Citation:

A. Pannekoek, The luminosity of stars of different types of spectrum, in:
KNAW, Proceedings, 9 I, 1906, Amsterdam, 1906, pp. 134-148

Astronomy. — *"The luminosity of stars of different types of spectrum."* By Dr. A. PANNEKOEK. (Communicated by Prof. H. G. VAN DE SANDE BAKHUYZEN).

The investigation of the spectra of stars which showed that, with a few exceptions, they can be arranged in a regular series, has led to the general opinion that they represent different stages of development gone through by each star successively. VOGEL's classification in three types is considered as a natural system because these types represent the hottest and earliest, the further advanced, and the coolest stage. This, however, does not hold for the subdivisions: the difference in aspect of the lines, the standard in this case, does not correspond to the different stages of development mentioned above. Much more artificial is the classification with letters, which PICKERING has adopted in his Draper Catalogue; it arose from the practical want to classify the thousands of stellar spectra photographed with the objective prism. After we have allowed for the indistinctness of the spectra which, arising from insufficient dispersion and brightness, influenced this classification, the natural affinity between the spectra will appear and then this classification has the advantage over that of VOGEL that the 2nd type is subdivided. The natural groups that can be distinguished are: class A: the great majority of the white stars (Sirius type), VOGEL's I*a*; class B: the smaller number of those stars distinguished by the lines of helium, called Orion stars, VOGEL's I*b*. In the continuous series the latter ought to go before the first type and therefore they are sometimes called type 0. Class F forms the transition to the second type (Procyon); class G is the type of the sun and Capella (the E stars are the indistinct representatives of this class); class K contains the redder stars of the 2d type, which approach to the 3d type, such as Arcturus (PICKERING reckons among them the H and I as indistinct representatives). The 3d type is called in the Draper Catalogue class M.

The continuity of the stellar spectra is still more evident in the classification given by Miss A. MAURY. (Annals Harv. Coll. Obs. Bd. 28). Miss MAURY arranges the larger number of the stellar spectra in 20 consecutive classes, and accepts groups intermediate to these. The classes I—IV are the Orion stars, VI—VIII constitute the first type, IX—XI the transition to the 2d type, XIII—XIV the 2d type itself such as the sun, XV corresponds to the redder Arcturus stars, XVII—XX constitute the third type. If we consider that from class I to III a group of lines is gradually falling out, namely the hydrogen lines of the other series, which are characteristic of the Wolf-Rayet stars or the so-called fifth type stars (VOGEL II*b*), it is obvious that we must place these stars at the head of the series, as it has also been done by Miss CANNON in her investigation of the southern spectra (H. C. O. Ann. Bd. 28) [1]).

Some of these stars show a relative intensity of the metallic lines different from that of the ordinary stellar spectra; VOGEL and SCHEINER have found this before in α Cygni and α Persei (Public. Potsdam Bd. 7, part 2). MAURY found representatives of this group in almost all the classes from III to XIII, and classed them in a parallel series designated by III*c*—XIII*c*, in contradistinction to which the great majority are called *a* stars.

According to the most widely spread opinion a star goes successively through all these progressive stages of development. It commences as an extremely tenuous mass of gas which grows hotter by contraction, and after having reached a maximum temperature decreases in temperature while the contraction goes on. Before the maximum temperature is reached, there is a maximum emission of light; past the maximum temperature the brightness rapidly decreases owing to the joint causes: fall of temperature and decrease in volume. That the first type stars are hotter than the stars of the second type may be taken for certain on the strength of their white colour; whether the maximum temperature occurs here or in the Orion stars is however uncertain.

