 07	— 05	+5.19
10.3	10	56	— 08	— 06	— 16	— 01	— 02	— 10	+5.19
10.5	9	60	— 04	— 05	— 04	— 03	— 05	— 01	+5.19
11.1	8	64	00	+ 06	— 07	— 29	— 12	+ 02	+7.21
11.3	9	67	— 04	— 06	— 23	— 37	— 01	+ 20	+7.21
11.5	8	70	— 07	— 17	— 26	00	+ 13	+ 07	+7.21
12.1	8	73	— 11	— 17	— 56	— 12	+ 06	+ 09	+7.27
12.3	8	75	— 03	+02	+ 06	+ 01	+ 05	+ 14	+7.27
12.5	8	75	+ 09	— 04	— 15	— 13	00	+ 01	+7.27
13.1	7	74	+ 01	+ 04	— 10	— 25	— 09	+ 03	+7.27
13.3	8	72	+ 05	— 02	— 16	— 18	— 04	+ 01	+7.33
13.5	7	68	+ 01	— 03	+ 13	— 02	— 05	— 02	+7.33
14.1	9	64	— 04	00	— 22	— 26	— 25	— 08	+7.33
14.3	9	60	— 04	— 02	+ 02	— 01	— 06	— 05	+7.33
14.5	10	56	— 13	— 14	— 22	— 03	+ 04	+ 03	+5.34
15.1	10	52	— 09	+ 01	+ 12	+ 02	— 01	— 04	+5.34
15.3	11	48	+ 07	+ 07	+ 15	— 02	— 23	— 16	+5.34
15.5	11	44	— 02	— 03	+ 03	— 05	— 02	+ 03	+5.34
16.1	13	39	— 15	— 08	+ 08	+ 01	+ 05	+ 12	+3.35
16.3	13	35	+ 01	— 09	— 23	— 16	— 11	— 07	+3.35
16.5	16	30	+ 08	+ 22	+ 15	— 06	— 09	— 07	+3.35
17.1	13	26	+ 03	— 17	— 34	— 22	— 15	— 11	+3.1
17.3	16	22	+ 02	+ 06	+ 06	— 07	— 09	— 03	+3.1
17.5	18	17	— 16	— 15	— 27	— 22	— 11	— 05	+1.1
18.1	26	13	— 04	— 06	+ 07	00	— 06	00	+1.1
18.3	24	8	— 14	00	+ 03	— 08	— 09	— 06	+1.1
18.5	30	4	— 10	+ 05	+ 06	+ 01	— 02	+ 01	+1.1
19.1	23	1	+ 02	+ 14	+ 08	00	— 08	— 10	—1.1
19.3	29	5	+ 11	+ 03	— 04	+ 02	+ 05	+ 07	—1.1
19.5	29	9	+ 07	+ 09	+ 10	00	+ 02	+ 12	—1.1
20.1	24	13	— 04	+ 09	+ 09	— 02	— 01	+ 06	—1.3
20.3	24	17	+ 21	+ 15	— 10	— 01	+ 10	+ 10	—1.3
20.5	20	21	— 02	— 11	+ 02	— 09	— 04	+ 12	—3.3
21.1	17	25	— 18	— 22	— 19	— 13	— 03	+ 02	—3.3
21.3	15	29	— 08	— 02	— 16	— 18	— 10	— 07	—3.3
21.5	14	32	— 04	— 02	— 05	— 16	— 10	— 05	—3.3
22.1	13	36	— 07	— 02	— 02	— 18	— 18	— 09	—3.5
22.3	14	39	+ 02	— 06	— 01	— 07	— 12	— 04	—3.5
22.5	12	42	— 02	— 13	— 37	— 12	— 13	— 10	—5.4
23.1	12	44	— 10	— 15	— 65	— 40	— 23	— 17	—5.4
23.3	11	46	— 14	— 24	— 112	— 32	— 08	— 13	—5.7
23.5	8	48	— 14	— 10	— 08	— 15	— 14	— 11	—5.7

+ 5° + 10°

AR	D	β	Δ′₁	Δ′₂	Δ₃	Δ₄	Δ₅	Δ₆	Region
0.1	10	54	— 05	— 05	— 02	— 13	— 26	— 11	—5.7
0.3	10	55	— 09	— 05	— 22	— 23	— 14	— 08	—5.7
0.5	11	55	+ 07	— 08	— 09	— 01	— 01	— 02	—5.10
1.1	10	54	— 05	+ 01	— 07	— 15	— 06	— 01	—5.10
1.3	11	53	— 09	00	— 08	— 31	— 15	+ 04	—5.10
1.5	10	51	— 13	— 11	— 02	— 02	— 12	— 06	—5.10
2.1	10	49	— 10	— 08	— 07	— 05	— 06	— 04	—5.13
2.3	9	46	+ 18	+ 02	— 19	— 14	— 06	— 06	—5.13
2.5	10	43	— 10	— 15	— 32	00	+ 08	— 01	—5.13
3.1	9	40	— 07	— 07	— 10	— 06	— 09	— 14	—5.13

AR	D	β	Δ'_1	Δ'_2	Δ_3	Δ_4	Δ_5	Δ_6	Region
3.3	9	36	— 08	— 08	— 04	— 07	— 18	— 17	—3.15
3.5	11	32	+ 12	+ 16	— 12	— 20	— 14	— 13	—3.15
4.1	11	28	+ 23	+ 27	— 05	— 34	— 31	— 26	—3.15
4.3	14	24	+ 02	+ 03	— 02	— 18	— 17	— 13	—3.15
4.5	18	20	— 03	+ 14	+ 02	— 09	— 08	+ 03	—3.15
5.1	20	16	+ 04	— 10	— 07	+ 08	+ 09	+ 06	—1.17
5.3	21	12	+ 03	00	— 09	+ 05	+ 08	+ 04	—1.17
5.5	23	8	+ 02	— 13	— 24	00	+ 07	+ 05	—1.17
6.1	29	3	+ 05	— 05	— 02	+ 06	+ 14	+ 15	—1.17
6.3	30	1	+ 05	+ 13	+ 16	+ 19	+ 18	+ 14	+1.17
6.5	29	5	+ 06	+ 08	+ 07	+ 10	+ 18	+ 18	+1.17
7.1	26	10	+ 07	— 04	+ 04	+ 18	+ 26	+ 26	+1.17
7.3	22	14	+ 04	— 13	— 18	+ 11	+ 17	+ 20	+1.17
7.5	19	19	— 03	— 04	— 04	+ 07	+ 16	+ 16	+1.19
8.1	17	23	— 14	— 14	+ 02	+ 10	+ 11	+ 13	+3.19
8.3	14	28	+ 11	+ 01	— 10	— 43	— 03	00	+3.19
8.5	12	32	+ 08	+ 06	— 13	— 40	— 03	00	+3.19
9.1	12	36	— 12	— 17	— 10	— 22	— 03	— 10	+3.19
9.3	9	41	— 03	— 01	— 21	— 22	— 13	— 07	+5.19
9.5	11	45	+ 02	00	— 09	— 20	— 13	— 04	+5.19
10.1	11	49	— 06	— 03	— 17	— 18	— 05	+ 02	+5.19
10.3	10	53	+ 03	— 11	— 33	— 09	— 02	— 01	+5.19
10.5	10	57	— 01	— 10	— 35	— 05	+ 02	— 04	+5.22
11.1	9	60	— 08	— 10	— 22	— 10	— 01	+ 02	+5.22
11.3	9	63	— 12	— 05	— 14	— 11	— 04	— 03	+7.21
11.5	8	66	+ 04	— 05	— 47	— 20	— 14	— 05	+7.21
12.1	8	68	— 04	— 17	— 39	+ 02	+ 08	+ 03	+7.27
12.3	8	70	— 04	+ 02	+ 11	+ 08	+ 02	— 01	+7.27
12.5	8	70	— 08	— 21	— 56	— 22	— 08	+ 04	+7.27
13.1	7	69	— 04	— 02	— 23	— 22	— 15	— 10	+7.27
13.3	10	67	— 04	— 01	— 10	— 22	— 10	+ 01	+7.33
13.5	10	64	+ 04	+ 03	— 06	— 23	— 10	+ 01	+7.33
14.1	9	61	00	— 10	— 11	— 03	— 11	— 05	+7.33
14.3	10	57	— 09	— 14	— 28	— 11	— 07	— 04	+5.31
14.5	11	53	— 13	— 09	— 04	— 13	— 16	— 10	+5.34
15.1	10	49	— 10	— 10	— 26	— 25	— 18	— 08	+5.34
15.3	12	45	— 10	— 06	— 22	— 08	+ 01	— 01	+5.34
15.5	12	41	— 03	— 12	— 29	— 28	— 46	+ 05	+5.34
16.1	14	37	+ 09	+ 07	— 02	— 07	+ 03	+ 04	+3.35
16.3	13	33	— 16	— 02	— 11	— 24	— 09	+ 02	+3.35
16.5	12	28	+ 03	+ 11	— 21	— 32	— 25	— 25	+3.35
17.1	14	24	— 10	— 10	— 06	— 07	— 09	— 10	+3.35
17.3	18	19	— 11	— 10	— 05	00	+ 02	+ 01	+1.35
17.5	20	15	— 12	+ 05	00	— 20	— 20	— 08	+1.1
18.1	27	10	— 09	— 03	— 08	— 10	— 06	— 01	+1.1
18.3	33	6	+ 02	— 06	+ 03	+ 04	+ 06	+ 11	+1.1
18.5	22	2	— 15	— 10	— 14	— 16	— 13	— 12	+1.1
19.1	23	3	— 11	— 06	+ 01	— 08	— 19	— 18	—1.1
19.3	29	7	— 18	— 03	— 12	— 05	+ 04	+ 07	—1.1
19.5	27	11	+ 11	— 03	— 12	— 01	00	— 01	—1.1
20.1	22	16	— 16	— 06	— 03	— 12	— 09	+ 03	—1.1
20.3	19	20	— 07	— 16	— 17	— 03	+ 01	+ 05	—1.1
20.5	18	24	— 02	+ 01	00	— 09	— 08	— 01	—3.3
21.1	15	28	+ 11	+ 11	— 08	— 13	— 13	— 10	—3.3
21.3	15	32	— 04	+ 01	— 04	00	+ 03	+ 05	—3.3
21.5	13	36	— 12	— 08	— 12	— 17	— 06	+ 06	—3.3
22.1	14	39	— 03	— 12	— 29	+ 03	+ 05	— 01	—3.3

AR	D	β	Δ'_1	Δ'_2	Δ_3	Δ_4	Δ_5	Δ_6	Region
22.3	10	43	— 14	+ 18	— 37	— 17	— 13	— 12	—5.4
22.5	12	46	+ 02	— 06	— 17	— 08	— 04	— 04	—5.4
23.1	10	49	— 02	— 11	— 42	— 19	— 12	— 04	—5.4
23.3	10	51	+ 03	+ 05	— 20	— 10	00	— 06	—5.4
23.5	9	53	+ 03	00	— 10	— 23	— 11	— 01	—5.7
					0° + 5°				
0.1	11	59	— 12	— 08	— 04	— 02	— 06	— 10	—5.7
0.3	12	60	— 12	— 16	— 20	+ 02	+ 08	+ 13	—7.9
0.5	13	59	— 08	— 01	+ 06	+ 03	00	+ 09	—7.9
1.1	10	59	— 04	+ 04	— 07	— 12	— 02	+ 05	—5.10
1.3	11	57	— 09	— 13	— 40	— 04	+ 05	+ 05	—5.10
1.5	11	55	+ 03	— 02	— 60	— 34	— 14	— 01	—5.13
2.1	10	53	— 13	— 19	— 13	— 06	— 06	— 05	—5.13
2.3	10	50	— 01	— 08	— 21	— 14	— 05	+ 04	—5.13
2.5	11	47	+ 06	— 08	— 03	+ 08	+ 05	+ 04	—5.13
3.1	11	44	— 06	— 16	— 36	— 13	— 04	— 02	—5.13
3.3	11	40	— 07	— 11	— 31	— 15	— 01	+ 05	—3.15
3.5	8	36	— 16	— 12	— 35	— 21	— 15	— 16	—3.15
4.1	11	31	— 20	— 24	— 19	— 06	— 03	— 03	—3.15
4.3	14	27	— 05	— 13	— 34	— 17	— 04	+ 01	—3.17
4.5	19	23	+ 10	+ 05	— 04	+ 01	+ 12	+ 15	—3.17
5.1	23	19	+ 13	+ 24	+ 11	— 05	— 03	+ 02	—1.17
5.3	22	14	+ 24	+ 02	— 02	— 04	00	+ 04	—1.17
5.5	18	10	+ 15	+ 01	+ 04	— 10	— 12	— 08	—1.17
6.1	25	6	+ 02	+ 09	+ 17	+ 09	+ 07	+ 07	—1.17
6.3	33	1	+ 29	+ 05	+ 14	+ 10	+ 20	+ 23	—1.17
6.5	34	3	— 07	— 11	— 08	+ 07	+ 24	+ 27	+1.17
7.1	25	8	— 18	— 12	— 11	+ 06	+ 22	+ 24	+1.19
7.3	22	12	+ 03	— 11	+ 04	+ 08	+ 24	+ 19	+1.19
7.5	19	16	00	— 02	— 09	— 01	+ 11	+ 16	+1.19
8.1	18	21	— 19	— 16	+ 01	+ 13	+ 19	+ 21	+3.19
8.3	15	25	— 02	— 12	00	+ 09	+ 09	+ 10	+3.19
8.5	16	29	— 17	— 16	+ 10	+ 13	+ 10	+ 11	+3.19
9.1	13	34	— 08	— 19	— 02	+ 08	+ 04	+ 03	+3.19
9.3	10	38	— 15	— 18	— 06	00	— 09	— 13	+3.21
9.5	9	42	+ 02	— 10	— 20	— 18	— 05	— 04	+5.19
10.1	8	46	— 06	— 02	— 07	— 12	— 10	— 10	+5.19
10.3	9	50	— 17	— 05	+ 02	— 03	— 08	— 13	+5.22
10.5	8	54	— 01	00	— 02	— 05	— 06	— 09	+5.22
11.1	9	57	+ 07	+ 01	— 11	— 08	— 03	— 04	+5.22
11.3	10	60	00	— 14	— 11	+ 01	— 03	— 03	+5.22
11.5	9	62	+ 04	— 03	— 13	— 12	— 09	— 08	+7.21
12.1	8	64	— 08	— 01	— 07	— 06	— 08	— 04	+7.27
12.3	9	65	— 08	— 13	— 31	— 08	+ 03	+ 02	+7.27
12.5	8	65	— 08	— 13	— 16	+ 02	— 02	— 09	+7.27
13.1	8	64	— 08	— 01	+ 01	00	— 03	— 06	+7.27
13.3	8	62	— 08	— 06	— 26	— 23	— 17	— 15	+7.27
13.5	8	60	— 04	— 10	— 13	— 10	— 06	+ 03	+7.33
14.1	8	57	— 01	+ 23	+ 12	— 10	— 06	— 06	+5.31
14.3	10	53	— 13	— 06	+ 05	— 03	— 04	+ 07	+5.31
14.5	10	50	+ 11	— 05	— 10	00	+ 02	— 02	+5.31
15.1	10	46	— 06	+ 01	— 12	— 43	— 32	— 17	+5.34
15.3	9	42	— 10	— 07	— 09	— 21	— 29	— 22	+5.34
15.5	9	38	— 03	— 07	— 12	— 13	— 18	— 22	+3.33
16.1	13	34	— 12	— 01	— 15	— 32	— 20	— 09	+3.35
16.3	13	30	— 01	+ 10	00	— 18	— 18	— 14	+3.35

AR	D	β	Δ'_1	Δ'_2	Δ_3	Δ_4	Δ_5	Δ_6	Region
16.5	15	26	— 10	—13	— 36	— 32	— 19	— 13	+3.35
17.1	13	21	— 11	+ 07	— 03	— 22	— 18	— 16	+3.35
17.3	16	17	00	+ 04	— 10	— 13	— 12	— 12	+1.35
17.5	19	13	+ 19	+ 28	+ 12	— 01	— 05	— 05	+1.35
18.1	21	8	— 02	+ 13	— 06	— 16	— 12	— 12	+1.35
18.3	20	4	— 10	— 09	— 08	— 05	— 13	— 23	+1.1
18.5	20	1	— 03	+ 03	+ 02	— 09	— 16	— 20	—1.1
19.1	22	5	— 06	+ 07	— 08	— 19	— 18	— 15	—1.1
19.3	21	10	+ 07	— 08	— 12	— 12	— 14	— 16	—1.1
19.5	22	14	— 08	— 12	— 17	— 18	— 10	— 03	—1.1
20.1	17	18	— 16	— 13	— 16	— 13	— 09	— 11	—1.1
20.3	16	23	— 14	— 20	— 28	— 18	— 08	+ 02	—3.1
20.5	13	27	— 01	+ 09	+ 04	— 02	— 03	— 04	—3.1
21.1	12	31	— 12	— 10	— 05	00	— 08	— 11	—3.3
21.3	11	35	— 04	— 01	— 04	— 13	— 15	— 08	—3.3
21.5	10	39	— 11	— 19	— 17	— 13	— 17	— 07	—3.3
22.1	8	43	— 14	— 14	— 32	— 35	— 24	— 11	—5.4
22.3	10	46	— 02	— 05	— 04	00	+ 02	+ 04	—5.4
22.5	9	50	+ 03	+ 17	+ 13	— 02	— 04	— 03	—5.4
23.1	9	53	— 01	— 02	— 09	— 03	+ 02	+ 08	—5.4
23.3	8	55	— 01	— 01	— 14	— 11	— 04	— 05	—5.4
23.5	9	57	— 03	— 14	— 13	+ 03	+ 05	+ 04	—5.7

0° — 2°

AR	D	β	Δ'_1	Δ'_2	Δ_3	Δ_4	Δ_5	Δ_6	Region
0.1	11	62	— 06	— 06	— 01	+ 04	+ 01	— 03	—7.9
0.3	10	63	+ 04	— 01	— 03	— 06	— 05	+ 05	—7.9
0.5	11	63	— 06	— 06	— 32	+ 01	+ 16	+ 10	—7.9
1.1	10	62	+ 14	— 02	— 06	— 03	— 07	— 12	—7.9
1.3	9	61	— 06	— 12	— 36	— 05	+ 16	+ 16	—7.9
1.5	10	59	+ 04	— 12	— 19	— 07	+ 02	+ 10	—5.13
2.1	9	56	+ 13	— 02	— 17	— 11	+ 09	+ 06	—5.13
2.3	12	53	+ 23	+ 17	— 07	— 14	+ 10	+ 20	—5.13
2.5	11	50	— 17	— 17	— 14	— 02	— 08	+ 03	—5.13
3.1	11	46	+ 02	— 04	— 20	— 17	— 05	+ 03	—5.13
3.3	8	42	+ 12	+ 19	— 04	— 11	— 08	— 22	—5.16
3.5	10	38	+ 11	+ 21	+ 11	— 04	+ 03	+ 12	—3.15
4.1	12	34	00	— 17	+ 15	+ 06	— 03	+ 08	—3.17
4.3	14	30	— 01	— 18	— 30	+ 06	+ 14	+ 16	—3.17
4.5	16	25	— 12	— 27	— 20	+ 19	+ 24	+ 16	—3.17
5.1	20	21	+ 57	+ 57	+ 21	— 05	— 03	+ 04	—3.17
5.3	26	17	+ 86	+ 43	+ 04	+ 16	+ 32	+ 37	—1.17
5.5	18	12	— 05	— 16	+ 10	+ 08	— 09	— 22	—1.17
6.1	25	8	— 16	+ 20	+ 23	+ 26	+ 25	+ 39	—1.17
6.3	26	3	+ 13	— 22	— 16	00	+ 18	+ 42	—1.17
6.5	35	2	+ 23	+ 08	+ 13	+ 12	+ 39	+ 63	+1.19
7.1	29	6	— 06	— 16	— 27	— 01	+ 27	+ 52	+1.19
7.3	21	11	— 15	— 21	— 15	— 04	+ 14	+ 22	+1.19
7.5	21	15	— 14	— 10	00	+ 14	+ 16	+ 15	+1.19
8.1	17	19	— 03	— 05	+ 04	— 02	+ 11	+ 20	+1.19
8.3	16	24	— 12	+ 02	— 08	— 01	+ 07	+ 04	+3.19
8.5	14	28	+ 09	— 28	— 19	+ 12	+ 10	+ 06	+3.19
9.1	12	32	— 20	+ 04	+ 02	— 08	— 01	+ 03	+3.19
9.3	9	36	+ 30	+ 14	— 39	— 44	— 16	— 07	+3.21
9.5	9	40	— 19	— 15	— 10	— 12	— 05	+ 06	+5.19
10.1	9	44	— 18	— 04	— 04	— 03	— 03	— 05	+5.22
10.3	8	48	+ 02	— 04	— 41	— 29	— 02	+ 10	+5.22
10.5	7	51	+ 33	+ 16	— 18	— 20	+ 04	+ 24	+5.22

AR	D	β	Δ'_1	Δ'_2	Δ_3	Δ_4	Δ_5	Δ_6	Region
11.1	8	54	— 17	— 14	+ 06	+ 10	+ 09	+ 03	+5.22
11.3	9	57	+ 13	— 03	— 17	— 03	— 03	— 09	+5.22
11.5	10	59	— 06	— 13	— 31	— 16	— 15	— 18	+5.25
12.1	9	61	+ 04	— 03	— 23	— 38	— 42	— 14	+7.27
12.3	9	62	+ 04	— 12	— 23	00	— 17	— 13	+7.27
12.5	10	62	— 16	— 22	— 106	— 32	+ 02	+ 20	+7.27
13.1	8	60	+ 04	— 22	— 10	— 10	— 20	— 08	+7.27
13.3	8	59	+ 14	— 12	— 15	+ 03	— 03	00	+5.28
13.5	9	57	— 07	— 12	— ∞	— 08	— 21	— 01	+5.31
14.1	9	54	+ 03	+ 08	— 09	— 22	— 16	— 09	+5.31
14.3	9	51	— 07	+ 07	— 12	— 22	— 18	— 08	+5.31
14.5	8	48	+ 12	— 04	— 43	— 24	— 04	+ 06	+5.31
15.1	8	44	+ 02	+ 18	+ 17	+ 06	+ 04	— 02	+5.31
15.3	8	40	— 09	+ 05	— 15	— 25	— 11	— 10	+5.34
15.5	8	36	— 10	— 15	— 07	— 01	— 21	— 26	+3.33
16.1	12	32	— 10	+ 05	+ 13	— 01	— 11	— 18	+3.33
16.3	12	28	— 11	+ 12	+ 05	— 06	— 05	— 13	+3.35
16.5	9	24	— 12	00	— 14	— 67	— 61	— 36	+3.35
17.1	10	20	+ 17	— 02	— 32	— 29	— 28	— 36	+3.35
17.3	13	16	— 04	— 13	— 30	— 20	— 24	— 33	+1.35
17.5	12	11	— 15	— 15	— 19	— 33	— 41	— 37	+1.35
18.1	11	7	— 16	— 26	— 23	— 22	— 37	— 46	+1.35
18.3	14	2	+ 03	+ 11	— 02	— 21	— 27	— 33	+1.35
18.5	19	3	— 07	— 07	— 02	+ 01	— 06	— 24	—1.1
19.1	18	7	+ 14	+ 03	— 17	— 40	— 30	— 19	—1.1
19,3	18	12	— 05	— 06	— 06	— 06	— 07	— 05	—1.1
19.5	14	16	— 14	— 04	— 25	— 47	— 32	— 26	—1.1
20.1	18	20	+ 17	+ 07	+ 04	+ 08	+ 09	+ 07	—1.1
20.3	15	25	— 02	+ 10	— 03	+ 02	+ 06	+ 10	—3.1
20.5	13	29	— 11	+ 22	+ 17	00	00	00	—3.1
21.1	12	33	— 20	+ 03	+ 13	— 04	— 09	— 01	—3.1
21.3	8	37	— 09	— 16	— 09	— 22	— 35	— 30	—3.3
21.5	9	41	— 09	+ 05	— 06	+ 02	+ 05	+ 02	—5.1
22.1	10	45	+ 12	— 14	— 19	+ 07	+ 08	+ 12	—5.1
22.3	11	49	+ 02	— 03	— 12	+ 15	+ 14	+ 13	—5.4
22.5	9	52	+ 03	— 23	00	— 01	— 12	— 04	—5.4
23.1	10	55	— 17	— 22	— 16	— 24	— 27	+ 01	—5.4
23.3	10	58	+ 04	— 14	+ 07	— 03	— 13	+ 04	—5.4
23.5	10	61	— 06	— 15	+ 08	+ 03	— 09	00	—7.9

— 2° — 7°

AR	D	β	Δ'_1	Δ'_2	Δ_3	Δ_4	Δ_5	Δ_6	Region
0.1	7	66	+ 04	+ 06	+ 10	+ 02	— 12	— 13	—7.9
0.3	7	67	— 04	+ 05	— 06	— 28	— 13	— 01	—7.9
0.5	8	67	— 08	— 01	+ 05	— 02	— 02	+ 01	—7.9
1.1	8	66	00	+ 07	+ 14	+ 04	+ 04	+ 02	—7.9
1.3	9	64	— 08	+ 01	00	— 06	— 02	+ 04	—7.9
1.5	9	61	— 08	— 07	+ 02	— 09	— 03	+ 11	—7.15
2.1	8	59	— 04	— 08	— 19	— 31	— 06	+ 07	—7.15
2.3	8	55	— 05	— 08	— 20	— 31	00	— 01	—5.13
2.5	8	52	+ 03	00	— 05	00	— 08	— 12	—5.13
3.1	10	48	— 06	— 07	— 15	— 21	— 09	— 03	—5.16
3.3	9	44	— 02	— 04	— 21	— 16	— 09	00	—5.16
3.5	11	40	— 07	— 08	— 09	— 08	— 04	+ 03	—5.16
4.1	13	35	— 08	— 10	+ 07	+ 07	+ 04	+ 07	—3.17
4.3	13	31	00	— 04	— 22	— 07	— 03	+ 01	—3.17
4.5	15	27	+ 15	+ 11	— 04	+ 41	— 02	+ 05	—3.17
5.1	16	22	+ 14	+ 02	— 06	00	+ 09	+ 13	—3.17

AR	D	β	Δ'_1	Δ'_2	Δ_3	Δ_4	Δ_5	Δ_6	Region
5.3	20	18	+ 72	+ 54	+ 19	+ 17	+ 23	+ 24	—1.17
5.5	17	14	+ 04	+ 17	— 11	— 14	+ 03	+ 08	—1.17
6.1	21	9	+ 22	+ 22	+ 13	+ 02	+ 09	+ 15	—1.19
6.3	22	5	+ 02	— 03	— 17	— 01	+ 12	+ 16	—1.19
6.5	23	0	+ 05	+ 06	— 02	00	+ 16	+ 23	—1.19
7.1	23	4	— 06	— 05	— 08	— 03	+ 15	+ 22	+1.19
7.3	22	8	— 10	— 10	— 07	+ 05	+ 14	+ 16	+1.19
7.5	22	13	— 05	+ 08	+ 05	+ 10	+ 21	+ 24	+1.19
8.1	20	17	— 04	+ 10	+ 12	+ 09	+ 20	+ 27	+1.19
8.3	14	22	+ 06	— 06	— 03	+ 01	+ 06	+ 07	+3.19
8.5	13	26	— 02	+ 05	— 06	— 06	+ 10	+ 09	+3.21
9.1	12	30	— 13	— 13	— 14	— 02	+ 04	+ 05	+3.21
9.3	12	34	+ 04	00	+ 09	+ 08	+ 10	+ 07	+3.21
9.5	11	38	— 15	— 09	+ 03	— 14	— 07	+ 06	+3.21
10.1	9	41	— 07	00	+ 04	+ 01	— 08	— 08	+5.22
10.3	8	45	— 06	— 12	— 36	— 40	— 31	— 12	+5.22
10.5	8	48	— 10	— 16	— 19	— 14	— 06	+ 01	+5.22
11.1	8	51	— 09	— 10	— 28	— 37	— 17	00	+5.22
11.3	8	53	— 09	— 05	— 29	— 14	00	— 01	+5.22
11.5	8	55	— 13	00	+ 01	— 07	— 17	— 07	+5.25
12.1	9	57	— 13	— 06	— 01	+ 01	— 04	— 04	+5.25
12.3	8	58	— 04	00	— 06	— 12	— 09	— 09	+5.25
12.5	9	58	+ 04	+ 01	+ 07	— 11	— 04	+ 03	+5.28
13.1	8	56	— 05	— 08	— 19	— 10	— 08	— 06	+5.28
13.3	7	55	— 01	— 01	— 06	— 21	— 16	— 08	+5.28
13.5	7	53	— 13	— 01	— 05	— 02	— 01	— 03	+5.28
14.1	8	51	— 01	+ 07	— 03	— 04	— 06	— 04	+5.31
14.3	8	48	— 06	— 09	— 11	+ 04	— 04	— 08	+5.31
14.5	7	45	— 02	— 06	— 27	— 18	— 23	— 19	+5.31
15.1	10	42	— 10	— 09	00	— 06	— 16	— 15	+5.31
15.3	9	38	— 15	— 15	— 36	— 10	— 09	— 13	+3.33
15.5	10	34	— 04	00	— 13	— 13	— 14	— 18	+3.33
16.1	10	30	+ 07	— 05	— 40	— 39	— 32	— 22	+3.33
16.3	8	26	— 14	— 19	— 36	— 38	— 35	— 40	+3.33
16.5	9	22	— 10	— 17	— 28	— 28	— 44	— 47	+3.35
17.1	7	18	— 08	— 12	— 26	— 46	— 51	— 58	+1.35
17.3	11	14	— 08	— 21	— 33	— 34	— 42	— 45	+1.35
17.5	10	9	— 14	— 12	— 34	— 41	— 38	— 43	+1.35
18.1	11	5	— 06	— 17	— 33	— 38	— 37	— 41	+1.35
18.3	15	0	— 07	— 12	— 22	— 40	— 43	— 36	+1.35
18.5	17	4	+ 02	— 11	— 27	— 37	— 34	— 23	—1.35
19.1	17	8	— 14	— 25	— 35	— 22	— 22	— 25	—1.35
19.3	17	13	— 17	— 23	— 42	— 23	— 17	— 16	—1.1
19.5	14	17	— 16	— 14	— 25	— 38	— 37	— 27	—1.1
20.1	15	22	— 22	— 11	— 17	— 14	— 16	— 16	—3.1
20.3	14	36	— 02	— 05	— 18	— 12	— 10	— 02	—3.1
20.5	13	30	+ 07	+ 04	— 08	— 20	— 17	— 11	—3.1
21.1	11	35	— 04	— 01	— 01	— 06	— 04	— 09	—3.1
21.3	11	39	— 07	— 06	— 24	— 26	— 05	+ 04	—3.1
21.5	10	43	+ 06	— 03	— 09	— 15	— 14	— 02	—5.1
22.1	10	47	+ 06	+ 01	+ 01	— 09	— 04	+ 02	—5.1
22.3	9	51	— 01	— 06	— 37	— 29	— 10	— 12	—5.1
22.5	9	55	— 01	+ 08	— 03	— 17	— 06	+ 04	—5.4
23.1	8	58	00	+ 05	— 12	— 28	— 10	+ 01	—5.4
23.3	8	61	— 08	00	— 09	— 28	— 14	+ 04	—7.3
23.5	9	64	+ 08	+ 05	— 03	+ 06	+ 04	+ 01	—7.3

