

DESCRIPTION OF PLATE 7

This plate illustrates the change of position of the single bright components of the hydrogen lines ($H\gamma$ and $H\delta$) from coincidence with the violet component (A and B) to coincidence with the red component (C and D).

The upper spectra A and C of each pair were taken on dates when both bright components were visible, namely, 1933 January 9 and 1932 July 20 respectively.

The lower of each pair B and D were taken on dates (1) *just after* the red component had disappeared, namely, 1934 April 9, and (2) *just before* the violet component appeared again, namely, 1934 November 2.

Each pair of spectra are so adjusted that all the lines other than hydrogen are made to coincide.

All the spectra indicate also the wide absorption band, midway between $H\gamma$ and $H\delta$, which is generally such a conspicuous feature in many Be-type stars, as was pointed out by me in a previous paper (*M.N.*, 89, 127, 1928).

The reproductions are made from direct enlargements from the original negatives magnified 5.5 times.

THEORETICAL COLOUR TEMPERATURES.

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At the meeting of the Society in 1934 April Mr. Greaves communicated the results of the Greenwich determinations of colour temperatures of stars, after the last comparisons with new absolute standards.* The startling result of colour temperatures of $18,000^\circ$ to $20,000^\circ$ for A stars, though in accordance with other investigations, was at variance with the general belief that the radiation of the stars did not deviate much from black-body radiation corresponding to the effective temperature. Dr. Woolley shortly afterwards cleared up this contradiction and showed that we have no reason to expect the stars to radiate like black bodies.†

If we denote by k the mass absorption coefficient for a frequency ν ; by \bar{k} the mean absorption coefficient for the total radiation (Rosseland mean), which determines the temperature increase with depth; by $-k\rho dh = d\tau$ the optical depth for ν radiation; by $E(\nu)$ and E the black-body radiation for frequency ν and integrated, then \bar{E} increases with depth as $E_0(1 + \frac{3}{2}\tau\bar{k}/k)$, and $E(\nu)$, in the first approximation, increases as $E_0(\nu)(1 + \frac{3}{2}c\tau\bar{k}/k)$, where the coefficient c denotes ‡

$$c = \frac{d \log E(\nu)/dT}{d \log E/dT} = \frac{1}{4} \frac{c_2}{\lambda T} (1 - e^{-c_2/\lambda T})^{-1}.$$

* *M.N.*, 94, 488.

† *M.N.*, 94, 713.

‡ Cf. Milne, *Handb. d. Astrophysik*, III, 1, p. 167.

Using the Eddington integrals of the radiation $\int I d\omega = 4\pi\mathfrak{J}$; $\int I \cos \vartheta d\omega = 4\pi H$ ($\vartheta =$ angle of the ray with the outward vertical), the equations for radiation of each ν of the continuous spectrum are

$$\frac{d\mathfrak{J}}{d\tau} = 3H, \quad \frac{dH}{d\tau} = \mathfrak{J} - E(\nu),$$

the solution of which (with surface condition $\mathfrak{J} = 2H$) gives for the surface radiation:

$$H_0(\nu) = \frac{E_0(\nu)}{2 + \sqrt{3}} \left(1 + \frac{\sqrt{3}}{2} c \frac{\bar{k}}{k} \right). \quad (1)$$

Hence the variation of $H_0(\nu)$, the background intensity in the integrated spectrum, depends on the variation of the ratio \bar{k}/k with wave-length.

The results for the continuous absorption coefficients k , derived at the Amsterdam Institute as part of a theoretical investigation of line intensities, may be used to derive the intensity distribution over the continuous spectrum and the corresponding colour temperature. Whereas for the details of the computations we must refer to a future publication of the whole investigation, we give here only the necessary foundations and chief results.

