

here, this may serve as a proof that our interpretation is right.

For ζ Geminorum and α Ursae minoris the conclusions are the same. From the periods we find their absolute magnitudes -3.18 and -1.85 , $\log I = 3.47$ and 2.94 ; with $\log \sigma = -0.32$ and -0.12 we have $\log R = 1.90$ and 1.53 ; thus for the first named star $\log g = \log \mu - 3.79$, for the other $\log g = \log \mu - 3.06$. With EDDINGTON's relation $\log g$ becomes -2.66 and -2.19 . If $\log g$ according to the linear formula for both is taken 0.6 smaller than for δ Cephei, their masses will be equal to and 4 times smaller than for this star. If, however, in figure 3 a curve is adopted, the difference of $\log g$ becomes smaller to an unknown extent.

The variation of $\log g$ accompanying the light variation of a Cepheid has the greatest importance in the comparison of theoretical explanations. Here, however, the scarceness of our data make clear decisions difficult. Expressed in the linear scale of the formula δ Cephei shows at minimum a smaller g than at maximum (amount 0.3), while ζ Geminorum shows on the contrary a larger g at minimum. With the curved relation of fig. 3 the real differences in $\log g$ become much smaller. From the pulsation theory we may compute

these changes, if we assume that in each phase the condition of the surface layers does not deviate appreciably from those prevailing in an ordinary star having the same T and R of that phase. During a period the point in the spectral diagram representing the spectrum of a Cepheid wanders through a kind of ellipse; at maximum and minimum the star is in the midst of the process of expansion or contraction, so that the extremes of temperature have both an intermediate value of g ; midway between these phases, with intermediate temperature, the star reaching its largest and its smallest volume, shows extreme values of $\log g$. Assuming EDDINGTON's values (Pulsations of a gaseous star., Table II A) these extreme values of $\log g$ must differ 0.29 for δ Cephei, 0.24 for ζ Geminorum, differences which lie near the limit of what can be observed in the spectrum. If, however, the curve in fig. 3 should be adopted, i. e. if the masses are not very small, these differences become much more marked in the spectral diagram. Thus it may be expected that an investigation of Cepheid spectra in other phases of the light variations than we could make use of, will throw much light upon the solution of the Cepheid problem.

The masses of the swift moving stars, by *A. Pannekoek*.

It is an open question whether for the velocities of the stars through space equipartition of energy between the stars of different mass may be assumed as a general principle. The cause of equipartition of energy in a mixture of gases, viz: the large number of collisions, is absent in the case of the stars; encounters of stars are, according to the computations of CHARLIER and JEANS, so very scarce that a system of stars is not at all comparable with a mass of gas. On the other hand SEARES has found that by the continuous decrease of mean mass along the dwarf branch parallel to the continuous increase of the mean velocity the mean kinetic energy of translation along this branch is approximately constant (*Aph. J.* 55, 167); and he makes use of this principle of equipartition to derive the mean mass of giant stars. It may be of interest to see whether this principle also holds for the stars with high velocities, which in this case must be expected to have small masses. If, however, these stars must be considered as foreign intruders in our system, because they have a special movement, relative to our environment, in the direction $120^\circ-60^\circ$, thus being connected with the large system of globular clusters and spirals, then there is no reason to expect that they will partake in the equipartition of energy among the stars of our system.

In order to investigate this matter the list of high velocity stars with known parallaxes, published by BOSS, RAYMOND and WILSON in Table III of their 3^d paper „On the real motions of the stars”. (*Astron. Journ.* 35, 28) has been used. In order to find their mass I have used the relation between mass, spectroscopic and trigonometrical parallax, deduced in *B. A. N.* 19 and tested in the tables p. 116 of that paper, viz: $M = (s/t)^2 M_0$, if M_0 denotes the mean mass of the spectral class to which the star belongs.