— 7° — 12°

AR	D	β	Δ'_1	Δ'_2	Δ_3	Δ_4	Δ_5	Δ_6	Region
0.1	9	70	+ 01	— 01	— 32	— 07	+ 05	+ 01	—7.9
0.3	9	72	— 11	— 14	— 41	— 18	— 08	+ 02	—7.9
0.5	8	72	+ 05	+ 05	— 14	— 20	— 20	— 14	—7.9
1.1	8	70	+ 21	+ 12	— 14	— 28	— 21	— 03	—7.9
1.3	8	68	+ 01	— 07	— 11	— 04	+ 04	00	—7.15
1.5	9	66	— 04	— 05	— 04	— 13	— 01	+ 06	—7.15
2.1	9	63	— 08	— 07	— 08	— 21	— 08	+ 02	—7.15
2.3	9	59	— 04	— 07	— 09	— 27	— 19	— 07	—5.13
2.5	9	55	+ 12	+ 13	00	— 21	— 18	— 15	—5.16
3.1	8	50	— 05	— 10	— 28	— 27	— 29	— 14	—5.16
3.3	10	46	+ 19	+ 17	+ 07	— 09	— 06	— 08	—5.16
3.5	10	42	— 14	— 14	— 30	— 28	— 01	— 02	—5.16
4.1	11	38	+ 10	+ 01	— 10	— 14	— 19	— 06	—3.17
4.3	11	34	+ 05	00	— 13	— 26	— 05	+ 02	—3.17
4.5	14	29	— 13	— 14	— 14	— 04	+ 04	+ 11	—3.17
5.1	15	25	+ 11	+ 03	— 09	+ 07	+ 10	+ 09	—3.17
5.3	14	20	+ 10	+ 10	+ 06	— 05	— 06	+ 02	—3.19
5.5	17	16	+ 09	+ 13	— 03	— 03	+ 03	+ 08	—1.19
6.1	19	11	+ 12	+ 19	+ 11	+ 02	+ 07	+ 11	—1.19
6.3	19	7	— 01	— 16	— 17	— 16	— 03	+ 06	—1.19
6.5	25	2	+ 06	+ 10	+ 04	00	+ 15	+ 26	—1.19
7.1	23	2	+ 10	— 11	— 08	— 07	+ 03	+ 08	+1.19
7.3	23	6	+ 02	00	— 09	— 18	— 03	+ 10	+1.19
7.5	23	10	— 13	— 14	— 21	— 27	— 03	+ 12	+1.19
8.1	21	15	— 04	—09	— 10	— 16	— 08	+ 02	+1.19
8.3	17	19	— 03	— 03	— 06	— 10	— 11	— 14	+1.21
8.5	13	23	— 18	— 26	— 20	— 12	— 13	— 12	+3.21
9.1	12	27	+ 03	00	— 11	— 12	— 12	— 11	+3.21
9.3	12	31	+ 12	+ 03	— 02	— 07	— 21	— 23	+3.21
9.5	12	35	— 12	— 06	— 14	— 24	— 08	— 02	+3.21
10.1	10	38	+ 14	+ 05	— 06	— 13	— 09	— 09	+3.21
10.3	10	41	— 06	— 08	— 03	— 09	— 04	— 02	+5.22
10.5	10	44	— 06	— 04	+ 01	00	+ 07	+ 08	+5.22
11.1	9	47	— 10	— 04	— 09	— 11	— 04	+ 03	+5.22
11.3	9	49	— 05	— 03	+ 01	— 18	— 20	— 06	+5.25
11.5	9	50	— 01	+ 01	+ 02	— 04	— 02	— 03	+5.25
12.1	9	52	— 13	— 07	— 07	— 05	— 02	+ 07	+5.25
12.3	10	53	— 05	+ 06	+ 02	— 05	+ 02	+ 08	+5.25
12.5	9	53	— 09	— 07	— 03	— 07	— 06	+ 02	+5.28
13.1	9	52	+ 03	+ 15	+ 21	+ 04	— 04	+ 03	+5.28
13.3	11	51	— 09	+ 03	— 03	+ 03	+ 12	+ 18	+5.28
13.5	9	49	— 05	+ 06	+ 11	+ 06	+ 04	+ 02	+5.28
14.1	10	47	+ 03	+ 11	+ 05	— 18	— 11	+ 01	+5.31
14.3	8	45	— 14	+ 01	— 11	— 27	— 08	+ 01	+5.31
14.5	9	42	+ 02	+ 01	+ 05	— 06	— 12	— 06	+5.31
15.1	10	39	— 11	— 10	— 13	— 07	— 08	— 02	+3.31
15.3	10	35	— 12	— 10	— 32	— 05	00	— 04	+3.33
15.5	11	31	— 12	— 14	— 29	— 14	— 05	+ 02	+3.33
16.1	11	28	— 05	+ 02	— 18	— 18	— 12	— 08	+3.33
16.3	8	24	+ 03	— 12	— 24	— 26	— 32	— 35	+3.33
16.5	8	20	— 11	— 22	— 53	— 36	— 35	— 35	+1.33
17.1	8	15	— 16	— 26	— 32	— 26	— 40	— 46	+1.33
17.3	10	11	— 05	— 17	— 43	— 44	— 39	— 42	+1.35
17.5	9	7	— 10	— 15	— 34	— 40	— 47	— 45	+1.35
18.1	11	2	— 10	— 15	— 20	— 28	— 43	— 45	+1.35
18.3	15	2	+ 02	— 02	— 32	— 49	— 37	— 21	—1.35

AR	D	β	△′₁	△′₂	△₃	△₄	△₅	△₆	Region
18.5	22	6	— 14	— 21	— 06	— 12	— 14	+ 04	—1.35
19.1	18	11	— 13	— 21	— 12	+ 18	— 07	— 10	—1.35
19.3	16	15	— 12	— 15	— 19	— 20	— 13	— 02	—1.35
19.5	16	20	+ 06	+ 01	+ 06	— 06	— 08	— 01	—1.35
20.1	15	24	— 10	— 01	— 08	— 07	+ 04	+ 11	—3.1
20.3	13	29	— 17	— 14	— 20	+ 03	+ 12	+ 12	—3.1
20.5	13	33	— 04	— 08	— 08	+ 03	+ 07	+ 08	—3.1
21.1	10	37	— 03	00	+ 09	+ 08	+ 03	+ 01	—3.1
21.3	10	42	— 06	— 03	— 16	+ 03	+ 12	+ 12	—5.1
21.5	10	46	— 06	— 07	+ 06	— 03	— 06	— 02	—5.1
22.1	10	50	— 01	+ 15	+ 02	— 08	+ 08	+ 01	—5.1
22.3	9	54	+ 08	+ 08	+ 15	— 02	00	+ 07	—5.1
22.5	9	58	+ 08	+ 03	— 05	— 12	— 04	+ 01	—5.1
23.1	9	62	+ 04	+ 02	— 21	+ 01	+ 09	+ 10	—7.3
23.3	8	65	— 12	— 11	— 04	+ 01	— 01	+ 02	—7.3
23.5	9	68	— 04	— 01	— 21	— 09	— 07	— 03	—7.3

— 12° — 17°

AR	D	β	△′₁	△′₂	△₃	△₄	△₅	△₆	Region
0.1	9	75	— 03	+ 02	+ 05	— 15	+ 01	+ 12	—7.3
0.3	8	77	— 07	— 06	— 12	— 26	— 19	— 11	—7.9
0.5	8	77	— 03	— 07	— 25	— 25	— 11	— 09	—7.9.
1.1	9	76	— 11	— 02	— 03	— 08	— 07	— 05	—7.9
1.3	8	73	+ 05	— 04	— 43	— 14	00	— 01	—7.15
1.5	8	69	— 12	— 09	— 14	— 13	— 01	+ 03	—7.15
2.1	8	66	— 16	— 09	— 09	— 10	— 11	— 11	—7.15
2.3	10	62	+ 08	— 04	— 10	— 02	— 01	+ 06	—7.15
2.5	9	58	— 12	— 13	— 24	— 12	— 15	— 16	—5.16
3.1	9	53	— 13	— 05	— 10	— 09	— 08	— 08	—5.16
3.3	9	49	— 09	— 02	+ 03	— 11	— 16	— 08	—5.16
3.5	9	44	+ 06	00	— 15	— 11	00	— 06	—5.16
4.1	10	40	— 15	— 11	— 18	— 08	— 01	— 02	—5.16
4.3	11	36	+ 05	+ 03	— 14	— 20	— 04	— 04	—3.17
4.5	12	31	+ 16	+ 15	— 02	+ 06	+ 12	+ 07	—3.19
5.1	13	27	+ 20	+ 22	00	+ 05	+ 10	+ 08	—3.19
5.3	14	22	— 18	— 20	— 01	+ 12	+ 15	+ 14	—3.19
5.5	14	18	+ 01	+ 11	+ 09	+ 04	+ 06	+ 07	—1.19
6.1	16	13	+ 12	+ 06	— 07	— 02	+ 10	+ 14	—1.19
6.3	17	9	— 01	00	+ 02	+ 05	+ 13	+ 17	—1.19
6.5	20	5	+ 06	— 05	— 08	+ 12	+ 17	+ 17	—1.19
7.1	22	0	— 06	— 02	+ 09	+ 05	+ 14	+ 20	—1.19
7.3	24	4	+ 06	+ 08	+ 01	+ 06	+ 19	+ 14	+1.19
7.5	24	8	+ 07	— 04	+ 03	+ 06	+ 15	+ 21	+1.21
8.1	22	12	+ 04	+ 16	+ 06	— 01	+ 08	+ 15	+1.21
8.3	20	16	— 04	— 03	— 12	+ 06	+ 13	+ 11	+1.21
8.5	14	20	— 07	— 03	— 11	— 10	+ 04	+ 04	+3.21
9.1	12	24	— 06	— 09	+ 04	— 02	— 06	— 07	+3.21
9.3	12	27	— 09	— 12	— 14	— 08	+ 03	+ 09	+3.21
9.5	12	31	— 16	00	+ 11	+ 04	+ 02	+ 09	+3.21
10.1	11	34	+ 01	+ 10	+ 08	— 02	+ 01	+ 10	+3.23
10.3	11	37	+ 01	+ 08	+ 02	+ 01	+ 03	+ 10	+3.23
10.5	9	40	+ 10	+ 05	+ 01	00	00	00	+5.22
11.1	8	42	— 14	— 15	— 29	— 12	— 12	— 06	+5.22
11.3	8	44	— 02	+ 05	00	— 15	— 09	— 08	+5.25
11.5	8	46	— 10	— 12	— 15	— 13	— 08	— 03	+5.25
12.1	9	47	— 05	— 07	— 16	— 06	— 02	— 13	+5.25
12.3	8	48	+ 11	+ 12	+ 06	— 04	— 07	— 10	+5.25
12.5	9	48	— 14	— 11	— 01	— 04	— 05	— 04	+5.28
13.1	9	47	— 14	+ 01	+ 04	— 12	— 10	— 09	+5.28
13.3	10	46	+ 03	— 03	— 19	— 23	— 07	+ 06	+5.28
13.5	8	45	— 06	— 11	— 05	— 07	— 17	— 18	+5.28
14.1	9	43	— 10	— 10	— 33	— 12	— 11	— 12	+5.28
14.3	9	40	— 19	— 18	— 31	— 38	— 13	— 02	+5.31
14.5	11	38	— 03	+ 03	— 09	— 03	— 07	— 10	+3.31
15.1	11	35	— 07	— 13	— 17	— 17	— 12	— 10	+3.31
15.3	11	31	+ 04	— 01	+ 02	— 14	— 13	— 15	+3.31
15.5	11	28	+ 04	— 03	— 20	— 21	— 18	— 19	+3.33
16.1	11	24	— 06	— 15	— 26	— 36	— 28	— 22	+3.33
16.3	9	20	— 15	— 22	— 39	— 31	— 28	— 32	+3.33
16.5	9	16	— 08	— 13	— 22	— 41	— 46	— 36	+1.33
17.1	15	13	— 12	— 01	— 10	— 22	— 28	— 19	+1.33
17.3	15	9	— 05	— 09	— 36	— 34	— 21	— 17	+1.33
17.5	16	5	— 18	— 32	— 50	— 19	— 23	— 22	+1.35
18.1	22	0	— 18	— 17	— 09	— 08	— 05	— 02	+1.35
18.3	19	4	— 02	— 08	— 01	— 03	— 03	— 03	—1.35
18.5	19	8	— 05	— 01	— 17	— 13	— 06	— 05	—1.35
19.1	20	13	— 04	— 02	+ 22	— 03	— 06	+ 10	—1.35
19.3	18	17	+ 01	— 04	+ 03	00	— 03	+ 01	—1.35
19.5	13	22	— 02	00	00	— 05	— 04	— 03	—3.35
20.1	15	26	+ 11	+ 10	— 04	+ 04	+ 05	+ 01	—3.35
20.3	14	31	+ 04	+ 06	+ 06	— 09	+ 03	+ 10	—3.35
20.5	12	35	— 07	00	+ 10	+ 01	+ 04	+ 04	—3.1
21.1	11	39	— 03	— 03	+ 11	+ 06	+ 01	+ 07	—3.1
21.3	11	44	— 06	+ 13	+ 07	— 06	+ 02	+ 06	—5.1
21.5	10	48	— 05	— 03	+ 02	— 08	— 09	+ 03	—5.1
22.1	11	53	+ 08	+ 05	— 06	— 01	+ 10	+ 14	—5.1
22.3	9	57	— 08	— 16	— 26	— 14	— 07	00	—5.1
22.5	10	61	+ 08	00	— 36	— 25	+ 04	+ 12	—7.3
23.1	9	65	00	+ 07	+ 08	— 06	— 04	00	—7.3
23.3	9	69	+ 13	+ 23	+ 07	— 05	— 03	— 06	—7.3
23.5	8	72	+ 05	+ 01	— 04	— 15	— 03	00	—7.3

— 17° — 22°

AR	D	β	△′₁	△′₂	△₃	△₄	△₅	△₆	Region
0.1	8	79	+ 06	+ 06	+ 11	+ 04	— 22	— 11	—7.3
0.3	9	81	— 03	+ 06	— 08	— 18	— 05	+ 08	—9
0.5	7	81	— 11	— 15	— 21	— 02	+ 04	— 02	—9
1.1	8	80	— 11	— 19	— 18	— 07	— 11	— 06	—9
1.3	8	76	— 11	— 15	— 07	— 02	— 06	— 03	—7.15
1.5	10	72	— 03	— 04	— 14	— 13	— 09	— 06	—7.15
2.1	8	68	+ 01	+ 04	— 05	— 17	— 04	+ 03	—7.15
2.3	8	64	— 08	— 17	— 19	— 17	— 16	— 06	—7.15
2.5	9	60	— 08	— 18	— 44	— 05	— 02	— 03	—5.16
3.1	9	55	— 13	— 09	— 07	— 05	— 07	— 08	—5.16
3.3	9	51	— 05	— 14	— 15	— 03	— 03	— 02	—5.16
3.5	11	46	— 05	— 05	+ 08	+ 08	+ 01	+ 02	—5.16
4.1	13	42	+ 02	+ 03	— 01	— 07	+ 11	+ 14	—5.19
4.3	12	38	— 02	+ 01	+ 02	+ 01	+ 04	+ 07	—3.19
4.5	12	33	— 07	— 14	— 13	— 16	— 11	— 02	—3.19
5.1	13	29	00	+ 02	+ 02	+ 01	— 01	— 01	—3.19
5.3	15	25	+ 16	+ 09	+ 05	+ 05	+ 11	+ 12	—3.19
5.5	15	20	— 10	— 14	— 20	+ 03	+ 09	+ 08	—3.19
6.1	17	16	+ 14	+ 17	— 06	— 09	+ 05	+ 06	—1.19
6.3	20	12	— 04	+ 07	+ 14	+ 13	+ 18	+ 22	—1.19
6.5	22	7	+ 12	+ 15	+ 12	+ 08	+ 16	+ 19	—1.19
7.1	25	3	— 06	+ 03	+ 14	+ 17	+ 26	+ 27	—1.21

AR	D	β	Δ'_1	Δ'_2	Δ_3	Δ_4	Δ_5	Δ_6	Region
7.3	25	1	+ 07	— 02	+ 04	+ 06	+ 22	+ 29	+1.21
7.5	25	5	— 10	— 03	+ 02	+ 09	+ 14	+ 13	+1.21
8.1	25	9	00	— 01	— 14	— 05	+ 17	+ 24	+1.21
8.3	21	13	— 04	— 08	— 03	+ 04	+ 13	+ 19	+1.21
8.5	15	17	— 07	— 11	— 24	— 18	00	+ 01	+1.21
9.1	13	20	— 15	— 20	— 17	— 18	— 15	— 06	+3.21
9.3	14	24	— 01	— 06	— 24	— 06	+ 04	+ 03	+3.21
9.5	12	27	— 09	— 14	— 26	— 02	— 02	— 09	+3.23
10.1	12	30	— 12	— 07	+ 06	— 02	— 04	— 01	+3.23
10.3	11	33	— 16	— 16	— 07	— 12	— 08	+ 02	+3.23
10.5	10	36	— 11	— 02	— 13	— 11	— 07	00	+3.23
11.1	11	38	— 06	— 01	— 01	+ 05	+ 08	+ 06	+3.25
11.3	10	40	— 06	— 10	— 20	— 14	— 06	— 01	+3.25
11.5	10	42	— 10	— 15	— 18	00	+ 06	+ 06	+5.25
12.1	10	43	— 06	00	+ 01	+ 11	+ 08	+ 05	+5.25
12.3	10	43	— 02	00	+ 10	00	+ 07	+ 09	+5.25
12.5	9	43	— 14	— 04	+ 02	+ 01	+ 01	+ 01	+5.28
13.1	9	42	+ 07	00	— 10	— 06	— 05	— 04	+5.28
13.3	9	41	— 14	— 10	— 13	— 07	— 02	— 02	+5.28
13.5	10	40	— 02	— 04	— 08	— 09	+ 03	+ 11	+5.28
14.1	10	38	— 02	— 08	+ 03	+ 06	00	00	+3.29
14.3	10	36	— 15	— 10	— 03	+ 03	00	— 05	+3.31
14.5	10	34	— 07	— 09	— 07	— 04	+ 02	+ 06	+3.31
15.1	11	31	00	+ 07	+ 02	— 20	— 14	— 03	+3.31
15.3	12	28	+ 04	— 04	— 05	— 14	— 09	— 03	+3.31
15.5	12	24	— 05	+ 07	+ 07	00	— 06	— 08	+3.31
16.1	10	20	+ 15	+ 09	— 04	— 26	— 34	— 38	+3.33
16.3	9	17	— 03	— 15	— 59	— 43	— 39	— 36	+1.33
16.5	15	14	— 08	— 03	— 06	— 13	— 08	— 11	+1.33
17.1	16	10	00	— 11	— 22	— 16	— 04	— 04	+1.33
17.3	15	6	— 22	— 18	— 03	— 34	— 18	— 19	+1.33
17.5	20	2	— 14	— 02	— 04	— 11	— 08	— 06	+1.33
18.1	25	2	+ 28	+ 01	— 10	— 04	+ 06	+ 11	—1.33
18.3	21	6	+ 03	+ 08	+ 06	— 13	— 10	— 03	—1.35
18.5	23	11	+ 17	+ 14	— 03	+ 05	+ 14	+ 16	—1.35
19.1	24	15	+ 14	+ 04	— 08	+ 13	+ 21	+ 17	—1.35
19.3	19	20	— 06	— 04	— 06	+ 04	+ 11	+ 12	—1.35
19.5	16	24	— 14	— 15	— 14	+ 02	+ 08	+ 08	—3.35
20.1	16	28	— 17	— 11	00	+ 16	+ 16	+ 12	—3.35
20.3	13	33	— 03	+ 05	+ 03	— 05	— 03	+ 08	—3.35
20.5	13	37	+ 02	+ 19	+ 14	+ 02	+ 10	+ 14	—3.35
21.1	11	41	+ 23	+ 19	+ 05	— 03	— 02	+ 02	—5.34
21.3	11	46	+ 07	+ 01	+ 04	+ 03	+ 03	+ 02	—5.34
21.5	9	50	+ 04	— 08	— 06	— 04	— 04	+ 02	—5.1
22.1	9	54	— 04	+ 04	00	— 05	+ 02	+ 10	—5.1
22.3	8	59	— 04	+ 05	— 09	— 02	+ 09	+ 10	—5.1
22.5	8	63	— 04	+ 03	— 10	— 10	+ 02	+ 04	—7.3
23.1	8	68	+ 01	+ 02	— 02	— 02	+ 10	+ 12	—7.3
23.3	7	72	+ 10	00	— 79	— 22	— 04	— 04	—7.3
23.5	9	76	+ 06	+ 07	+ 01	— 01	— 01	+ 07	—7.3

— 22° — 27°

AR	D	β	Δ'_1	Δ'_2	Δ_3	Δ_4	Δ_5	Δ_6	Region
0.1	12	82	+ 02	— 01	+ 16	+ 13	00	— 10	—9
0.3	14	86	+ 11	+ 19	+ 16	— 03	+ 04	+ 08	—9
0.5	13	86	+ 11	+ 03	+ 02	+ 09	+ 14	+ 01	—9
1.1	12	83	— 06	— 06	— 03	+ 08	+ 04	— 03	—9
1.3	12	78	— 02	+ 15	+ 14	— 13	— 07	— 04	—7.15
1.5	11	74	— 07	— 11	— 21	— 19	— 01	+ 03	—7.15
2.1	11	70	— 07	— 02	— 19	— 18	— 01	+ 03	—7.15
2.3	12	66	— 12	— 07	00	+ 05	+ 05	— 03	—7.21
2.5	14	61	+ 14	+ 13	+ 07	— 01	+ 02	+ 05	—7.21
3.1	13	57	+ 19	+ 13	— 03	— 16	— 02	— 03	—5.19
3.3	12	52	— 13	— 09	— 16	— 22	— 08	— 01	—5.19
3.5	14	48	00	+ 06	+ 05	— 04	— 04	— 02	—5.19
4.1	15	43	— 05	+ 07	+ 08	—10	— 14	— 06	—5.19
4.3	14	39	— 10	— 07	+ 06	— 02	— 08	— 19	—3.19
4.5	13	35	— 06	— 08	— 30	— 12	— 13	— 21	—3.19
5.1	17	30	— 03	— 04	— 14	— 16	— 14	— 15	—3.19
5.3	15	26	— 17	— 19	— 16	— 24	— 26	— 22	—3.19
5.5	19	22	+ 13	— 02	— 26	— 39	— 21	— 13	—3.19
6.1	25	17	+ 12	+ 07	+ 10	— 05	— 08	— 04	—1.19
6.3	24	13	+ 02	+ 03	— 16	— 25	— 15	— 14	—1.21
6.5	31	9	+ 23	+ 15	+ 17	— 01	+ 01	00	—1.21
7.1	29	5	+ 66	+ 46	+ 12	— 10	— 04	00	—1.21
7.3	26	1	+ 39	+ 46	+ 06	— 09	— 07	— 13	—1.21
7.5	25	3	+ 09	+ 06	— 08	— 06	— 08	— 13	+1.21
8.1	22	6	— 13	— 07	— 27	— 22	— 18	— 21	+1.21
8.3	26	10	+ 05	— 05	— 13	— 07	— 09	— 14	+1.21
8.5	20	14	— 11	— 17	— 19	— 19	— 15	— 18	+1.21
9.1	19	17	— 06	— 13	— 24	— 25	— 29	— 25	+1.21
9.3	23	21	— 01	+ 07	— 04	— 18	— 20	— 10	+3.23
9.5	22	24	+ 04	+ 04	00	— 19	— 20	— 14	+3.23
10.1	19	27	— 08	— 07	— 02	— 10	— 20	— 16	+3.23
10.3	19	29	+ 01	— 05	— 11	— 24	— 16	— 08	+3.23
10.5	14	31	— 16	— 10	— 27	— 13	— 15	— 23	+3.23
11.1	12	33	— 07	— 22	— 39	— 10	— 19	— 27	+3.25
11.3	14	35	— 11	— 09	— 12	— 20	— 23	— 11	+3.25
11.5	15	37	— 06	— 10	— 01	— 15	— 24	— 17	+3.25
12.1	17	38	+ 03	— 08	— 13	— 14	— 16	— 02	+3.27
12.3	13	38	— 06	— 12	— 52	— 28	— 16	— 06	+3.27
12.5	14	38	— 06	— 07	+ 03	— 10	— 19	— 20	+3.27
13.1	15	37	— 06	— 12	— 15	— 02	— 01	— 06	+3.27
13.3	13	36	— 06	— 07	— 08	— 15	— 20	— 18	+3.29
13.5	13	35	+ 07	+ 01	— 07	— 10	— 13	— 11	+3.29
14.1	16	34	+ 06	— 02	— 08	— 10	+ 01	+ 03	+3.29
14.3	15	32	— 16	— 10	+ 13	— 01	— 12	— 05	+3.29
14.5	14	30	+ 14	+ 02	— 08	— 07	— 12	— 16	+3.31
15.1	13	27	+ 01	— 01	00	— 12	— 10	— 19	+3.31
15.3	14	24	00	— 06	+ 09	— 10	— 20	— 18	+3.31
15.5	16	21	+ 26	+ 28	+ 08	— 09	— 25	— 18	+3.31
16.1	13	18	+ 20	+ 06	+ 04	— 15	— 24	— 26	+1.31
16.3	10	14	— 07	— 14	— 24	— 37	— 44	— 42	+1.33
16.5	16	11	— 12	+ 02	+ 07	— 19	— 20	— 18	+1.33
17.1	16	7	— 04	+ 07	— 04	— 22	— 29	— 21	+1.33
17.3	11	3	— 09	— 22	— 53	— 35	— 44	— 46	+1.33
17.5	23	1	+ 04	— 01	+ 04	+ 04	— 02	— 06	+1.33
18.1	18	5	— 09	— 20	— 32	— 29	— 15	— 13	—1.33
18.3	17	9	+ 18	00	00	— 19	— 16	— 11	—1.33
18.5	19	13	+ 24	+ 22	+ 05	— 24	— 09	— 05	—1.33
19.1	18	17	+ 07	+ 09	— 05	— 19	— 06	— 04	—1.35
19.3	18	21	— 05	— 06	— 13	— 22	— 08	+ 02	—3.35
19.5	13	25	00	— 02	— 31	— 18	— 08	— 14	—3.35
20.1	14	30	— 16	— 22	— 26	— 20	— 10	— 08	—3.35
20.3	17	34	+ 02	+ 10	+ 17	— 14	— 16	+ 03	—3.35