We assume the continuous absorption to be due for the lesser part to free-free transitions of electrons with ions, and for the main part to ionization; the formulæ of Kramers and Gaunt are used. In the case of hydrogen we write for the latter part:

$$k(\nu) = CT^{-5/2}\nu^{-3}SP; \quad S = x \sum_n \frac{2n}{(n^2 - \frac{1}{4})^2} e^{h\nu_n/kT}; \quad \nu_n = \nu_1/n^2,$$

where

$$C = \frac{8\pi^2 \sqrt{2\pi}}{3\sqrt{3}} \frac{\epsilon^{10}}{ch^3 m^{1/2} k^{5/2} m_H},$$

P is the electron pressure, x the degree of ionization, ν_1 is the lowest, ν_n some higher level or band edge, and the summation is extended over all n , up to ∞ , for which the absorption bands cover this ν , hence $\nu_n < \nu$.* With increasing ν the absorption coefficient $k(\nu)$ continually decreases as ν^{-3} , but then at each band edge jumps up to a higher value, because a term with the next lower value of n is added.

For the hotter stars we have only to consider the hydrogen absorption. Since the number of hydrogen atoms exceeds the metal atoms nearly a thousand times, the contribution of the latter to the absorption is entirely negligible. When, however, the temperature falls below $10,000^\circ$ the ionization of hydrogen and with it the absorption coefficient rapidly decreases. Then the strongly ionized metal atoms replace hydrogen as the chief source of absorption, which is then 100 to 1000 times smaller than in the hotter atmospheres. With decreasing temperature the metals with highest ionization potential fall out one after another; but by the T factor the absorption

* That the symbol k is used for Boltzmann's constant as well as for the absorption coefficient will cause no confusion.

coefficient gradually increases. If the metal atoms in absorbing radiation when ionizing behaved just like the hydrogen atoms, we should see conspicuous band edges (of *Mg*, *Na*, *K*) in the continuous spectra of class G and K stars. Since such edges are not observed, we have to assume that some smoothing process is at work with these atoms. Hence a smoothed absorption coefficient has been computed in the manner explained in my former paper * by replacing the large number of irregularly situated band edges by a quasi-infinite number, each of which has $d\nu$ instead of the finite $\Delta\nu$ in Kramers' formula. Then the sum becomes an integral.

$$\sum_n \frac{p}{n^2} e^{-h(\nu_1 - \nu_n)/kT} \Delta\nu = \int_0^{\nu} e^{-h(\nu_1 - \nu_n)/kT} d\nu_n = \frac{kT}{h} e^{-h\nu_1/kT} (e^{h\nu/kT} - 1),$$

from which, after eliminating the number of neutral atoms by means of the ionization formula, we get

$$k(\nu) = C' T^{-3/2} \nu^{-3} x P (e^{h\nu/kT} - 1),$$

where

$$C' = \frac{4\sqrt{2\pi}}{3\sqrt{3}} \frac{\epsilon^6}{ch(mk)^{3/2} m_H}.$$

For the mean ionization x the ionization of each element in the Russell mixture had been computed separately.

For temperatures below 6000° this value increases with higher values of ν , in consequence of the increasing number of lower, hence strongly occupied levels in these atoms. There is a domain of transition temperatures (between 10,000° and 6000°, depending on the pressure) where the absorption is a combined effect of hydrogen and metal atoms.

To this ionization part of the absorption the free-free transition part has been added, corrected for Gaunt coefficients; though these coefficients are rather uncertain this has not much influence upon the results. Then the Rosseland mean has been computed; corrections for the absorption by Fraunhofer lines were applied to the mean coefficient. Now by means of formula (1) the intensity of the continuous spectrum was computed for wave-lengths 4000 Å., 5000 Å., 6000 Å. Colour temperatures are derived by comparing intensity ratios within the visible spectrum, situated between the 2nd and the 3rd hydrogen edge. We will derive theoretical colour temperatures from the computed intensities, treating them by the same procedure as is followed with observed spectrum intensities.