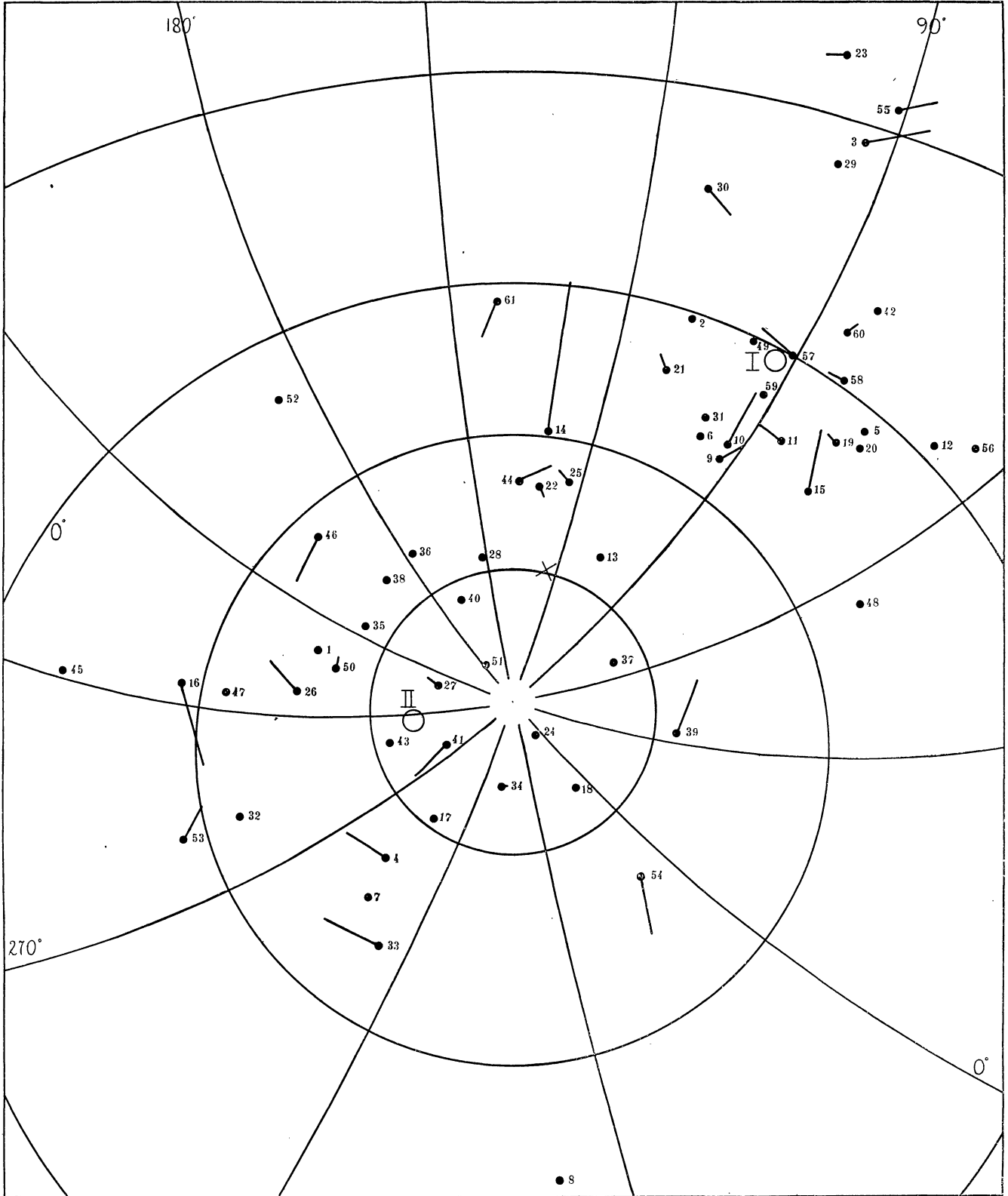
The different data involved in these computations are contained in the following table. Column 2 contains the BOSS or the Cincinnati number of the star, columns 3–5 the usual name, the magnitude, the spectral class according to *Mt. Wilson Contr.* 199, columns 6–7 amount and direction of the proper motion, 8 the parallax assumed by BOSS (in $0''.001$), column 9 the radial velocity. From these data the rectangular velocities and the apex and velocity relatively to the sun (in km) for each star (columns 10–11) have been derived. The next column contains the *Mt. Wilson* absolute magnitudes, indicating whether the star is a giant or a dwarf. Then follow the data for the mass: the trigonometrical and the *Mt. Wilson* spectroscopic parallax (columns 14–15); the first one is taken from *Mt. Wilson Contrib.* 199, completed by some later series. Since in the