AR	D	β	Δ'_1	Δ'_2	Δ_3	Δ_4	Δ_5	Δ_6	Region
20.5	13	38	— 02	— 07	+ 03	— 02	— 15	— 11	—3.35
21.1	13	43	— 05	+ 06	+ 11	00	— 12	— 13	—5.34
21.3	13	47	+ 13	+ 03	+ 01	+ 08	— 04	— 17	—5.34
21.5	11	52	— 17	— 19	— 30	— 22	— 07	+ 01	—5.34
22.1	11	56	+ 10	+ 12	+ 17	+ 02	— 09	— 13	—5.34
22.3	13	61	— 08	— 09	— 15	— 05	— 05	— 07	—5.34
22.5	13	65	+ 02	+ 04	+ 15	+ 06	+ 08	+ 08	—7.33
23.1	12	69	— 07	— 13	— 47	— 12	— 05	+ 03	—7.3
23.3	13	74	— 11	— 04	+ 04	+ 01	— 06	— 09	—7.3
23.5	11	78	— 02	— 03	— 04	— 12	— 08	+ 01	—7.3
— 27° —32°									
0.1	13	83	— 01	+ 03	— 07	— 14	— 05	— 02	—9
0.3	15	87	— 11	— 05	— 11	— 04	+ 08	+ 08	—9
0.5	12	87	— 01	+ 05	— 14	— 25	— 14	00	—9
1.1	11	83	— 06	— 10	— 49	— 16	+ 01	00	—9
1.3	11	79	— 06	— 10	+ 12	+ 04	— 10	— 15	—9
1.5	17	75	+ 03	+ 06	+ 07	00	+ 08	+ 15	—7.21
2.1	15	70	— 06	— 13	— 24	+ 11	+ 05	+ 03	—7.21
2.3	13	66	+ 02	+ 03	+ 08	+ 03	+ 03	— 06	—7.21
2.5	13	62	+ 07	+ 02	+ 04	— 14	— 21	— 08	—7.21
3.1	14	58	+ 02	+ 10	+ 08	— 02	— 09	— 06	—5.19
3.3	15	53	— 03	— 08	+ 05	— 08	— 06	— 01	—5.19
3.5	17	49	— 04	— 08	— 06	— 07	— 01	+ 01	—5.19
4.1	16	44	— 14	— 18	— 41	— 15	— 06	— 01	—5.19
4.3	18	40	00	— 06	— 05	+ 02	— 02	— 02	—5.19
4.5	17	36	— 06	— 12	— 19	— 17	— 13	— 08	—3.19
5.1	18	32	— 11	— 12	— 19	— 22	— 14	— 08	—3.19
5.3	22	28	+ 02	+ 03	— 10	— 15	— 07	— 06	—3.21
5.5	23	24	— 04	+ 09	+ 06	— 06	— 09	— 07	—3.21
6.1	22	20	00	00	— 04	— 08	— 12	— 19	—1.21
6.3	30	16	— 06	— 03	+ 03	— 03	— 02	— 06	—1.21
6.5	35	12	+ 40	+ 23	+ 13	+ 07	+ 03	00	—1.21
7.1	39	8	+ 20	+ 34	+ 23	+ 09	+ 04	+ 04	—1.21
7.3	31	4	+ 15	+ 03	— 12	— 10	— 11	— 13	—1.21
7.5	28	0	— 18	— 22	— 36	— 11	— 08	— 18	—1.21
8.1	26	4	— 06	— 22	— 10	— 17	— 21	— 22	+1.21
8.3	33	7	— 08	— 17	— 07	+ 02	— 02	— 08	+1.21
8.5	33	11	— 02	— 03	+ 08	+ 08	+ 02	— 05	+1.23
9.1	32	14	— 15	— 15	— 05	+ 02	+ 01	— 02	+1.23
9.3	31	17	— 01	+ 02	00	— 05	— 07	+ 01	+1.23
9.5	26	20	— 18	— 23	— 29	— 14	— 14	— 08	+1.23
10.1	25	23	— 08	— 14	— 16	— 03	— 02	— 05	+3.23
10.3	20	25	— 03	— 05	— 13	— 30	— 14	— 13	+3.23
10.5	25	27	— 12	— 14	— 25	— 07	+ 03	+ 03	+3.25
11.1	24	29	— 02	+ 02	— 03	— 03	+ 03	— 09	+3.25
11.3	18	31	+ 02	— 04	— 25	— 18	— 19	— 18	+3.25
11.5	15	32	— 11	— 14	— 36	— 25	— 32	— 26	+3.25
12.1	19	33	— 20	— 14	— 27	— 16	— 16	— 08	+3.27
12.3	17	33	— 06	— 12	— 13	— 21	— 17	— 15	+3.27
12.5	17	33	— 15	— 23	— 05	— 06	— 24	— 26	+3.27
13.1	17	32	— 16	— 21	— 45	— 33	— 25	— 16	+3.27
13.3	19	32	— 06	— 09	— 16	— 26	— 29	— 18	+3.29
13.5	20	31	+ 06	— 03	+ 04	— 02	— 12	— 16	+3.29
14.1	19	30	— 12	— 14	— 12	— 15	— 12	— 14	+3.29
14.3	21	28	— 07	— 13	— 19	— 04	— 09	— 10	+3.29
14.5	19	26	+ 02	+ 04	+ 01	— 16	— 22	— 18	+3.31
15.1	16	23	— 04	— 15	— 26	— 29	— 47	— 33	+3.31
15.3	19	20	00	— 01	— 03	— 17	— 35	— 27	+3.31
15.5	20	17	+ 13	+ 03	— 12	— 27	— 21	— 21	+1.31
16.1	18	14	+ 03	— 04	— 18	— 15	— 15	— 26	+1.31
16.3	22	11	— 11	— 12	— 08	— 10	— 11	— 10	+1.31
16.5	27	8	— 21	— 10	+ 11	+ 10	+ 09	— 02	+1.33
17.1	23	4	— 08	— 03	+ 16	— 02	— 16	— 16	+1.33
17.3	23	1	— 13	— 26	— 26	— 15	— 13	— 10	+1.33
17.5	32	3	+ 06	+ 06	— 01	+ 11	+ 11	+ 10	—1.33
18.1	25	7	+ 15	+ 19	+ 12	00	— 11	— 09	—1.33
18.3	24	11	— 11	— 15	— 06	+ 05	+ 02	— 08	—1.33
18.5	23	15	— 06	+ 05	+ 05	+ 01	— 08	— 15	—1.33
19.1	21	19	00	— 04	00	— 13	— 19	— 21	—1.33
19.3	22	23	— 13	— 05	— 16	— 13	— 07	— 08	—3.33
19.5	20	27	— 03	— 04	— 08	+ 03	— 07	— 13	—3.33
20.1	24	31	— 02	— 06	— 01	— 10	00	+ 06	—3.33
20.3	21	36	— 06	— 05	+ 21	+ 15	— 01	— 06	—3.35
20.5	21	40	+ 04	+ 14	+ 16	00	+ 03	+ 04	—3.35
21.1	17	44	— 04	— 11	+ 02	+ 06	— 04	— 06	—5.34
21.3	15	49	— 18	— 10	— 55	— 10	— 09	— 12	—5.34
21.5	15	53	— 03	— 05	— 04	+ 08	— 08	— 12	—5.34
22.1	14	57	— 07	— 12	— 28	— 13	— 23	— 11	—5.34
22.3	14	62	+ 11	+ 04	— 01	— 07	— 04	+ 05	—7.33
22.5	12	66	+ 02	00	— 05	— 10	— 02	— 02	—7.33
23.1	15	70	+ 03	+ 15	+ 11	— 10	+ 02	+ 05	—7.33
23.3	12	75	— 15	— 11	— 22	— 09	+ 01	— 05	—7.33
23.5	16	79	+ 12	+ 10	+ 07	+ 07	— 01	— 01	—7.33
— 32° — 37°									
0.1	13	80	+ 04	+ 05	+ 11	— 01	— 08	+ 02	—7.33
0.3	10	82	— 10	— 03	— 30	— 23	— 18	— 18	—9
0.5	11	82	— 10	— 08	— 11	— 28	— 24	00	—9
1.1	13	80	— 15	— 07	+ 05	— 02	— 04	— 03	—9
1.3	10	78	— 06	— 03	— 07	— 25	— 18	— 08	—7.21
1.5	12	74	— 11	— 10	— 24	— 13	— 08	— 03	—7.21
2.1	11	69	— 16	— 13	— 08	— 02	— 11	— 15	—7.21
2.3	12	65	+ 04	+ 02	+ 10	+ 06	— 04	— 10	—7.21
2.5	9	61	+ 08	+ 06	— 04	— 28	— 23	— 20	—7.21
3.1	10	57	— 12	— 07	— 11	— 18	— 14	— 12	—5.19
3.3	13	53	+ 03	+ 11	+ 10	— 10	— 15	— 04	—5.19
3.5	14	49	— 03	— 03	— 10	— 23	— 21	— 05	—5.19
4.1	13	45	+ 02	+ 02	+ 06	+ 02	— 02	— 05	—5.19
4.3	14	41	— 09	— 13	— 15	— 02	— 06	— 14	—5.19
4.5	14	37	— 14	00	— 05	— 09	— 20	— 21	—3.21
5.1	16	33	+ 14	— 04	— 03	— 10	— 09	— 08	—3.21
5.3	18	29	+ 14	— 22	+ 01	— 11	— 13	— 09	—3.21
5.5	18	25	+ 08	+ 15	+ 04	— 06	— 14	— 14	—3.21
6.1	20	21	+ 12	— 25	+ 09	— 05	— 05	— 11	—3.21
6.3	21	17	+ 35	— 26	+ 08	— 07	— 13	— 15	—1.21
6.5	20	13	+ 05	+ 09	00	— 01	— 16	— 23	—1.21
7.1	25	10	+ 14	+ 06	— 03	— 07	— 05	— 08	—1.21
7.3	24	6	+ 18	+ 11	+ 03	— 09	— 20	— 22	—1.21
7.5	28	2	+ 22	+ 27	+ 20	+ 08	— 07	— 14	—1.21
8.1	32	1	+ 47	— 26	+ 18	+ 06	+ 03	— 01	+1.23
8.3	26	4	+ 08	+ 04	00	00	— 07	— 15	+1.23
8.5	24	7	— 11	— 10	— 08	— 08	— 07	— 14	+1.23
9.1	24	10	— 20	— 12	— 04	— 04	— 11	— 18	+1.23

AR	D	β	Δ'_1	Δ'_2	Δ_3	Δ_4	Δ_5	Δ_6	Region
9.3	21	13	+ 05	— 04	— 01	— 09	— 12	— 19	+1.23
9.5	23	16	— 04	— 01	+ 05	— 16	— 28	— 19	+1.23
10.1	24	19	+ 06	— 01	— 02	— 21	— 13	— 09	+1.23
10.3	25	21	— 18	— 09	+ 05	— 03	— 08	— 06	+3.25
10.5	25	23	— 08	— 02	— 11	— 11	— 09	— 05	+3.25
11.1	25	24	— 07	— 08	— 01	— 08	— 04	— 03	+3.25
11.3	21	26	+ 18	+ 03	— 01	— 06	— 07	— 11	+3.25
11.5	23	27	— 02	+ 04	+ 07	— 04	— 18	— 13	+3.25
12.1	21	28	+ 03	— 02	+ 05	+ 02	— 11	— 10	+3.27
12.3	21	28	— 06	— 17	— 26	— 15	— 08	— 02	+3.27
12.5	22	28	— 06	+ 03	— 05	— 10	— 11	— 11	+3.27
13.1	22	27	— 02	— 12	+ 06	+ 04	— 07	— 03	+3.27
13.3	20	27	— 16	— 09	00	— 09	— 11	— 07	+3.29
13.5	27	26	+ 13	+ 12	+ 10	+ 08	+ 06	+ 03	+3.29
14.1	26	25	— 02	— 06	— 10	— 08	— 05	— 04	+3.29
14.3	29	23	— 08	— 07	— 03	+ 06	+ 01	+ 02	+3.29
14.5	27	21	+ 07	+ 16	+ 20	+ 03	— 04	— 03	+3.29
15.1	17	19	+ 01	— 07	— 18	— 23	— 27	— 24	+1.31
15.3	18	16	— 04	— 08	— 24	— 20	— 27	— 32	+1.31
15.5	20	14	00	+ 04	+ 04	— 12	— 20	— 23	+1.31
16.1	19	11	— 01	— 09	— 19	— 39	— 41	— 28	+1.31
16.3	20	8	— 06	— 11	— 06	— 19	— 39	— 25	+1.31
16.5	19	5	+ 08	— 08	— 11	— 19	— 40	— 28	+1.31
17.1	21	1	+ 12	+ 02	+ 02	— 07	— 15	— 28	+1.31
17.3	26	2	— 02	+ 07	+ 11	+ 06	+ 06	— 09	—1.33
17.5	30	6	+ 52	+ 22	+ 32	+ 16	+ 10	+ 01	—1.33
18.1	28	9	+ 09	+ 14	+ 17	+ 15	+ 12	— 01	—1.33
18.3	23	13	— 05	— 05	— 04	+ 06	— 02	— 09	—1.33
18.5	25	17	— 24	— 17	+ 08	+ 14	+ 09	+ 05	—1.33
19.1	19	21	— 13	— 14	— 03	00	— 06	— 07	—1.33
19.3	15	25	— 17	— 13	— 06	— 08	— 15	— 18	—3.33
19.5	18	29	+ 04	+ 05	00	— 07	— 11	— 03	—3.33
20.1	21	33	— 10	— 02	+ 08	+ 06	+ 04	+ 01	—3.33
20.3	20	37	— 05	+ 01	— 05	— 01	+ 02	00	—3.33
20.5	18	41	+ 06	+ 01	+ 08	— 05	— 10	+ 02	—3.33
21.1	18	45	+ 02	— 08	— 10	+ 02	+ 03	+ 02	—5.34
21.3	18	49	+ 02	+ 07	+ 05	— 03	— 03	+ 05	—5.34
21.5	17	53	— 17	— 18	— 02	+ 05	+ 05	+ 01	—5.34
22.1	18	57	+ 08	+ 03	— 08	— 05	— 02	+ 08	—5.34
22.3	16	61	— 02	00	+ 03	+ 10	+ 01	+ 01	—5.34
22.5	16	65	+ 28	+ 17	+ 02	— 18	— 09	00	—7.33
23.1	15	69	— 06	— 12	+ 21	— 08	+ 10	+ 08	—7.33
23.3	14	73	— 06	— 01	+ 03	— 09	— 10	+ 02	—7.33
23.5	15	77	— 10	+ 01	+ 01	00	— 23	+ 03	—7.33
				— 37° — 42°					
0.1	9	76	— 15	— 04	— 06	— 17	— 25	— 18	—7.27
0.3	11	77	+ 05	+ 03	+ 10	— 06	— 07	— 05	—7.27
0.5	12	77	— 10	— 15	— 05	— 16	— 10	+ 07	—7.27
1.1	11	76	— 10	— 04	+ 01	— 01	— 14	— 17	—7.27
1.3	13	73	+ 05	+ 06	— 24	— 11	+ 05	+ 02	—7.21
1.5	10	71	00	— 03	— 06	— 14	— 07	— 09	—7.21
2.1	8	68	— 11	+ 06	+ 08	— 24	— 24	— 21	—7.21
2.3	10	64	+ 10	+ 01	— 41	— 42	— 16	— 09	—7.21
2.5	12	61	+ 04	+ 06	+ 01	— 03	— 09	— 08	—7.21
3.1	13	57	— 17	— 16	+ 12	+ 16	+ 07	— 04	—5.22
3.3	11	53	+ 04	+ 02	— 08	— 15	— 18	— 21	—5.22

AR	D	β	Δ'_1	Δ'_2	Δ_3	Δ_4	Δ_5	Δ_6	Region
3.5	12	49	— 07	— 07	— 07	— 09	— 10	— 10	—5.19
4.1	11	45	— 13	— 13	— 03	— 20	— 30	— 19	—5.19
4.3	14	42	— 13	+ 03	+ 11	+ 04	— 06	— 10	—5.19
4.5	13	38	+ 07	+ 01	+ 04	— 10	— 13	— 12	—3.21
5.1	12	34	— 15	— 16	— 10	— 24	— 29	— 21	—3.21
5.3	15	30	+ 05	— 01	— 20	— 32	— 24	— 14	—3.21
5.5	17	27	+ 05	— 03	— 04	— 02	— 16	— 12	—3.21
6.1	14	23	+ 04	— 03	— 23	— 13	— 24	— 26	—3.21
6.3	22	19	+ 24	+ 25	+ 08	+ 08	+ 06	— 07	—1.21
6.5	22	16	— 08	— 09	— 09	— 13	— 08	— 05	—1.21
7.1	22	12	+ 12	+ 10	+ 04	— 07	— 16	— 13	—1.21
7.3	25	9	+ 32	+ 41	+ 18	— 04	— 03	— 04	—1.23
7.5	34	5	+ 52	+ 44	+ 26	+ 17	+ 09	+ 10	—1.23
8.1	31	2	+ 10	+ 08	— 01	+ 04	+ 06	+ 06	—1.23
8.3	30	1	+ 15	+ 12	+ 27	+ 18	— 03	— 06	+1.23
8.5	29	4	+ 26	+ 17	+ 18	+ 14	— 02	— 07	+1.23
9.1	23	7	+ 26	+ 18	— 06	— 20	— 23	— 14	+1.23
9.3	23	10	+ 06	— 05	— 25	— 19	— 09	— 11	+1.23
9.5	20	12	— 20	— 31	— 10	— 03	— 24	— 26	+1.23
10.1	20	15	+ 23	+ 15	— 06	— 22	— 18	— 18	+1.25
10.3	16	17	— 13	— 26	— 26	— 15	— 30	— 41	+1.25
10.5	22	19	— 13	— 18	+ 01	— 01	— 08	— 12	+1.25
11.1	22	20	— 18	— 02	+ 06	— 09	— 15	— 11	+1.25
11.3	18	21	— 07	— 14	+ 03	— 07	— 30	— 26	+3.25
11.5	17	22	— 07	— 15	— 30	— 04	— 14	— 28	+3.25
12.1	19	23	+ 19	+ 15	+ 04	— 27	— 27	— 25	+3.27
12.3	17	24	+ 04	— 03	— 07	— 35	— 29	— 21	+3.27
12.5	22	24	— 06	— 13	— 12	— 02	— 06	— 11	+3.27
13.1	26	23	— 07	— 07	— 04	— 05	— 04	+ 04	+3.27
13.3	23	22	+ 19	+ 03	— 05	— 03	— 14	— 14	+3.27
13.5	22	21	+ 08	+ 01	— 14	— 04	— 15	— 17	+3.29
14.1	20	20	— 07	— 10	— 15	— 26	— 25	— 17	+3.29
14.3	23	18	+ 28	+ 12	— 04	— 28	— 22	— 15	+1.29
14.5	30	16	+ 23	+ 12	— 02	— 05	— 02	— 04	+1.29
15.1	37	14	+ 22	+ 30	+ 12	+ 03	+ 05	+ 13	+1.29
15.3	27	12	+ 12	+ 37	+ 18	— 09	— 14	— 10	+1.31
15.5	22	10	+ 21	— 02	— 14	— 14	— 18	— 24	+1.31
16.1	16	7	00	— 13	— 33	— 36	— 49	— 48	+1.31
16.3	27	4	+ 05	— 03	— 04	— 06	— 18	— 21	+1.31
16.5	38	1	+ 41	+ 36	+ 31	+ 26	+ 07	— 10	+1.31
17.1	30	2	— 01	— 03	+ 03	— 03	— 06	— 09	—1.31
17.3	34	5	+ 10	+ 18	+ 09	+ 10	+ 10	+ 03	—1.31
17.5	35	8	+ 21	— 01	+ 17	+ 22	+ 10	+ 10	—1.31
18.1	25	12	— 04	— 11	— 20	— 02	+ 03	— 04	—1.33
18.3	28	15	+ 12	+ 03	+ 11	+ 16	+ 09	+ 08	—1.33
18.5	25	19	+ 18	+ 22	+ 07	+ 10	+ 06	+ 05	—1.33
19.1	22	22	+ 13	— 08	— 10	+ 04	+ 02	+ 03	—3.33
19.3	15	26	— 11	— 13	— 25	— 10	— 04	— 11	—3.33
19.5	17	30	— 05	+ 01	+ 15	+ 02	— 13	— 06	—3.33
20.1	14	33	— 15	— 13	— 05	+ 11	+ 03	— 06	—3,33
20.3	19	37	+ 02	— 03	+ 08	— 27	+ 01	+ 16	—3.33
20.5	20	41	+ 07	+ 04	— 02	+ 09	+ 08	+ 11	—5.31
21.1	15	45	+ 08	+ 12	+ 02	— 05	+ 02	+ 01	—5.31
21.3	15	49	— 02	— 12	+ 05	— 05	— 03	+ 05	—5.31
21.5	16	53	+ 04	+ 04	+ 01	— 02	+ 05	+ 09	—5.31
22.1	11	56	— 01	— 15	— 32	— 06	— 11	— 11	—5.31
22.3	10	60	+ 04	00	— 15	— 20	— 22	— 12	—5.31

AR	D	β	Δ'_1	Δ'_2	Δ_3	Δ_4	Δ_5	Δ_6	Region
22.5	10	64	— 11	+ 03	+ 12	+ 01	— 05	— 15	—7.33
23.1	10	68	+ 10	— 04	— 07	— 32	— 12	— 04	—7.33
23.3	10	71	— 10	— 10	— 07	— 14	— 27	— 19	—7.33
23.5	10	74	— 05	— 13	+ 10	00	— 28	— 09	—7.33

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AR	D	β	Δ'_1	Δ'_2	Δ_3	Δ_4	Δ_5	Δ_6	Region
0.1	7	71	— 04	— 03	— 06	— 11	— 10	+ 02	—7.27
0.3	8	72	+ 07	+ 08	+ 02	+ 06	+ 01	— 02	—7.27
0.5	7	72	+ 01	— 17	— 50	— 08	00	+ 06	—7.27
1.1	9	71	— 04	+ 11	+ 09	+ 03	+ 09	+ 08	—7.27
1.3	7	69	+ 07	— 02	— 16	— 04	— 03	+ 01	—7.27
1.5	9	67	+ 12	+ 08	+ 10	+ 13	— 01	+ 05	—7.21
2.1	8	65	+ 06	+ 08	— 01	— 14	— 04	+ 08	—7.21
2.3	6	62	— 05	— 15	— 16	+ 01	— 04	— 02	—7.21
2.5	6	59	— 16	— 21	00	+ 03	— 11	— 16	—5.22
3.1	8	56	— 06	— 04	— 47	— 20	— 02	+ 10	—5.22
3.3	7	52	— 12	+ 04	00	— 07	— 06	— 13	—5.22
3.5	8	49	— 18	— 22	— 25	+ 06	+ 06	+ 03	—5.22
4.1	9	45	+ 10	+ 11	— 20	— 06	+ 01	— 03	—5.22
4.3	10	42	+ 10	+ 05	00	— 01	— 03	+ 04	—5.22
4.5	10	38	— 19	— 19	— 17	+ 11	+ 10	+ 07	—3.21
5.1	8	34	— 20	— 26	— 40	— 19	— 11	— 04	—3.21
5.3	8	31	— 09	— 09	— 29	— 20	— 11	— 19	—3.21
5.5	8	28	+ 12	— 03	— 20	— 23	— 21	— 08	—3.21
6.1	13	24	+ 17	+ 26	+ 10	— 21	+ 03	+ 03	—3.21
6.3	12	21	— 17	— 20	— 26	— 07	+ 02	+ 07	—3.23
6.5	13	18	+ 10	+ 06	+ 02	— 01	— 01	— 03	—1.23
7.1	12	14	+ 15	+ 10	— 16	— 20	— 12	— 04	—1.23
7.3	13	11	— 03	— 12	— 08	00	— 04	— 11	—1.23
7.5	17	8	+ 58	+ 48	+ 22	+ 14	+ 04	+ 02	—1.23
8.1	20	5	+ 30	+ 38	+ 22	+ 10	+ 08	+ 18	—1.23
8.3	18	2	+ 35	+ 37	+ 14	+ 09	+ 05	+ 08	—1.23
8.5	17	1	+ 35	+ 16	+ 06	+ 12	+ 03	+ 03	+1.23
9.1	14	3	+ 41	+ 35	+ 01	— 12	— 14	+ 01	+1.23
9.3	12	6	— 21	— 28	— 18	— 10	— 14	— 09	+1.23
9.5	12	8	+ 13	+ 06	— 26	— 17	— 10	— 08	+1.25
10.1	14	10	— 14	— 21	— 17	— 02	— 06	— 06	+1.25
10.3	13	12	+ 09	— 27	+ 10	— 02	— 11	— 09	+1.25
10.5	13	14	— 19	— 15	— 13	+ 01	— 08	— 10	+1.25
11.1	12	15	— 02	+ 02	— 02	— 09	— 09	— 11	+1.25
11.3	12	16	— 07	— 09	— 05	— 03	— 08	— 12	+1.25
11.5	9	17	— 07	— 15	— 33	— 36	— 27	— 17	+1.27
12.1	11	18	— 07	— 14	— 27	— 16	— 09	— 06	+1.27
12.3	10	19	— 12	— 19	— 31	— 21	— 12	— 08	+1.27
12.5	11	19	— 12	— 19	— 16	— 07	— 04	— 02	+1.27
13.1	10	18	+ 04	— 04	— 08	— 33	— 20	— 12	+1.27
13.3	13	17	— 12	— 07	— 07	— 05	— 05	— 01	+1.27
13.5	11	16	+ 10	+ 07	— 16	— 18	— 18	— 16	+1.29
14.1	11	15	+ 21	+ 13	— 09	— 11	— 21	— 14	+1.29
14.3	13	14	+ 09	+ 20	— 04	— 13	— 11	— 08	+1.29
14.5	13	12	+ 03	— 10	— 16	— 14	— 13	— 03	+1.29
15.1	12	10	+ 08	+ 01	— 16	— 02	— 13	— 12	+1.29
15.3	14	8	+ 02	00	— 25	— 01	— 06	— 12	+1.29
15.5	13	6	— 09	— 04	+ 04	— 07	— 24	— 17	+1.31
16.1	11	4	— 04	— 13	— 17	— 13	— 16	— 25	+1.31
16.3	14	1	+ 07	+ 07	— 06	+ 04	— 08	— 13	+1.31
16.5	17	2	+ 07	+ 24	+ 12	+ 05	— 01	+ 02	—1.31
17.1	17	5	+ 07	+ 20	+ 10	+ 04	— 26	— 01	—1.31
17.3	18	7	+ 08	+ 01	+ 06	+ 11	+ 07	+ 13	—1.31
17.5	11	10	— 09	— 12	— 25	— 10	— 14	— 09	—1.31
18.1	13	14	+ 09	— 03	— 06	— 03	— 01	+ 02	—1.31
18.3	13	17	+ 04	+ 03	— 10	— 06	— 03	+ 05	—1.31
18.5	9	20	+ 22	+ 04	— 19	— 14	— 13	— 09	—1.31
19.1	10	24	+ 17	+ 10	— 23	— 22	— 11	— 04	—3.33
19.3	9	27	— 04	— 07	— 16	— 23	— 13	+ 01	—3.33
19.5	9	30	— 04	— 11	— 07	— 02	— 03	00	—3.33
20.1	9	34	— 03	— 04	— 04	— 09	— 04	— 03	—3.33
20.3	7	38	— 14	— 05	— 08	— 15	— 12	— 06	—3.33
20.5	8	41	— 07	00	+ 14	+ 01	— 16	— 06	—5.31
21.1	10	45	— 12	— 03	+ 09	+ 20	+ 14	+ 02	—5.31
21.3	7	48	— 01	— 01	— 08	— 12	— 09	00	—5.31
21.5	8	52	— 17	— 19	— 17	— 01	+ 04	+ 02	—5.31
22.1	11	55	00	+ 09	+ 14	+ 12	+ 16	+ 13	—5.31
22.3	8	59	00	+ 01	+ 08	+ 02	+ 03	+ 04	—5.31
22.5	9	62	— 11	— 10	+ 13	+ 24	+ 11	00	—7.33
23.1	8	65	+ 12	+ 02	— 13	— 09	+ 07	+ 09	—7.33
23.3	9	68	+ 12	+ 15	+ 22	+ 06	+ 04	+ 06	—7.33
23.5	8	69	— 10	— 09	— 19	00	+ 14	+ 09	—7.27

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AR	D	β	Δ'_1	Δ'_2	Δ_3	Δ_4	Δ_5	Δ_6	Region
0.1	8	67	— 10	— 20	— 08	+ 15	+ 06	00	—7.27
0.3	7	68	+ 09	+ 14	+ 04	— 10	+ 02	— 01	—7.27
0.5	6	68	+ 03	+ 06	— 09	— 09	— 01	— 10	—7.27
1.1	8	66	— 10	— 08	— 24	— 10	— 02	+ 05	—7.27
1.3	8	65	+ 03	— 03	+ 03	+ 10	+ 01	— 09	—7.27
1.5	7	64	+ 09	+ 05	— 32	— 17	— 08	— 04	—7.27
2.1	9	62	+ 02	— 09	— 31	+ 09	+ 08	+ 02	—7.27
2.3	6	59	+ 02	+ 04	— 17	— 18	— 15	— 11	—5.22
2.5	6	57	— 04	— 09	— 27	— 03	— 03	— 10	—5.22
3.1	7	54	+ 02	— 10	— 17	— 03	+ 06	— 15	—5.22
3.3	8	51	— 05	— 03	+ 10	00	+ 03	— 06	—5.22
3.5	7	48	+ 07	+ 02	+ 08	+ 02	— 13	— 12	—5.22
4.1	8	45	— 12	— 16	— 16	— 12	— 03	00	—5.22
4.3	7	42	— 06	+ 01	— 13	— 31	— 20	— 05	—5.22
4.5	10	39	— 13	— 17	— 16	+ 10	+ 04	— 07	—3.23
5.1	8	35	— 01	— 13	— 61	— 02	— 02	— 13	—3.23
5.3	9	32	— 08	— 06	— 10	— 07	— 09	— 14	—3.23
5.5	9	29	— 02	— 07	— 11	— 16	— 08	— 14	—3.23
6.1	10	25	— 03	00	— 16	— 23	— 12	— 12	—3.23
6.3	12	22	+ 09	+ 12	— 03	— 07	— 02	— 09	—3.23
6.5	10	19	+ 14	+ 09	— 13	— 16	— 20	— 28	—1.23
7.1	15	16	+ 13	+ 07	— 12	+ 01	+ 01	— 04	—1.23
7.3	10	13	00	+ 05	— 13	— 18	— 23	— 23	—1.23
7.5	17	10	+ 30	+ 43	+ 28	+ 12	— 06	— 09	—1.23
8.1	21	8	+ 30	+ 27	+ 16	+ 08	+ 08	+ 08	—1.23
8.3	20	5	+ 17	+ 13	+ 10	+ 15	+ 08	+ 03	—1.23
8.5	18	3	+ 11	— 04	— 06	— 02	— 04	+ 04	—1.23
9.1	16	0	— 02	— 02	— 18	— 14	— 10	— 04	+1.23
9.3	17	2	+ 17	+ 06	— 16	— 07	— 06	— 05	+1.25
9.5	14	4	— 02	— 10	— 20	— 08	— 11	— 15	+1.25
10.1	19	6	+ 11	+ 15	+ 07	+ 12	+ 06	— 02	+1.25
10.3	16	7	— 14	— 20	— 17	— 06	— 05	— 05	+1.25
10.5	17	9	— 07	— 02	— 10	00	+ 02	00	+1.25
11.1	14	10	+ 12	+ 04	— 05	— 29	— 08	— 06	+1.25

RA	D	β	Δ'₁	Δ'₂	Δ₃	Δ₄	Δ₅	Δ₆	Region
11.3	16	11	—06	—08	—36	—11	+01	00	+1.25
11.5	15	12	—12	—23	00	—10	—05	—05	+1.27
12.1	16	13	+06	+14	—03	00	+03	—02	+1.27
12.3	12	14	+19	00	—23	—14	—13	—15	+1.27
12.5	14	14	+13	+14	—17	—01	—10	—12	+1.27
13.1	13	13	+31	+26	+10	—07	—11	—19	+1.27
13.3	14	13	+19	+13	+07	—08	—11	—11	+1.27
13.5	15	12	+06	—12	—08	—02	00	—02	+1.29
14.1	11	11	—06	—14	—26	—15	—17	—28	+1.29
14.3	11	10	+06	+10	00	—18	—29	—28	+1.29
14.5	11	8	+05	—02	—19	—18	—19	—20	+1.29
15.1	16	7	+30	+19	—03	—08	—10	+02	+1.29
15.3	18	5	—08	+01	+15	+03	—06	+06	+1.29
15.5	15	3	—02	—03	—16	—10	—07	—08	+1.29
16.1	16	0	+04	—03	—09	+06	—06	—11	+1.31
16.3	15	2	—14	—16	—01	+05	—06	—15	—1.31
16.5	11	5	—08	—16	—11	—16	—21	—25	—1.31
17.1	17	7	+11	00	+03	+09	+02	—03	—1.31
17.3	14	10	+06	+04	—28	—05	—07	—12	—1.31
17.5	11	13	—18	—27	—14	—08	—18	—16	—1.31
18.1	16	16	—05	+03	—08	+09	+09	+03	—1.31
18.3	13	19	+02	+05	+19	+17	—05	—12	—1.31
18.5	13	22	—10	+10	+20	+11	00	—11	—3.31
19.1	11	25	—16	—03	—10	—08	—01	—11	—3.31
19.3	10	28	—15	—21	—40	+02	+06	—07	—3.31
19.5	10	31	—08	—08	—10	—19	—07	+01	—3.31
20.1	9	35	—01	+04	+06	+05	—06	—04	—3.31
20.3	7	38	—13	—16	—17	+05	—16	—25	—3.31
20.5	6	41	00	—05	+01	00	—22	—30	—5.31
21.1	10	45	—06	00	+18	+13	+04	+01	—5.31
21.3	8	48	—18	—17	—13	—02	—07	—04	—5.31
21.5	9	51	—11	—06	—01	—01	+03	—01	—5.31
22.1	9	53	—05	—10	—24	—09	+05	+05	—5.31
22.3	9	56	+02	+10	+14	+13	+07	—05	—5.31
22.5	6	59	+08	+20	+19	—02	—29	—33	—5.31
23.1	6	61	+02	+05	+02	—12	—14	—15	—7.33
23.3	7	63	—04	—02	—13	—18	—05	+04	—7.27
23.5	7	65	+03	+13	+12	—10	—20	—09	—7.27

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RA	D	β	Δ'₁	Δ'₂	Δ₃	Δ₄	Δ₅	Δ₆	Region
0.2	9	62	—03	+01	—05	+01	+15	+06	—7.27
1.0	9	62	—09	—14	—49	—18	+13	+10	—7.27
1.4	7	60	+01	00	—02	+05	—03	—10	—7.27
2.2	9	57	—06	—04	—10	+03	+09	+05	—5.25
3.0	7	53	—14	—13	—44	—46	—03	—03	—5.22
3.4	9	48	—11	—13	00	+18	+05	—08	—5.22
4.2	9	42	—12	—17	—34	—02	+01	—04	—5.22
5.0	11	36	—02	—15	—06	+11	+10	+03	—3.23
5.4	11	31	+10	—03	—01	+04	+02	—07	—3.23
6.2	13	25	+09	00	+02	+02	+01	—06	—3.23
7.0	14	20	+11	—06	—03	+01	00	—05	—1.23
7.4	16	14	+23	+10	+17	+10	—02	—09	—1.23
8.2	17	9	+29	+21	+17	+09	—08	—14	—1.23
9.0	23	5	+18	+11	+02	+10	+15	+14	—1.25
9.4	24	1	+14	+05	+11	+14	+14	+15	—1.25
10.2	26	2	00	+13	—02	+10	+17	+19	+1.25
11.0	21	5	—06	—10	—05	00	+03	+05	+1.25
11.4	19	7	—02	—17	—22	—08	+04	+02	+1.27
12.2	19	8	—02	—03	—01	+03	+02	—03	+1.27
13.0	20	8	+12	+08	—04	—11	+04	+08	+1.27
13.4	18	7	+05	—03	—10	—15	—03	—03	+1.27
14.2	17	5	+11	—02	—12	—07	—02	—13	+1.29
15.0	18	2	—16	—28	—24	—08	+04	—03	+1.29
15.4	21	1	—03	+02	—06	+05	+10	—01	+1.29
16.2	26	5	—02	—07	+02	+19	+21	+15	—1.29
17.0	17	9	+05	00	—17	—11	00	—04	—1.31
17.4	15	14	+03	—09	—13	—04	00	—11	—1.31
18.2	16	19	—13	—13	—04	+02	+06	—02	—1.31
19.0	15	24	+02	—01	—09	+06	+12	—04	—3.31
19.4	14	30	+10	00	—04	+05	+15	00	—3.31
20.2	11	36	—09	—15	—68	—26	+08	+05	—3.31
21.0	11	42	—05	—03	00	+04	+06	—01	—5.31
21.4	8	47	+03	+01	+02	—03	—02	—13	—5.31
22.2	9	52	—10	—12	—32	+03	+09	—07	—5.31
23.0	10	57	—03	—02	—12	00	+15	+07	—5.28
23.4	9	60	—10	—11	—10	00	+02	+02	—5.28

— 57° — 62°

AR	D	β	Δ'₁	Δ'₂	Δ₃	Δ₄	Δ₅	Δ₆	Region
0.2	8	57	—01	—01	—23	—22	—06	+03	—5.28
1.0	10	57	—05	+01	+03	+01	+03	+03	—5.25
1.4	10	56	+03	—01	—10	—12	+07	+04	—5.25
2.2	9	54	—13	—14	—27	—06	+01	—01	—5.25
3.0	9	50	—10	—12	—47	—06	+01	—04	—5.25
3.4	10	46	—02	—07	—17	+01	+06	—06	—5.25
4.2	8	41	—03	—07	—20	—16	—03	—06	—5.22
5.0	10	36	—08	—10	—24	—10	+06	+02	—3.23
5.4	10	31	—05	—15	—12	—10	—08	—11	—3.23
6.2	15	26	+02	+09	+04	+07	+14	+02	—3.23
7.0	14	21	—03	—04	+07	+11	00	—06	—3.23
7.4	19	16	+15	+03	+19	+16	+09	+03	—1.23
8.2	14	12	+10	—05	—22	—12	—10	—12	—1.25
9.0	24	8	+37	+30	+09	+08	+14	+12	—1.25
9.4	23	5	+24	+19	+04	+11	+12	+07	—1.25
10.2	30	2	+67	+40	+26	+26	+27	+38	—1.25
11.0	30	0	+28	+19	+35	+39	+39	+48	+1.25
11.4	30	2	+24	+27	+26	+28	+33	+34	+1.27
12.2	25	3	+20	+08	+03	+09	+14	+12	+1.27
13.0	27	4	+24	+17	+05	+06	+17	+20	+1.27
13.4	24	2	—07	—18	—01	+12	+13	+10	+1.27
14.2	20	0	+01	—07	—13	+01	+01	—02	+1.29
15.0	21	2	+08	+05	+05	00	—04	00	—1.29
15.4	22	4	—07	—02	—07	—05	+04	+06	—1.29
16.2	22	8	+09	+07	—05	—03	+03	+09	—1.29
17.0	22	12	+02	—02	+10	+11	+07	+14	—1.29
17.4	20	16	—08	—06	—09	—07	+08	+15	—1.31
18.2	17	21	—07	—17	—12	—11	+06	+11	—3.31
19.0	12	26	—18	—21	—53	—13	00	00	—3.31
19.4	10	31	+07	—02	—11	—09	—07	—10	—3.31
20.2	8	36	—04	—04	—53	—19	—11	—16	—3.31
21.0	10	41	—15	—16	—29	—06	+06	+02	—5.31
21.4	10	45	—02	—05	—19	—03	+07	—03	—5.31
22.2	11	49	+06	+02	—11	—04	+14	+10	—5.28
23.0	14	53	—01	+06	+23	+26	+23	+11	—5.28
23.4	15	56	—13	—13	+22	+33	+30	+22	—5.28

The irregularities in the surface distribution.

