Investigation of the coarse-grating in use with the ZEISS 15 cm UV-Triplet.

by

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This coarse-grating consists of about 130 steel wires of 0.6 m.m. thickness, with clear spaces of 0.6 m.m. between them.

In order to investigate its errors a series of measurements of the grating wires were made by means of the measuring arrangements of the stereocomparator constructed by ZEISS. A wire cross inclined at 45° was pointed at the limits of the dark and the bright spaces; while one observer set the wire cross another read the scale with the micrometer. The readings were all made in microns. The grating is mounted in the stereo comparator so that the wires were vertical in the microscope. All the measurements were made by VOÛTE and PANNEKOEK in the following series:

1st On each 10 wires, the left edges of wires no 25, 35 etc, a number of points 10 m.m. apart were pointed.

2nd In three horizontal levels, one across the centre, the others 38 m.m. above and 37 m.m. below, all the limits of bright and dark spaces, were pointed (on the central line numbering 2 × 126).

The readings of the first series are collected in Table I; only the fractions of a millimetre are given. The readings of the second series have been reduced to the zero-point of Table I; then by subtracting 0.6 1.2 1.8.....m.m., they have been reduced to nearly the same values; the results are found in Table II. The vertical argument (scale reading in m.m.) runs from 120 to 260 m.m., the three horizontal lines have a vertical scale reading 188, 151 and 226. The horizontal argument is the number of the wire, counted from the left hand side. In Table II two readings are given of each wire of the first and second borders respectively.

TABLE I.

Vertical \ Wire	25	35	45	55	65	75	85	95	105	115
260	—	—	—	114	128	095	101	—	—	—
250	—	500	480	130	146	069	100	119	—	—
240	497	500	482	142	149	070	103	132	160	—
230	465	493	478	152	173	094	115	133	168	173
220	494	486	470	171	200	096	130	146	168	190
210	453	471	450	212	289	096	144	137	171	217
200	429	460	430	224	203	140	170	136	180	207
190	438	453	424	245	259	191	194	150	215	220
180	414	450	419	258	278	165	206	157	205	234
170	390	461	400	248	259	149	220	207	219	237
160	378	432	393	260	251	135	249	234	228	249
150	355	414	382	267	251	165	210	229	224	257
140	335	373	388	281	273	214	215	244	236	—
130	—	383	385	285	292	223	238	273	251	—
120	—	—	340	315	302	262	277	—	—	—

TABLE II.

WIRE	LINE 151		LINE 188		LINE 226		WIRE	LINE 151		LINE 188		LINE 226	
	br-d	d-br	br-d	d-br	br-d	d-br		br-d	d-br	br-d	d-br	br-d	d-br
4			473	468			67	227	234	201	220	141	128
5			456	418			68	250	252	230	218	166	176
6			479	456			69	245	231	230	215	169	164
7			462	425			70	247	242	204	206	158	159
8			424	381			71	256	243	215	204	128	111
9			417	380			72	232	226	212	221	134	138
10			448	421			73	208	206	196	207	117	098
11			441	430			74	202	183	212	203	113	100
12		396	438	407	508	483	75	156	156	196	196	122	107
13	365	356	406	389	477	466	76	184	176	189	181	116	114
14	395	402	427	413	502	479	77	199	177	254	252	134	138
15	371	370	422	380	478	458	78	196	181	223	209	144	113
16	369	356	393	378	458	439	79	196	177	203	181	126	116
17	350	341	384	357	452	435	80	218	223	199	207	149	149
18	354	350	381	355	467	449	81	207	197	199	190	135	123
19	371	367	397	367	455	449	82	227	210	184	176	122	120
20	376	393	422	403	499	485	83	230	217	195	197	143	145
21	405	407	438	415	492	466	84	210	208	187	177	142	136
22	396	391	449	425	494	468	85	216	203	204	190	139	122
23	400	390	455	427	514	487	86	219	212	199	182	118	112
24	390	397	436	433	503	483	87	201	187	168	150	128	113
25	361	370	439	407	486	453	88	253	224	201	188	172	154
26	384	383	457	433	512	486	89	267	246	183	178	138	130
27	409	421	462	429	498	474	90	231	215	194	188	141	124
28	401	396	432	413	483	466	91	273	259	230	224	148	159
29	451	438	453	417	499	488	92	261	236	219	207	185	170
30	385	410	429	411	491	467	93	290	296	234	231	187	164
31	409	392	444	428	483	477	94	229	223	217	202	163	149
32	394	399	417	395	479	456	95	229	218	169	152	139	136
33	399	397	447	425	490	465	96	245	208	216	201	163	150
34	388	383	441	414	493	460	97	192	184	176	156	139	130
35	430	425	479	445	507	483	98	210	214	207	190	149	136
36	383	381	429	401	487	461	99	212	213	192	178	150	136
37	386	380	418	401	464	447	100	225	198	233	210	183	172
38	362	367	426	401	483	454	101	252	225	217	201	206	186
39	366	386	394	384	450	450	102	242	217	209	192	204	197
40	395	359	410	403	487	475	103	225	212	202	192	168	159
41	380	367	417	398	485	475	104	226	206	212	199	161	158
42	371	359	411	402	478	475	105	228	213	226	217	160	167
43	373	370	412	393	470	459	106	250	230	234	221	177	169
44	373	392	400	400	468	456	107	267	249	257	229	190	175
45	399	401	427	429	494	488	108	241	228	224	198	170	158
46	353	360	403	393	482	460	109	226	222	223	214	161	152
47	377	375	438	415	485	466	110	233	214	221	205	194	163
48	328	328	389	370	463	469	111	242	235	251	248	189	183
49	361	348	406	396	499	475	112	258	241	226	212	199	179
50	360	352	401	387	466	436	113	238	229	223	214	173	154
51	292	302	340	321	347	322	114	245	233	239	227	192	174
52	305	320	315	301	343	335	115	250	250	247	233	206	196
53	277	251	275	255	213	198	116	226	217	222	200	184	165
54	286	286	225	220	205	195	117	268	260	239	228	186	185
55	267	271	246	244	174	149	118	245	249	224	220	182	181
56	284	258	215	203	177	160	119	269	260	248	235	203	191
57	278	278	238	222	173	154	120	264	252	240	225	187	174
58	250	244	205	199	173	138	121	278		239	233	196	192
59	233	219	227	203	152	118	122			250	241		
60	251	238	206	176	155	121	123			244	231		
61	249	237	216	193	152	137	124			258	239		
62	241	249	224	193	175	156	125			257	246		
63	254	253	252	258	217	208	126			242	228		
64	247	229	225	221	163	145	127			259	246		
65	245	256	258	271	185	187	128			260	241		
66	231	223	211	192	161	139	129			289	277		