In the table the highest lines give $\log \bar{k}/k$ for these three wave-lengths as functions of the effective temperatures (for which the Russell values $5040/T = 0.2, 0.3, \dots$ were taken). The next lines show the logarithm of the factors $(1 + \frac{1}{2}\sqrt{3} c\bar{k}/k)$. They must be multiplied by $E_0(\nu)$, the black-body radiation corresponding to the surface temperature T_0 , for

* *M.N.*, 91, 163. The derivation given there contains an error; by the omission of the weight p in the Boltzmann sum it was not seen that the denominator factor n^2 , which means the same weight, was cancelled; hence the formula given there deviates from the present one.

which $1/1.23$ times the effective temperature T_1 was taken. In colour-temperature determinations the relative gradients are derived from $\log(e^{c_2/\lambda T} - 1)^{-1}$; hence the relative intensities $E_0(\nu)/E_1(\nu)$ are derived by this function, and by combining them with the above factors we get the relative intensity of the star spectrum to the black-body spectrum of the effective temperature. Since $1/\lambda$ for the three wave-lengths is $2.5, 2, 1\frac{2}{3}$, we find the gradient $\phi = (c_2/T)(1 - e^{-c_2/\lambda T})^{-1}$, relative to the black-body gradient, by multiplying the differences of two consecutive lines by 2×2.30 and 3×2.30 . So the relative gradients $\Delta\phi$ are found. Adding them to the gradients of the black-body radiation $\phi(T_1)$ we find the apparent gradients ϕ , as given in the last lines of the table.

$5040^\circ/T$	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.5
$\log \frac{\bar{k}}{k}$ 4000 A.	-.73	-.36	+.09	+.38	+.48	+.33	+.02	-.19	-.35	-.71	-1.0
$\log \frac{\bar{k}}{k}$ 5000 A.	-.98	-.65	-.20	+.09	+.19	+.19	+.15	+.06	-.03	-.26	-.5
$\log \frac{\bar{k}}{k}$ 6000 A.	-1.22	-.89	-.44	-.15	-.05	-.01	+.15	+.16	+.12	-.02	-.3
$\log \left(1 + \frac{\sqrt{3} \bar{k}}{2 c_2 \bar{k}} \right)$	+.032	.090	.256	.463	.583	.521	.360	.278	.227	.133	.0
	.016	.041	.129	.256	.339	.373	.381	.358	.333	.258	.1
	.009	.022	.070	.146	.199	.237	.340	.371	.374	.338	.2
$\log \frac{E_0}{E_1} \left(1 + \frac{\sqrt{3} \bar{k}}{2 c_2 \bar{k}} \right)$	-.146	-.145	-.041	+.101	+.158	+.022	-.209	-.362	-.483	-.719	-1.0
	-.142	-.159	-.118	-.041	-.010	-.029	-.076	-.155	-.236	-.423	-.6
	-.137	-.156	-.147	-.110	-.098	-.104	-.045	-.059	-.103	-.231	-.4
$\Delta\phi$	+.02	-.06	-.35	-.65	-.77	-.23	+.61	+.95	+1.14	+1.36	+1.6
	+.04	+.02	-.20	-.48	-.61	-.52	+.21	+.66	+.92	+1.33	+1.6
$\phi(T_1)$.79	1.00	1.24	1.48	1.75	2.02	2.29	2.57	2.85	3.42	3.9
	.88	1.08	1.30	1.54	1.79	2.05	2.31	2.58	2.86	3.42	3.9
ϕ app.	.81	.94	.89	.83	.98	1.79	2.90	3.52	3.99	4.78	5.5
	.92	1.10	1.10	1.06	1.18	1.53	2.52	3.24	3.78	4.75	5.5

The results are illustrated in fig. 1, where ϕ for each of the wave-length intervals (denoted as 4500 and 5500) is given as a function of the effective temperature (abscissa c_2/T). The value of ϕ for black-body radiation is given by two curves, and the scales of colour temperature at the left-hand side correspond to these curves.

We see that when the effective temperature decreases from $20,000^\circ$ to $10,000^\circ$ the gradient in this part of the spectrum remains nearly constant, and the colour temperature is in the vicinity of $20,000^\circ$ or $18,000^\circ$. This is just what was found by the Greenwich observers; their high colour temperatures for class A stars could have been foretold by our absorption coefficients. When, then, the effective temperature falls below $10,000^\circ$ the colour temperature falls more rapidly, and below 7000° it becomes considerably lower than the effective temperature. It will be difficult to test this result, because in the spectra of stars of later type the wings of the numerous absorption lines overlap, and nowhere can the real continuous background of the spectrum be observed and measured.