53. Before making use of the deviations $\triangle$ of Table 57 for deducing the space distribution, it will be necessary first to study the irregularities in the surface distribution. As each irregularity in the space density affects a great number of magnitude classes in a slowly varying degree the irregularities in the surface density will show a gradually varying picture for many consecutive classes. Thus it is not necessary to represent separately each of our magnitude groups; it is sufficient to condense them into three groups:

$$^1/_2\,(\triangle'_1+\triangle'_2),\quad ^1/_3\,(\triangle_3+2\,\triangle_4),\quad ^1/_5\,(2\,\triangle_5+3\,\triangle_6),$$

representing the distribution for the magnitudes 5.7, 7.4, and 8.6. These values are given on the charts I, II, III, accompanying this work; the underlined numbers denote positive deviations, the others negative ones. They were smoothed by taking means of every 4 adjacent values (cf. § 57, Note); the curves drawn on the charts for every 0.10 depend almost entirely on these smoothed means, which themselves have been omitted from the printed charts. On charts III for the faintest class, where the accidental errors are smallest, also dotted lines for 0.05, 0.15... have been added. The regions with an excess or shortness in starnumber, indicated by red and blue colour, immediately strike the eye. The galactic circle is inserted, with dashes for each 10° of longitude, as well as the declination circles for the even hours and the parallels of 30° and 60° for the northern, 32° and 62° for the southern-hemisphere.

54. On charts III, giving the distribution for 8.6, we see at once that the greatest deviations occur in the galactic zone, while outside this zone they are much smaller. In the galactic zone we find three strong condensations.

1. The *Cygnus condensation.* Its densest parts, around a centre at $20.1^h + 37°$ (galactic coordinates $42° + 0°$) near γ Cygni, extend from β Lyr—β Cyg to α—ε Cyg. Its shape is, roughly, a triangle elongated along the galaxy reaching from, say, $18^h + 30°$ and $20^h + 15°$ as a basis towards $0^h + 50°$ as its top (gal. $35° + 20°$, $35° - 10°$, $85° - 12°$). It is inclined towards the galactic circle, extending in Lyra and Hercules far to the northern side and ending, with decreasing density, in a broad tail along the southern border of the galaxy.[1] Streamers go to ε and ι Her, δ Dra and the cluster χ—h Per to the north, to Aquila—Delphinus, β Peg and γ And to the south.

2. The *Monoceros condensation.* Around a centre at $6.8^h + 0°$ (gal. $181° + 2°$) it extends chiefly along the aequator, filling Monoceros and Orion, where around $5.5^h - 4°$ (gal. $175° - 15°$) a secondary maximum is seen, and reaching as far as Hydra and Eridanus (4^h to 9.5^h). Along the galaxy its extension is far less, reaching from $6^h + 30°$ to $8^h - 20°$ (λ 150° to 205°). At this side it ends rather abruptly at the parallel of $-20°$. The great richness of faint *DM* stars in the hours 6 and 7 has already been remarked by SCHOENFELD.

3. The *Carina condensation.* It is a small but strong agglomeration around $11^h - 60°$ (gal. $257° + 0°$) near η Car; on account of the missing south polar cap we cannot see its extension in southern latitude, but probably it extends chiefly in longitude, from ε Car to β Cen (λ $240° - 280°$). A less dense appendix stretches along the galactic circle to λ 220°, around γ Vel—ζ Pup, with centre $8.1^h - 44°$ (gal. $228° - 4°$).

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[1] We must bear in mind, however, that in a chart representing the densities itself and not the deviations from normal values decreasing with latitude, the extension in latitude for such condensations would be smaller.

As minor condensations in the galactic zone may be noted:

4. a small region in *Auriga* ($5.3^h + 39°$, gal. $136° + 1°$).

5. a region covering *Cauda Scorpii* and Corona australis, extending from ϑ Sco to γ CrA and γ Sgr. ($18^h - 38°$, gal. $320° - 10°$).

6. a region in *Ara,* from the galactic circle at λ 290° to the south.

Outside of the galactic zone several regions of surplus density are seen: a long extended region from Sagittarius through Capricornus and Aquarius, a smaller one in Cetus and a region Phoenix-Tucana ($0^h - 60°$).

Alternating with these condensations there are two regions with a strong deficiency of stars caused by absorbing nebulosities.

1. The *Taurus nebulae,* so called because its strongest core is found at $4.4^h + 28°$ (gal. $137° - 13°$) in Taurus. Of course the complicated minor structure of these dark regions, shown by the chart of DYSON and MELOTTE (*M.N.* **80.**) and still more by the photographs of BARNARD, cannot be revealed in our smoothed values of the density over great areas; our chart is, however, adapted to reveal their general distribution over the sky at large. It shows that the absorbing nebulosities extend far beyond the Taurus constellation; they cross the galactic circle between λ 110° and 130°, extend northward through Camelopardalus and go as far as the North pole and β Cep. A secondary core at $4.3^h + 50°$ (gal. $122° + 2°$) is seen on Milky Way pictures and photographs as a black spot between ε Aur and λ Per. The shortness of stars extends also from Taurus along the parallel of $+20°$ through Aries and Pisces.

2. The *Ophiuchus nebulae.* The strongest absorption occurs between η and β Oph, extending over a long streak from η Ser to ξ Oph (16^h to 18.5^h, $-2°$ to $-12°$, centre gal. $345° + 14°$). Adjoining we find a region with on the average less dense nebulosity, stretching over a part of Scorpius and Lupus, as far as γ Sco — γ Nor — ε Sco, with centres at $15.3^h - 30°$ and $16.1^h - 37°$ (gal. $307° + 22°$ and $312° + 10°$); another centre of absorption is seen at $17.4^h - 24°$ (gal. $330° + 5°$), following ϑ Oph.

From these regions of strong absorption a broad streak with a shortness of stars runs nearly parallel to the Galaxy ($\beta + 15°$ to $+35°$) through Lupus, Centaurus, Vela, Antlia; in Puppis ($8^h - 28°$, λ 215°) it crosses the Milky Way. On the southern side we find a poor region covering Columba and a part of Eridanus; on the northern side a poor streak runs through Leo.

There can be no doubt that the more striking irregularities shown by these charts, are real features in the distribution of the stars. As to the minor details, however, there are some phenomena indicating that the elimination of scale errors has not quite succeeded. The abrupt southern limit of the Monoceros condensation at $-20°$ to $-25°$ of declination is certainly partly due to the transition of Bonn to Cordoba data. The first Cordoba zone $-22°$ to $-27°$ shows nearly everywhere a much more negative deviation than the last Bonn zone $-17°$ to $-22°$. Probably the fault is with Cordoba; as the Cordoba scale was first adapted to the Bonn scale and afterwards to GOULD's scale it may be that the magnitudes in the first degrees $-22°$ and 23° deviate rather strongly from the zone $-24°$ 50′ to 60′, observed photometrically by BAILEY and used here to deduce corrections for the whole zone. In this case a deviation of this very kind may well be expected. Furthermore there is a tendency in the coloured regions to extend more widely along the parallels than in a north-south direction; the great extension of the Monoceros surplus and the Ophiuchus deficiency along the aequator is only partially an effect of projection; the blue streaks through Centaurus, Leo and Aries and the red streak through Capricornus are other instances. They may be readily explained by the fact that the observations were all made in zones of declination; remaining zone errors, not eliminated in our reduction, will have the tendency to cause deviations following the parallels. A third point of uncertainty in our scale reductions must be kept in mind : if the correction of Cordoba zone VIII for magnitudes 8—9 should be more positive than has been assumed, the positive deviations in this most southern zone must be diminished. Then the Carina condensation would become somewhat less accentuated, while the surplus regions in Ara and Tucana will disappear or become less clear.

55. The distribution of the brightest groups, the naked eye stars, represented on charts I, offers several differences with the faint stars. We find here:

1. a large condensation embracing Cygnus, Lyra, Vulpecula, Sagitta, and the adjacent parts of Aquila, Delphinus, Cepheus and, separated by a gap, of Cassiopeia. Its densest parts extend from γ Lyr over η—γ—α to π Cyg; a secondary maximum is seen between κ and δ Cep. A surplus region μ Her—κ Oph may belong to it. Where the condensation of faint stars extends along the southern border of the Milky Way, these bright stars are lacking; in its stead they show a condensation farther south, from $22.2^h + 35^\circ$ to $1.5^h + 45^\circ$, through Andromeda and the southern part of Lacerta.

2. a small, strong condensation α—δ Per, corresponding with the densest part of the well-known moving cluster of *B*-stars.

3. a strong condensation in Taurus, caused by the Hyades cluster and the Pleiades.

4. a condensation in the head of Monoceros at $6.2^h + 12^\circ$, coinciding with a secondary maximum of the faintest class.

5. a strong condensation in the south of Orion at $5.5^h - 4^\circ$, the first of the dense clusters of *B*-type stars, which are situated on Gould's belt of bright stars inclined 19° to the galactic circle. The next groups of this kind are

6. a cluster at $7.1^h - 25^\circ$, around δ, ε, η Canis majoris.

7. a cluster about $7.9^h - 42^\circ$, around ζ Pup and γ Vel, extending to λ Vel.

8. a cluster from ι Car to α Cru, around ϑ Car at $10.6^h - 63^\circ$ 1).

The first is not, the second faintly, the third strongly represented among the faint stars. They are the maxima of a long streak of great density, extending from ο CMa through Puppis, Vela, Carina, Crux to ε Cen. They are not situated exactly on Gould's great circle, for they run nearly parallel to the Milky Way along its southern border, not in line with the Orion cluster. They continue in

9. a condensation in Lupus, $15^h - 40^\circ$, from α to φ Lup, and

10. a condensation β—δ—π Sco, $16^h - 25^\circ$, both less strong, but also accumulations of *B*-type stars. They are projected on a region of absorbing nebulae, effacing the faint stars.

11. a large condensation covering the tail of Scorpius, Corona austrina and the chief star-groups of Sagittarius. It shows several centres of density: $16.8^h - 40^\circ$ (μ Sco), $17.8 - 35^\circ$ (λ Sco), $18.8^h - 40^\circ$ (Cor. austr.) and $18.8^h - 29^\circ$ (σ Sgr).

Other surplus regions of less importance are found in Coma, Cancer, Bootes on the northern, in Capricornus, Grus, Eridanus on the southern hemisphere.

Regions with a strong deficiency of bright stars are scarce. The poorest region is Ophiuchus, where the influence of the absorbing nebulae is visible; in Scorpius it is neutralised by the condensation of stars already mentioned. The Taurus nebulae, however, are not visible, because the Hyades cluster with its numerous bright stars is projected before them; only remnants of their northern parts appear in Perseus and Camelopardalus.

56. The deviations for the stars of medium brightness 7.4, represented in Charts II, are on the

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1) According to a representation of the Harv. Ann. 50 stars, extending to the south pole.

whole more negative than on the other charts; the regions with positive excess are smaller, those with negative excess generally more extended than on charts III. Taking this into account their distribution corresponds on the whole with that of the faintest class 8.6; the differences are mostly in the direction of the distribution of the bright class. Some points of difference may be noted especially.

The Cygnus condensation shows in its densest part another shape; it does not cease at α Cyg but extends as far as π Cyg; the deep bay at $21^h + 50°$ and the extension to δ Dra, shown on III, are missing here. The eastern tail extends along the parallel of 45°, more south than on III, thus showing a transition between the nothern position for the faint and the southern for the bright stars. The Taurus nebulosities are shown nearly as for the faint stars; the influence of the Hyades has disappeared, only the Pleiades are visible (as they are also on III). The Monoceros condensation is less accentuated, especially in its central and southern part; its northern part, with a maximum $6.5^h + 5°$, is separated from a small southern surplus region (centre $7^h - 20°$) which extends also over the Canis-major cluster of bright stars. The condensation around γ Vel is stronger than on III, the Carina condensation bears the same character as on I and III. The Cauda Scorpii condensation penetrates farther into the nebulous region to the West, just as with the bright stars, but the σ Sgr maximum of these bright stars has disappeared. The Ophiuchus nebulae have here nearly the same extension and density in Ophiuchus itself; but in Scorpius-Lupus they are weakened, especially in the regions of the bright star clusters.

DISCUSSION OF RESULTS.

Results for curvature and luminosity function.

57. The separate areas of 20—25 square degrees contain far too few stars to derive the coefficients of quadratic formulae from them. If we wish to find the coefficient c with a mean error 0.010 from three values of log A for magnitudes 5.8, 7.4, 8.6, we want areas with nearly 150 stars of the brightest class, thus covering 1000 square degrees. For this reason the data of table 57 have been condensed into a smaller number of regions, each nearly 400 sq. degrees, arranged between galactic latitudes 0°, 20°, 40°, 60°, 80°, and extending 20°, 20°, 30,° 60° in longitude. In the region e.g. $\beta = +30°$, $\lambda = 90°$ (designated + 3.9) all the areas are included whose centres lie between β + 20° and + 40°, λ 80° and 100°. [1]) The region to which each area is assigned, has been given in the last column of Table 57.

For the brightest classes from $\triangle'_1$ and $\triangle'_2$, denoting $\triangle A$ for the separate areas, the mean $\overline{\triangle A}$ was computed, giving $\bar{A}$, log $\bar{A}$ and $\triangle$ log $\bar{A}$ for the whole region. From $\triangle_3$, denoting $\triangle$ log A (6.9), $\triangle A$ was computed for each area and used in the same way. For $\triangle_4$, $\triangle_5$, $\triangle_6$ a shorter method was followed; their mean $\overline{\triangle \log A}$ was computed directly and corrected according to the formula:

$$\triangle \log \bar{A} = \overline{\triangle \log A} + {}^1/_2 \times 2.30\, \overline{x^2}$$

where $x = \log A - \overline{\log A}$ represents the deviations of the separate log A from their mean. [2]) The areas of zone 0° to —2° have been given half weight.

The resulting values of $\triangle$ log A for the six magnitudes are contained in Table 58 under the headings $\triangle_1 \ldots \triangle_6$. They are represented by $\triangle a + \triangle b\ (m - 7.5) - \triangle c\ (m - 7.5)^2$, and the coefficients $\triangle a$, $\triangle b$, $\triangle c$ are computed by least square solutions. Weighing the errors originating from the scale of magnitudes against the mean error of a number of stars $(\sqrt{n})$, we arrive at weights 1, 1, 2, 4, 6, 8 for the six $\triangle$ in each area; for each region, according to the greater number of stars, much less different weights: 2, 2, 3, 3, 4, 4 were adopted. With these weights the solution is given by

$$\begin{aligned}
\triangle a &= -.06\,\triangle_1 + .02\,\triangle_2 + .43\,\triangle_3 + .41\,\triangle_4 + .31\,\triangle_5 - .11\,\triangle_6 \\
\triangle b &= -.10\,\triangle_1 - .11\,\triangle_2 - .16\,\triangle_3 - .06\,\triangle_4 + .09\,\triangle_5 + .34\,\triangle_6 \\
\triangle c &= -.13\,\triangle_1 - .07\,\triangle_2 + .19\,\triangle_3 + .18\,\triangle_4 + .07\,\triangle_5 - .24\,\triangle_6
\end{aligned}$$

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[1]) The abacus of Dr. Nort, Plate I in „Recherches astronomiques de l'Observatoire d'Utrecht, VII" was used.

[2]) The means of 4, used on the charts as described in § 53 were corrected by the same formula and hus represent the true logarithmic deviations for the fourfold areas.

The results for $\triangle c$ are contained in Table 58 under the heading „First Computation". From the remaining differences the mean error of unit weight was found 0.044 and the mean error of a value of $\triangle c$ 0.0096.

58. The values found for $\triangle c$ are for the greater part negative. For the zones between $+90°$, $+40°$, $+20°$, $0°$, $-20°$, $-40°$, $-90°$ the means are $-.011$, $-.005$, $-.013$, $-.021$, $-.009$, -012; the general mean is -0.012. Some positive values occur, collecting in three regions (about $6^h + 70°$, $12^h - 20°$, 17^h to $20^h - 40°$). In some parts of the sky the negative corrections become very great; the regions in Cygnus have nearly -0.030, in Boötes they amount to 0.02 and 0.04; at both sides along the aequator from 22^h to 8^h stretches a region of strong negative values, reaching -0.044. It is clear that such great corrections, giving strong negative curvatures in the $\log A$ curve, cannot represent real features of space distribution. If we compute h, k, l and the density D we find a very low value of h, a lack of stars for the mean distance ρ_1, and increasing surplus of stars for smaller and greater distances. The low value of h can be increased (as shown in § 3) by assuming these values of h, k, l only to prevail over a limited realm of ρ while outside these limits D is normal; but in this case just the outlying surplus densities causing the strong curvature in $\log A$ are cut off and the strong negative $\triangle c$ disappears. If we take e.g. $\triangle a = +0.069$, $\triangle b = +0.036$: $\triangle c = -0.032$ (calculated for region $+1.3$ with somewhat different weights) we have for a, b, c, 0.101, 0.535, -0.020; for ρ_1, h, k, l: 12.56, 9.777, -0.065, -0.0127; for $\log D$ ($\rho = 10 \ldots 14$) 9.986, 9.869, 9.777, 9.711, 9.670, giving the deviations $\triangle D$ $-.030$, $-.213$, $-.245$, $-.145$, $+.016$, showing a relative voidness of stars in space, just where we see a condensation. If now we limit the domain of this correction by $\rho = 9.5$ and 14.5, we must increase $\log D$ (according to § 3, last lines) by 0.10, making $\triangle D$ ($10 \ldots 14$) now $+.221$, $-.022$, $-.090$, $-.012$, $+.137$. Now computing $\log A(m)$ back with these values (and zero outside these limits) we find $\triangle \log A$ for $m = 5$, 7, 9 $+0.013$, $+0.014$, $+0.018$, showing an almost imperceptible change of the normal curvature and of c.

59. If these negative c and l cannot have a real existence in the density distribution of the stars, whence do they come? The first idea offering itself is to seek for an explanation in remaining systematic errors in the scale of magnitudes. For each zone we have determined the mean scale of magnitudes from photometric measures, and also the remaining correction depending on AR for each hour within each zone; but this correction was assumed for the Bonn zones to be the same for all magnitudes. If this supposition is not right, the correction for AR vaying with magnitude and the correction curve for magnitude varying with AR, systematic errors must be caused in b and c. Thus the deviations of the hourly means H—B from the average of the whole zone, already used in deriving the corrections depending on AR (§ 28 and § 32), were discussed once more, now separating the different magnitude classes. Making up the differences: deviation of hourly means for each magnitude class *minus* (magnitude correction + correction depending on AR), and calling them A for classes 6.5 and 7.0, B for classes 7.5, 8.0, 8.5 and C for the mean of classes 8.8—9.0 and 9.0—9.5, it is the value of $A + C - 2B$ that causes an error in $\triangle c$. Representing A, B and C by a quadratic function of m and computing the resulting corrections of $\log N(m)$ and of the six differences $\triangle_1 \ldots \triangle_6$, we find that the error caused in $\triangle c$ is nearly equal to $-0.10\ (A + C - 2B)$.

On the whole the values of $A + C - 2B$ manifestly exceed their mean error (estimated 0.08 a priori), indicating that real irregularities of the scale occur. But only in some parts of the sky they show some regularity of sign over greater areas. So over and around Cygnus the signs are nearly all $+$; the mean value for the regions $+1.3$, $+1.5$, -1.3, -1.5, being $+0.04$, is,

Table 58. Results for the regions.

Region	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6	1st. Comp.	Corr. fr.	Second Comput.		ρ_1	Δ log D (10—14)	
							Δc	sc. err.	Δa	Δb		in 0.001	
+ 9	+ 058	— 034	—057	— 090	— 104	— 082	— 20	+ 4	— 86	— 10	10.8	— 8	— 62...
+ 7.3	— 041	— 057	— 051	— 084	— 099	— 044	— 12	— 1	— 72	+ 5	11.1	.— 45	— 71..
+ 7.9	— 069	— 067	— 051	— 048	— 048	— 048	+ 4	— 1	— 49	+ 1	11.0	.— 22	— 56..
+ 7.15	— 100	— 155	— 099	— 104	— 119	— 102	+ 2	+ 2	— 107	— 2	11.0	.— 80	— 119..
+ 7.21	— 148	— 146	— 156	— 134	— 086	— 045	— 20	+ 3	— 102	+ 56	11.8	.— 117	— 48..
+ 7.27	— 144	— 201	— 175	— 088	— 038	— 002	— 19	+ 7	— 69	+ 79	12.1	..— 14	+ 98.
+ 7.33	— 038	— 052	— 056	— 079	— 054	— 010	— 18	+ 4	— 50	+ 27	11.4	.— 33	— 18..
+ 5.1	— 028	— 063	— 124	— 139	— 072	— 021	— 41	— 6	— 87	+ 58	12.0	..— 48	+ 23.
+ 5.4	— 028	— 012	— 026	— 099	— 048	— 017	— 17	— 10	— 49	+ 17	11.4	.— 25	— 29..
+ 5.7	— 101	— 059	— 048	— 030	+ 020	— 032	+ 12	— 1	— 20	+ 11	11.4	.+ 7	— 7..
+ 5.10	000	— 016	— 067	— 080	— 073	— 054	— 18	— 6	— 69	+ 8	11.3	.— 39	— 59..
+ 5.13	— 077	— 072	— 068	— 043	— 043	— 035	— 1	+ 8	— 46	+ 13	11.4	.— 19	— 30..
+ 5.16	— 010	— 068	— 090	— 100	— 094	— 066	— 20	— 7	— 88	+ 13	11.4	.— 61	— 72..
+ 5.19	— 148	— 172	— 174	— 119	— 068	— 044	— 17	— 4	— 96	+ 62	12.1	..— 57	+ 21.
+ 5.22	— 127	— 132	— 119	— 085	— 042	— 014	— 12	+ 15	— 62	+ 51	11.9	.— 81	— 23..
+ 5.25	— 170	— 044	— 028	— 053	— 037	— 019	+ 12	+ 1	— 35	+ 8	11.3	.— 5	— 25..
+ 5.28	— 159	— 059	— 038	— 059	— 040	— 003	+ 5	— 8	— 35	+ 21	11.5	.— 16	— 12..
+ 5.31	— 111	— 022	— 076	— 104	— 089	— 039	— 14	— 8	— 78	+ 22	11.5	.— 59	— 54..
+ 5.34	— 050	— 055	— 072	— 120	— 109	— 046	— 23	+ 5	— 89	+ 18	11.5	.— 66	— 68..
+ 3.1	— 028	— 035	— 076	— 074	— 039	— 017	— 20	+ 5	— 50	+ 32	12.0	..— 26	— 4.
+ 3.3	— 054	— 030	— 033	— 045	— 023	+ 042	— 17	+ 3	— 14	+ 39	12.1	..+ 9	+ 43.
+ 3.5	— 056	— 022	— 035	— 037	— 023	— 027	— 1	+ 4	— 30	+ 5	11.6	.+ 9	— 15..
+ 3.7	— 011	— 049	— 074	— 046	— 011	+ 004	— 20	+ 6	— 29	+ 38	12.1	..— 6	+ 27.
+ 3.9	— 054	— 083	— 100	— 067	— 048	— 067	— 5		— 68	+ 14	11.8	.— 39	— 48..
+ 3.11	— 134	— 127	— 053	— 051	— 061	— 059	+ 17	+ 3	— 56	— 4	11.5	.— 8	— 48..
+ 3.13	— 073	— 070	— 061	— 020	— 022	— 055	+ 9	— 10	— 38	— 1	11.5	.+ 7	— 27..
+ 3.15	— 123	— 157	— 116	— 104	— 093	— 058	— 7	— 13	— 92	+ 28	12.0	.— 83	— 68..
+ 3.17	— 028	— 089	— 048	— 074	— 026	+ 003	— 15	+ 14	— 36	+ 32	12.0	..— 12	+ 10.
+ 3.19	— 109	— 194	— 051	— 020	+ 042	+ 054	+ 4	— 4	+ 10	+ 54	12.3	..+ 27	+ 87.
+ 3.21	— 117	— 109	— 073	— 076	— 034	— 011	— 4	+ 4	— 47	+ 34	12.1	..— 23	+ 3.
+ 3.23	— 146	— 065	— 067	— 102	— 088	— 049	— 1		— 78	+ 14	11.8	.— 49	— 58..
+ 3.25	— 137	— 140	— 091	— 088	— 109	— 107	+ 14		— 100	— 10	11.4	.— 46	— 96..
+ 3.27	— 073	— 155	— 091	— 114	— 141	— 105	— 3		— 115	— 7	11.5	.— 64	— 109..
+ 3.29	— 049	— 070	— 024	— 055	— 089	— 075	+ 8		— 64	— 25	11.2	.— 3	— 79..
+ 3.31	— 002	— 019	— 027	— 103	— 132	— 113	— 5	+ 9	— 99	— 36	11.0	.— 35	— 130..
+ 3.33	— 115	— 153	— 218	— 192	— 187	— 192	— 17	+ 11	— 196	+ 10	11.7	.— 163	— 178..
+ 3.35	— 120	+ 002	— 088	— 180	— 138	— 101	— 19	+ 5	— 130	+ 7	11.7	.— 92	— 113..
+ 1.1	— 113	— 023	— 035	— 065	— 053	— 008	— 4	— 15	— 41	+ 17	12.3	..— 18	— 21.
+ 1.3	+ 129	+ 085	+ 059	+ 077	+ 127	+ 170	— 30	+ 5	+ 111	+ 56	12.9	..+ 94	+ 153.
+ 1.5	+ 137	+ 115	+ 084	+ 030	+ 067	+ 136	— 33	+ 6	+ 78	+ 34	12.6	..+ 88	+ 112.
+ 1.7	+ 007	— 053	— 045	— 035	— 032	— 039	— 5	— 1	— 37	+ 2	12.1	..— 8	— 35.
+ 1.9	— 063	— 057	— 030	— 063	— 070	— 078	+ 9	— 6	— 62	— 20	11.8	.+ 28	— 34..
+ 1.11	— 065	— 102	— 138	— 142	— 138	— 144	— 12	— 10	— 140	— 2	12.1	..— 111	— 145.
+ 1.13	— 089	— 092	— 084	— 061	— 065	— 067	+ 2	— 8	— 68	+ 5	12.2	..— 40	— 62.
+ 1.15	— 005	— 102	— 068	— 047	— 015	+ 035	— 22	— 8	— 21	+ 50	12.8	..— 29	+ 21.
+ 1.17	+ 051	— 005	+ 001	+ 047	+ 136	+ 164	— 28	+ 4	+ 93	+ 82	13.3	...+ 131	+ 232
+ 1.19	— 059	— 064	— 039	+ 007	+ 146	+ 208	— 34	— 10	+ 88	+ 128	14.0	...+ 80	+ 254
+ 1.21	— 045	— 080	— 079	— 047	+ 019	+ 037	— 19		— 13	+ 59	13.0	..— 35	+ 29.
+ 1.23	+ 078	— 036	+ 004	— 033	— 086	— 084	+ 2		— 54	— 44	11.5	.+ 61	— 40..
+ 1.25	— 007	— 058	— 040	— 019	— 020	— 019	— 2		— 24	+ 8	12.2	..+ 3	— 15.
+ 1.27	+ 081	— 010	— 043	— 043	+ 001	+ 004	— 27		— 18	+ 27	12.5	..— 2	+ 11.