On a scrutiny of table I the chief irregularity of the grating immediately strikes the eye. Going down along the vertical wires 25, 35, 45 the reading decreases, while on the other wires, 55 to 115, it increases; — the two parts of the grating are somewhat inclined towards one another. From the readings in Table II we see that there are two jumps, one between the wires 50 and 51, another between 52 and 53. The wires 4–50 constitute one group of parrallel wires, 53 – 129 another, mutually inclined, while the wires 51 and 52 occupy an intermediate position.

From the readings in table I the inclination of the vertical wires (relative to the vertical movement of the measuring instrument) was determined. The values found are (in 0.0001 mm. per 10 mm.).

— 158 — 101 — 103 + 138 + 114 + 120 + 127 + 126 + 082 + 097

We may assume two parallel systems of wires, with an average inclination of — 121 and + 115, thus having a mutual inclination of 0.0236 per 10 mm., i.e. of 0.00236 per mm., corresponding to an angle of 0° 8.′1.

For each horizontal line the readings of the limits bright-dark and dark-bright in table II averaged over 11 to 14 values (3 averages for the first part, 5 or 6 for the cecond part); from the gradual change in these averages the excess of the period over 1.2 mm. was derived; in this way the exact value of the period was found to be 1.20013 mm. (for the first part 1.19956, for the second part 1.20025). The average difference between the values for * br.-d. and d.-br. is — 15 for the first part, — 11 for the second part, — 12 for the whole grating; thus the average breadth of the dark spaces is 0.588, of the bright spaces 0.612 mm. Reducing all the readings, by means of the excess of the period 0.00013, to the same vertical line (e.g. wire 50), we get:

horiz. line 151:	first part	385 (b-d)	383 (d-b);	second part	233 (b-d)	223 (d-b);	jump 156
„ „ 188:	„ „	432 „	410 „	„ „	218 „	207 „	„ 208^5
„ „ 226:	„ „	487 „	468 „	„ „	159 „	147 „	„ 324^5

The jumps between the first and the second part reveal the relative position of the two systems of wires as well as their inclination. From the difference between the upper and the lower row the mutual inclination is found $(324^5 - 156) : 75 = 0.00225$. Combining it with the value found above we may adopt 0,00230. Smoothing the jumps, in accordance with this inclination, to 145, 230. 317, we get the smoothed values for the reduced readings of the above table

384	372	239	227
440	428	210	198
479	467	162	150

From these values and the period 1,20013 we compute the readings for a perfectly regular grating (in this case two half gratings) with the adopted constants. The deviations of the real from these computed readings are the accidental irregularities of the grating. Assuming the limit between 51 and 52 these deviations for the wires 51 and 52 themselves become very large.

2. If a grating is placed before the objective, the intensity in a point of the focal plane (focal distance f) at a distance af from the central image wil be given by

$$I = C^2 + S^2 \qquad C = \int \cos 2\pi \frac{a x}{\lambda} d x \qquad S = \int \sin 2\pi \frac{a x}{\lambda} d x$$

* bright-dark

where the amplitude integrals C and S are taken over the whole of the bright spaces. Supposing a quite regular grating, then if n is the number of wires, l and d the breadth of the bright and the dark spaces, $L = l + d$ the period, $\frac{(l-d)}{L} = \frac{2\psi}{\pi}$ and $\frac{2\pi \alpha x}{\lambda} = \varphi$, the intensity at the point, for which $\alpha_1 = \frac{\lambda}{L}$ (first diffraction point) and $\varphi = \frac{2\pi x}{L}$, will be given by

$$C = n\frac{L}{2\pi}\int_{-\psi}^{\pi+\psi} \cos\varphi\, d\varphi = 0 \qquad S = n\frac{L}{2\pi}\int_{-\psi}^{\pi+\psi} \sin\varphi\, d\varphi = n\frac{L}{\pi}\cos\psi \qquad I = n^2\frac{L^2}{\pi^2}\cos^2\psi.$$