This development of a tenuous mass of gas into a dense and cold body, of which the temperature first increases and then decreases is in harmony with the laws of physics. In how far, however, the different spectral types correspond to the phases of this evolution is a mere hypothesis, a more or less probable conjecture; for an actual transition of a star from one type into the other has not yet been

1) According to CAMPBELL's results (Astronomy and Astrophysics XIII, p. 448), the characteristic lines of the Wolf-Rayet stars must be distinguished in two groups and according to the relative intensity of the two groups these stars must be arranged in a progressive series. One group consists of the first secondary series and the first line of the principal series of hydrogen: $H\beta'$ 5414, $H\gamma'$ 4542, $H\delta'$ 4201, principal line 4686); it is that group which in MAURY's classes I—III occurs as dark lines and vanishes and which in the classes towards the other side (class O*e*—O*b* CANNON) is together with the ordinary H lines more and more reversed into emission lines. The other group, which as compared with the hydrogen lines becomes gradually stronger from this point, consists of broad bands of unknown origin of which the middle portions according to CANNON's measurements of γ Velorum have the wavelengths 5807, 5692, 5594, 5470, 4654, 4443. The brightest band is 4654; its relative intensity as compared with the H line 4689 gradually increases in the series: 4, 47, 5, 48, 42 (CAMPBELL's star numbers).

observed. The hypothesis may be indirectly tested by investigating the brightness of the stars. To answer to a development as sketched here the brightness of a star must first increase then decrease; the mean apparent brightness of stars, reduced to the same distances from our solar system must vary with the spectral class in such a way that a maximum is reached where the greatest brightness is found while the apparent brightness decreases in the following stages of development.

§ 2. For these investigations we cannot make use of directly measured parallaxes as a general measure for the distance because of the small number that have been determined. Another measure will be found in the proper motions of the stars when we assume that the real linear velocity is the same for different spectral classes. In 1892 W. H. S. MONCK applied this method to the Bradley-stars in the Draper Catalogue [1]). He found that the proper motions of the B stars were the smallest, then followed those of the A stars; much larger are the mean proper motions of the F stars [2]) which also considerably surpasses that of the G, H and K stars and that of the M stars. He thence concluded that these F stars (the 2^d^ type stars which approach to the 1^st^ type) are nearest to us and therefore have a smaller radiating power than the more yellow and redder stars of the 2^d^ type. "Researches on binary stars seem to establish that this is not due to smaller average mass and it would therefore appear, that these stars are of the dullest or least light-giving class — more so not only than the Arcturian stars but than those of the type of Antares or Betelgeux" (p. 878). This result does not agree with the current opinion that the G, K and M stars have successively developed from the F stars by contraction and cooling.

It is, however, confirmed by a newly appeared investigation of EJNAR HERTZSPRUNG: Zur Strahlung der Sterne [3]), where MAURY's classification of the spectra has been followed. He finds for the mean magnitude, reduced to the proper motion 0",01, the values given in the following table where I have added the corresponding proper motions belonging to the magnitude 4.0.

Here also appears that for the magnitude 4,0 the proper motion is largest and hence the brightness smallest for the classes XII and

[1]) Astronomy and Astrophysics XI p. 874.

[2]) He constantly calls them incorrectly "Capellan stars" because in the Dr. Cat. Capella is called F, though this star properly belongs to the sun and the G stars.

[3]) Zeitschrift für wissenschaftliche Photographie Bd. III. S. 429.

Spectrum		Magn. for	P. M. for
Maury	Draper C.	P. M. 0″.01	Magn 4.0
			″
II—IV	B	4.37	0.012
V—VI	B—A	7.25	0.045
VII—VIII	A	8.05	0.065
IX—XI	F	9.06	0.103
XII—XIII	F—G	11.23	0.279
XIII—XIV[1])	G	7.93	0.061
XV	K	9.38	0.119
XV—XVI	K—M	7.77	0.057
XVII—XVIII	M	8.28	0.072

XIII that form the transition from F to G; for the later stages of development the brightness again increases.