Region	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6	1st. Comp.	Corr. fr. sc. err.	Second Comput.		ρ_1	$\Delta \log D$ (10—14) in 0.001	
							Δc		Δa	Δb			
+ 1.29	+ 112	+ 029	— 079	— 075	— 076	— 072	— 34		— 75	+ 3	12.2	..— 46	— 72.
+ 1.31	+ 086	— 001	— 040	— 105	— 193	— 216	0		— 145	— 87	10.9	+ 151	— 19...
+ 1.33	— 201	— 153	— 117	— 192	— 190	— 193	+ 13		— 177	— 29	11.7	.— 76	— 153..
+ 1.35	— 143	— 143	— 191	— 235	— 255	— 264	— 4	— 20	— 237	— 33	11.6	.— 134	— 217..
— 1.1	— 085	— 054	— 075	— 096	— 086	— 062	— 7	— 14	— 80	+ 9	12.2	..— 54	— 70.
— 1.3	+ 030	+ 019	— 018	+ 002	+ 067	+ 106	— 28	+ 1	+ 43	+ 64	13.0	...+ 86	+ 158
— 1.5	+ 013	— 005	+ 003	+ 028	+ 077	+ 127	— 21	+ 4	+ 62	+ 62	13.0	...+ 105	+ 174
— 1.7	+ 086	+ 037	— 005	+ 035	+ 071	+ 112	— 31	+ 4	+ 57	+ 56	12.9	..+ 40	+ 99.
— 1.9	+ 036	— 018	— 006	+ 029	+ 046	+ 084	— 17	+ 1	+ 41	+ 41	12.7	..+ 44	+ 79.
— 1.11	+ 007	— 007	— 055	— 059	—039	— 041	— 15	— 3	— 48	+ 9	12.2	..— 22	— 38.
— 1.13	— 073	— 073	— 132	— 206	— 238	— 202	— 17	+ 4	— 200	— 29	11.7	.— 99	— 176..
— 1.15	+ 063	— 010	— 122	— 135	—102	— 070	— 44	— 5	— 107	+ 30	12.5	..— 93	— 75.
— 1.17	+ 218	+ 095	+ 044	+ 053	+ 112	+ 144	— 44	— 18	+ 91	+ 53	12.9	..+ 78	+ 132.
— 1.19	+ 096	+ 076	+ 022	+ 015	+ 100	+ 145	— 38	+ 14	+ 74	+ 68	13.1	...+ 116	+ 194
— 1.21	+ 208	+ 115	+ 052	— 027	— 043	— 070	— 16		— 27	— 52	11.4	.+ 93	— 21..
— 1.23	+ 270	+ 175	+ 095	+ 045	—003	— 012	— 19		+ 27	— 51	11.4	.+ 147	+ 35..
— 1.31	+ 031	— 014	— 018	+ 015	— 021	— 025	+ 2		— 12	— 10	12.0	.+ 64	+ 18..
— 1.33	+ 092	+ 023	+ 039	+ 026	+ 005	— 031	+ 5		+ 8	— 34	11.6	.+ 114	+ 29..
— 1.35	— 019	— 065	— 062	— 065	— 052	— 005	— 20	+ 10	— 46	+ 29	12.5	..— 31	— 15.
— 3.1	— 182	— 042	— 043	— 047	— 015	+ 015	+ 6	— 6	— 21	+ 31	12.0	..+ 3	+ 24.
— 3.3	— 152	— 115	— 091	— 110	— 093	— 031	— 9	— 3	— 82	+ 32	12.0	..— 58	— 36.
— 3.5	— 091	— 107	— 049	— 104	— 070	— 003	— 13	+ 2	— 58	+ 31	12.0	..— 34	— 13.
— 3.7	— 155	— 067	— 041	— 106	— 089	— 029	— 1	— 3	— 69	+ 14	11.8	.— 41	— 49..
— 3.9	— 088	— 054	— 056	— 076	— 046	— 003	— 12	— 1	— 45	+ 30	12.0	.— 40	— 21..
— 3.11	— 037	— 046	— 071	— 063	— 064	— 051	— 8	+ 2	— 62	+ 9	11.7	.— 27	— 44..
— 3.13	+ 046	+ 002	— 104	— 127	— 115	— 134	— 25	— 16	— 121	— 11	11.4	.— 67	— 119..
— 3.15	+ 018	+ 057	— 104	— 173	— 155	— 132	— 36	— 2	— 144	— 5	11.5	.— 95	— 136..
— 3.17	+ 096	— 022	— 081	+ 017	+ 024	+ 063	— 35	+ 2	+ 11	+ 60	12.4	..+ 25	+ 96.
— 3.19	— 019	— 054	— 069	— 054	— 023	— 016	— 15	+ 8	— 38	+ 27	11.9	.— 29	— 15..
— 3.21	+ 022	— 076	— 060	— 115	— 115	— 099	— 14		— 100	— 13	11.4	.— 45	— 100..
— 3.23	— 050	— 113	— 050	— 014	+ 005	— 056	+ 16		— 26	— 4	11.5	.+ 23	— 17..
— 3.31	— 158	— 129	— 131	— 040	+ 008	— 046	+ 10		— 46	+ 36	12.1	..— 23	+ 6.
— 3.33	— 083	— 070	— 042	— 058	— 050	— 021	— 1		— 43	+ 12	11.7	.— 12	— 24..
— 3.35	— 068	— 005	+ 010	— 025	— 004	+ 022	+ 2		0	+ 11	11.7	.+ 32	+ 18..
— 5.1	+ 002	+ 003	— 041	— 062	+ 001	+ 042	— 29	— 26	— 14	+ 48	11.9	.— 28	+ 24..
— 5.4	— 080	— 061	— 160	— 120	— 075	— 026	— 37	— 2	— 92	+ 65	12.1	..— 54	+ 29.
— 5.7	— 221	— 155	— 124	— 132	— 116	— 077	+ 2	+ 4	— 112	+ 25	11.6	.— 96	— 85..
— 5.10	— 086	— 107	— 157	— 179	— 111	— 024	— 46	— 2	— 116	+ 73	12.3	..— 80	+ 18.
— 5.13	— 034	— 174	— 162	— 099	— 042	— 017	— 32	— 26	— 74	+ 69	12.2	..— 37	+ 53.
— 5.16	— 124	— 102	— 115	— 107	— 072	— 057	— 9	+ 7	— 86	+ 30	11.6	.— 75	— 55..
— 5.19	— 108	— 042	— 022	— 084	— 073	— 043	+ 3		— 58	— 3	11.2	.— 23	— 63..
— 5.22	— 162	— 142	— 104	— 046	— 030	— 057	+ 14		— 56	+ 18	11.5	.— 33	— 35..
— 5.31	— 089	— 052	— 007	+ 008	+ 006	— 017	+ 20		— 2	— 6	11.1	.+ 33	— 12..
— 5.34	— 026	— 052	— 033	— 003	— 043	— 036	+ 6		— 29	— 8	11.1	.+ 6	— 43..
— 7.3	+ 157	+ 028	— 073	— 072	— 021	+ 024	— 38	0	— 33	+ 52	11.7	.— 43	+ 19..
— 7.9	— 079	— 041	— 070	— 078	— 042	— 001	— 17	— 6	— 47	+ 38	11.5	.— 40	— 4..
— 7.15	— 156	— 134	— 103	— 129	— 041	+ 011	— 20	— 14	— 63	+ 66	11.9	.— 95	— 7..
— 7.21	+ 011	+ 004	— 031	— 057	— 053	— 035	— 13		— 45	+ 1	11.0	.— 18	— 52..
— 7.27	— 038	— 048	— 075	— 037	— 006	— 004	— 12		— 27	+ 33	11.5	.— 14	+ 12..
— 7.33	+ 035	+ 022	+ 056	— 035	— 038	— 004	— 4		— 10	— 20	10.7	+ 84	+ 11...
— 9	— 152	— 076	— 050	— 055	— 026	— 017	+ 7		— 36	+ 19	11.2	.— 14	— 14..

however, far too small to account for the great negative $\triangle c = -0.028$. For the regions of Table 58, so far as they are covered by the Bonn *DM*, the average values of $A + C - 2B$ have been deduced as well as possible (the limits of the zone-hours do not coincide with the limits of the regions, so that also some smoothing was necessary now and then) and the results (in 0.01^m) put down in Table 58 next to the column of $\triangle c$. These values thus represent also the corrections of $\triangle c$ (in 0.001) caused by these scale errors. Comparing them we see that in some instances the remaining scale errors may account for part of the extreme negative values of $\triangle c$. But in other regions this is not the case, and there are as many cases where these scale errors would give a positive $\triangle c$, so that taking them into account would increase the deviations.

Remaining scale errors of this kind can of course only explain values of $\triangle c$ with average zero. The mean $\triangle c$ for all regions, however, is negative, just as is the greater part of the single values; this phenomenon cannot be explained by local deviations from a mean magnitude scale that is itself correct. By collecting the deviations of the observed values from the curves in a great number of the diagrams p. 40, 41 and 50 into a single diagram, it is seen that the average curve must be correct within 0.01 or 0.02 magnitude. If, however, it is allowed to leave the results of argument *H* (crosses) out of consideration, and to draw somewhat nearer to the other points, it appears possible to get a curve with positive curvature, represented by a term $+0.012\,m^2$ for the zones 15° to 60°, and by $+0.004\,m^2$ for the *SDM*. In this way a negative $\triangle c = -0.005$ and -0.002 could result; but these values are certainly too small to account for the mean correction shown by the stardensities. Thus as a result of this discussion it does not seem possible to explain the result for $\triangle c$ by remaining errors of the magnitude scale.

60. If neither errors of the magnitude scale nor real peculiarities in the space distribution of the stars can give an explanation of the phenomenon, we may try to find its origin in the luminosity curve used in our computations. From the charts I we learn that the strongest positive deviations for the bright stars coincide with clusters of *B*-type stars; these stars have a special distribution of their own in space, not coinciding with the accumulation of other stars. Thus it may be questionable whether they should be included in a computation of space density based on the KAPTEYN luminosity curve. Therefore another computation was made after excluding the *B*-stars 0—6.5 (*B* 0—*B* 5 according to *Harv. Annals* **50**). As to the surface density we find the condensations in Orion and Argo considerably weakened now, though still perceptible, while those in Cygnus and Sagittarius have also diminished somewhat. Computing the change in $\triangle c$ in such a way that the change of log $N(6.5)$ was applied to both $\triangle_1$ and $\triangle_2$ while $\triangle_3 \ldots \triangle_6$ were left unchanged, we find for the zones $+30°$ $+10°$ $-10°$ $-30°$ $+.002$, $+.006$, $+.012$, $+.003$; for the galactic zone this is nearly half the amount of $\triangle c$ itself. For some separate regions the result of a computation, made in the same way, was: for the four Cygnus regions $+.010$, $+.009$, $+.005$, $+.006$; for region -1.17 (Orion) $+.028$, for -1.23 (Carina) $+.029$ (here $\triangle c$ itself was small). All these values are too great, because the change of $\triangle_2$ has been taken too great, while the diminishing of $\triangle_3$ and $\triangle_4$ by an omission of the *B*-type stars has been neglected. Thus we may say that the *B*-type stars cannot explain the whole phenomenon expressed in the negative $\triangle c$, only somewhat less than half of it.

This specially arranged class of luminous stars causes a surplus of bright (naked eye) stars outside the regular relation between star number and magnitude. But the real surplus of bright stars is still greater; it suggests the existence of another class of stars of high luminosity, with a special space distribution, not confined to the galactic zone. Such stars of the same or greater luminosity than *B*-type stars must be classed as supergiants. It might seem that their number is much too small for our purpose, as we want a class somewhat comparable with class B in numerousness. A recent investigation, however, of K. F. BOTTLINGER at Berlin (*Lichtelektrische Farbenindices von* 459 *Sternen*)[1]) has revealed the existence of a group of supergiants, recognizable by a great colour-excess, of mean absolute magnitude between $M = -8$ and -9. Together with the c-stars their number amounts to 52 among 459 stars (27 G- and K-stars with $E > 0.090$, 14 Fc stars or composite spectra, 11 with $E < 0.090$ but very small proper motion). Their concentration to the Milky Way is less strong than for the Cepheids, as among the first group there are 4, and among the last group 5 with $\beta > 30°$. Stars of absolute magnitude $M = -8.5$ within a distance of 1000 parsecs will appear brighter than 6.5; as the space density outside this limit is small already, such stars will contribute only an insignificant percentage to the fainter classes. The existence of these highly luminous stars thus gives a sufficient explanation of the strong negative values of $\triangle c$.

61. It follows that $\triangle a$, $\triangle b$, $\triangle c$ computed as in § 57, cannot be used to find the space density by means of our formulae; for this reason the results of $\triangle a$ and $\triangle b$ have been omitted from Table 58. As the number of stars in the brightest classes is materially changed by these supergiants it must be excluded in deriving the space density. Then the remaining $\triangle_3 \ldots \triangle_6$ are too few to compute quadratic formulae. Thus the only way left is to use these $\triangle_3 \ldots \triangle_6$ as representing two data, represent them by linear formulae $\triangle a + (m - 8.0)\triangle b$, and thus keep the mean c of the whole zone. The results for $D(\rho)$ will then be found for a much smaller extent in ρ. With the weights 2, 3, 4, 4 we have:

$$\triangle a = +.20\triangle_3 + .26\triangle_4 + .29\triangle_5 + .25\triangle_6$$
$$\triangle b = -.38\triangle_3 - .20\triangle_4 + .10\triangle_5 + .48\triangle_6.$$

The results are put down in Table 58 under the heading „second computation". They too show a systematic irregularity, the values $\triangle b$ being for the greater part positive, the $\triangle a$ negative. It may be expressed in this way that not only the number of bright stars $N(6.5)$ is too great, but also the number of stars of the 7th and 8th magnitudes is too small. It is the same fact that appeared already in the comparison of statistical and photometric magnitudes (p. 42). It also presented itself in the deviations on Charts II being more negative than on Charts I and III. Therefore a somewhat closer consideration is needed, before we can use our results for the deduction of space densities.

62. The results for the separate regions were at first collected into means for the zones[2]) (Table 59). Coefficients $\triangle a$, $\triangle b$, $\triangle c$ were derived and used to compute $\triangle \log A(m)$ for magnitudes 6.0—9.0; they may be considered as interpolated values, expressing the $\triangle \log A$ given by observation for round magnitudes.

In order to compare them with a wider range of magnitudes, the differences between $\log A(m)$, derived from the tables $\log N(m)$ of VAN RHIJN, and the schematical universe were computed for $\beta = 10°$ (mean of 5°, 10°, 20°), $\beta = 30°$ (mean of 20°, 30°, 40°) and $\beta = 40$—$90°$. They are contained in Table 60 for $m = 3$ to 13, accompanied by our values $m = 6$ to 9 for the zones 0° to 20°, 20° to 40°, 40° to 90°.

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[1]) Veröffentlichungen der Universitäts Sternwarte Berlin—Babelsberg, Band III, Heft 4, 1923.

[2]) By a mistake in forming the means the regions -1.23, -3.23, -5.22 have been omitted.

Table 59. Results for the zones.

Gal. lat.		Nr.	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6	Δa	Δb	Δc	6.0	7.0	8.0	9.0
+ 90	+ 60	7	— 69	— 102	— 92	— 90	— 77	— 48	— 93	+ 15	— 11	— 90	— 97	— 83	— 46
+ 60	+ 40	12	— 84	— 65	— 78	— 85	— 58	— 33	— 79	+ 17	— 11	— 79	— 84	— 68	— 29
+ 40	+ 20	18	— 79	— 87	— 74	— 80	— 68	— 52	— 77	+ 10	— 5	— 81	— 81	— 71	— 51
+ 20	+ 0	18	— 6	— 42	— 49	— 56	— 39	— 24	— 54	+ 5	— 13	— 32	— 53	— 49	— 18
— 0	— 20	14	+ 50	+ 8	— 24	— 27	— 7	+ 15	— 28	+ 4	— 21	+ 13	— 25	— 21	+ 25
— 20	— 40	14	— 61	— 52	— 67	— 77	— 58	— 33	— 72	+ 11	— 11	— 63	— 74	— 64	— 31
— 40	— 60	9	— 85	— 82	— 91	— 86	— 58	— 28	— 86	+ 23	— 13	— 91	— 94	— 72	— 23
— 60	— 90	7	— 52	— 35	— 49	— 66	— 33	— 4	— 56	+ 16	— 13	— 51	— 61	— 45	— 03

Table 60. Deviations from the schematical universe.

m		3	4	5	6	7	8	9	10	11	12	13
10°	Gron.	+ 155	+ 101	+ 55	+ 20	— 2	— 11	— 10	— 6	— 6	— 10	— 12
	P				— 10	— 39	— 35	— 03				
30°	Gron.	+ 74	+ 22	— 22	— 53	— 69	— 71	— 67	— 60	— 54	— 48	— 41
	P				— 72	— 78	— 66	— 41				
40°—90°	Gron.	— 18	— 37	— 47	— 48	— 41	— 30	— 19	— 12	— 9	— 15	— 20
	P				— 78	— 84	— 67	— 25				

In the figure on the next page these values are represented at the lefthand side. It appears that an analogous depression is visible in the Groningen results, but in a much smaller degree. The table of Van Rhijn is identical (after correction of the magnitudes) with table 1 for log $N(m)$ derived by Kapteyn in *Groningen Public.* 18. The original data on which it rests, are compared with the adopted curves in schema 35 of this work (p. 37); from the column „$O-C$ in fraction of computed number" we find the difference $O-C$ in log. of number, in three decimals, for

$m =$ 2.5	3.5	4.5	6.75	7.62	8.12	8.67	9.10	9.25	9.96	10.86
+ 41	+ 13	+ 13	0	— 9	— 32	— 22	— 4	+ 57[1]	+ 49	— 81
Harvard Photometry				*BDM.* Seeliger			*CPD*	*BDM*	var. star-fields	

Thus it appears that the number of stars of 7^m and 8^m given by observation is smaller than has been acccpted in the curves. In the curves of Kapteyn this irregularity has been smoothed away, because the errors that were left in the observational data by the adopted curve — which shows its effect still in a broad concave curvature — were not greater here than for other magnitudes. In our more detailed discussion it now returns.

It is not possible to speak with complete certainty about the reality of this irregularity of the $A(m)$ curve. It is based on a rather unhomogeneous complex of Durchmusterung estimates, corrected afterwards by photometric measures of a small part taken as a specimen of the whole. Complete certainty can only be obtained when the brightness of all the stars down to magnitude 9 or 10 has been measured photometrically or photographically — a work well within the reach of modern astronomical resources, and also extremely desirable in view of the remaining uncertainties in the surface distribution due to irregularities in the scale error. Considering the existing data,

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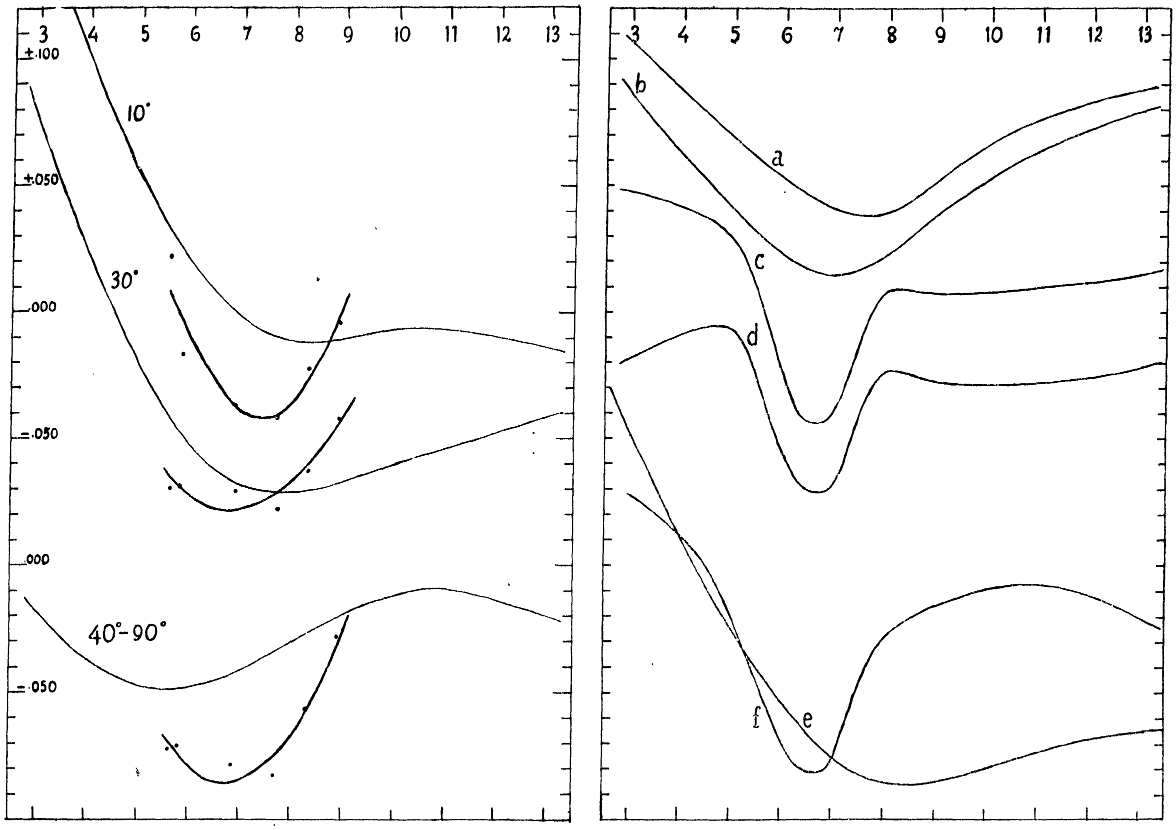
[1] This difference is too great, because the limit 9.0 — 9.1 DM has been adopted only 0.05 below the photometric aequivalent of 9.0.



however, we think it highly improbable that the irregularity found here can be explained by some remaining systematic error of the scales.[1] It must rather be assumed to be a real phenomenon, a real shortness of stars of the 7th and 8th magnitudes, followed by a greater normal number in the fainter classes.

63. Such a short-period irregularity in the function A (m) cannot be explained by any irregularity in the space distribution $D(\rho)$ alone, because owing to the great dispersion of the luminosity curve we shall always have a great dispersion in the A curve. It is not compatible with the KAPTEYN luminosity curve at all and points to irregularities in the luminosity curve itself. Now from other researches it is probable that the real luminosity function is composed of different luminosity functions of the separate spectral classes with their giant- and dwarf divisions, so that it cannot have the regular Gaussian form. The method used by KAPTEYN in deriving it — distributing the stars of a certain m and μ over a number of sections in distance according to a probability function — while admirably adapted to show its general course, must needs efface all minor irregularities.

An irregularity in the luminosity function alone is also insufficient to cause a depression in the A curve of this steep character, if the density function $D(\rho)$ is smooth and regular. Irregularities in both curves must work together. This may be shown by computing the values of $A(m)$ corresponding to different arbitrarily adopted irregularities in the functions $D(\rho)$ and $\Phi(M)$. For

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[1] An irregularity of the Harvard scale itself, amounting to 0.1^m, affecting only this small range of magnitudes, is wholly inadmissible.

these computations mechanical integration is necessary; it is easily performed by computing, instead of $A(m)$ itself, the function

$$A(m).10^{-0.6m} = [\bar{6}.147] \int D(\rho)\, \Phi(m-\rho)\, 10^{-0.6\,(m-\rho)}\, d\rho$$

by means of a table $D(\rho)$ and another table $[6.147]$ $\Phi(M)$ $10^{-0.6\,M}$. Using the $D(\rho)$ of the schematical universe and simply adding the products for ρ increasing with unity, the results $A(m)$ of Table 4 are obtained, now resolved in the contributions of the separate ρ. Thus it is easy to compute the effect of arbitrary irregular changes in the primary functions $D(\rho)$ and $\Phi(M)$. The result of some of these computations (made with the data of β 40°—90°) is shown on the right hand side of the preceding figure. Curve a is obtained by doubling $\Phi(-6)$ and $\Phi(-7)$ and omitting $\Phi(-4)$ and $\Phi(-5)$; curve b by doubling $D(7)$ and omitting $D(10)$; they show a similar wave, much smoother than is shown by the empirical curve. Curves c and d result from the combinations: $\Phi(-4)$ and $\Phi(-5)$ increased by half their amount and $D(11)$ omitted in the first case, $D(10)$ and $D(11)$ increased by half their amount and $\Phi(-4)$ omitted in the second. Here we have those rapid changes that were wanted. The somewhat curious result that nearly opposite changes in the primary functions cause nearly identical variations of the $A(m)$ is only a corollary of the fact that from $A(m)$ alone it is not possible to find both $D(\rho)$ and $\Phi(M)$, and the same A can be satisfied by an infinity of suppositions about these functions. We may try to explain it in this case by the irregularity in $\Phi(M)$ already spoken of, the great surplus of supergiants, compensated by a lack of somewhat smaller stars (cf. the remark of BOTTLINGER, that his supergiants of -8^m seem to be separated by a gap from the ordinary giants). If we change the luminosity curve in this way: $2\,\Phi(-9)$, $4\,\Phi(-8)$, $2\,\Phi(-7)$, $\frac{1}{2}\,\Phi(-6)$, $\frac{1}{2}\,\Phi(-5)$ we get curve e, where the wave is again too broad. Combining it with $2\,D(12)$ and $\frac{1}{2}\,D(10)$ we get curve f, which shows a rapid fall and rise. These specimens of computations show the possibility of short period variations in the A curve; they do not pretend, of course, to represent the real distributions, as other irregularities may occur for lower M, and moreover the supergiants may have a distribution of their own, not coinciding with the distribution of the bulk of the stars.

In studying this last we are obliged to use the smooth luminosity curve of KAPTEYN. Then it is necessary to use also smoothed values of $A(m)$, where the short period irregularity has been removed. This was effectuated by reducing our values to the curve of VAN RHIJN; from table 60 we find the differences -26, -30, -19, $+11$ for $m = 6$, 7, 8, 9. In the reductions (by another arrangement of data) somewhat more negative values -30, -38, -27, $+03$ have been used, from which the reductions to VAN RHIJN for 6.9, 7.7, 8.3, 8.9: $+38$, $+33$, $+20$, 00 were derived; they give the correction to Δa $+0.022$, to Δb -0.019. Thus the results of the second computation in Table 58 have all been corrected by this amount, before using them to find the space density.

The space density.

64. In deducing the space density the following course was adopted. From log $A(m)$ for 6^m, 8^m and 10^m for the schematical universe (§ 4) values a_0, b_0, c_0, $\rho_1{}^0$, h_0, k_0, l_0 and $D_0(\rho)$ (this last of course somewhat deviating from D in table 4) were computed for $\beta = 10°$, 30°, 50°, 70°, 90°. Then by means of Δa and Δb the values $a = a_0 + \Delta a$, $b = b_0 + \Delta b$, $c = c_0$ were formed, h, k, l and $D(\rho)$ were computed and the differences log $D(\rho)$—log $D_0(\rho)$ were added to the densities log D of Table 4.

Before treating the separate regions the mean distribution after galactic latitude was derived by taking the average $\triangle a$ and $\triangle b$ for each zone. They are given in Table 61 (already corrected); the resulting ρ_1 (giving the distance for which D is determined by the mean surface density for 8.0, independent of $\triangle b$) is put in the 4th column; the 5th contains in stead of $\triangle h$ and $\triangle k$ the $\triangle \log D = \triangle h + \triangle k \; (\rho - \rho_1)$ for $\rho = 10$ and $\rho = 14$. The distances ρ, for which $\log D = 0.0$, 9.95, 9.90 etc., derived graphically, are given in the last section. If we assume the values $A(m)$ to be trustworthy between the limits 8.0 ± 0.9 the values $D(\rho)$ will be so (according to formulae 12, p. 7) between the limits $\rho_1 \pm 0.5$ (the factor $(r - c)/r$ varying between 0.54 and 0.62). The values of ρ outside these limits are put in parentheses.

Table 61. Results for the zones, second computation.

β	Number	$\triangle a$	$\triangle b$	ρ_1	$\triangle \log D$(10,14)		$D = 9.95$	9.90	9.85	9.80	9.75	9.70	9.60	9.50
+90	1	−64	−29	10.82	−8	−224				10.39	10.65	10.89	11.32	(11.71)
+70	6	−53	+9	11.40	−75	−7				(10.14)	(10.61)	11.00	11.61	(12.12)
+50	12	−41	+6	11.56	−58	−14				(10.75)	11.17	11.52	(12.08)	
+30	18	−46	−7	11.73	−25	−73			(11.18)	11.59	11.93	12.22	(12.76)	
+10	18	−19	−5	12.31	−1	−33	(11.43)	12.08	12.57	(12.95)				
−10	15	+14	−3	12.34	+25	+5	11.88	12.38	12.82	(13.20)				
−30	15	−34	−2	11.80	−29	−41			(11.23)	11.66	12.01	(12.31)	(12.85)	
−50	10	−42	+12	11.65	−77	+11				(10.60)	(11.10)	11.48	12.11	(12.60)
−70	6	−15	+9	11.40	−37	+31				(10.50)	10.91	11.24	11.80	(12.29)
−90	1	−14	0	11.24	−14	−14				(10.50)	10.84	11.12	11.63	(12.09)

For each of these surfaces of equal density $r \sin \beta$, the vertical distance of the computed points from the galactic plane, is given in Table 62 (in parsecs).

Table 62.

$\log D$	$\beta = 90°$			$\beta = 70°$			$\beta = 50°$			$\beta = 30°$		
	N	S	S—N	N	S	S—N	N	S	S—N	N	S	S—N
9.60	184	212	+28	198	216	+18	200	202	+2			
9.70	151	168	+17	149	167	+18	154	151	−3	139	145	+6
9.75							131	127	−4	122	126	+4
Weight			1			6			12			18

Thus the southern side seems to be somewhat more convex, the northern side somewhat more plane or concave. The mean of the differences is $+5 \pm 2.4$. It is only by this small amount that the surfaces of equal density on the northern side are nearer to us than those on the southern side. Thus we may say that the sun is situated perhaps somewhat but not more than 3 parsecs to the northern side of the plane of symmetry of the Durchmusterung stars. It could suggest the idea that the thinning out of the stars perpendicular to the galactic plane has an optical rather than a geometrical cause; but it is more likely that it is mere chance.

SHAPLEY (*Harvard Circular* 239) finds the sun 50 light years = 15 parsecs to the north of the plane of the faint and remote B stars (7.25—8.25); thus these stars lie 12 parsecs to the south of the plane of symmetry of the central parts of our star system. A summary of KAPTEYN (*Groningen*

Publ. **18**. S. 21, p. 27), stating that for $N(5.5)$ the *S*. hemisphere is 15 %, for $N(9.2)$ 11 % richer than the *N*. hemisphere, seems to contradict our result. The summaries 20, 21, 23, however, show that these differences are great for low, insignificant for high latitudes. As our Table 59 also shows zone —10° richer than zone + 10°, all evidences agree that the more remote starmasses and groups in the galactic plane are found somewhat to the southern side of the plane of symmetry of the star system in our surroundings.

65. For the computation of $D(\rho)$ for the separate regions somewhat simplified formulae may be used. If c and l are taken constant $=$ c_0 and l_0, we find from formulae 12, p. 7:

$$b = b_0 + \triangle b; \quad \rho_1 - 8 = (b - q)/2r; \quad \triangle k = \triangle b\,(r + l)/r;$$
$$\triangle \log D(12) = \triangle a - (l + r)\,\{(\rho_1 - 12)^2 - (\rho_1{}^0 - 12)^2\}, \text{ or}$$
$$\triangle \log D(\tfrac{1}{2}\,(\rho_1 + \rho_1{}^0)) = \triangle a;$$
$$\triangle \log D(\rho) = \triangle \log D(12) + \triangle k\,(\rho - 12).$$

The normal values b_0, $\rho_1{}^0$, and the coefficients used, corresponding to the schematical universe, are found in Table 62.

Table 62.