For the second diffraction image we have to put $\alpha_2 = \frac{2\lambda}{L}$, $\varphi = \frac{4\pi x}{L}$ thus

$$C = n\frac{L}{4\pi}\int_{-2\psi}^{2\pi+2\psi} \cos\varphi\, d\varphi = n\frac{L}{2\pi}\sin 2\psi \qquad S = 0 \qquad I_2 = n^2\frac{L^2}{4\pi^2}\sin^2 2\psi.$$

For the third image we find in the same way $I_3 = n^2\frac{L^2}{9\pi^2}\cos^2 3\psi$. The central image has an intensity $I_c = n^2 l^2$, while without grating the intensity would be $I_O = n^2 L^2$. Compared with the normal image we have the relative intensities:

$$\frac{I_c}{I_O} = \left(\frac{l}{L}\right)^2; \qquad \frac{I_1}{I_O} = \frac{1}{\pi^2}\cos^2\psi; \qquad \frac{I_2}{I_O} = \frac{1}{4\pi^2}\sin^2 2\psi; \qquad \frac{I_3}{I_O} = \frac{1}{9\pi^2}\cos^2 3\psi. \qquad A)$$

If the dark and the bright spaces have equal width $\psi = 0$ and $I_2 = 0$.

In the case of an objective and a grating of unlimited extension the diffraction images would be restricted to these points; the limited size of the objective, however, gives a certain extension, determined by the aperture nL. If we consider a point at a distance $r\frac{\alpha}{n}$ outside or inside the first diffraction poit, the limits of the amplitude integrals become:

$$-\psi \text{ to } \pi\left(1+\frac{r}{n}\right)+\psi; \qquad 2\pi\left(1+\frac{r}{n}\right)-\psi \text{ to } 3\pi\left(1+\frac{r}{n}\right)+\psi\ldots\ldots$$

Taking $\psi = 0$ for simplicity the integrals become:

$$C = -\frac{L}{2\pi}\left(\sin 0 + \sin\frac{r}{n}\pi + \ldots\ldots\ \sin\frac{2n-1}{n}r\pi\right) = -\frac{n}{r\pi}\frac{L}{2\pi}\left(\cos\frac{1}{2n}r\pi - \cos\left(2-\frac{1}{2n}\right)r\pi\right)$$

$$S = +\frac{L}{2\pi}\left(\cos 0 + \cos\frac{r}{n}\pi + \ldots\ldots\ \cos\frac{2n-1}{n}r\pi\right) = -\frac{n}{r\pi}\frac{L}{2\pi}\left(\sin\frac{1}{2n}r\pi - \sin\left(2-\frac{1}{2n}\right)r\pi\right)$$

for which we may also write

$$C = -\frac{n}{r\pi}\frac{L}{2\pi}\left(1-\cos 2r\pi\right); \quad S = \frac{n}{r\pi}\frac{L}{2\pi}\sin 2r\pi; \quad I = \frac{n^2}{r^2}\frac{L^2}{\pi^2 4\pi^2}\left(2-2\cos 2r\pi\right) = n^2\frac{L^2}{\pi^2}\frac{\sin^2 r\pi}{r^2\pi^2}$$

This formula, giving the distribution of intensity horizontally over the first diffraction image, is the same formula that determines the distribution of light over a normal image formed by an objective of aperture nL; vertically we have the same distribution. According to this formula the (monochromatic) diffraction image is a disc bounded by a dark ring $r = 1$, thus having a radius $\frac{\alpha}{n}$, and surrounded by rings. Since the distribution of light is precisely the same in the central image and in the diffraction images, the total light of these images is proportional to their central intensities in the diffraction points, and the relation between these total intensities is given by the same formulae A). It is this total light, which in HERTZSPRUNG's method is expanded into an extrafocal disc, usually having a diameter somewhat smaller than the distance of these images. The relative intensities of the centres of these discs, which are measured with the microphotometer, will be expressed by the same formulae A).

In the Lembang grating, if we neglect the accidental irregularities, we have the more complicated case of two half gratings (number of wires n_1 and n_2), mutually inclined at an angle 0.0023 and in the central horizontal line leaving a clear space of l — 0.2085 between them. The diffraction image is now formed by the concurrence of waves coming from both half gratings. We consider at first only the jump in the phase, amounting to $\gamma = \frac{0{,}2085}{0{,}6}\pi = 62\overset{\circ}{.}5$, without the inclination. Then at the first diffraction point the limits of the amplitude integrals will be 0 and π for the n_1 first spaces, $-\gamma$ and $\pi - \gamma$ for the n_2 other spaces. Thus we have

$$C = \frac{L}{\pi} n_2 \sin\gamma \qquad S = \frac{L}{\pi}(n_1 + n_2 \cos\gamma)\,;$$

$$I = \frac{L^2}{\pi^2}(n_1^2 + n_2^2 + 2\, n_1 n_2 \cos\gamma) = \frac{L^2}{\pi^2} n^2 \left(1 - \frac{4\; n_1\; n_2}{n^2} \sin^2 \tfrac{1}{2}\gamma\right).$$