§ 3. A better measure than the proper motion for the mean distance of a group of stars is the parallactic motion. This investigation was rendered easy by means of N°. 9 of the "Publications of the astronomical Laboratory at Groningen", where the components τ and v of the proper motion are computed with the further auxiliary quantities for all the Bradley-stars. Let τ and v be the components of the proper motion at right angles with and in the direction of the antapex, λ the spherical distance of the star-apex, then

$$q = \frac{\Sigma\, v \sin \lambda}{\Sigma \sin^2 \lambda}$$

is the parallactic motion for a group of stars, i.e. the velocity of the solar system divided by the mean distance of the group. The mean of the other component $\frac{1}{n}\, \Sigma\, \tau$ is, at a random distribution of the directions, equal to half the mean linear velocity divided by the distance.

The mean magnitudes of the different groups are also different. Because we here especially wish to derive conclusions about the brightness, and as both the magnitude and the proper motion depend on the distance the computation was made after the reduction to

[1]) The Roman figures in italics in MAURY's classification designate the transition to one class higher.

magnitude 4.0; that is to say, we have imagined that every star was replaced by one which in velocity and in brightness was perfectly identical with the real one, but placed at such a distance that its apparent magnitude was 4.0. If the ratio in which we then increase the proper motion is

$$p = 10^{0.2\,(m-4)}$$

we have

$$q_{4.0} = \frac{\Sigma\, pv \sin\lambda}{\Sigma \sin^2\lambda} \quad \text{and} \quad \tau_{4.0} = \frac{\Sigma\, p\tau}{n}.$$

In this computation we have used MAURY's classes as a basis. We have excluded 61 Cygni on account of its extraordinary great parallax, while instead of the whole group of Ursa Major ($\beta\,\gamma\,\delta\,\varepsilon\,\zeta$) we have taken only one star (ε). In the following table are combined the results of the two computations.

Spectrum MAURY	Spectrum Dr Cat.	Typical star	n	mean m	mean τ	q	$\tau_{4.0}$	$q_{4.0}$
					"	"	"	"
I—III	B	ε Orionis	33	3 57	0.007	0 018	0 007	0 013[5]
IV—V	B—A	γ Orionis	48	4 31	0 011	0 035	0 014	0.036
VI—VIII	A	Sirius	93	3.92	0.040	0 054	0.038	0 061
IX—XII	F	Procyon	94	4.14	0.089	0.153	0 095	0.136
XIII—XIV	G	Capella	69	4 08	0 141	0.157	0 160	0 199
XV	K	Arcturus	101	3 90	0 123	0 119	0 120	0 096
XVI—XX	M	Betelgeuze	61	3.85	0 049	0.068	0 050	0 061

In both the series of results the phenomenon found by MONCK and HERTZSPRUNG manifests itself clearly. I have not, however, used these numbers $\tau_{4\,0}$ and $q_{4\,0}$, but have modified them first, because it was not until the computation was completed that I became acquainted with HERTZSPRUNG's remark that the above mentioned c stars show a very special behaviour; their proper motions and parallaxes are so much smaller than those of the a stars of the same classes that they must be considered as quite a separate group of much greater brilliancy and lying at a much larger distance[1]). We have

[1]) In his list of parallaxes HERTZSPRUNG puts the question whether perhaps the bright southern star α Carinae (Canopus) belongs to the c stars; but he finds no indication for this except in its immeasurably small parallax and small proper motion. In her study of the southern spectra Miss CANNON has paid no regard