β	b_0	$\rho_1{}^0$	$(l + r)$	$(l + r)/r$
0°	0.498	12.52	0.0529	1.53
10°	0.488	12.38	0.0554	1.60
20°	0.468	12.10	0.0574	1.66
30°	0.450	11.83	0.0592	1.72
50°	0.425	11.47	0.0624	1.81
70°	0.411	11.26	0.0644	1.87
90°	0.409	11.24	0.0644	1.87

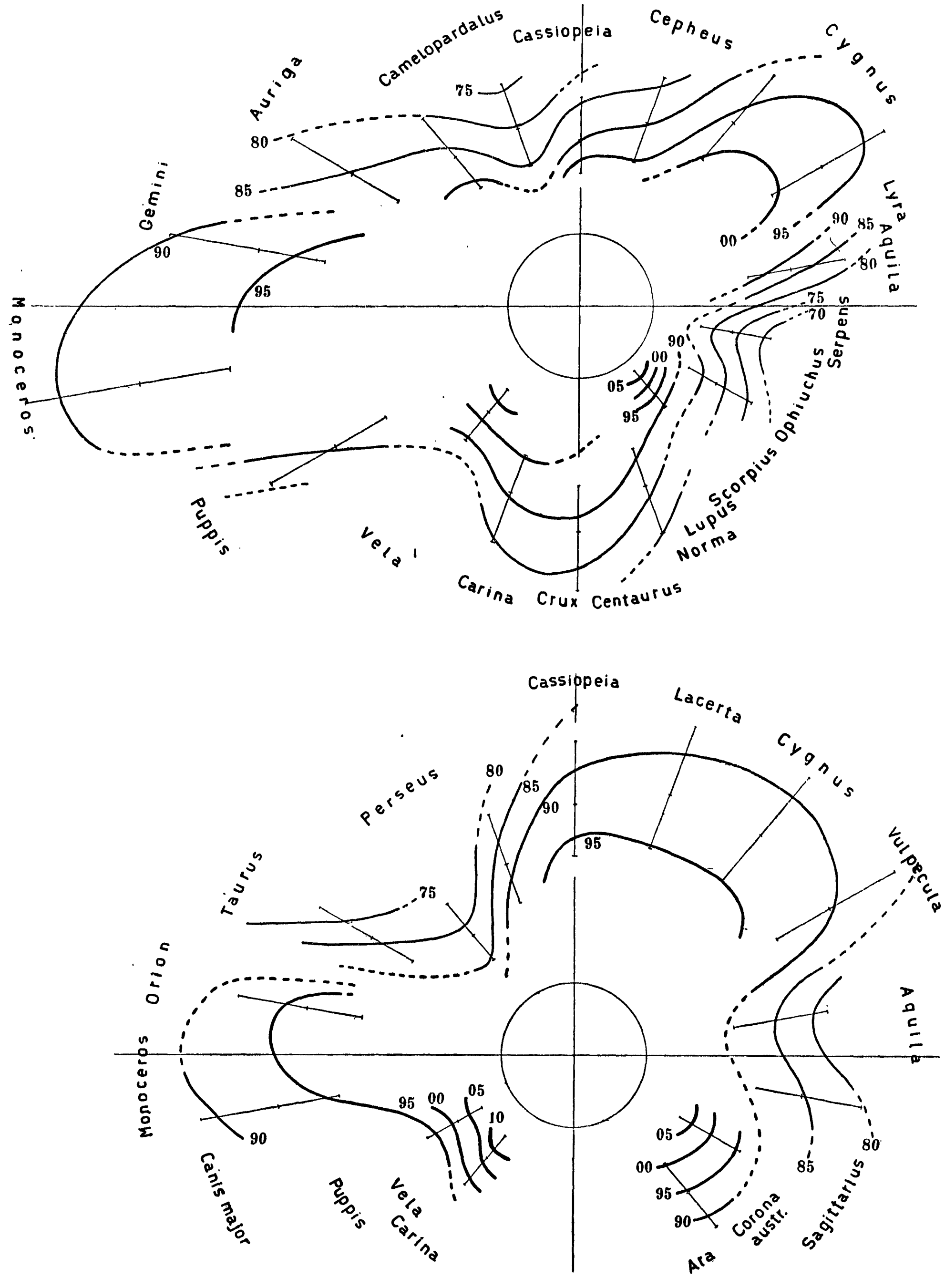
The last columns of Table 58 contain the results in this way, that after ρ_1, instead of $\triangle \log D$ (12) and $\triangle k$, two values $\triangle \log D$ are given for two integer values of ρ closest to ρ_1; the three dots and two numbers in the last column denoting $\rho = 10$ to 14, it is easily seen for what values of ρ these numbers stand. By adding $\triangle \log D$ for these and adjacent ρ to the values $\log D_0$ of Table 4, we get $\log D\,(\rho)$; drawing curves through them we may read the distances ρ for which $\log D$ takes the values 0.00, 9.95, 9.90 .... 9.7, 9.6, 9.5 ..... The results are summarized in the drawings p. 104 and 105, showing the distribution of space density by lines of equal density decreasing in strength with the density they represent. Because our DM data cover a limited range in magnitude only, the space density is also determined over a limited range in ρ, which (as in § 64) is assumed $\rho_1 \pm 0.5$. In the drawings this range is indicated by radial lines, with a dash for ρ_1; the lines of equal density are drawn in full only so far as they lie in these well determined realms. The drawings on p. 104, deduced from the regions between 0° and + 20°, and between 0° and —20°, represent horizontal sections through + 10° and —10° galactic latitude (strictly speaking cone-shaped sections), those on the next page show three vertical sections through galactic longitudes 30°—210°, 90°—270° and 150°—330°.

These drawings show clearly the irregular shape of the local starsystem. In Cygnus and Monoceros a great density, little below 1, extends to a great distance on the northern side of the galactic aequator; the Cygnus density continues on the southern side through Lacerta to Cassiopeia, from λ 40° to 90°. Deep gulfs of small density at 350° on the southern, at 140° on the northern side are caused by the absorbing nebulae of Ophiuchus and Taurus. Regions of great density at small distance are indicated at 210° (*S*) or 230° (*N*, *S*) in Canis major and Vela, at 310° (*N*) or 330° (*S*) in Scorpius. The lines of highest density $\log D = 0.00$ or 9.95 cannot be traced without breaks; truly we cannot say at all that they will have a continuous course around the sun and the central parts of the drawing. It may be as well that the parts drawn in the figures lie all apart and must be closed separately, surrounding separate condensations, which are situated in the directions 40°, 185° at distances 300 and 400 parsecs, and 230°, 320° at 100 to 150 parsecs. The starcounts of the DM catalogues can give no information on the space density within a sphere of 100 parsecs radius, nor can the brighter stars do so owing to the presence and behaviour of the supergiants. From a discussion of proper motions SCHOUTEN (*Thesis* „On the Determination of the Principal Laws of Statistical Astronomy" Tables 52—56) found a smaller density within this sphere. The data given by SHAPLEY in Harvard Circular 240, however, point to the contrary (*cf* § 76).

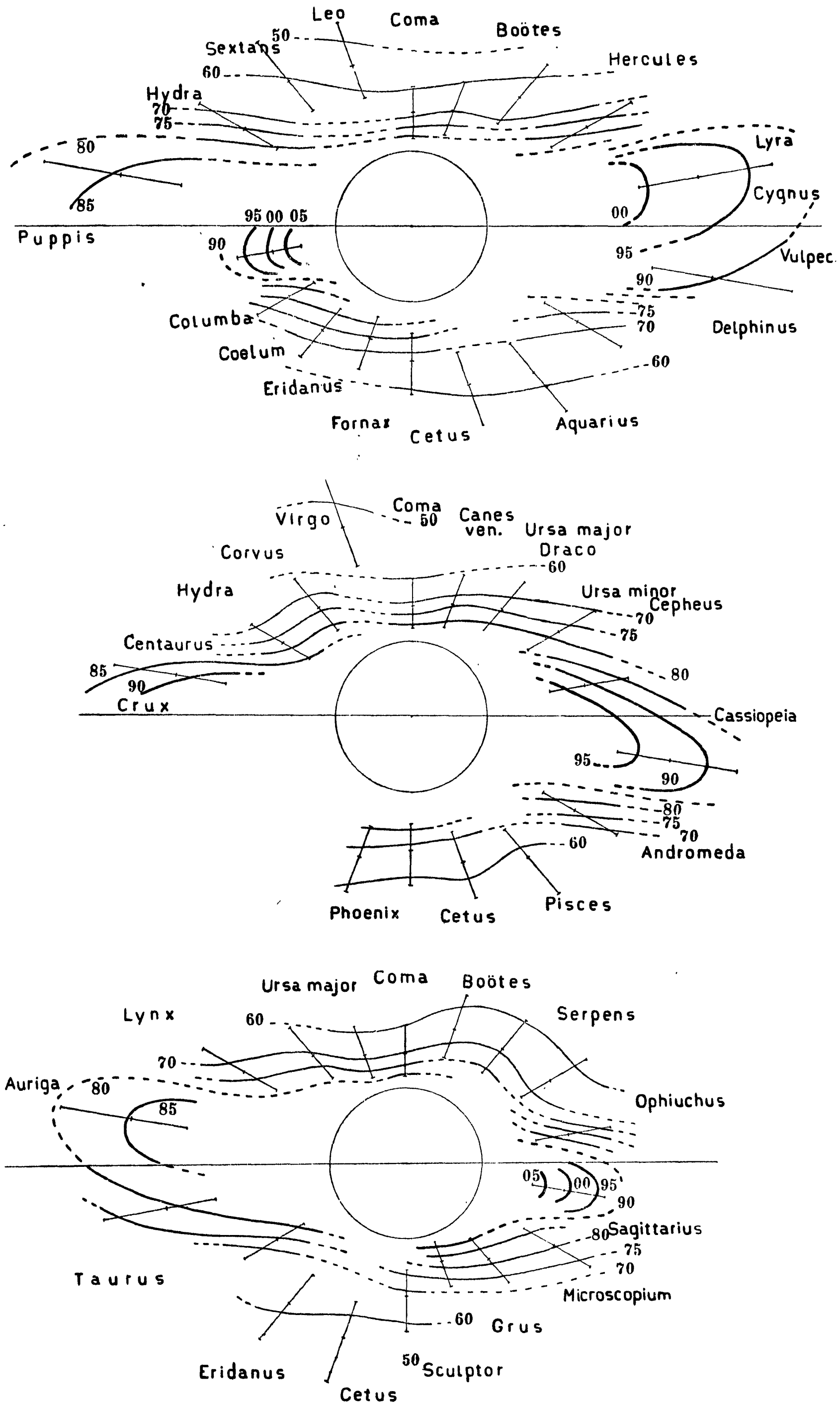
These drawings, based on average values for great regions of 20° square, can represent the distribution of density only in its largest features. First because all minor irregularities are smoothed and made invisible. And then because the limits of the regions are artificially defined by circles at round values of latitude and longitude without any regard to the real coherence of regions of surplus or small density, cutting these natural regions into pieces and distributing them over several of the regions used before. Thus their character, e.g. their excess of density, is weakened in the average of these regions, and the resulting space densities do not show the real extreme values. A striking instance is offered by the Carina condensation; by its small extension in latitude diluted in a large region of ordinary density, it appears in the drawing as a protuberance of secondary importance.

66. If we suppose that the lines and coloured fields on the charts I, II and III, obtained by smoothing the observed deviations from the schematical universe $\triangle$, represent the real fluctuations of surface density over the sky, we may deduce space densities in the direction of each point of the sky. Assuming the densities on chart II and III to stand for $m = 7.4$ and 8.6, we have $\triangle a = {}^1/_2\ (\triangle_{\rm II} + \triangle_{\rm III})$, $\triangle b = {}^5/_6\ (\triangle_{\rm III} - \triangle_{\rm II})$; after correcting them $+0.022$ and -0.019 (corresponding to corrections $+0.03$ to all values on chart II, $+0.01$ on chart III) we find $\triangle k$, ρ_1 and D ($\rho_1 \pm 0.05$) by the formulae of § 65. As it is questionable whether the fluctuations indicated on the charts for higher latitudes are sufficiently warranted, we have used this method only for β 0°, + 10° and — 10°; the results for the first will be given here. In the diagram p. 106 curves are constructed with the galactic longitude as absciss, representing the deviations of density for $m = 7.4$ and 8.6. They are based partly on the intersections of the lines for $\triangle = 0.05$, 0.10 .... on the charts with the galactic circle, partly on the points and crosses representing the averages of 4 at the cornerpoints (which had been used also in drawing the lines on the charts) situated between $\beta \pm 5°$. Between 250° and 290° where cornerpoints at — 62° are lacking, the values for the areas themselves are indicated by points and crosses with white centres. From these curves the values $\triangle_{\rm II} = \triangle \log A$ (7.4) and $\triangle_{\rm III} = \triangle \log A$ (8.6) could be read for each 10° of longitude or for other interesting points.

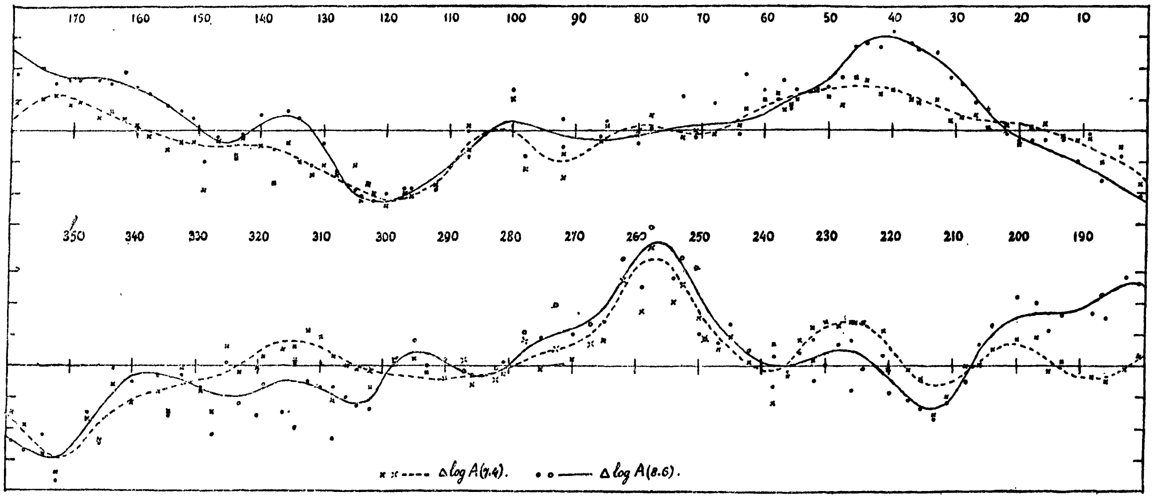
From the formulae in § 65 it is easy to see how ρ_1 and D are connected with these quantities. For $\triangle_{\rm III} > \triangle_{\rm II}$ (full curve above the dotted curve) we get values for D at a greater distance than $\rho_1{}^{\circ} = 12.5$ (300 parsecs), for $\triangle_{\rm III} < \triangle_{\rm II}$ values for smaller distances. A surplus of stars greater on charts III than on charts II, indicates a condensation at great distance, greatest for the greatest difference between them. On the contrary a surplus greatest on chart II indicates, in normal con-



Curves of equal density for zones 0° to + 20°, and 0° to — 20°.



Curves of equal density for polar sections of longitude 30° and 210°, 90° and 270°, 150° and 330°.

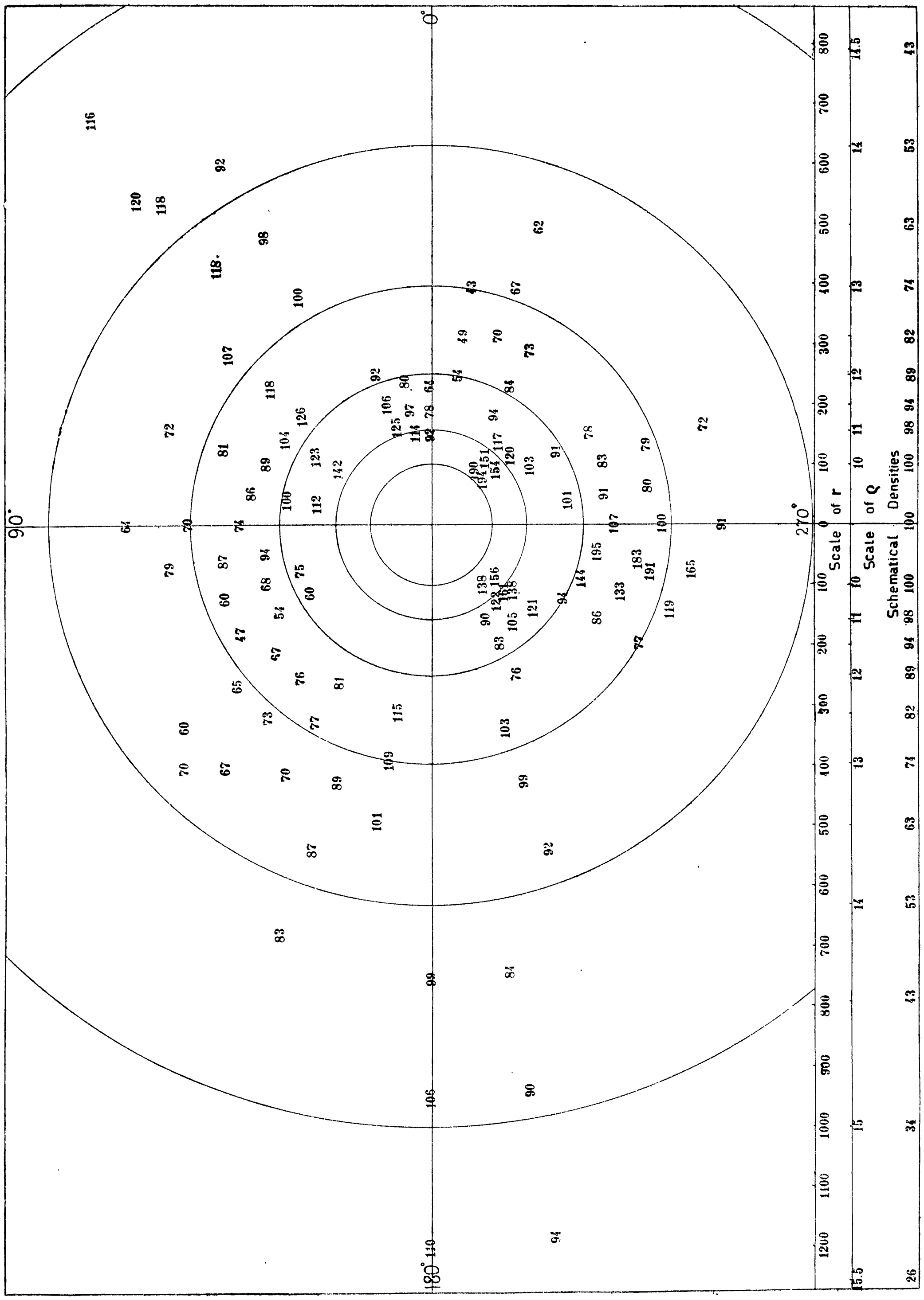


ditions, an accumulation at small distance. But such a difference may be due also to absorbing nebulae diminishing the number of faintest stars more than the brighter classes. In this case the $\triangle$ usually will be negative and the resulting densities small. But if these obscured regions are mixed with dense fields, the resulting $\triangle$ will be positive and diminishing, thus making the impression of a condensation nearby.

The results for D obtained in this way are collected in the drawing p. 107. It represents the galactic plane around the sun S; circles are drawn with radius 100, 158, 251, 400, 631, 1000 parsecs, corresponding to $\rho = 10$ to 15; scales at the bottom give ρ, r and D_0 of the schematical universe. The densities are put down in unit 0.01.

We find here again in the directions of Cygnus and Monoceros (λ 42° and 93°) parts of space with high densities > 1 at great distances, which may be estimated 700 ($\rho = 14.2$) and 1200 ($\rho = 15.4$) parsecs. A still greater density is found in the direction of Carina (λ 257°), at 300 parsecs ($\rho = 12.2$). In the directions 227° (Vela) and 315° (Cauda Scorpii) great densities occur at small distances; from the decreasing values on the chart these may be estimated 130 ($\rho = 10.5$) and 110 ($\rho = 10.2$) parsecs. Low values rapidly decreasing with distance are found at λ 350° and 120°; they are no real densities at those places, but are due to absorbing nebulae before them. Where in the same part of the sky they are intermingled with dense starmasses, we get high densities at small distances, which are not real either; this is the case for λ 60°, the region of large dark patches Cygnus-Cepheus, and also for 20° and 170°; perhaps the high densities at 310° and 320° are partly caused by the Scorpius nebulae.

67. The density numbers in this drawing show a radial structure: laterally there are great jumps in density, while radially the differences are smaller. It indicates that also without nebulae, the numbers cannot represent real densities. The formulae with which they have been computed presuppose slow fluctuations in density, extending over many units in the modulus of distance. If, however, condensations of small extent occur, they will appear in our results (*cf.* p. 11) dissipated over a great range of ρ; and this drawing out of every condensation in radial direction must give the appearance of a radial structure. Thus there is every reason to assume a structure of the starsystem consisting of separate condensations of moderate extension, situated at different distances in or near the galactic plane. On the real depth of such accumulations of stars our results can teach us


0°
90°
180°
270°
Scale of r
0
100
200
300
400
500
600
700
800
900
1000
1100
1200
Scale of Q
10
11
12
13
14
14.5
15
15.5
Schematical Densities
100
98
94
89
82
74
63
53
43
34
26

nothing. We may, however, safely assume that their radial dimensions will correspond to their lateral dimensions, or, if they are flattened perpendicular to the galactic plane, at least to their extension in galactic longitude. Then we find that

for an angular diameter of 10, 20, 30, 40, 60, 90 degrees
the radial diameter becomes 0.17, 0.35, 0.52, 0.68, 1.00, 1.41 times the mean distance,
extending 0.4, 0.8, 1.1, 1.5, 2.4, 3.8 in ρ.

The density in the schematical universe decreasing with distance must then be considered as an average of a large realm of less density and some dense condensations of smaller volume. Besides the condensations observed by us laterally, there may be another condensation filling up the innermost sphere around our sun.

68. We will now combine to „natural" regions all the areas that according to the charts belong together. They are indicated in Table 63 by the constellation as well as by the galactic coordinates of their centres, while the number of areas used (of 22 square degrees) gives their extension. At the bottom of the table the areas included in each are given in this way that the first area of each zone is indicated by the median declination and three figures for the Rectascension, and is followed in parentheses by the number of consecutive areas in this zone. From the mean $\triangle_3 \ldots \triangle_6$ (corrected as indicated in § 57) the values $\triangle a$ and $\triangle b$ are computed and corrected $+.022$ and $-.019$; then follows the average β used for the schematical values, the distance ρ_1, the density D and the deviation of density from the schematical values, for ρ_1 and $\rho_1 \pm 0.5$, in unit 0.01.

If the density is decreasing from $\rho_1 - 0.5$ to $\rho_1 + 0.5$, it means that the real distance is smaller, if increasing, greater. This rule cannot afford a rigid determination of distance, for, by the simplified formulae used (with c_0 adopted as the real c), D must be of necessity increasing for great ρ, decreasing for small ρ. While thus only more extensive data allowing us to find c can give more precise information on the real situation of the condensations, the results so far obtained in Table 63 may indicate the most probable values of their distances. Of course the densities in this table on account of the wide dissipation of the condensations are not real; in proportion to their contraction into a narrower range of ρ the excess of density $\triangle D$ will be greater.

Table 63. Special regions.

	Region	Areas	Centre gal.	$\triangle_3$	$\triangle_4$	$\triangle_5$	$\triangle_6$	$\triangle a$	$\triangle b$	βad.	ρ_1	D ($\rho_1 \pm 0.5$)			$\triangle$ D ($\rho_1 \pm 0.5$)		
1	Cygnus	19	38° + 2°	+ 077	+ 110	+ 192	+ 258	+ 186	+ 072	5°	13.5	110	107	101	+ 39	+ 46	+ 53
2	Lyr-Her.	28	25° + 20°	— 039	— 017	+ 022	+ 068	+ 033	+ 034	20°	12.6	74	68	59	+ 3	+ 7	+ 9
3	Lyr-Cyg N.	10	50° + 15°	+ 039	— 058	— 020	+ 059	+ 024	+ 004	15°	12.3	87	78	67	+ 4	+ 5	+ 4
4	Aql-Del.	11	30° — 10°	— 017	— 028	+ 022	+ 087	+ 039	+ 037	15°	12.8	78	72	64	+ 5	+ 9	+ 11
5	Cyg-Peg.	18	55° — 15°	— 053	— 010	+ 035	+ 081	+ 039	+ 046	15°	12.9	74	71	64	+ 3	+ 10	+ 14
6	Aql.-Vul.	9	20° + 0°	+ 023	— 018	— 042	— 028	+ 003	— 042	0°	11.9	108	95	82	+ 13	+ 5	— 2
7	Cyg N-Cep.	11	60° + 4°	+ 088	+ 068	+ 057	+ 063	+ 089	— 030	5°	12.0	124	110	96	+ 30	+ 22	+ 15
8	Lac N-Cas.	9	67° — 8°	— 018	+ 061	+ 123	+ 178	+ 114	+ 073	10°	13.4	89	86	81	+ 21	+ 29	+ 34
9	Lac S-And.	7	73° — 17°	+ 054	— 006	— 019	+ 018	+ 030	— 031	15°	11.8	104	91	76	+ 15	+ 9	+ 3
10	Cas-Per	15	93° — 10°	+ 016	+ 047	+ 062	+ 095	+ 079	+ 017	10°	12.6	97	90	80	+ 14	+ 16	+ 16
11*a*	Taurus	7	140° — 15°	— 197	— 378	— 434	— 412	— 345	— 110	15°	10.7	76	60	46	— 22	— 35	— 45
11*b*	Taurus	22	140° — 15°	— 128	— 283	— 312	— 295	— 241	— 087	15°	11.0	80	65	52	— 17	— 28	— 35
12	Per-Cam.	10	120° + 5°	— 180	— 223	— 226	— 217	— 192	— 033	5°	12.0	66	58	50	— 28	— 30	— 31
13	Auriga	7	135° + 0°	— 071	— 019	+ 014	+ 076	+ 026	+ 050	0°	13.2	82	79	73	+ 3	+ 9	+ 14
14	Monoceros	12	180° + 3°	— 003	+ 071	+ 207	+ 266	+ 166	+ 116	5°	14.2	96	97	95	+ 39	+ 51	+ 58
15	Gem-Mon	11	163° + 7°	+ 001	+ 018	+ 128	+ 170	+ 106	+ 071	5°	13.5	91	89	84	+ 20	+ 28	+ 34

	Region	Areas	Centre gal.	Δ_3	Δ_4	Δ_5	Δ_6	Δa	Δb	βad.	ρ_1	D ($\rho_1 \pm 0.5$)			Δ D ($\rho_1 \pm 0.5$)		
16	C Mi-Mon.	14	190° + 15°	— 035	+ 020	+ 114	+ 165	+ 093	+ 081	15°	13.4	77	74	70	+ 16	+ 24	+ 29
17	Mon-C Ma.	14	195° + 0°	+ 023	+ 050	+ 152	+ 193	+ 132	+ 070	5°	13.5	97	94	88	+ 26	+ 33	+ 38
18*a*	Orion	3	175° — 20°	+ 101	+ 085	+ 170	+ 205	+ 165	+ 041	20°	12.7	98	90	79	+ 29	+ 31	+ 31
18*b*	Orion	12½	170° — 20°	+ 000	+ 080	+ 083	+ 107	+ 094	+ 025	20°	12.5	88	80	69	+ 15	+ 17	+ 17
19	Pup-Vel.	9	228° — 3°	+ 141	+ 105	+ 046	+ 064	+ 107	— 058	5°	11.6	149	129	109	+ 55	+ 37	+ 22
20*a*	Carina	3	258° + 0°	+ 292	+ 313	+ 333	+ 404	+ 359	+ 035	0°	13.0	183	174	159	+ 101	+ 100	+ 96
20*b*	Carina	10	258° + 0°	+ 109	+ 149	+ 194	+ 221	+ 194	+ 035	5°	13.0	122	115	105	+ 41	+ 44	+ 44
21	Sco-Lup.	18	315° + 17°	— 102	— 208	— 289	— 275	— 205	— 110	15°	10.7	104	82	64	+ 6	— 13	— 27
22	Scorpius	7	315° — 2°	+ 099	+ 036	— 087	— 105	— 001	— 123	5°	10.7	184	146	114	+ 84	+ 48	+ 18
23	Sco-Sgr.	12	325° — 5°	+ 064	+ 094	+ 055	+ 025	+ 081	— 045	10°	11.7	129	112	95	+ 35	+ 24	+ 14
24*a*	Ophiuchus	8	344° + 15°	— 343	— 364	— 417	— 447	— 374	— 072	15°	11.2	53	44	35	— 42	— 47	— 48
24*b*	Ophiuchus	27	344° + 15°	— 234	— 291	— 317	— 320	— 272	— 057	10°	11.6	61	52	43	— 33	— 37	— 40

1 + 47201(1), 42193(4), 37191(5), 32185(5), 27191(4); **2** + 42173(4), 37173(5), 32171(5), 27171(6), 22173(5), 17181(3); **3** + 57190(2), 52190(2), 47185(4), 42185(2); **4** + 22201(3), 17195(4), 12195(4); **5** + 37205(6), 32203(7), 27203(5); **6** + 22191(3), 17191(2), 12185(2), 7185(2); **7** + 62213(1), 57202(3), 52202(3), 47203(3), 42205(1); **8** + 52222(3), + 47221(6); **9** + 42221(7); **10** + 57234(5), 52002(4), 47001(5), 42013(1); **11*a*** + 32043(2), 27041(3), 22041(2); **11*b*** + 37041(2), 32033(5), 27035(5), 22041(4), 17035(4), 12041(2); **12** + 62043(1), 57034(3), 52034(3), 47041(3); **13** + 42045(3), 37051(2), 32051(2); **14** + 7063(3), 2063(4), — 1063(4), 5063(3); **15** + 22061(3), 17061(4), 12061(4); **16** + 12073(2), 7073(2), 2075(2), — 1075(2), 5073(3), 10073(3), 15081(1); **17** — 10063(3), 15063(5), 20063(6); **18*a*** — 1051(2), 5051(2); **18*b*** + 7051(2), 2045(3), — 1045(3), 5045(3), 10045(3); **19** — 40075(3), 45075(3), 50081(3); **20*a*** — 60102(3); **20*b*** — 55102(3), — 60090(7); **21** — 20161(2), 25153(4), 30151(4), 35151(5), 40155(3); **22** — 35165(3), 40165(2), 45165(2); **23** — 30175(1), 35175(4), 40173(5), 45183(2); **24*a*** — 5165(4), 10165(4); **24*b*** — 1161(10), 5161(10), 10163(7), 15163(5).

69. The following remarks on the separate regions of Table 63 may indicate their character.

1. Core of Cygnus condensation, extending 20 degrees square, around η Cyg, from α — ε Cyg to β Cyg — β Lyr; the distance is found nearly $\rho = 13{,}5$ or 500 parsecs.

2, 3, 4, 5, constitute the outer less dense parts of this condensation. Their distance is found smaller, probably by the influence of field stars decreasing outward (*cf* also § 70). The first, N°. 2, between α Lyr — ι — ε — 109 Her and 3. between α Lyr — 33 Cyg, occupy the areas along the N. border, while 4. between α Aqu — Delphinus — 31 Vul and 5. between 42 — ν Cyg — η Peg form the S. border.