The intensity at this point is diminished, and for the case of $n_1 = n_2$, $\gamma = 180^\circ$ would become even zero. But at other points the intensity is increased. At a distance $\frac{r\,\alpha}{n}$ from this point we have

$$C = -\frac{L}{2\pi}\left\{ 0 + \sin\frac{r}{n}\pi + \ldots\ldots + \sin\frac{2\, n_1 - 1}{n} r\,\pi + \sin\left(\frac{2\; n_1}{n} r\;\pi - \gamma\right) + \ldots\ldots + \right.$$
$$\left. + \sin\left(\frac{2\;n - 1}{n} r\;\pi - \gamma\right)\right\}$$

$$S = +\frac{L}{2\pi}\left\{ 1 + \cos\frac{r}{n}\pi + \ldots\ldots + \cos\frac{2\, n_1 - 1}{n} r\,\pi + \cos\left(\frac{2\; n_1}{n} r\;\pi - \gamma\right) + \ldots\ldots + \right.$$
$$\left. + \cos\left(\frac{2\;n - 1}{n} r\;\pi - \gamma\right)\right\}$$

which may be written

$$C = \frac{n}{r\,\pi}\frac{L}{2\pi}\left\{ -1 + \cos\frac{n_1}{n} 2\; r\;\pi - \cos\left(\frac{n_1}{n} 2\, r\,\pi - \gamma\right) + \cos\;(2\; r\;\pi - \gamma)\right\} =$$
$$= \frac{n}{r\,\pi}\frac{L}{\pi}\left\{ -\sin^2 (r\,\pi - \tfrac{1}{2}\gamma) - \sin\tfrac{1}{2}\gamma \sin\left(\frac{n_1}{n} 2\, r\,\pi - {}^1/_2\,\gamma\right)\right\}$$

$$S = \frac{n}{r\,\pi}\frac{L}{2\pi}\left\{ \sin\frac{n_1}{n} 2\; r\;\pi - \sin\left(\frac{n_1}{n} 2\, r\,\pi - \gamma\right) + \sin\,(2\, r\,\pi - \gamma)\right\} =$$
$$= \frac{n}{r\,\pi}\frac{L}{\pi}\left\{ \sin\,(r\;\pi - \tfrac{1}{2}\gamma) \cos\,(r\;\pi - {}^1/_2\,\gamma) + \sin\tfrac{1}{2}\gamma \cos\left(\frac{n_1}{n} 2\; r\;\pi - {}^1/_2\;\gamma\right)\right\}$$

$$I = \frac{n^2}{r^2\,\pi^2}\frac{L^2}{\pi^2}\left\{ \sin^2 (r\,\pi - {}^1/_2\,\gamma) + \sin^2\tfrac{1}{2}\gamma + 2\,\sin\tfrac{1}{2}\gamma \sin\,(r\,\pi - \tfrac{1}{2}\gamma) \cos\left(\tfrac{1}{2} - \frac{n_1}{n}\right) 2\; r\;\pi\right\}.$$

For a given value of $\frac{n_1}{n}$ and γ this more irregular distribution of intensity in the region of the first diffraction image may be computed. But it is not necessary. From the denominator r^2 we see that the whole energy is confined to a limited area around the diffraction point of the order of the magnitude of a regular diffraction image, and at a greater distance it becomes imperceptible.

Now by the inclination of the two parts of the grating the diffraction images produced by the second part would be situated on a line inclined 0.0023 to the line through the diffraction images of the first part, but at nearly the same distances, because the mean period of the two parts differ only $\frac{1}{1800}$. The vertical distance at the place of the first diffraction image is 0.0023 αf;

since the size of this image is given by $\frac{af}{n} = 0.008\ af$ the two images would overlap and the real image is formed by interference. We get the values of the amplitude integrals, going along a horizontal line:

$$C_1 = \frac{n}{r\pi}\frac{L}{2\pi}\left(-1 + \cos\frac{n_1}{n} 2\, r\, \pi\right); \qquad C_2 = \frac{n}{r\pi}\frac{L}{2\pi}\left\{-\cos\left(\frac{n_1}{n} 2\, r\, \pi - \gamma\right) + \cos(2\, r\, \pi - \gamma)\right\}$$

(similary for S_1 and S_2). Going vertically over a distance $\beta \times af$, these values should be multiplied by $\frac{(\sin\ \beta\, n\, \pi)}{\beta\, n\, \pi;}$ thus on a line $\beta \times af$ above the middle line of the image of the first part of the grating these values should be multiplied by

$$\frac{\sin \beta\, n\, \pi}{\beta\, n\, \pi} \quad \text{and} \quad \frac{\sin(\beta - 0.0023)\, n\, \pi}{(\beta - 0.0023)\, n\, \pi}$$

and then be added. In the same way S_1 and S_2 are treated, and then I for each point may be computed. Here again we find that the intensity becomes imperceptible for βn rising above some units; the whole phenomenon is confined to the immediate vicinity of the first diffraction point. In expanding this image to a large extrafocal disc the special minute distribution of energy over a region of the order of the size of the image produced by the aperture of the objective $\left(\frac{af}{n}\right)$ becomes irrelevant; only the total intensity over this region matters, and this is equal to the total intensity of the diffraction image in the case of a regular grating.