	Boss Cincinnati.	Name	<i>m</i>	Sp. <i>MW</i>	Prop. Mot.	Pos. Angle	Boss Parall.	Rad. Vel.	<i>x</i>	<i>y</i>	<i>z</i>	Apex star	Veloc.	Abs. magn.	Tr. Par.	Sp. Par.	log <i>M</i> log <i>M'</i>	Mass	Stream	
1	130	ε Andr.	4.5	G 3	0.336	222	32	-83	-50	-41	-72	220	-48	97	0.2	53	14	-1.21 -79	sm.	II
2	177 C	Ll. 2450	7.6	G 1	524	91	26	0	-31	+89	-2	109	-1	96	4.7	17*	22	+20 -17		I
3	229 C	Ll. 3022	7.8	G 8	502	72	22	+57	-8	+108	+56	94	+28	134	5.4	47*	32	-47 +19		I
4	274 C	Ll. 3922	7.5	G 1	466	218	9	-41	+38	-154	-192	284	-51	248	4.1	-42*	28	- +98	pr. la	II
5	388 C	W ₁ 2. 927	8.5	K 1	667	104	71	+67	+19	+78	-4	76	-3	81	5.7	96*	32	-1.11 -86	sm.	I
6	764	ε Erid	4.4		3.168	76	158	+87	-16	+119	-44	98	-20	129						I
7	984	40 Erid	4.5	K 1	4.085	213	200	-42	+22	-71	-74	288	-45	106	6.2	196	219	-08 -09		II
8	709 C	Gr. 990	7.8	K 2	0.563	281	37	-44	+65	-47	-26	324	-18	84	6.9	36	48	+06 +04		II
9	1373	40 Orion	4.3	G 7	321	163	24	+99	-5	+109	-44	93	-22	118	1.6	29	28	-16 00		I
10	1627	6 Lync	6.1	G 8	330	182	10	+36	-9	+152	-52	93	-19	160	2.9	4	24	+1.42 +62	la	I
11	1783	δ Can Ma	4.3	K 4	138	264	22	+98	+8	+100	-24	85	-13	103	0.8	17*	21	-01 -23		I
12	873 C	Ll. 14146	8.3	F 9	513	288	47	+57	+27	+71	+3	68	+2	77	3.9	51	17	-91 -83	sm.	I
13	949 C	D'Ag 1415	7.5	G 7	1.170	188	53	+14	-17	+62	-83	106	-52	106	5.9	66	63	-18 +02		II
14	2148	Br. 1159	6.7	K 3	0.073	171	3	+38	-70	+81	-59	131	-29	121	0.8	-30*	8			I
15	2247	ο Urs Ma	3.4	G 2	166	226	2	+20	+92	+363	-112	76	-17	394	-0.8	2	14		pr. la	I
16	2742	γ Leon.	2.3	G 8	346	119	2	-36	-401	-605	-354	236	-27	821	-0.6	3	30			II
17	2822	Y 4514	6.1	F 7	734	159	45	-9	+11	-35	-69	288	-62	76	3.9	20	42	+77 +07	la	II
18	2935	Ll. 21185	7.8	Ma	4.779	187	415	-87	+38	-4	-95	354	-68	103	10.5	411	380	+31 -31	sm.	II
19	2942	γ Leon.	4.7	F 5	0.350	262	11	+6	+30	+147	-19	78	-7	151	0.8	12*	17	+50 +58	la	I
20	3069	Gr. 1812	6.7	F 8	583	272	28	-18	+26	+95	-11	75	-6	101	4.1	38*	36	+06 +32	la	I
21	1452 C	Gr. 1822	7.7	F 9	644	244	18	+24	-59	+159	-32	110	-11	172	4.1	24*	17	+25 -01	sm.	I
22	3145	Pi 218	7.0	G 7	629	214	20	-13	-75	+84	-98	132	-41	149	3.6	17	23	+12 -02		I, II
23	1522 C	W ₁ 12. 69	7.3	G 6	732	306	23	+11	-18	+122	+89	99	+35	151	5.0	11	33	+75 +19	la	II
24	1524 C	Ll. 22908	8.0	G 4	589	179	33	-30	+13	-2	-88	352	-82	89	5.5	49	33	-44 -10	sm.	II
25	3342	γ Comae	4.6	K 2	122	225	1	+4	-242	+390	-361	123	-39	578	0.7	2*	17		pr. la	I, II
26	3326	Br. 1706	5.9	K 0	531	149	20	+51	-57	-75	-97	233	-46	136	4.2	1	46	+56	la	II
27	1687 C	Ll. 24414	7.4	G 4	682	174	24	+22	-30	-25	-130	219	-73	169	4.1	29	28	-12 +04		II
28	3662	α Boot.	0.0	K 0	2.282	209	87	-5	-60	+33	-104	151	-57	124	1.2	77	158	+46 +36	la	II
29	3734	Pi 127	6.5	F 4	0.946	292	41	-70	-16	+117	+55	97	+25	130	3.2	51	25	-38 -18	sm.	I