6. The areas between 23—6 Vul — α Sae — ε Aqu — ϑ Ser, are combined, because most of them have Δ (8.6) smaller than Δ (7.4), thus indicating absorbing nebulae mixed with star masses. This region covers the rift between the two branches of the Milky Way.

7. The region following the core of the Cygnus condensation in longitude, between α — ξ Cyg and α — ζ Cep. Also here we have extended dark nebulae (e.g. the black space between α Cyg and α Cep) intermingled with dense starclouds; by their influence the Cygnus condensation on chart III extends less far in longitude than on chart II, and we get ρ_1 smaller and *D* rapidly decreasing.

8. The continuation of the Cygnus condensation through Lacerta—Andromeda, between ρ Cyg and σ Cas — λ And, S. of the galactic circle; its distance is found the same as for 1.

9. The southern border of the preceding region (1 H Lac — ν And), taken separately because for the brighter classes the stream lies farther south. Therefore its distance is found much smaller (100—200 parsecs). It will be interesting to test this result (that the most southern border of this stream stretches far toward the sun) by exact photometric data.

10. The further continuation of this condensation through Cassiopeia to Perseus (β Cas

— υ And — η Per); its distance is found somewhat smaller ($\rho = 12$), between 200 and 300 parsecs. The clusters h and χ Persei, however, which make the impression of being strongly condensed cores in these star masses, have been found by **Parvulesco**[1]) to be situated at a distance of 500 parsecs.

11 and 12 are the regions of the Taurus nebulae and the Perseus—Camelopardalus nebulae.

13. A feeble accumulation in Auriga (β — ε Aur — β Tau); the distance may be 300—400 parsecs.

14. Core of the Monoceros condensation, between 15—11—22 Mon — α CMi. A constant density nearly 1 is found at a distance $\rho = 14$ to 14.5, at 600—800 parsecs.

15. (η Gem — β CMi), 16. (β CMi — 26 Mon), 17 (11 Mon — β CMa — ι Pup.) constitute the outer less dense parts of this condensation, situated at the N, E, and S side of the core; the distances are found somewhat smaller (*cf.* 2—5).

18. The Orion condensation occupies the southern half of the constellation, around η Ori; it coincides with the cluster of B-type stars on chart I. Its distance, after the D of the table, may be estimated $\rho = 12$ or 250 parsecs. **Kapteyn**[2]) finds the distance of the „nebula-group" of B stars 160—180 parsecs, not much different. This suggests that both are identical and the accumulation of B stars contains at the same time a greater density of small stars.

19. The condensation between c — ζ Pup and γ — δ Vel must be situated at a small distance, $\rho < 11.6$, between 100 and 150 parsecs; therefore it becomes less striking for the fainter stars. It coincides with the bright star group on chart I (chiefly B stars) except for the part around λ Vel. The two bright O-type stars ζ Pup and γ Vel. ($m = 2.3$ and 1.9) ,if they have the absolute magnitude — 9 found by J. S. **Plaskett**[1]) for this class of stars, belong also to this condensation.

20. The Carina condensation (q Car — ϑ Cru), extending 10° in longitude, 5° in latitude, around two dense clusters (η Car and u—x Car). Its distance ρ must lie somewhat below 13, thus may be put at 300 parsec. Its density is greater than any other condensation; but its dimensions are small.

21. The region α — β — γ Sco — γ Lup is the domain of the strong, interesting Scorpius-nebulosities, with perhaps a feeble condensation of stars at 100 parsecs before them. The B-type stars of these region are situated according to **Kapteyn** at distances of 50—140 parsecs.

22. The region ε — η — λ Sco. has a surplus on chart II, a shortness of stars on chart III; thus ρ_1 is found small and the density rapidly decreasing. Though the high density for ρ_1 —0.5 is only a computing effect, there seems to be a condensation at distance 100 (also visible in the B-type stars), mixed with absorbing nebulae.

23. For the condensation between λ — ϑ Sco and γ — ζ Sgr — γ CrA, contiguous to 22, with fewer nebulous parts, we find a distance $\rho = 11$ or 150 parsecs. Probably the Sagittarius condensation of bright stars is connected with it.

24. The region of the Ophiuchus absorbing nebulae.

10. In order to find the real densities by means of the luminosity curve, the number of stars for each magnitude was computed for a condensation (at different distances ρ) filling the space between $\rho \pm 0.5$ with a constant density 1; this corresponds to a total number of $6300 \times 10^{0.6(\rho - 15)}$ stars per square degree (*cf.* § 4). The observed number of stars for $m = 7$, 8, and 9 was obtained by adding $\triangle a - \triangle b$, $\triangle a$ and $\triangle a + \triangle b$ to $\log A_0$ for 7, 8, and 9 of the schematical universe. Now we do not know what part of these stars must be ascribed to the field stars occupying the space before and behind the condensation. We cannot assume the condensation simply superposed

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[1]) Bulletin Astronomique **1923**, 410.

[2]) Astrophysical Journal **47**, 175, 176.

[1]) The O Type stars, Publications Victoria B. C. II. Nr. 16.

upon the stars of the schematical universe, because the latter are found as an average to which the condensations themselves have strongly contributed. Thus by ascribing only the differences $A—A_0$ to the condensations, we certainly underrate them. The other extreme would be to ascribe the whole number A to the condensations. The truth must lie between these extremes, and as an average of reasonable assumptions we may take half the values of the schematical universe to be due to the field stars outside the condensations, and log $(A—\frac{1}{2}A_0)$ to be due to the condensation.

Comparing the observed number of stars with the number computed for a certain ρ and density 1 for these three assumptions, the two extremes and the average one, we get the densities D_1. At the same time we get the distance; for if this ρ is smaller than the real distance, the differences will increase, if larger, decrease with m. Indeed by this method we do nothing but look for what interval $M \pm 1$ the luminosity curve shows the same gradient as the number of stars of the condensation shows between $m = 7$ and 9; then $\rho = 8 - M$. That the validity of KAPTEYN's luminosity curve must be assumed, is no new hypothesis, since all our computations here are based on it. For the rest it does not seem too risky to adopt its general validity for the local starsystem, as it is exactly these stars from which it has been deduced; and the irregularity spoken of in § 62 has been acounted for by our empirical corrections of $\triangle a$ and $\triangle b$. Table 64 contains for some of the regions of the preceding table on each of the three hypotheses the log. of the star number for m 7.0, 8.0, and 9.0, the results for the distance ρ and for the density D_1.

Table 64. Results for distance and density.

Region		log A	log $(A - {}^1/_2 A_0)$	log $(A - A_0)$	ρ			log D_1			D_1(II)	D
					I	II	III	I	II	III		
1	Cyg	9.916 0.495 1.049	9.705 0.324 0.908	9.279 0.037 0.700	13.5	14.1	15.6	0.73	0.55	0.34	3.5	3.5
2–5	surr.	9.754 0.275 0.769	9.457 0.005 0.524	—	12.7	13.0	—	0.56	0.26	—	1.8	1.0
14	Mon	9.852 0.475 1.073	9.596 0.294 0.942	8.940 0.143 1.034	14.1	15.0	>16	0.71	0.55	—	3.5	4.3
20*a*	Car	0.138 0.683 1.204	0.020 0.576 1.106	9.859 0.433 0.980	13.0	13.2	13.4	0.94	0.83	0.68	6.8	17
18*a*	Ori	9.840 0.362 0.857	9.635 0.180 0.695	9.235 9.862 0.434	12.7	13.0	14.0	0.65	0.44	0.08	2.8	9
19	Vel	9.967 0.416 0.840	9.785 0.201 0.583	9.467 9.755 9.868	11.6	11.1	<10	0.84	0.71	—	5.1	8.5
23	Sgr	9.905 0.361 0.791	9.702 0.128 0.523	9.306 9.592 9.692	11.7	11.3	<10	0.77	0.60	—	4.0	7

The last hypothesis, i.e. to ascribe only the surplus over the schematical universe to the condensations, gives the most deviating results, which must be judged least probable. The densities found vary only little with the distance ρ assumed; thus in so far they are rather trustworthy. But of course they vary considerably with the assumption as to what part of the stars belongs to the condensation. According to the second hypothesis we may say that a density 3.5 extending over a range unity in ρ will represent the observational data for the core of the Cygnus condensation; and so for the others the other values of the last columns. These D_1 are not yet the real densities; if the radial depth of a condensation is less, the space density of the stars must be greater in the same proportion. Taking the lateral and the corresponding radial extension in ρ of the condensations of Table 64

25°	50°	20°	10°	8°	15°	15°
1.0	1.9	0.8	0.4	0.3	0.6	0.6

we get for the real space densities of these condensations the values D of the last column.

Thus the small condensations of the southern hemisphere, recognizable as clusters of B stars,

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have the greatest density; in the Carina condensation we have as its core a mean density of 17 over a rectangular part of space 160 parsecs broad and deep, and of half that amount perpendicular to the galactic plane. The great northern condensations, in Cygnus and Monoceros, are less dense, but much larger; surrounding the core they have a still larger volume of surplus density, which in the Cygnus case is of the order unity. The relative importance of these condensations may be expressed by the number of stars they contain; it is given by

$$(\text{surface in squ. d.}/57.3^2) \times 0.47\ D_1\ r^3 \times 0.045$$

where 0.045 is the number of stars per cubic parsec for density 1 and 0,47 r is the radial depth between $\rho \pm 0.5$. Assuming the surface and the distance for the same condensations of Table 64:

Cygnus	surr.	Monoc.	Carina	Orion	Vela	Sagitt.	
420	1600	260	65	65	200	260	squ. d. and
600	500	800	400	300	160	170	parsecs, we find
200.10^4	230.10^4	300.10^4	18.10^4	32.10^3	27.10^3	33.10^3	stars.

A sphere of 100 parsecs radius with density 1 contains 19.10^4 stars. Thus also in the case that around our sun we have a density not smaller than 1, this cannot be considered as a condensation of similar importance as the others ones. Compared with the Cygnus and Monoceros condensations it is wholly negligible, while the Carina group has the same number of stars concentrated in a much smaller volume of space.

Computing the surface brightness caused by these condensations by means of the formula $L = 0.00756\ D_1 \times r/1000$ (*cf.* § 4), we find the values

0.016, 0.007, 0.021, 0.020, 0.006, 0.006, 0.005 (in stars 0.0 per sq.d.),

which owing to the field stars, for which we adopted half the number of the schematical universe at $\beta = 0°$, must be increased by 0.010. For the three chief condensations this total surface brightness corresponds (according to *Astron. Nachr.* 5132) to 2.3, 3.0 and 2.9 steps in the scale used in our charts of the Milky Way (*Die nördliche Milchstrasse.* Tafel VII—IX)[1]; for the others it corresponds to less than 1. Thus the small condensations of the southern hemisphere will not be visible as a perceptible increase of the galactic light. For the Monoceros region the observed brightness, 2 to 3.5 steps, is explained almost entirely by the light of this condensation. For Cygnus, however, only the feeble light of the eastern branch can be due to this condensation; the brilliant cloud between β and γ Cygni, though covering a great part of the condensation, must be a separate gigantic object at great distance behind it. The same holds for the bright parts of the Galaxy in Carina and Crux; the near Carina condensation is seen projected against a more remote galactic stream.

It may be noticed, that the two principal nests of *O*-type stars are situated at $6^h\ 29^m + 5°$ (av. m 7.0) and at $20^h\ 5^m + 36°$ (av. m 7.4), not far from the centres of the Monoceros and the Cygnus condensation. With PLASKETT'S mean absolute magnitude —9 for the *O*-type stars we find distances, however, for these stars appreciably greater than those found here for the condensations.

71. For the distances of the chief condensations we have now obtained the values collected in the following table. Their differences are due to a different treatment of the same data. In the curves used in § 66 we may expect a preponderance of extreme values, which in the averages of large regions in § 68—69 are smoothed. The results of § 69 bear the character of somewhat un-

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[1] Annalen van de Sterrewacht te Leiden. XI. 3.

certain estimates; but in the results of § 70 the same uncertainty is contained in the supposition on the fraction of A_0 ascribed to the condensation. Thus the differences may give a measure of the unavoidable uncertainties of results founded on a rather narrow range of Durchmusterung magnitudes.

Table 65. Results for distances.

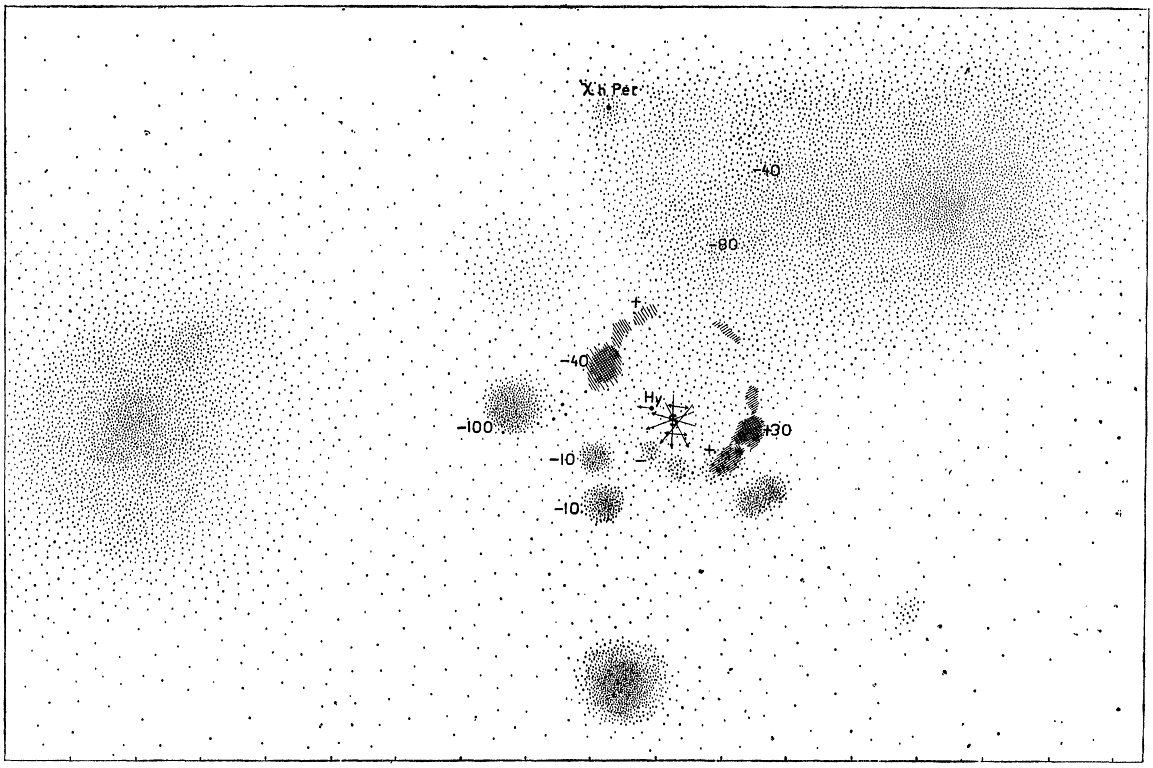
	§ 66	§ 69	§ 70	Adopted ρ	Distance r
Cygnus	14.2	13.5	14.1	13.8	600
Monoceros	15.4	14.2	15.0	14.6	800
Carina	12.2	<13.0	13.2	13.0	400
Orion	—	12.0	13.0	12.5	300
Vela	10.5	<11.6	11.1	11.0	160
Sagittarius	10.2	<11.7	12.3	11.2	170

The degree of reliability of these distances is not only determined by these differences due to different methods of treatment, but depends also on the systematic and accidental errors of the data themselves, on which the discussion is founded. The diagram of p. 106 is useful in that it shows at a glance how the distances of the condensations are wholly determined by the vertical distances of the two curves and are thus influenced by errors in the adopted magnitudes. Therefore everything that has been said in former chapters about doubts and errors remaining in the scale reductions of the DM magnitudes pertains here to the question of the distances of the condensations.

Especially injurious is a variation with magnitude of the scale corrections depending on AR, which were assumed in § 28 the same for all magnitudes. Thus in the new discussion of this matter, mentioned in § 59, the differences A (for $m = 6.5$ and 7.0), B (for $m = 7.5$ to 8.5) and C (for $m = 8.8$ to 9.5) have been computed for every hour of AR in every zone of declination. Their influence on $\triangle b$ (derived by computing a quadratic formula for $\triangle m$ and its influence on $\triangle_3 \ldots \triangle_6$) was found to be expressed by $\triangle b = -0.25\ (C-B)$. The deviations $C-B$ vary irregularly over the sky; but just for all the fields covering the Cygnus and the Monoceros condensations — the coincidence is not exact because they extend full hours in AR — they are positive, on an average $+0.083$ and $+\ 0.116$. Thus $\triangle b$ must be diminished here by 0.021 and 0.029, giving corrections to ρ_1 -0.3 and -0.4. Thus instead of 14.1 and 15.0, which may be taken as the most probable result from Table 65, we will adopt 13.8 and 14.6, or in round numbers 600 and 800 parsecs.

For Carina there remains a much greater uncertainty. Firstly because there was some doubt (*cf.* § 43) on the course of the magnitude correction curve for the southernmost Cordoba zone. Secondly on account of our ignorance as regards the density correction to be applied in this region of excessive surface density of stars. Corresponding to the observed differences between the photometric and the DM magnitudes just in this region, for D a lower value than the real density has been used here in computing the correction. It is doubtful whether in such dense regions with brilliant clusters eye estimates can be corrected with sufficient accuracy to derive a trustworthy value of the slope of log A. On the degree of this uncertainty in the distance of the Carina condensation of course nothing can be said.

The distances finally adopted are contained in the last columns of Table 65. This result of our investigation may be represented tentatively by the picture on the next page. The different condensations are given projected on the galactic plane; the vertical distances in parsecs are written beside some of them (e.g. — 100 for Orion). A scale where each division represents 100 parsecs is given at the bottom-line. Of course there is much that is hypothetical in this picture; especially in the outlying parts of the great dominating condensations, their extension towards the sun, and in the filling up of the remaining parts of space. It must be remarked that in consequence of



the spreading of the luminosity curve two condensations in the same line of sight are blended, so that in every direction only one is found and others situated before or behind them are made imperceptible. Thus there may be other secondary condensations not remarked by us in this part of space, left blank in the drawing. The Hyades and the *h* χ Persei clusters are represented by dots; on the stars within 100 parsecs and the absorbing nebulae compare the next paragraphs.

The absorbing nebulae.

72. For the regions showing a deficiency of stars owing to absorbing nebulae the ρ_1 and D computed in Table 63 have no real meaning. In order to compare the deviations of star number $\triangle_3 \ldots \triangle_6$ with suppositions on these nebulae, we first reduce these separate $\triangle$ to the scale of Groningen, by applying the corrections $+0.038$, $+0.033$, $+0.020$, 0.000 (*c.f.* § 63). The results are given in the first columns of Table 66 for 11*a*, 11*b*, 24*a*, 24*b*, the densest core and the larger domain of the Taurus and the Ophiuchus nebulae.

Comparing them with the curves computed in Table 3 for absorbing screens (directly for distances $\rho_2 = 9$ and 12, and interpolated for 10 and 11) we find the Taurus results run much steeper than the curves, while for Ophiuchus *a* and *b* there is a sufficient concordance with the curves

(12.1) (*i.e.* $\rho_2 = 12$, $\varepsilon = 1$) and (12.2). This, however, cannot be taken as an indication of the real distance of the screen. For in the areas of 5° × 5°, and still more so in the large regions, we have a mixture of unobscured starfields and parts with moderate or strong absorption. In the average starnumber of such regions the effect of the nebulae is different from a pure region with constant absorption everywhere. By the mixture with unobscured regions the curves become less steep, and in some cases get a resemblance to curves of somewhat greater distance. Therefore it is not possible to derive the distance of the nebulae from the results of Table 66; for this purpose we should separate the dark parts of these regions from the surrounding starry parts, as has been done for the Taurus nebulae in our former investigation. Thus we will assume here the distance $\rho_2 = 11$, found there, and we will compare our data with the case of such absorbing nebulae, mixed and thus attenuated in different degrees with unobscured regions (column $^3/_5$ (11.4) expresses the decrease in log. A, if $^3/_5$ of the region is covered by an absorbing screen at $\rho_2 = 11$, $\varepsilon = 4$, and $^2/_5$ is unobscured).

Table 66. Comparison of nebulous regions (Unit 0.001).

m	Tau a	Tau b	Oph a	Oph b	(12.1)	(12.2)	(11.1)	$^3/_5$(11.4)	$^3/_4$(11.4)	T.a — $^3/_4$	T.b — $^3/_5$	O.a — $^3/_4$	O.b — $^3/_5$
6.9	— 159	— 090	— 305	— 196	211	278	318	230	315	+ 156	+ 140	+ 10	+ 34
7.7	— 345	— 250	— 331	— 258	256	354	363	268	372	+ 7	+ 18	+ 41	+ 10
8.3	— 414	— 292	— 397	— 297	289	417	389	293	413	— 1	+ 1	+ 16	— 4
8.9	— 412	— 295	— 447	— 320	323	487	407	316	452	+ 40	+ 21	+ 5	— 4

The residuals for some cases are contained in the last columns. By varying the distance ρ_2, the absorption ε, the percentage of mixture, and the unobscured density (causing a constant positive or negative deviation) we may get a closer concordance in different ways. The great positive residuals for the brightest class in the Taurus nebulae, however, are due to the Hyades stream of bright stars projected in front of the nebulae. They are the cause why on charts I, where the Ophiuchus nebulae show clearly, the Taurus nebulae are hardly visible.

From the concordance of the remaining figures may be inferred that the distance of the Ophiuchus nebulae must be of the same order as for the Taurus nebulae, viz. between 100 and 200 parsecs. In *Harvard Circular* 239 SHAPLEY estimates the distance of the nebulosities in the Ophiuchus and Scorpius region as between 200 and 300 parsecs. The basis of this statement is not given in extenso. Comparing his diagrams we find that on the first diagram already, containing the class B stars brighter than 5.26 (corresponding to a limit of 180 parsecs) the Ophiuchus region shows a shortness of stars; the great number of these bright stars in Scorpius is due to a local group nearer than this limit. Furthermore in *Harvard Circular* 229 it is stated that the class A stars brighter than 6.5 show a scarcity north of the galactic circle in Ophiuchus; this points to absorbing matter nearer than 130 parsecs, the average limit of these stars. According to these data the distance of the Ophiuchus nebulae would sooner appear to be somewhat smaller than the Taurus nebulae.

On the picture p. 114 these absorbing nebulae are represented by shaded areas.

Comparison with other results.

73. The irregularities of the surface distribution have been studied first by W. Stratonoff in his ,,*Etudes sur la structure de l'univers*'' 1). For each half magnitude charts of the relative density were constructed, and these pictures were considered to represent a series of spherical shells at increasing distances. This, of course, would only be true if the spreading of the luminosity function was small. With the existing great spreading of the luminosities the distribution of classes differing half a magnitude must practically be indentical; and the differences found must chiefly be ascribed to accidental and systematic errors. Moreover no systematic scale corrections have been applied to the DM magnitudes; this deprives the charts of their chief value. Many apparent features of the distribution of the stars therefore are nothing but peculiarities of the magnitude scale errors; so e.g. the small density in the neighbourhood of the aequator. Nevertheless the chief features, the condensations in Cygnus, in Auriga and in Monoceros, come out clearly. The first named is considered by Stratonoff as the richest and most important agglomeration of the whole sky, including even our sun and its nearest surroundings. For the southern hemisphere, where below — 18° the magnitudes of the Cape Photographic Durchmusterung are used without corrections, many minor details are found, some of them coinciding with those of our charts.

74. In our paper ,,*The Local Starsystem*'' 2) the formulae developed in the first chapter of the present work were applied to DM data reduced with preliminary formulae. For the N. hemisphere results from the ,,Selected Areas'' were added to find the density at greater distances; for the S. hemisphere only roughly reduced counts of the C.P.D. could be used. By taking regions between ± 20° of latitude, extending 30° in longitude, the distribution over a galactic disc was represented in a diagram of lines of equal density. The Cygnus condensation is shown here by a far extension of the dense parts in this direction. The Monoceros condensation is indicated in the same way with less distinctness. The agglomerations of Scorpius and Sagittarius are combined here in sector 300°—330° showing a condensation at distance 100—200 parsecs. The necessity of taking together large regions did not allow the isolation of the separate condensations, and by the method used they were smoothed into a single system of irregular shape decreasing outwardly in density.

75. In his articles on ,,*Individual Parallaxes of Galactic Helium Stars*'' and ,,*On Parallaxes and Motion of the Brighter Galactic Helium Stars* 150°—216°'' 3) Kapteyn discusses the arrangement in space of the *B* stars contained in Boss' Preliminary General Catalogue, on the basis of proper motions and stream motion. On a plate accompanying the first article the distribution of the stars used (mostly brighter than 6.0) along the galaxy is given in two diagrams, for $\mu > 0''.017$ and for $\mu < 0''0.17$. They may be considered to represent a nearer and a more remote class of stars; a chart we made of all the *B* stars down to 6.50 (where the condensations correspond with those of our southern chart I) may be taken to picture the distribution of a still more remote group. The large proper motion stars show a dense group of brilliant stars around Crux, at 270°, situated at distance 60 to 80 parsecs, not distinctly separated on the following side from similar groups in Lupus and the head of Scorpius, for which Kapteyn finds distances 70—150 parsecs, and on the preceding side from a group in Vela-Carina, at distances 40—80 parsecs. Together they form a broad band from 220° to 330° longitude, inclined to the galaxy, the main part of Gould's belt. The small proper motion stars are scarce in this region, but they are numerous in the preceding

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1) Publications de l'observatoire de Tachkent, No. **2** et **3** (1900, 1901).

2) Proceedings of the Amsterdam Academy, Vol. XXIV. 56 (1921).

3) Astrophysical Journal **40**, 43; **47**, 104, 146, 255; (Mount Wilson Contributions 82, 147).

longitudes, which is only partly due to the vicinity of antapex and vertex. They crowd in two groups, coinciding with the Orion cluster (175°—16°) and the Canis major group (200° to 215°, 0° to — 10°); Kapteyn finds their distances 180 and 120—130 parsecs. There is a discordance between the first value and our result 250—400 parsecs; if the latter is diminished to 180, the $\triangle_3 \ldots \triangle_6$ must decrease instead of increasing as they do.

On our charts I the condensations of large proper motion stars in Scorpius and Lupus are visible, while the other stars may contribute to the long streak of surplus density. The condensations of small proper motion stars are already more striking; but the Canis major group is lacking on the charts of the fainter stars. The other condensations visible on chart I, which are also visible on the other charts and thus situated at a greater distance (Vela and still more so Carina) are wholly imperceptible among the stars discussed by Kapteyn. Thus his investigations and ours are complementary and deal with different depths of the universe, meeting in the regions between 100 and 200 parsecs. The groups discussed by Kapteyn have also been inserted on our picture p. 114.

76. The completion of the *Henry Draper Memorial* Catalogue of stellar spectra at Harvard College Observatory has opened a wide field of investigation of the nearer parts of our starsystem, as it enables us to study separately the different spectral classes. In several important papers, published in *Harvard Circulars*, Harlow Shapley has already given some first results of the discussion of these data.

In „*The Distribution of Stars of Spectral Class B*" (H. C. 239) a series of figures shows the distribution of the *B*-stars of different magnitude classes. The stars brighter than 5.25 show the inclined belt from Orion to Scorpius; for 5.26 to 6.25 the Lupus and Scorpius stars disappear, but from Orion to Crux the mass of the stars, among which the Canis major and the Vela group appear indistinctly, show still the same inclination. Among the stars 6.26—7.25 the Vela group at λ 230° β 0° to — 10°, is most distinct, while in the faintest class 7.25—8.25, where the inclination to the galactic circle has nearly disappeared, the Carina condensation comes out most strongly. Thus we have here the same sequence of distances as found above. Shapley's faintest class shows besides the Carina cluster a dense group at λ 315°, indicating a more remote condensation behind our Cauda Scorpii-Sagittarius condensation.

In another paper „*On the Spectral Constitution of the Nearer Parts of the Milky Way*" (H. C. 240) the number of stars of each spectral class is given in areas 10° square having their centres at regular points of the circles $\beta = 0°$, — 10° and + 10°, for the limits 6.0, 7.0 and 8.25. Assuming the absolute magnitude of the class *A* stars — 4 and of the class *B* stars — 6 (+ 1.0 and — 1.0 on the usual scale) and neglecting the spreading of this magnitude, these limits correspond to the distances $\rho = 12$, 13 and 14.25 for class *B*, to $\rho = 10$, 11 and 12.25 for class *A*. Dividing the numbers counted by the relative volumes 1, 3, 18 for class *A*, and 16 times these values for class *B*, we may find the relative densities for the whole zone as well as for some interesting areas, covering our condensations (Table 67).

The class *A* stars show a small general decrease from the inner sphere to the outer shell not much different from the decrease adopted in the schematical universe; thus a somewhat larger density than the average of the space between the condensations may be assumed for the inner sphere. The class *B* stars show a much stronger decrease than the bulk of the stars, indicating a general condensation of these stars in the central parts of the system. In the direction of the Cygnus condensation (λ 30° to 60°) the density of the *A* stars (average of the 4 fields 8.7, 8.7, 7.1) remains constant, and greater than the average of the inner sphere. Thus it seems that the space between us and the densest condensation is filled up by its outer parts and has a surplus density throughout.[1]) This densest core itself is not reached by the class *A* stars of the Draper Catalogue; and the class *B* stars show only a small condensation in this region. This is true in a still higher degree for the Monoceros condensation; the densities in the field 180° 0° show no trace of a remote accumulation of stars.

Table 67. Relative Densities of Class A and B stars.

	Class *A* stars			Class *B* stars		
	I. $\rho < 10$	II. 10 to 11	III. 11 to $12^1/_4$	I. $\rho < 12$	II. 12 to 13	III. 13 to $14^1/_4$
zone β 0°	6.0	5.0	4.3	0.29	0.13	0.07
zone $\beta + 10°$	4.0	3.8	2.8	0.09	0.015	0.010
zone $\beta - 10°$	3.4	4.6	3.6	0.28	0,07	0.023
30° 0°	10	9·3	7.0	0.3	0.12	0.045
40 0	7	9.7	5.5	0.4	0.10	0.07
50 0	10	9.7	6.3	0.25	0.08	0.07
60 0	8	6.3	6.4	0.4	0.10	0.03
180 0	4	5.7	3.4	0.3	0.10	0.05
210 —10	2	5.7	3.4	0.6	0.40	0.03
230 —10	3	5.0	4.4	0.9	0.40	0.08
260 0	9	5.3	6.4	0.75	0.33	0.25
290 + 10	7	3.3	2.3	0.7	0.08	0.04

For the southern condensations the information is more positive. The rapid decrease of density from *B* II to *B* III for Vela, and still more for the Canis major condensation is in full accordance with the small distance $\rho = 11$ or 10 to 11 found above. Class *A* showing a maximum density in II for the latter, between II and III for the former confirms that at the place of these condensations of class *B* stars we have accumulations also of the other spectral classes. For the Carina condensation there is an increase from *A* II to *A* III, while the small difference between *B* II and *B* III points to a distance of nearly 13. The strong fall of density from *B* I to *B* II is only apparent, for the Carina condensation occupies only part of the area of 100 square degrees, while the near bright stars are spread over the whole area.