Thus we find that the chief abnormality of the Lembang grating, viz that it consists of two parts somewhat inclined and displaced to one another, has no influence upon the brightness of the extrafocal images produced by the grating.

3. The influence of the accidental irregularities of a grating has been treated in the B.A.N. 110 Vol.IIIp.209, which article is reprinted here in full, to conjoin the whole investigation of the grating.

We suppose only variations in one dimension x; physically this corresponds to strict parallelism of all the limits between the dark and bright spaces, and to a rectangular aperture. The average breadth of the dark spaces is d, of the bright spaces l; the average value of a period is $L = l + d$; their number is n. The deviations of the real limits from an ideal grating, where the breadth is everywhere exactly d and l, are $e_1\, e_2 \ldots e_{2n}$; then by our definitions $e_1 + e_2 + \ldots e_{2n} = 0$; $e_1 - e_2 + e_3 - e_4 + \ldots - e_{2n} = 0$.

If we put for the amplitude integrals

$\int \cos 2\pi \frac{x\alpha}{\lambda}\, dx = C$ and $\int \sin 2\pi \frac{x\alpha}{\lambda}\, dx = S$, the integrals being taken over the whole of the bright spaces, then the intensity at a distance αf from the central image will be given by $I = C^2 + S^2$. For the first diffraction image $\alpha_1 = \frac{\lambda}{L}$. Putting $2\pi\frac{\alpha_1 x}{\lambda} = 2\pi\frac{x}{L} = \varphi$, we have $C = \frac{L}{2\pi}\int \cos\varphi\, d\varphi$, $S = \frac{L}{2\pi}\int \sin\varphi\, d\varphi$. Putting $\frac{1}{2}\frac{(l-d)}{L} = \frac{\psi}{\pi}$ and $\frac{2\pi e}{L} = \varepsilon$, the limits of the integrals are

$-\psi + \varepsilon_1$ to $\pi + \psi + \varepsilon_2$; $2\pi - \psi + \varepsilon_3$ to $3\pi + \psi + \varepsilon_4$; ... etc., and the integrals become

$$C = \frac{L}{2\pi}\left\{+\sin(\psi - \varepsilon_1) - \sin(\psi + \varepsilon_2) + \sin(\psi - \varepsilon_3) - \sin(\psi + \varepsilon_4) + \ldots\right\}$$

$$= \frac{L}{2\pi}\sin\psi\left\{\cos\varepsilon_1 - \cos\varepsilon_2 + \cos\varepsilon_3 - \cos\varepsilon_4 + \ldots\right\} - \frac{L}{2\pi}\cos\psi\left\{\sin\varepsilon_1 + \sin\varepsilon_2 + \ldots\right\}$$

$$S = \frac{L}{2\pi}\left\{ -\cos(\psi - \varepsilon_1) - \cos(\psi + \varepsilon_2) - \cos(\psi - \varepsilon_3) - \cos(\psi + \varepsilon_4) - \dots \right\}$$

$$= -\frac{L}{2\pi}\cos\psi \left\{ \cos\varepsilon_1 + \cos\varepsilon_2 + \cos\varepsilon_3 + \dots \right\} - \frac{L}{2\pi}\sin\psi \left\{ \sin\varepsilon_1 - \sin\varepsilon_2 + \dots \right\}$$

If the deviations ε are not large, such that ε^3 may be neglected, while the second term of each formula disappears because $\Sigma\,\varepsilon = 0$ and $\Sigma\,(\varepsilon_1 - \varepsilon_2) = 0$, then

$$C = \frac{L}{2\pi}\,\frac{\sin\psi}{2}\left\{ -\varepsilon_1^2 + \varepsilon_2^2 - \varepsilon_3^2 + \dots \right\}$$

$$S = -\frac{L}{2\pi}\cos\psi \left\{ 2n - \tfrac{1}{2}(\varepsilon_1^1 + \varepsilon_2^2 + \dots) \right\}$$

In the intensity $I = C^2 + S^2$ the first term becomes of the 4[th] order and unless $\cos\psi$ is very small, i. e. the wires are very thin compared with the bright spaces, may be omitted. Putting $\Sigma\,\varepsilon^2 = 2\,n\,\mu^2$ (thus μ being the mean value of the deviations) we get

$$S = -\frac{nL}{\pi}\cos\psi\,(1 - \tfrac{1}{2}\mu^2) \text{ and}$$

$$I = \frac{n^2\,L^2}{\pi^2}\cos^2\psi\,(1 - \mu^2).$$

In this deduction it is not supposed that the deviations e and ε behave as accidental errors; they may show any systematic course up and down.