30	2024 C	Ll. 27742	6.9	G 6	668	296	25	-35	-48	+116	+41	112	+18	132	4.2	6	33	+1.35 +09	la	I
31	2027 C	Ll. 27744	7.0	K 0	1.376	248	66	-70	-19	+114	-35	99	-17	121	5.8	70	52	-42 -36	sm.	I
32	3895	5 Serp.	5.2	F 6	0.644	145	52	+54	-12	-64	-46	260	-35	80	3.5	55	46	00 +04		II
33	3952	37 Libr.	4.9	K 1	389	130	19	+49	+35	-75	-70	295	-40	109	2.9	-	42	+51	la	II
34	2163 C	Ll. 29437	7.6	K 0	760	163	21	-2	+35	-39	-163	312	-72	142	4.4	14	48	+90 +56	la	II
35	2248 C	Ll. 30694	7.0	K 0	1.664	207	86	+41	-52	-26	-81	207	-54	100	5.8	99	63	-56 -42	sm.	II
36	4342	Par. 21575	8.0	K 5	1.465	220	53	+27	-85	+3	-104	177	-50	134	7.9	60	100	+24 +36	la	II
37	4403	72 Herc.	5.5	F 8	1.060	173	95	-78	+13	+35	-87	69	-67	94	5.5	85	105	+28 +18	la	II
38	4638	η Serp.	3.3	G 8	898	219	45	+10	-58	-10	-74	190	-51	96	2.5	63	66	-10 +20		II
39	4656	109 Herc.	4.0	K 1	324	144	18	-57	+49	+32	-85	33	-56	102	1.5	15	33	+51 +35	la	II
40	2420 C	Mu 16281	8.4	G 5	55	205	33	-22	-35	+8	-75	167	-64	82	5.5	33	33	--12 -12		II
41	4705	ι Aquil.	4.0	K 4	0.318	184	19	+36	-3	-25	-83	264	-73	88	0.6	-	20	-15		II
42	4950	31 Aquil.	5.4	G 8	961	49	70	-98	+11	+115	+23	84	+11	118	5.1	67	95	+16 +12	la	I
43	4961	Br. 2459	6.4	F 6	658	196	34	-5	-13	-39	-82	252	-63	92	3.9	60	35	-32 +18		II
44	4976	α Vulpec	4.6	Ma	170	227	18	-86	-56	+49	-63	138	-40	97	0.1	-1*	13			I, II
45	2554 C	Gr. 2875	6.7	K 4	664	233	42	+12	-38	-65	-13	239	-10	76	6.7	36	100	+69 +57	la	II
46	5044	56 Sagitr.	5.1	K 0	165	234	3	+20	-206	-60	-150	196	-35	262	1.7	-	21		pr. la	II
47	2620 C	Ll. 38383	7.1	K 2	1.369	229	67	+12	-45	-68	-54	236	-34	98	6.6	59	76	+04 +08		II
48	5180	Pi 29	5.9	K 2	1.256	99	61	-53	+60	+85	-38	55	-20	112	5.9	42	110	+66 +32	la	I
49	2709 C	Fed. 3638	7.6	G 6	0.691	36	27	-30	-17	+123	-2	98	-1	125	5.3	15	30	+47 -03		I
50	2728 C	W ₁ 20. 1454	7.7	F 6	485	218	12	-9	-84	-83	-152	224	-52	191	3.9	-15	14			II
51	2749 C	W ₂ 21. 97	7.2	F 4	915	187	44	-46	-19	+1	-107	177	-80	109	4.0	37	22	-22 -36	sm.	II
52	2769 C	Ll. 41363	6.7	G 8	665	238	43	-43	-83	-9	-15	186	-10	85	5.6	-100*	63	+20	la?	II
53	2885 C	Gr. 3689	8.6	K 1	629	238	15	-36	-27	-177	-93	261	-27	202	4.8	51	24	-83 +23	pr. sm.	II
54	5790	35 Peg.	5.0	K 0	325	167	20	+55	+62	-8	-71	352	-49	77	1.8	33*	24	-44 00		II
55	5940	β Peg.	2.3	Mb	234	55	5	+9	-2	+190	+116	91	+32	222	0.0	0	30		la	I
56	3014 C	Lc. 9352	7.5	-	6.900	79	290	+12	+49	+101	+8	64	+5	114	-	-	-			I
57	5981	91 Aquar.	4.4	K 0	0.367	92	15	-27	-3	-120	+1	91	0	119	1.4	-	24	+24	la?	I
58	5988	γ Pisc.	3.8	G 5	753	88	23	-13	+20	+155	+3	83	+1	156	0.5	21	22	-08 -16		I
59	3081 C	AOε 25685	7.0	K 0	1.06	85	44	-25	-4	+115	-17	92	-8	117	5.6	75*	58	-40 +08		I
60	3085 C	AOε 25734	7.4	G 4	0.42	83	17	+8	+7	+115	+14	86	+7	117	4.0	-12*	21	+08		I
61	3115 C	Pi. 164	7.0	G 2	610	38	25	-68	-107	+83	-8	142	-3	134	4.7	-62	35	+23		I