Class	n	τ_{40}	q_{40}	$2\tau/q$
I	5	″ 0.009	″ 0 022	0 8
II	13	005	009	1.1
III	14	006	015	0.8
IV	18	014	023	1 2
IV	16	016	044	0 7
V	11	009	042	0.4
VI	16	030	068	0 9
VII	30	040	086	0 9
VIII	41	043	055	1 6
IX	25	050	064	1 6
X	16	070	171	0.8
XI	22	103	061	3 3
XII	23	170	282	1 2
XIII	18	297	346	1 7
XIV	21	192	305	1 3
XIV	20	077	025	6 2
XV *A*	26	234	148	3 2
XV *B*	35	105	070	3 0
XV *C*	40	059	087	1 4
XVI	19	049	071	1 4
XVII	19	049	032	3 1
XVIII	16	050	075	1 3
XIX—XX	7	057	078	1 5

to the difference between the *a* and the *c* stars. Yet all the same this question may be answered in the affirmative; on both spectrograms of this star occurring in her work, we see very distinctly the line 4053.8, which in Capella and Sirius is absent and which is a typical line for the *c* stars. Hence follows that *α* Carinae is indeed a *c* star.

therefore repeated the computation after exclusion of the *c* and the *ac* stars.

The table (see p. 139) contains the results for all the classes of MAURY separately; class XV is divided into three subdivisions: XV *A* are those whose spectra agree with that of α Boötis, XV *C* are those which agree with the redder α Cassiopeiae, while XV *B* embraces all those that cannot with certainty be classed among one of the other two groups.

The values for $\tau_{4.0}$ and $q_{4.0}$ differ very little from those of the preceding table. If we take the value of the velocity of the solar system = 4.2 earth's distances from the sun, the q's divided by 4.2 yield the mean parallax of stars of different spectral classes for the magnitude 4.0 ($\pi_{0.4}$). Reversely, we derive from the q's the relative brightness of these stellar types, for which we have here taken the number which expresses how many times the brightness exceeds that of magnitude 4.0 when placed at a distance for which $q = 0''.10$, hence with the parallax 0''.024. Finally the last column $2\tau/q$ contains the relation between the mean linear velocities of the group of stars and our solar system.

In the following table we have combined these values in the same way as before.

Spectrum MAURY	Spectrum Dr. Cat.	Typical star	n	$\tau_{4.0}$	$q_{4.0}$	$\pi_{4.0}$	L for $q=0''.10$	$2\tau/q$
				"	"	"		
I—III	B	ε Orionis	32	0.005[5]	0 014	0 0033	51	0 8
IV—V	B—A	γ Orionis	45	0.013	0.036	0.0086	7.7	0.7
VI—VIII	A	Sirius	87	0.040	0.063	0 015	2.5	1.3
IX—XII	F	Procyon	86	0.101	0.141	0.034	0.50	1.4
XIII—XIV	G	Capella	59	0 182	0 224	0.053	0.20	1.6
XV	K	Arcturus	101	0.120	0 096	0 023	1.1	2.5
XVI—XX	M	Betelgeuze	61	0.050	0 061	0.015	2.7	1.6

§ 4. Conclusions from this table. The numbers of the last column are not constant but show a systematic variation. Hence the mean linear velocity is not constant for all kinds of stars but increases as further stages of development in the spectral series are reached. (Whether the decrease for the 3rd type, class M, is real must for the present be left out of consideration). That the linear speed of the Orion stars is small is known and appears moreover from the radial velocities. While CAMPBELL found 19.9 kilometres for the velocity of the solar motion, and 34 kilometres for the mean velocity of all the stars, FROST and ADAMS derived from the radial velocities of 20 Orion stars measured by them; after having applied the correction for the solar motion: 7.0 kilometres as mean value [1]), hence for the actual mean speed in space 14 kilometres, whence follows the ratio 0.7 for $2\tau/q$. Hence the Orion stars are the particularly slow ones and the Arcturian stars (class XV) are those which move with the greatest speed.

§ 5. When we look at the values of $q_{4.0}$ or those of $\pi_{4.0}$ or $L_{0.10}$, derived from them, we find, as we proceed in the series of development from the earliest Orion stars to the Capella or solar type G, that the brightness constantly decreases. That q was larger for the 2[d] type as a whole than for the first (the Orion stars included) has long been known; some time ago KAPTEYN derived from the entire Bradley-Draper material that on an average the 2[d] type stars (F G K) are 2,7 times as near and hence 7 times as faint as the 1[st] type stars (A and B). This result perfectly agrees with the ordinary theory of evolution according to which the 2[d] type arises from the 1[st] type through contraction and cooling.