The intensity at the central image is found in the same way ($\cos\varphi$ being 1) $I_c = n^2\,l^2$, and the intensity without grating $I_0 = n^2 L^2$. Thus $\frac{I}{I_0} = \frac{1}{\pi^2}\cdot\cos^2\psi\,(1 - \mu^2)$ and $\frac{I_c}{I_0} = \frac{l^2}{L^2} = \left(\frac{1}{2} + \frac{\psi}{\pi}\right)^2$

For the second and the higher diffraction images ($\alpha = 2\frac{\lambda}{L}$, $3\frac{\lambda}{L}$, etc.) the limits of the integrals become twice, 3 times, etc. the former values; we find

$\frac{I_2}{I_0} = \frac{1}{\pi^2}\cdot\sin^2 2\,\psi\,(1 - 4\,\mu^2)$; $\frac{I_3}{I_0} = \frac{1}{\pi^2}\cdot\cos^2 3\,\psi\,(1 - 9\mu^2)$.

For equal breadth of the bright and the dark spaces I_2 vanishes. The brightness of all these diffraction images is diminished in consequence of the irregularities in the breadth of the dark and the bright spaces.

Since, however, the light that is lacking in these images, must be dispersed somewhere else in the focal plane beside them, it may be that part of it is gathered up in the extrafocal images. The brightness in the centre of an extrafocal image is determined by the sum total of the light falling in the focal image within a circle of the size of the extrafocal image around this centre. Thus the distribution of the light in the focal plane (coordinate α) must be determined.

We consider a point at the outer (or the inner) side of the first diffraction image at distance $\frac{r}{n}\alpha$; then the the phase angle $2\pi\frac{\alpha x}{\lambda} = \left(1 + \frac{r}{n}\right)\varphi$, and the limits of the integrals over the bright spaces are $-\psi + \varepsilon_1$, $\pi\left(1 + \frac{r}{n}\right) + \psi + \varepsilon_2$, $2\pi\left(1 + \frac{r}{\pi}\right) - \psi + \varepsilon_3$, etc.

We will take $\psi = 0$ in order to avoid complicated formulae. The integrals now become

$$C_r = -\frac{L}{2\pi}\left\{ \sin\varepsilon_1 + \sin\left(\frac{r}{n}\pi + \varepsilon_2\right) + \sin\left(\frac{2\,r}{n}\pi + \varepsilon_3\right) + \dots \right\}$$

$$S_r = -\frac{L}{2\pi}\left\{ \cos\varepsilon_1 + \cos\left(\frac{r}{n}\pi + \varepsilon_2\right) + \cos\left(\frac{2\,r}{n}\pi + \varepsilon_3\right) + \dots \right\}$$

Since there are $2\,n$ terms, the periodic argument goes r times through the circumference $2\,\pi$.

We suppose the deviations ε small; thus the formulae may be written, neglecting the terms with ε^2:

$$C_r = -\frac{L}{2\pi}\left\{ \varepsilon_1 + \varepsilon_2 \cos\frac{r\pi}{n} + \varepsilon_3 \cos\frac{2r\pi}{n} + \dots \right\}$$

$$S_r = +\frac{L}{2\pi}\left\{ \qquad \varepsilon_2 \sin\frac{r\pi}{n} + \varepsilon_3 \sin\frac{2r\pi}{n} + \dots \right\}$$

Now the $2n$ values $\varepsilon_1 \dots \varepsilon_{2n}$ can always be represented by a FOURIER series

$$\varepsilon_{s+1} = a_0 + a_1 \cos\frac{s\pi}{n} + a_2 \cos\frac{2s\pi}{n} + \dots a_{n-1} \cos\frac{(n-1)s\pi}{n} + b_1 \sin\frac{s\pi}{n} + b_2 \sin\frac{2s\pi}{n} + \dots b_{n-1} \sin\frac{(n-1)s\pi}{n}$$

Introducing these values in the formulae for C_r and S_r the coefficient of each a or b becomes a series of trigonometric products whose sum total is zero, except for a_r and b_r where it is a sum of $2n$ squares of sinus or cosines, evenly distributed over the circumference.

The value of this sum is n; thus we have $C_r = -\frac{L}{2\pi} n a_r$ and $S_r = +\frac{L}{2\pi} n\, b_r$ and the intensity at this point $I_r = \frac{L^2}{4\pi^2} n^2 (a_r^2 + b_r^2)$.

Thus proceeding from the place of the first diffraction image to that of the second one with n equal steps, we find intensities just corresponding to the squares of the amplitudes of the consecutive FOURIER terms up to the nth, the mechanism of diffraction performing here the harmonic analysis of the ε values. The same values we find at the other side, from the first diffraction image towards the central one. The diffraction figure of the central image and each of the others, caused by the whole aperture, corresponds in size to one of these steps. Thus it is easily seen that between the points taken above the intensities have intermediate values.

If we are able to collect the sum total of these intensities into one image, it will have

the brightness $\Sigma I_r = 2 \times \frac{L^2}{4\pi^2} n^2 (\Sigma a_r^2 + \Sigma b_r^2)$.

Now we have $\quad \Sigma \varepsilon^2 = n (\Sigma a^2 + \Sigma b^2)$;

thus $\quad \Sigma I_r = \frac{L^2}{4\pi^2} 2n \; \Sigma \varepsilon^2 = \frac{n^2}{\pi^2} L^2 \mu^2$.

This is exactly the amount by which I was diminished in consequence of the irregularities; thus in such an image extending from the centre to the place of the second diffraction image the whole theoretical intensity would be collected, just as if there were no irregularities.