For the high-temperature stars where the continuous background of the spectrum itself can be measured, the computation may be tested by observa-

tion. And here it must be remarked that, though in a first approximation theory is able to explain the Greenwich results, still there are notable discrepancies. At the high temperatures theoretical computation shows a flat wave, a minimum at $T_1 = 16,000^\circ$, a maximum at $10,000^\circ$. The Greenwich measures of relative colour show a continuous though at last slow increase from A5 and A0 through B5 to B0 (*cf.* fig. 10, p. 36, of the Greenwich

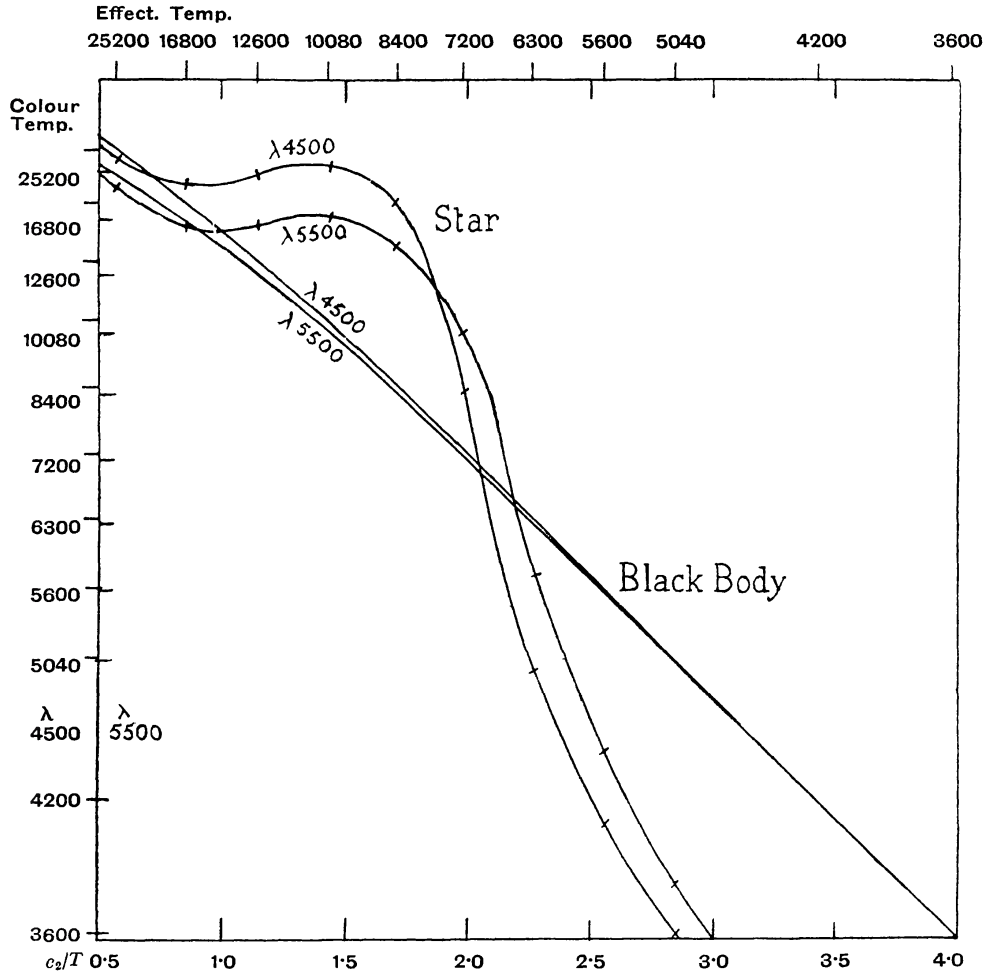


FIG. 1.

publication). At least this holds for the maximum values; the lower colour temperature for a number of B stars may safely be ascribed to space reddening, and cannot give an indication of really lower colour temperature. For the Sun, the difference between effective temperature (5800°) and colour temperature (6300°) is given by theory in the wrong direction; we should wish to have the curve displaced to the right-hand side. It must be left to further research to clear up these discrepancies and to see what changes have to be made in the absorption coefficients.