A look at the subdivisions shows us first of all that the Orion stars greatly surpass the A stars in brightness, and also that among the Orion stars those which represent the earliest stage greatly surpass again in brightness those of the later stages. As compared with the solar type G the Sirius stars are 12 times, the stars which form the transition to the Orion stars 38 times and lastly the ε Orionis type 250 times as bright. This result is in good harmony with the hypothesis that one star goes successively through the different conditions from class I to class XIV; we then must accept that the density becomes less as we come to the lower classes. Whether the temperature of the Orion stars is higher than that of the Sirius stars or lower cannot be derived from this result; even in the latter case it may be that the larger surface more than counterbalances the effect of smaller radiation. This must be decided by photometric measurements of the spectra. As the Wolf-Rayet stars follow next to class I, an investigation of their proper motion, promised by KAPTEYN, will be of special interest.

Past the G stars, the solar type of the series, the brightness again increases. The values obtained here for q confirm in this respect the results of MONCK and HERTZSPRUNG.

1) Publications Yerkes Observatory. Vol. II. p. 105.

Against the evidence of the q's only one objection can be made, namely that these classes K and M might have a proper motion in common with the sun, so that q would not be a good measure for the distance. A priori this objection is improbable but it may be tested by material, which, though otherwise of small value, may for this kind of investigations yield very valuable conclusions on this point, namely the directly measured parallaxes. HERTZSPRUNG gives mean values of the measured parallaxes reduced to magnitude 0,0; by the side of these we have given the values for somewhat different groups derived from our $\pi_{4.0}$:

	Observed $\pi_{0\,0}$		Derived from q $\pi_{0.0}$
II—IV	0″.0255 (6)	I—III	0″.021
IV—VI	0 .106 (5)	IV—V	0 .054
VII—VIII	0 .153 (10)	VI—VIII	0 .094
IX—XI	0 .226[1] (6)	IX—XII	0 .21
XII—XIII	0 .442 (2)		
XIV	0 .567 (5)	XIII—XIV	0 .33
XV	0 .151 (8)	XV	0 .14
XVI	0 .171 (3)	XVI—XX	0 .096
XVII—XVIII	0 .115 (3)		

In general HERTZSPRUNG's numbers are somewhat larger, this can be easily explained by the circumstance that many parallaxes measured in consequence of their large proper motions will probably be above the mean. It appears sufficiently clear from this, at any rate, that also the directly measured parallaxes markedly point at an increase of brightness past class XIV, and that there is not the least ground to assume for the other groups a motion in common with the sun.

It is therefore beyond doubt that the K and M stars have a greater intrinsic brilliancy than the F and G stars. MONCK derives from this fact that they have a greater radiating power, because about the same value for the masses is derived from the double stars.

That the latter cannot be derived from the double stars will appear hereafter. Moreover MONCK's conclusion of the greater radiating power of the K and M stars is unacceptable. In incandescent bodies this radiating power depends on the temperature of the radiating layers and of the atmospheric absorptions. With unimpaired radiance a greater amount of radiation is accompanied with bluer light (because the maximum of radiation is displaced towards the smaller wave-lengths) as both are caused by the higher temperature. The general absorption by an atmosphere is also largest for the smaller wave-lengths, so that when after absorption the percentage of the remaining light is less, the colour of the radiated light will be redder.

Therefore it is beyond doubt that a redder colour corresponds at any rate with a less degree of radiance per unit of surface.

Then only one explanation remains: *the K and M stars (the redder 2nd type stars like Arcturus and the 3rd type) possess on an average a much larger surface and volume than the other 2nd type stars of the classes F and G.* This result is at variance with the usual representation of stellar evolution according to which the redder K and later the M stars are developed from the yellow-white F and G stars by further *contraction* and cooling.