The values of the density show a rather strong spreading in ρ_1 not reconcilable with a small extension in depth of these clusters. It must be attributed to the considerable spreading in absolute magnitude of the class *B* stars; according to ADAMS and JOY it is — 3.0 for *B*0, — 0.5 for *B*5. This cannot explain, however, all the divergences; the stars below 7.0 at 230°—10° (showing a density far in excess of other fields at $\beta - 10°$), if belonging to a condensation at $\rho = 11$, must have an absolute magnitude fainter than + 1.0 (common scale). SHAPLEY mentions the same case for the Orion cluster and he suggests absorption by parts of the great nebula (*H. C.* 239).

Thus for the near condensations the Harvard results are in accordance with our conclusions. It will be possible to apply the same method to the more remote parts of our starsystem as soon as the extension of the catalogue of spectra for special galactic fields to a fainter limit of magnitude, which is now in execution at Harvard College Observatory, shall be completed.

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[1]) The surplus for the first group is hardly reconcilable with the supposition that these stars occupy a sector of the inner sphere. We may explain it by assuming a perceptible spreading in luminosity of the *A* stars, some *A* giants from more remote denser parts of space appearing brighter than 6.0.

CONCLUSIONS.

77. The structure of the universe around us from our observing point must be studied in layers at different depths of distance, determined by different sets of data and methods of treatment. The starcounts of the Durchmusterung Catalogues have allowed us to study the density of stars at distances that are somewhat different, but all contained between 100 or 150 and 600 to 1000 parsecs. The distribution of the stars in the nearer parts of space, within a sphere of 100 to 150 parsecs radius, must be investigated by other means, by spectral class, by proper motions and individual parallaxes. On the other hand the more remote parts of space, the domain of the galactic clouds proper, must be investigated by a special study of the faintest star classes.

It must be borne in mind that the number of stars observed and counted in these catalogues, is smaller than the real number. For part of them is obscured by the large clouds of absorbing matter, situated on one side of us, towards Taurus, at nearly 140 parsecs, extending with less density through Perseus and Camelopardalus, and at the other side, at perhaps somewhat smaller distance, towards Ophiuchus and Scorpius. Probably the dark rift between the branches of the Milky Way through Aquila as far as β Cygni is due to an appendix of these nebulae, and the absorbing nebulae in the northern part of Cygnus, in Cepheus, Lacerta, Cassiopeia are perhaps a continuation of them. Owing to these obscuring masses the density in the space behind them cannot be found directly from the stars counted. The average densities computed from the tables of Groningen must, owing to this same cause, be too low.

The part of space considered here shows several larger and smaller condensations of stars. The most remote that can be detected in the DM stars is situated at 800 parsecs towards Monoceros; the largest is the Cygnus condensation at 600 parsecs, extending with a broad tail over 90° in longitude, as far as the clusters $h\,\chi$ Persei and filling the space down to the surroundings of our sun with surplus density. Whether the central sphere around the sun has a density 1, the maximum density assumed in the centre of the schematical universe of revolution, or less, it has still a secondary importance compared with the great condensations just named. The third condensation, situated towards Carina at 400 parsecs, is smaller, but much more concentrated than these. It is the most important of a series of minor condensations at smaller distances (100—200 parsecs) filling the southern half of the galaxy. They are all smaller and more concentrated than the large northern condensations, and at the same time contain a great number of massive *B*-type stars; probably there is a causal connection between these different characteristics.

In his „*First Attempt at a Theory of the Arrangement and Motion of the Sidereal System*” 1) Kapteyn explains a schematical universe, where the surfaces of equal density are flattened ellipsoids of revolution, as a system in dynamical equilibrium under its own attraction and two opposite rotational movements. The rotations appear to us as the phenomenon of star streaming with the

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1) Astrophysical Journal **55**, 302; Mount Wilson Contributions N^{o}. 230.

vertices at λ 167° and 347°. The condition that the relative velocity of rotation for stars in the vicinity of our sun should reach the value 40 km, while at the same time in the mean of all longitudes still a nearly constant central density must be found, fixes the centre of the system at 700 parsecs, in a direction perpendicular to the vertices. KAPTEYN prefers the direction λ = 77° (in Cassiopeia) for the centre; SHAPLEY thinks it more probable that it must be looked for in the opposite direction, and he considers the Carina condensation, the densest among the great visible accumulations, as the central core of the local starsystem. Our results confirm neither the first nor the other conception; according to them our starsystem has no definite core or central accumulation, but contains several extensive accumulations comparable with each other in dimensions and number of stars. Whether some regularity exists in the arrangement of these condensations cannot be decided from our first imperfect results. Much more accurate photometric and spectral data will be necessary to disentangle the complicated structure of the universe.

KAPTEYN'S admirable explanation of the decrease of density perpendicular to the galactic plane by analogy with an atmosphere will not be seriously affected if the attracting masses, instead of forming a flat ellipsoid of revolution, are distributed more irregularly over the galactic plane. But we cannot explain star streaming any longer now by a rotational movement of the whole universe. It must be considered as a more local phenomenon (local taken in the sense of reaching to many hundreds of parsecs distance), in some way connected with the arrangement of the most important masses over the starsystem. In our sketch on p. 114 the direction of the vertices is indicated (by two doubly pointed arrows), as well as the prominent directions of the high velocities (by single arrows). It appears that the centre of the antapices of the high velocities coincides with the Cygnus condensation; SHAPLEY thinks there is a real causal connection.¹) The vertices of stream I and II are directed to and from the border of the Monoceros condensation, and perpendicular to the Carina condensation. Considering the relative magnitude of the forces from the different condensations, (which are easily found from the data in § 70), it is not likely, however, that the relation between the masses and the motions can be expressed in this simple way. The galactic condensations outside the space here considered will play a role too. Thus it may be expected that with the structure also the motions in our universe are more complicated and irregular than they might seem in KAPTEYN'S first attempt.

The hypothesis that the universe has already reached a steady state must also be abandoned now. Our results concord with the conceptions developed 1916 by J. H. JEANS in his theoretical study on star streaming.²) He thinks the universe, in the earlier stages of evolution, at least in part, as a chaos of moving star clusters; afterwards these clusters, although partially disintegrated, may still be recognisable as star-streams. Our results point to the star system being still in the earlier stage. If we suppose the central parts of the universe to have originated by the encountering and partially intermingling of the great condensations of Cygnus and Monoceros, we might be inclined to connect with them the two streams I and II, in which case the southern clusters with their numerous *B*-type stars would be represented in HALM 's stream O. But the purely hypothetical character of such ideas is obvious, if we remember that hardly any of the stars situated in the core of the great condensations occurs in our catalogues of proper motions. Thus it will be very difficult to find their systematic motions. And only by determining the groupmotions of all these condensations may we hope to make progress in the dynamics of the universe and to find the meaning of starstreaming.

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¹) The Distribution of the Stars (The Scientific Monthly May, 1924.)

²) On the Theory of Star Streaming, and the Structure of the Universe; 2d Paper. (Monthly Notices **76**, 552).

ADDENDA AND CORRIGENDA.

§ 1 p. 4 line 17 in formula for k: for rc read r.

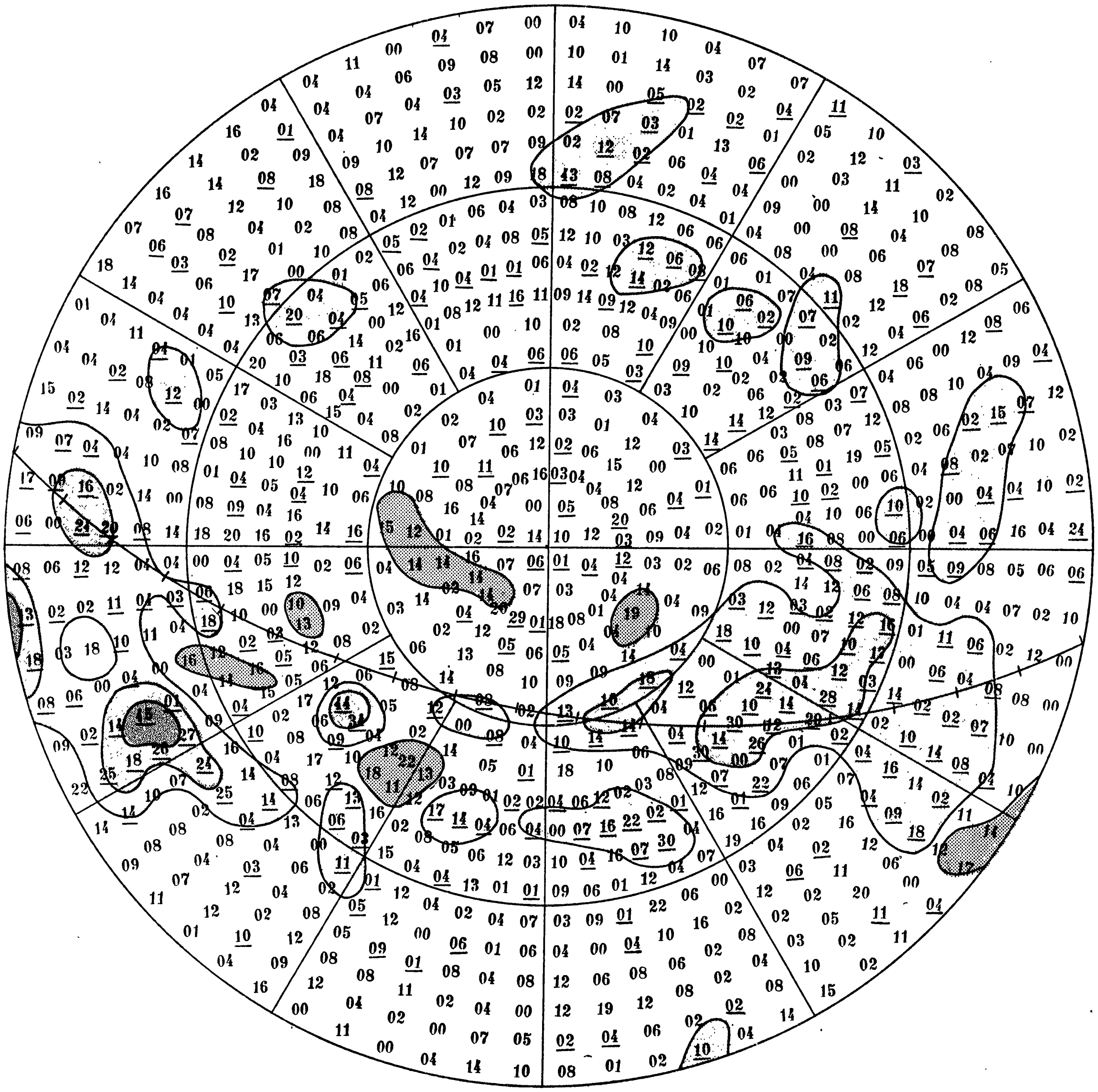
§ 7 p. 14 line 12 for „the coefficients k and l, and also b and c" read „the coefficients l and c".

§ 28 p. 57. Through the kindness of Prof. Harlow Shapley, the present director of Harvard College Observatory, I had an opportunity to consult some letters of Thomé to Pickering, preserved at this observatory. In a letter dated May 1. 1894 Thomé speaks of his visit to the United States and Cambridge, from which he just returned, and writes „I have begun the succeeding ten degrees, and if my progress continues to be as rapid as up to now, shall finish the observing during the next year". In a letter dated June 25 1901 he writes „Also if you have any later photometer observations of southern stars than those already published, especially of the fainter ones. I should be greatly obliged if you would send me an early copy. Since beginning Vol. XVIII (42° to 52°), my estimates have followed your photometric scale, and I only regret that it is not more extensive." This confirms the hypothesis that he brought with him from Cambridge a copy of the catalogue or advance sheets of Volume 34, which appeared a short time afterwards, and used them from the beginning of his observations of zone 42°—52°.

§ 70 p. 112. From a discussion of the proper motions R. E. Wilson (Astron. Journal **36**. 7) finds the distance of these O type stars 800 parsecs, not much different from our results for the condensations.

The numbers inscribed on the charts represent for fields of nearly 5° square the deviations of the real surface density of stars from the density in the schematical universe, described in § 7. Charts I give the deviations of densities themselves for $m = 5.7$, charts II and III give the deviations of their logarithms (in two decimals) for $m = 7.4$ and 8.6. The underlined numbers denote positive deviations, the others negative ones. Curves of equal deviations 0.10, 0.20 *etc.* are drawn; regions of surplus density are indicated by red colour, regions with a shortness of stars greater than 0.10 by a blue colour. On the charts of the southern hemisphere (right hand side) the polar regions beyond declination 62° are lacking. For more particulars on the computation of the numbers and the construction of the charts *cf.* § 48—53.

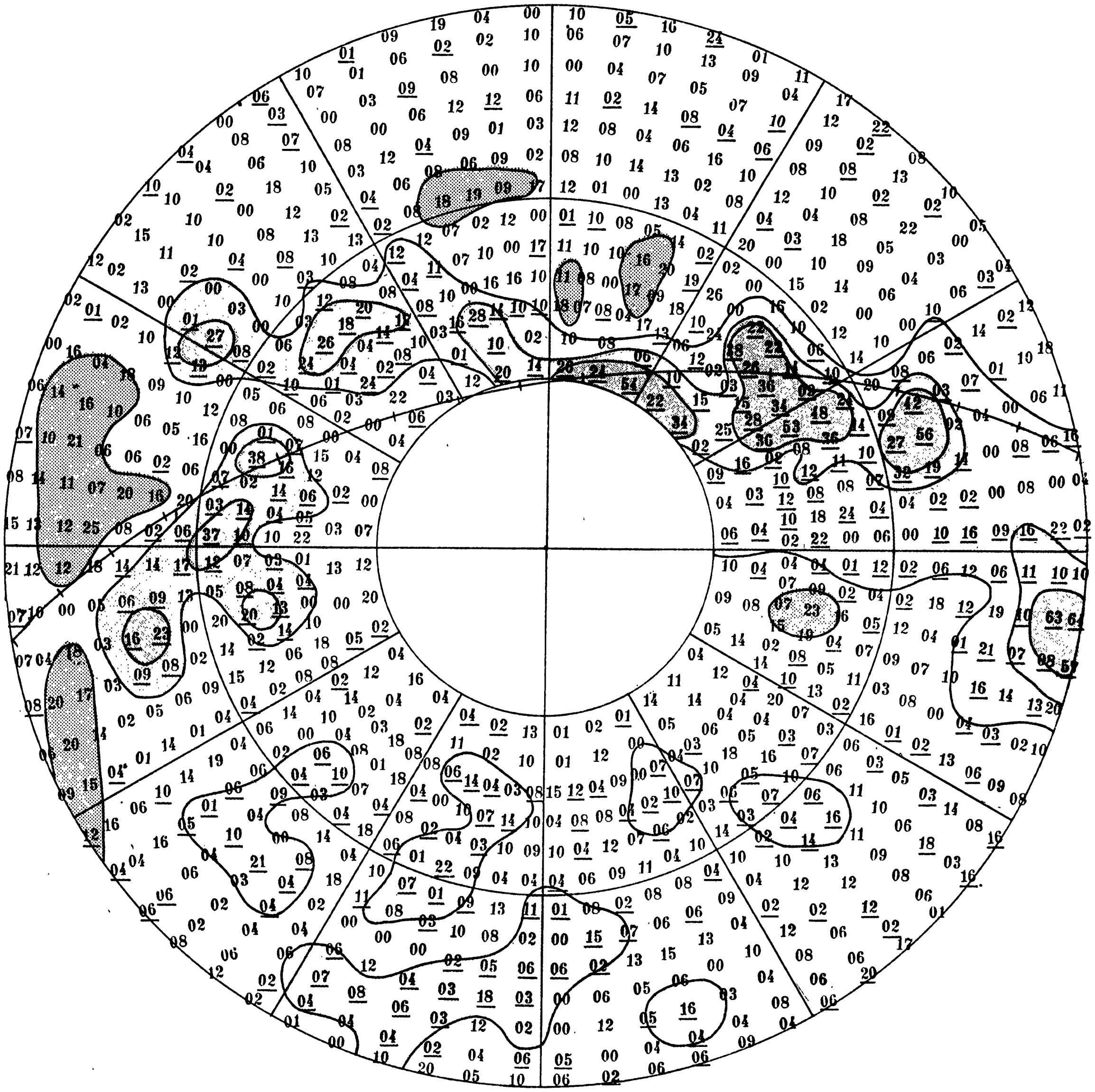
I. NORTH.



Deviations of star density for *m* 5.7.

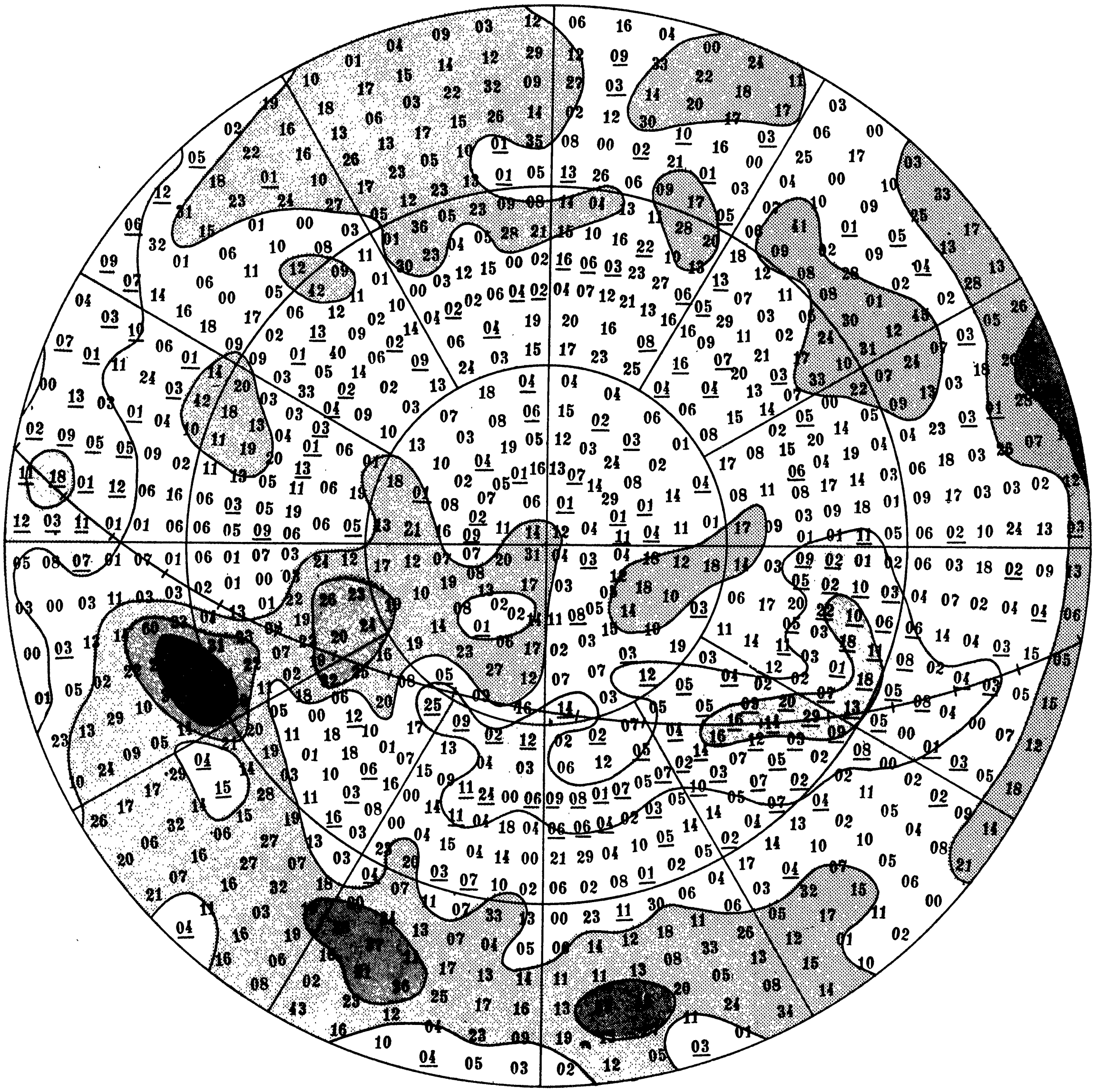
1924PUAms...1....1P

I. SOUTH.



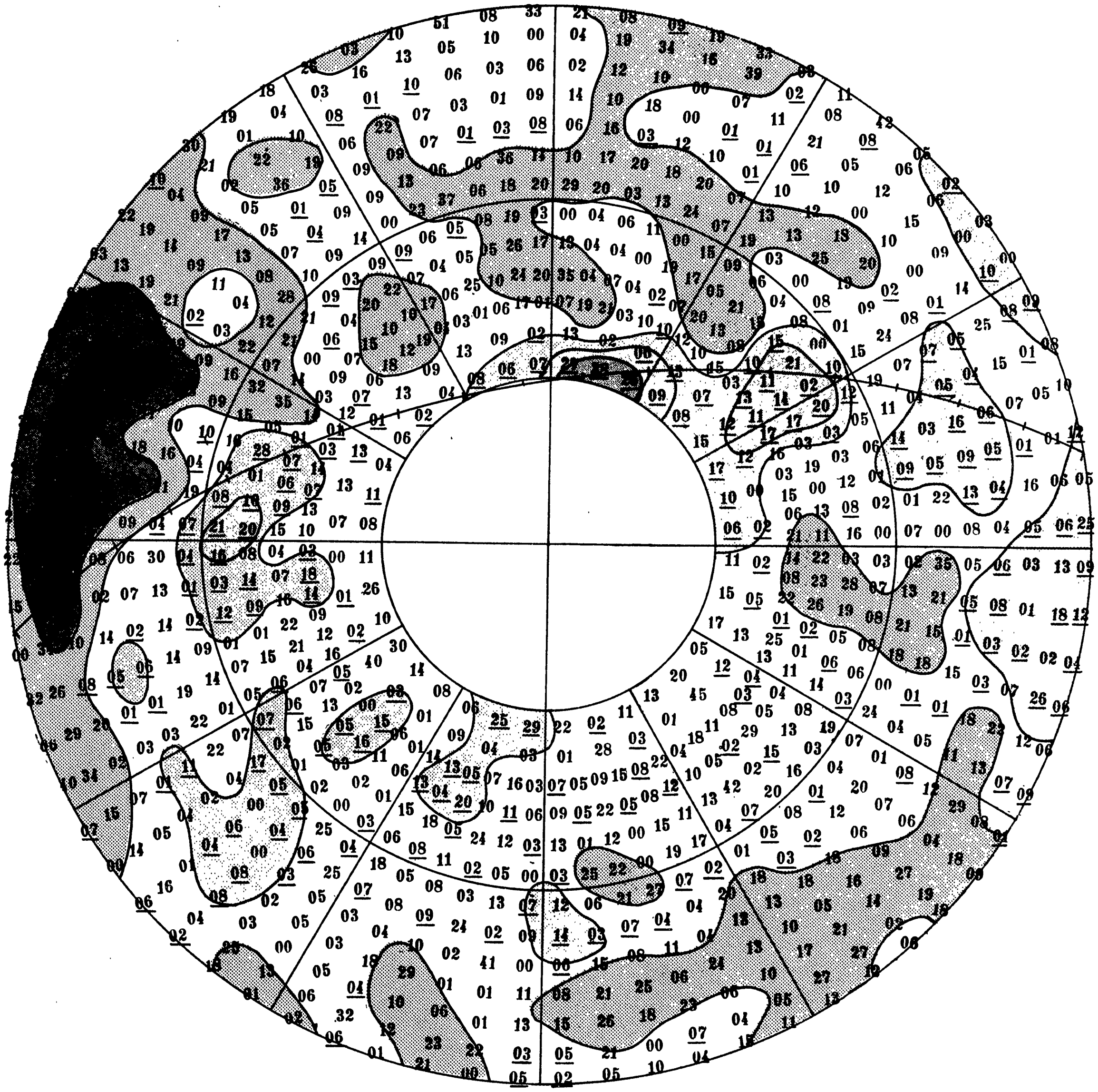
Deviations of star density for m 5.7.

II. NORTH.



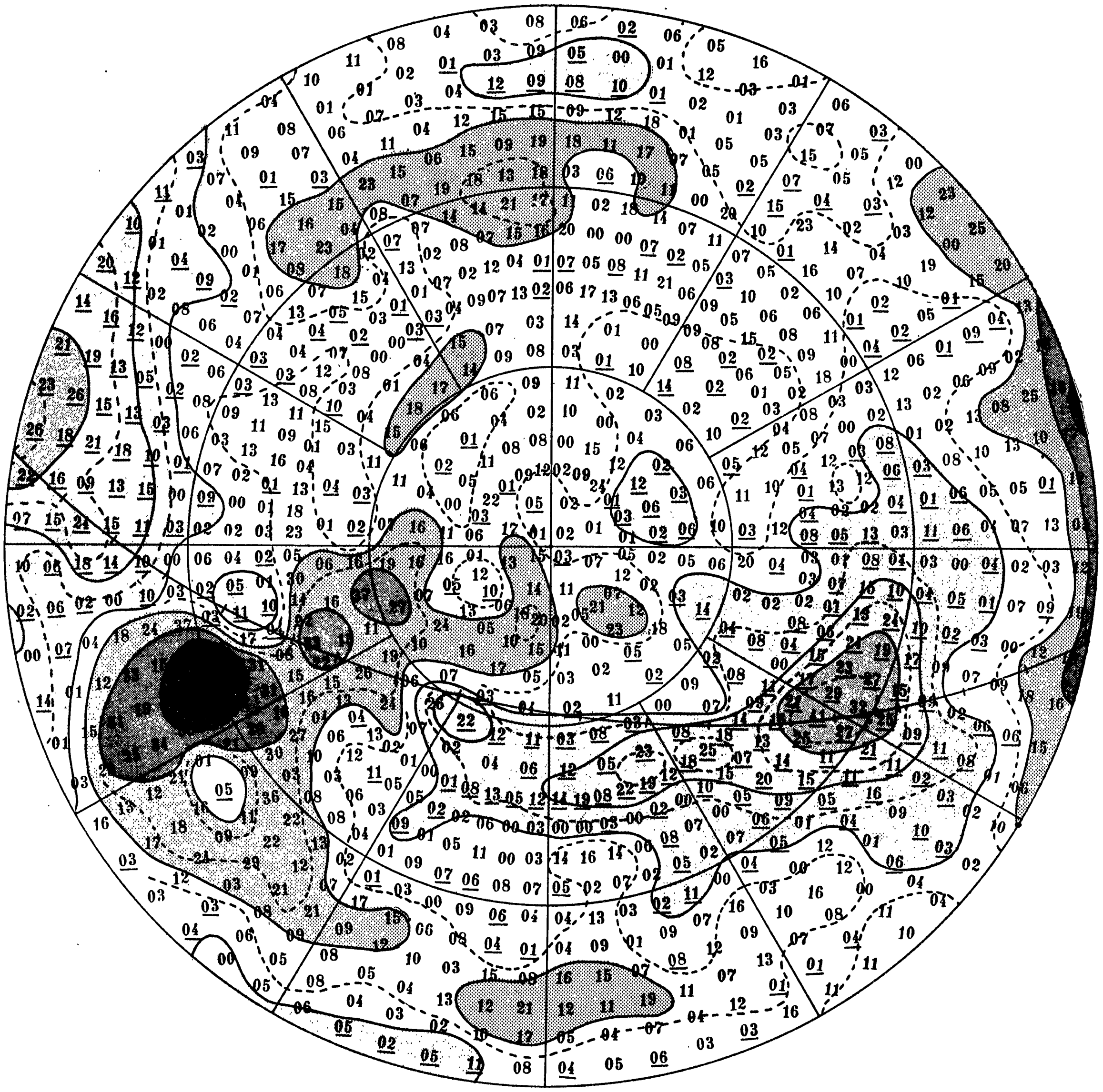
Deviations of star density for *m* 7.4.

II. SOUTH.



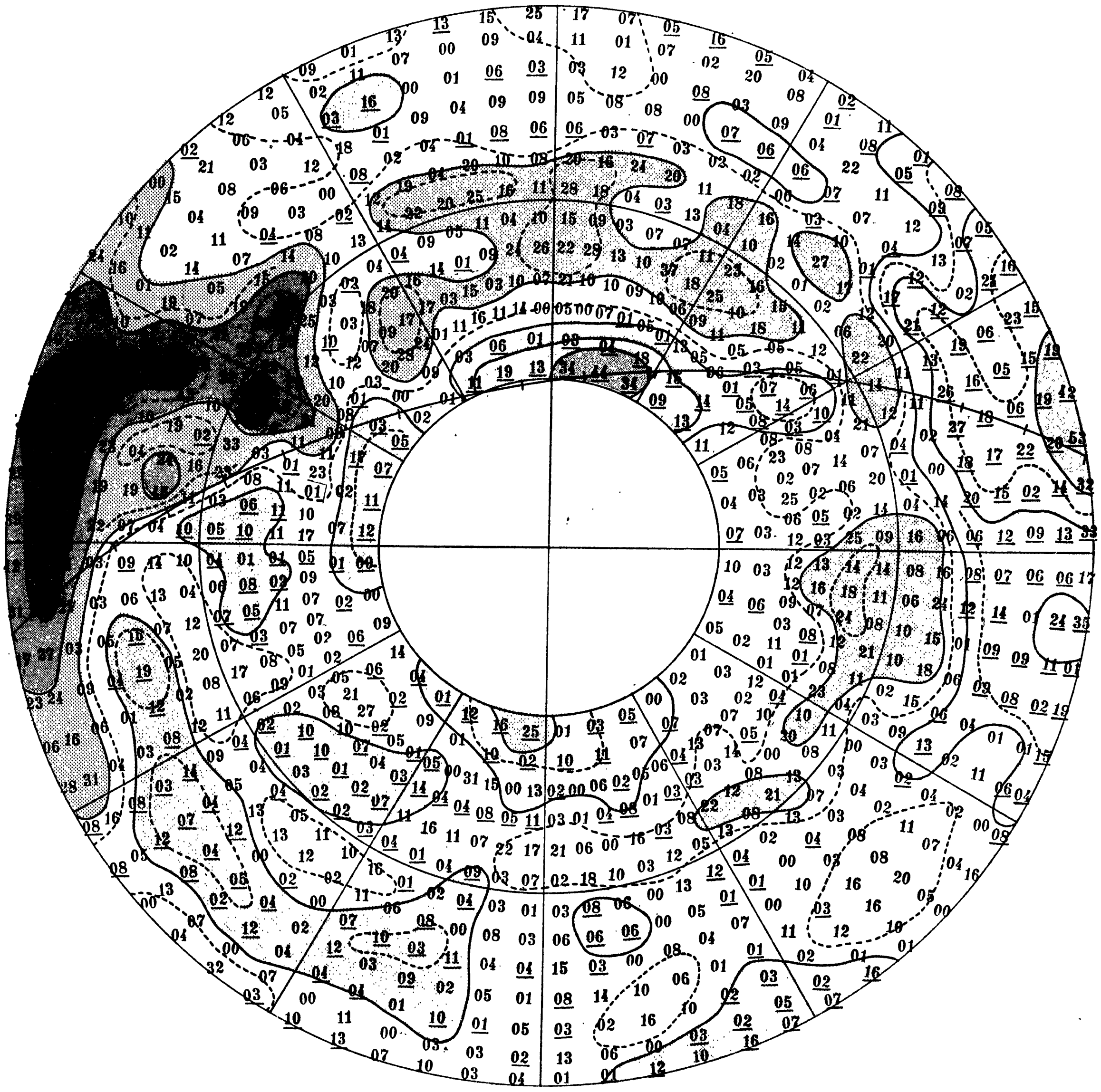
Deviations of star density for *m* 7.4.

III. NORTH.



Deviations of star density for *m* 8.6.

III. SOUTH.



Deviations of star density for m 8.6.