But we are not able to collect all this light into a single image. In applying this method the extrafocal central and first diffraction images are just separated; thus the irregularly dispersed light is only gathered up at most as far as $\frac{1}{2}\, n$; the higher coefficients $a_{1/2n}$ to a_n and $b_{1/2n}$ to b_n are even contributing to the extrafocal central image.

We can make an estimate of the values of these two groups of terms by computing the means and the differences of every two consecutive values of ε:

$$\varepsilon_s = \Sigma (a_r \cos r s \varphi + b_r \sin r s \varphi);$$

$$\varepsilon_{s+1} = \Sigma (a_r \cos r (s+1) \varphi + b_r \sin r (s+1) \varphi)$$

$$\tfrac{1}{2} (\varepsilon_{s+1} + \varepsilon_s) = \Sigma a_r \cos (s + \tfrac{1}{2}) r \varphi \cos \tfrac{1}{2} r \varphi + \Sigma b_r \sin (s + \tfrac{1}{2}) r \varphi \cos \tfrac{1}{2} r \varphi$$

$$\tfrac{1}{2} (\varepsilon_{s+1} - \varepsilon_s) = \Sigma a_r \sin (s + \tfrac{1}{2}) r \varphi \sin \tfrac{1}{2} r \varphi + \Sigma b_r \cos (s + \tfrac{1}{2}) r \varphi \sin \tfrac{1}{2} r \varphi.$$

$$\Sigma \left(\frac{\varepsilon_{s+1} + \varepsilon_s}{2} \right)^2 = n \, \Sigma \, (a_r^2 + b_r^2) \cos^2 \tfrac{1}{2} r \varphi$$

$$\Sigma \left(\frac{\varepsilon_{s+1} - \varepsilon_s}{2} \right)^2 = n \, \Sigma \, (a_r^2 + b_r^2) \sin^2 \tfrac{1}{2} r \varphi.$$

Here the square amplitudes $(a^2 + b^2)$ are multiplied with factors, which for $r = 1$ to n decrease from 1 to 0 for the means, increase from 0 to 1 for the half differences. Separating them into the groups $r = 1$ to $\frac{1}{2}n$, and $\frac{1}{2}n$ to n, the coefficients in these groups are for the means 1 to $\frac{1}{2}$, (average 0.82) and $\frac{1}{2}$ to 0 (av. 0.18), for the half differences 0 to $\frac{1}{2}$ (average 0.18) and $\frac{1}{2}$ to 1 (average 0.82). Thus putting

$$\Sigma\left(\frac{\varepsilon_s + {}_1 + \varepsilon_s}{2}\right)^2 = 2n\,(0.82\,\mu_1^2 + 0.18\,\mu_2^2)$$

$$\Sigma\left(\frac{\varepsilon_s + {}_1 + \varepsilon_s}{2}\right)^2 = 2n\,(0.18\,\mu_1^2 + 0.82\,\mu_2^2)$$

$2\mu_1^2$ may be taken for $\sum_{0}^{1/2n} (a^2 + b^2)$, and $2\mu_2^2$ for $\sum_{1/2n}^{n} (a^2 + b^2)$.

Then the light collected in the first diffraction image wil be

$$I = \frac{n^2 L^2}{\pi^2}\{\cos^2\psi\,(1 - \mu^2) + \mu_1^2\}$$

and the central image $I_c = n^2 l^2 + \frac{n^2 L^2}{\pi^2}\mu_2^2$; thus

$$\frac{I}{I_o} = \frac{1}{\pi^2}\{\cos^2\psi\,(1 - \mu^2) + \mu_1^2\} \qquad \frac{I_c}{I_o} = \frac{l^2}{L^2} + \frac{\mu_2^2}{\pi^2}.$$

Since as a rule the large values of ε will appear in the longer waves, we may expect that μ_1^2 will not differ very much from μ^2, and μ_2^2 will be small. But only exact measures of the grating can decide whether the deviation from the simple theory is relevant.

4. From the numerical data given in div. 1 pag. B 7, we find $\frac{\psi}{\pi} = 0.010$ $\psi = 1\overset{\circ}{.}8$; and $\log\frac{\cos^2\psi}{\pi^2} = 9.0053$, corresponding to 2.487 magnitudes for the difference $\frac{I_1}{I_o}$; $\log\left(\frac{l}{L}\right)^2 = 9.4152$, corresponding to **1.462** magnitudes for the difference $\frac{I_c}{I_o}$. The difference in brightness between the central and the first diffraction image therefore is 1.025 magnitudes.

From the values of the accidental irregularities computed as in 1) we find $\frac{1}{2}n\,\Sigma\, e^2 = 895$ (unit 1 micron), from which is derived $\mu^2 = 0.000895 \times \left(\frac{2\pi}{1.20}\right)^2 = 0.0246$. Computing now for the concecutive values of the deviations half the sum and half the difference, we find from them

$$\tfrac{1}{2}n\,\Sigma\,\tfrac{1}{4}\,(\varepsilon_s + \varepsilon_{s+1})^2 = 735 \text{ and } \tfrac{1}{2}n\,\Sigma\,\tfrac{1}{4}\,(\varepsilon_{s+1} - \varepsilon_s)^2 = 146.$$

The fluctuations are chiefly of large period, wich also becomes manifest by inspection of a graphical representation of the deviations; the differences between consecutive values are small. This means that the far larger part of the light is dispersed at small distance from the diffraction image and takes part in the formation of the extrafocal diffraction image. Computing values of μ_1^2 and μ_2^2 in the way as indicated in 3) even brings out a negative value of μ_2^2, indicating that the mean factor assumed 0.82 is too small in this case. Thus it is not possible to find a correction for the irregularities; the values found indicate that the brightness of the extrafocal image is exactly the same as it would have been in the case of a grating without accidental irregularities.