§ 6. A further examination of the constitution of these stars shows us that it is improbable that they should possess a very small density; the low temperature, the strongly absorbing vapours point to a stage of high condensation. These circumstances lead to expect greater (with regard to the F and G stars) rather than less density. From the larger volumes it then follows *that the K and M stars have much larger masses than the F's and G's.* This result is the more remarkable in connection with the conclusion derived above about their greater mean velocity. If the stars of our stellar system form a group in the sense that their velocities within the group depend on their mutual attraction, we may expect that on an average the velocities will be the greater as the masses are smaller. No difficulty from this arises for the Orion stars with small speed, because the same circumstances which allow us to ascribe to them a mass equal to that of the A, F and G stars, enable us likewise to ascribe to them a larger mass. The K stars which have both a greater mass and a greater velocity are characterized by this thesis as belonging to a separate group, which through whatever reason must *originally* have been endowed with a greater velocity. Arcturus with its immeasurably small parallax and large proper motion is therefore through its enormously great linear velocity and extraordinary luminosity an exaggerated type of this entire class, of which it is the brightest representative. Therefore it would be worth while to investigate separately the systematic motions of the K stars which hitherto have been classed without distinction with the F and G stars as 2nd type.

If this result with regard to the greater masses of the K and M stars should not be confirmed, the only remaining possibility is the supposition *that the density of these star is extremely small.* In this case their masses might be equal to that of other stars and they may represent stages of evolution of the same bodies. Where they ought to be placed in the series of evolution remains a riddle. There is a regular continuity in the series F—G—K—M; and according as we suppose the development to take place in one direction or in the other we find in the transition G—K either cooling accompanied with expansion, or heating accompanied with contraction. The puzzling side of this hypothesis can also be expressed in the following way: while in the natural development of the celestial bodies, as we conceive it, the temperature has a maximum but the density continuously increases, the values obtained here would according to this interpretation point at a maximum density in the spectral classes F and G.

In Vol. XI of Astronomy and Astrophysics MAUNDER has drawn attention to several circumstances, which indicate that the spectral type rather marks a difference in constitution than difference in the stage of development. "There seems to me but one way of reconciling all these different circumstances, viz.: to suppose that spectrum type does not primarily or usually denote epoch of stellar life, but rather a fundamental difference of chemical constitution"[1]). One of the most important of these facts is that the various stars of the Pleiades, which widely differ in brightness and, as they are lying at the same distance from the sun, also in actual volume show yet the same spectrum. The result found here confirms his supposition.

One might feel inclined to look for a certain relation between these K and M stars and the *c* stars, which, according to HERTZSPRUNG, have also a much greater luminosity, hence either less density or greater mass than the similar *a* stars; and the more so as these *c* stars reach no further than class XIII. Yet to us this seems improbable; the K stars are numerous, they constitute 20 % of all the stars, while the *c* stars are rare. Moreover the spectra of all the K stars are with regard to the relative intensity of the metallic lines perfectly identical with the *a* stars of preceding classes such as the sun and Capella. Therefore it as yet remains undecided to which other spectra we have to look for other phases in the K star lives and to which spectra for those in the *c* star lives. The *c* stars, except a few, are all situated in or near the Milky Way: this characteristic feature they have in common with the Wolf-Rayet stars and also with the 4th type of SECCHI (Vogel's III*b*), although these spectra have no lines in common which would suggest any relation between them.