derivation of BOSS' values of the parallax the spectroscopic results have been included with different weight, they should not be used in deriving the mass; since however, thereby only the ratio of M to M_0 is some-

what diminished and on the other hand partly other data in another combination with other weights are used, we have still made a second computation of the mass by comparing the spectroscopic absolute magni-

Figure 4.



tudes with BOSS' values ($\log M$ and $\log M'$ in column 16—17). For $\log M_0$ the values + 0.40 for the F_0 , 0 for the G_0 , - 0.16 for the K_0 , - 0.24 for the M stars have been used. In some cases no precise values but only qualitative designations are given.

The data are not altogether satisfying, many trigonometric parallaxes (indicated by an asterisk) resting on but one authority. Nevertheless they point clearly to the result, that *among the high velocity stars there are a number with small masses, but still more have large masses surpassing the sun's mass several times.*

It will be of interest to see, whether a relation exists between the mass and the motion. The suggestion presents itself that under the common character of swift moving stars two different kinds of stars are combined; foreign intruders having a common large velocity relatively to the sun, because they belong to the larger cluster system, and having intermediate or large masses; and stars of small mass belonging to our system and having got large velocities in various directions because of their small masses. For this purpose the velocity components, the velocities and the apices of each of these stars have been computed in the first columns of the table. In the accompanying diagram these apices have been represented in stereographic projection. They are almost entirely situated on one hemisphere having its centre at 122° ; -62° , not far from the antapex of the solar system and in the nearest vicinity of the apex of the clusters and spirals 126° ; -60° .

But the diagram shows at once that the apices of these high velocity stars are not concentrated around this point; *they cluster in two groups roughly coinciding with the vertices of KAPTEYN's two star streams.* The stars assigned to these groups are indicated by I and II in the last column of the table.

The apices and the velocities depend on the parallaxes assumed for each star. There are some parallaxes of a few thousandths of a second, for which the tangential velocity is quite uncertain and probably far too great. In order to see what the uncertainty caused in the position of the apices is for the stars having a parallax below $0''.026$, the displacement of the apex for an increase of $0''.007$ in the parallax has

been computed and represented in the diagram by a line proceeding from the apex point. It appears that the fact of their clustering in two groups is not affected by such corrections. Still it is probable that some of these stars will disappear from the list of swift moving stars if exact values of the parallax are available. Thus e.g. for α Ursae (15) and γ Comae (25) the velocity falls below 60 km. if the parallax exceeds $0''.010$, and for Br. 1159 (14) the same takes place for a parallax $> 0''.018$. The clustering of the apices in two groups may also be seen by projecting them on the YZ plane, which does not much deviate from the galactic plane. The stars 4, 16, 53, 55, which by their large velocities are falling far outside the groups, may be brought within them by increasing their parallaxes.

By taking the mean of the rectangular components for the stars of each group (excluding 14, 15 and 55 from I, 4, 16, 46, 53 from II and omitting 22, 25, 44) we obtain for

$$\begin{array}{l} \text{stream I} \quad x = -7 \quad y = +119 \quad z = -4; \\ \quad \quad \quad \text{vertex } 93^\circ, -2^\circ; \quad v = 119 \text{ km. (25 stars)} \\ \text{stream II} \quad x = -15 \quad y = -30 \quad z = -81; \\ \quad \quad \quad \text{vertex } 243^\circ, -68^\circ; \quad v = 88 \text{ km. (27 stars)} \end{array}$$

These vertices are indicated on the diagram by open circles. Combining the streams with equal weight we find

$$\begin{array}{l} \text{mean} \quad x = -11 \quad y = +45 \quad z = -42; \\ \quad \quad \quad \text{antapex } \odot \quad 104^\circ, -42^\circ; \quad v = 63 \text{ km.} \\ \text{difference} \quad x = +8 \quad y = +149 \quad z = +77; \\ \quad \quad \quad \text{true vertex } 87^\circ, +27^\circ; \quad v = 168 \text{ km.} \end{array}$$

Thus these stars do not form one moving group foreign to the system of the bulk of the stars surrounding our sun. They show the same motion in two streams as the slow moving stars, only on a larger velocity scale.

As might be expected the large and the small masses do not show any preference for either stream; they seem to be equally distributed over them. Thus their presence and distribution is an indication that the equipartition of energy is not a real principle valid for stellar movements.