The difference in brightness between the central and the first diffraction image therefore will be adopted **1.025** *magnitudes.*

Sources of error. In photographic photometry it is important to make the observations as homogeneous, and, to this end, as differential, as possible. A serious error results if the variable and its comparison stars is not taken always in the same relation to the optical axis of the camera. To secure this an arrangement was made having a milled screw by which to shift the plate-holder in the R.A. direction, so that, by making several exposures on the one plate, (a procedure mostly employed for short-period variables) the star chosen as guiding star could be used in the same position of the wire cross.

Another source of errors, which cannot be eleminated, is the liability of the plate to diversities in (photographic) sensitivity over its surface. This was examined by making a series of exactly equal exposures of a bright star, intra-focally, and with use of the coarse grating; shifting the plate two mm. after each exposure. The scale readings in the HARTMANN Micro-photometer ought to be similar for all the central images and for those of the first order, but sometimes differences occur which cannot be traced to errors of measurement, and are therefore only attributable to irregularities in photographic sensitivity over the area of the plate. A larger number of exposures is the only counteractive

The photographic plate. The inevitable deterioration (more or less), of photographic plates available in parts distant from the great factories of Europe and America, is a great han-dicap in their use for scientific work in general, but especially so for astronomy. Gevaert's „Sensima" plates, first used, were quite good in the beginning, but were later found to be unreliable on account of their different sensitivities of different emulsion numbers, thanks to which, no less than forty-three of these plates, exposed one fine night, were discovered useless after development. „Agfa" plates were then tried, but they are too slow. Than an arrangement has been completed with the Eastman Kodak Co., who now supply us monhtly regularly with fresh Eastman No. 40 plates, which are very clear, and, (which is specially important), every lot of plates presents an equal sensitivity.

In 1927 the use of „Hauff Ultra-Rapid" plates was limited to two stars, as the use of Eastman plates had at the time exceeded their supply.

In 1925 and 1926 the plates were developed during four minutes in metol-hydroquinon, but in 1927 only Rodinal $^1/_{20}$ was used.

Time. Throughout, the observations and reductions have been on „G. M. T." as was in use generally, in astronomy, before 1925 — that is the day beginning at noon.

Measuring and reduction of the plates. A full description of the HARTMANN micropho-tometer, used for the measurements, and the method of reduction, will be given in Vol. II part 3 of these Annals; this instrument can be used equally well as an ordinary HARTMANN wedge-microphotometer or as a thermopile-photometer.

All plates, which were taken intra-focally, have been measured with the ordinary HARTMANN wedge-microphotometer.

The scale readings of the micrometer wedge were corrected by an amount which would represent the measurements as if made with an ideal wedge. These corrections were obtained in a way slightly differing from that of PANNEKOEK's *) and will be given in the next part of these Annals. After applying these corrections the brightness of the variable was logarithmically interpolated between two comparison stars of which one was brighter and the other fainter than the variable. It is of prime importance that the difference between the comparison stars is not in excess of one magnitude. This consideration should, therefore, whenever possible, be kept uppermost when selecting

*) PANNEKOEK B.A.N. No. 44 Vol. II p. 19

them. But we were occasionally compelled to use comparison stars having a much greater difference. It has been found greatly advantageous to put the two microscopes of the photometer (one for the wedge and one for the plate) out of focus, so that the grain of the photographic plate and of the photographic wedge cannot be seen.*) This out-of-focus setting of the microscopes must be so as to render both their images equally smooth in appearance. Care has also been taken that no difference of colour appears in the microphotometer between the star image and the surrounding field of the wedge. The star image generally appears redder than the surrounding field of the wedge and, to eliminate this a blue glass was placed in the path of light rays which pass through the photographic plate. Further, these measurements have shown, that it is almost impossible to observe exact equalness of darkening of the star image and the wedge. Coming from darker parts of the wedge one measures as a rule the blackness of the star too small, coming from lighter parts it is measured too large; but this phenomena is often reversed, depending if we are working with the dark or light part of the wedge. Therefore a wholly symmetrical arrangement of the measures would enable higher accuracy to be obtained. Four settings were made upon each star image (throughout all the plates) with the wedge of the photometer, — two from the darker to lighter parts (of the wedge) and two in the opposite direction.

To save time, and not to fatigue the eyes by observing on the microscope, the readings of the wedge were recorded by a Malay writer.

Abbreviations of names of observers at the telescope or the measuring instrument will be as follows: Vo. for J. VOÛTE; tB. for P. ten BRÜGGENCATE and Wx. for A. WITLOX.

November 1927. *J. Voûte.*

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*) This proceeding has, to my knowledge, been followed by Pannekoek and by Jordan (Publ. Allegheny Obs. VII p. 9.)