§ 7. The constitution found here for the Arcturian stars among the third type stars may perhaps be tested by means of other

[1]) Stars of the first and second types of spectrum. p. 150.

data, namely by those derived from the double stars. The optically double stars cannot however teach us anything about the masses of the stars themselves as will appear from the following consideration (also occurring in "The Stars" by NEWCOMB). Let us suppose that a binary system is n times as near to us, while all its dimensions become n times as small, but that the density and the radiation remain the same. Then the mass will diminish in the proportion of n^3 to 1, the major axis of the orbit a in the proportion of n to 1 and hence the time of revolution remains the same; the luminosity becomes n^2 times as small, therefore the apparent brightness remains the same as well as the apparent dimensions of the orbit, in other words: it will appear to us exactly as it was before. Hence the mass cannot be found independently of the distance. Let a be the angular semi-major axis, M the mass, P the time of revolution, δ the density, λ the radiating power, π the parallax and ϱ the radius of the spherical volume of the star, then we shall have: $\pi^3 M = \frac{a^3}{P^2}$; the mass M is a constant value $\times \varrho^3 \delta$, the apparent brightness H is a constant $\times \pi^2 \varrho^2 \lambda$. Eliminating from this the parallax and the radius, we find

$$H^3 \frac{P^4}{a^6} = c \frac{\lambda^3}{\delta^2}.$$

Thus from the known quantities: elements of orbit and brightness, we derive a relation between the physical quantities: density and radiating power, independently of the mathematical dimensions. This relation has been derived repeatedly. In the paper cited before MAUNDER gives values for the density $\delta = c \left(\frac{\lambda}{H}\right)^{3/2} \frac{a^3}{P^2}$ in the supposition of equal values of λ; he found for the Sirius stars (1st type) 0,0211, for the solar stars (all of the 2nd type) 0,3026, hence 14 times as large on an average; we can also say that when we assume the same density the radiating power of the Sirius stars would be 6 times as large; the exact expression would be that the quotient λ^3/δ^2 is 200 times as large for the Sirius stars as for the solar stars.

In a different form the same calculation has been made by HERTZSPRUNG by means of AITKEN's list of binary system elements [1]). By means of $-2{,}5 \log H = m$ he introduces into his formula the stellar magnitudes; if we put in the logarithmical form

[1]) Lick Observatory Bulletin Nr. 84.

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$$3 \log H + 4 \log P - 6 \log a = const. + 3 \log \lambda - 2 \log \delta$$

$$m - {}^{10}/_{3} \log P + 5 \log a = m_r$$

then we have $m_r = const. - 2{,}5 \log \lambda + {}^{5}/_{3} \log \delta$.

If we arrange the values of m_r after the spectra according to the Draper Catalogue (for the Southern stars taking CANNON; according to the brightest component α Centauri was reckoned to belong to class G), we find as mean values:

Class A	— 2.92	(9 stars — 4.60 to — 1.09)
„ F	— 1.32	(19 „ — 3.61 „ + 0.14)
„ G and E	— 0.49	(11 „ — 1.60 „ + 1.28)

The 3 stars of the type K (with H) give — 4.88 (γ-Leonis), — 1.05 and + 0,87, hence differ so widely that no valuable result is to be derived from them. To the extraordinarily high value for λ^3/δ^2 given by γ Leonis attention has repeatedly been drawn. While for a great number of stars of the other classes the extreme values of m_r differ by 3.5 magnitudes we find that γ Leonis differs by 5 magnitudes from the mean of the two other values, that is to say: its radiating power is a hundred times as large, or its density is a thousand times as small as for these other stars. For the classes A and F we find that λ^3/δ^2 is 640 and 8 times respectively as large as for class G; conclusions about class K as a whole, such as are especially wanted here, cannot be derived from it. It may be that an investigation of binary systems with partially known orbit motion (for which we should require auxiliary hypotheses) would yield more results.

About the mass itself, however, something may be derived from the spectroscopic binary systems. The elements derived from observation $a \sin i$ and P directly yield $M \sin^3 i$; as it is improbable that there should be any relation between the type of spectrum and the angle between the orbit and the line of sight we may accept the mean of $\sin^3 i$ to be equal for all groups. For systems of which only one component is visible, the element derived from observation contains another unknown quantity, viz. the relation β of the mass of the invisible to that of the visible star. If a is the semi major axis of the orbit of the visible star round the common centre of gravity, we have

$$\frac{a^3 \sin^3 i}{P^2} = M \frac{\beta^3}{(1+\beta)^2} \sin^3 i.$$

It is not perfectly certain, of course, that on an average β is the same for all classes of spectrum; if this is not the case the M's may behave somewhat different from the values of $\frac{a^3 \sin^3 i}{P^2}$ computed here.