If we try to trace the origin of the strong systematic deviations of colour from effective temperature, we find it in the manner in which the mean absorption in a hydrogen atmosphere takes place. What is averaged in the Rosseland mean is the transparency. An energy stream driven by a tem-

perature gradient breaks through the different wave-lengths according to their transparency. In fig. 2 the absorption coefficient is given as a function of wave-length (both on a logarithmic scale; one vertical division is $\log 10$), as well as the black-body energy, for three temperatures. The vertical distance of the energy curve above the k line is the logarithm of the energy passing through that wave-length. At $25,200^\circ$ it is just above the Lyman band edge that most of the energy finds a passage. Hence the Rosseland mean corresponds with k in that corner and falls far below the value of k at 4000 \AA . For lower temperature the maximum of the energy curve is displaced towards larger wave-length; gradually the interval between the 1st and the 2nd band edge takes a smaller part, whereas an increasing part

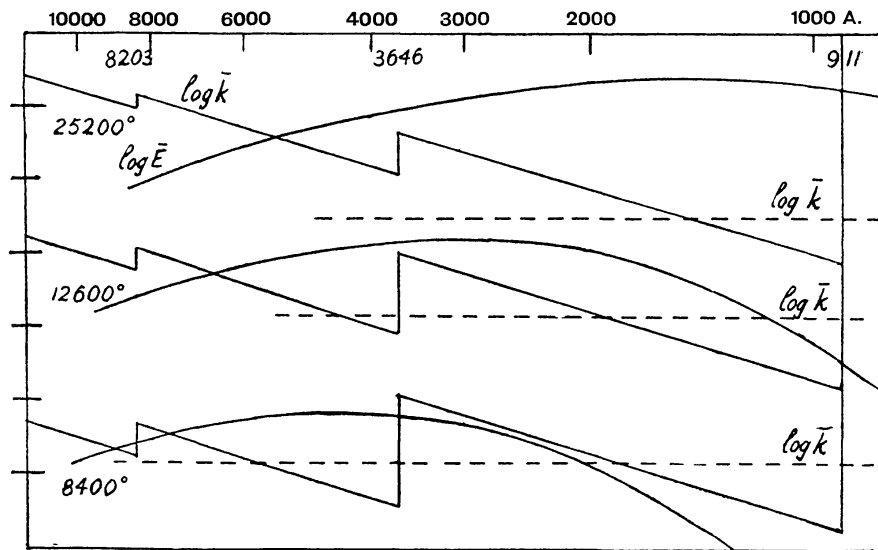


FIG. 2.

of the energy passes through the wave-lengths above the Balmer edge, the region of our measures. Then the mean \bar{k} rises at last above the k (4000), and now the slope of k produces a strong relative gradient. With still lower temperatures the fourth (infra red) interval above the Paschen edge comes into action; but now hydrogen loses its influence upon the absorption coefficient. For high temperatures the mean absorption, which determines the increase of temperature with depth, is small compared with the visual absorption, which determines from what depth the emitted radiation is coming; hence the latter is radiation of little more than surface temperature. While for $10,000^\circ$ the mean absorption is large compared with the visual absorption, we then look into deep layers of strongly increased temperature, producing a great intensity gradient in the visual part of the spectrum.

The limb darkening depends on the same absorption coefficients, since for oblique rays the intensity is $E_0(1 + \frac{3}{2}c\bar{k}/k \cdot \cos \vartheta)$. It is small at high temperatures; it is large and strongly variable with wave-length at $10,000^\circ$. For the three wave-lengths used we find the coefficient of darkening 0.12 , 0.06 , 0.03 at $25,200^\circ$; 0.78 , 0.58 , 0.41 at $10,080^\circ$.

Colour temperature is not, like effective temperature, a quality or physical