Unfortunately, of the great number of spectroscopic double stars discovered as yet (in Lick Observatory Bulletin N°. 79 a number of 147 is given) the orbit elements of only very few are known, They give, arranged according to their spectra:

Group II—IV (B) Orion type	
o Persei	0.61
η Orionis	2.51
δ Orionis	0.60
β Lyrae	7.85
α Virginis	0.33
V Puppis	34.2

Group VI—VIII (A) Sirius type	
β Aurigae	0 56
ζ Ursae	(3.41) [1]
Algol	0.72
α Androm.	0.36 [2]
α_2 Gemin.	0.002

Group XII—XIV *a* (F—G) Solar type	
α Aurigae	0.185
χ Draconis	0.120
(W Sagittarii	0.005)
(X Sagittarii	0.001)
ι Pegasi	0.117
η Pegasi	0.234

Group XII—XIV *ac*	
α Ursae min.	0.00001
ζ Geminorum	0.0023
η Aquilae	0.0029
δ Cephei	0.0031

Group XV (K)	
β Herculis	0.061

Of the K stars only one representative occurs here, so neither this material offers anything that could help us to test the results obtained about this stellar type. But all the same, some remarkable conclusions may be derived from this table. It appears here that notwithstanding their small number the Orion stars evidently surpass the others in mass, while the Sirius stars seem also to have a somewhat greater mass than the solar stars. Very striking, however, is the small mass of the *c* stars approaching towards *a*. Hence *the c stars combine a very great luminosity with a very small mass, and consequently their density must be excessively small.* If it should be not merely accidental that the three regularly variable stars of short period, occurring in Maury, all happen to show *c* characteristics and a real connection should exist between this particularity of spectrum and the variability, we may reasonably include into the

[1]) In the case of *ζ* Ursae *a* has been taken equal to the semi major axis of the relative orbit; hence this number is proportionally too large by an unknown number of times.

[2]) Assumed period 100 days, velocity in orbit 32.5 kilometres.

group W and X Sagittarii which also yield small values; as has been remarked, for the southern stars no distinction is made between the *a* and the *c* stars [1]).

We may expect that within a few years our knowledge of the orbits of the spectroscopic double stars will have augmented considerably. Then it will be possible to derive conclusions like those found here from much more abundant material, and also to arrive at some certainty about the mean mass of the K stars. With regard to the latter our results show at any rate that in investigations on grouping of stars and stellar motions it will be necessary not to consider the 2nd type as one whole, but always to consider the F and G stars apart from the redder K stars.

1) In this connection may be mentioned that in 1891 the author thought he detected a variability of α Ursae minoris with a period of a little less than 4 days. The small amplitude and the great influence of biased opinions on estimations of brightness after ARGELANDER's method in cases of short periods of almost a full number of days, made it impossible to obtain certainty in either a positive or a negative sense. CAMPBELL's discovery that it is a spectroscopic binary system with a period of $3^d\,23^h\,14^m$ makes me think that it has not been wholly an illusion.

ERRATA.

In the Proceedings of the Meeting of June, 1905, p. 81:
line 7 from top, read: "cooled by conduction of heat",
„ 16 „ „ for: "*Exh'* Pl. IV" read: "*Exh'* Pl. VI".

In Plate V belonging to Communication N°. 83 from the physical laboratory at Leiden, Proceedings of the Meeting of February 1903, p. 502, the vacuum glass B'_0 has been drawn 18 cm. too long.

(August 21, 1906).