Huygens Institute - Royal Netherlands Academy of Arts and Sciences (KNAW)

Citation:

A. Pannekoek, Calculation of Dates in the Babylonian Tables of Planets, in:
KNAW, Proceedings, 19 I, 1917, Amsterdam, 1917, pp. 684-703

This PDF was made on 24 September 2010, from the 'Digital Library' of the Dutch History of Science Web Center (www.dwc.knaw.nl) > 'Digital Library > Proceedings of the Royal Netherlands Academy of Arts and Sciences (KNAW), http://www.digitallibrary.nl'

If the two points of discontinuity $x = x_b$ and $x = x_c$ are very near each other, they may appear as a single accumulation in the rough frequency-curve.

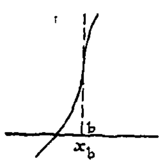

Fig 18a.

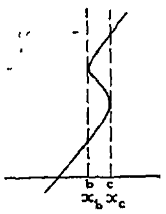

Fig. 18 b.

The former (simplified) analysis then required a very steep slope in the curve $z = f(x)$ at this point, by which the smooth character of the curve is often disturbed (fig. 18*a*).

Considering however this accumulation as a fusion of two discontinuities, we may assume that the function is three-valued in the immediate vicinity of $x = x_b$ (fig. 18*b*) Usually the smooth transition may be obtained by freehand drawing. Care must however be taken that the three-valued zone remains as narrow as possible.

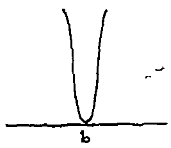

Fig. 19 a.

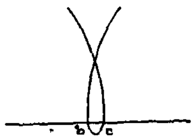

Fig. 19 b

The reaction-curve must now be modified in such a way that in the point b the reaction becomes neither very small and positive, nor zero, but negative.

So instead of the shape of fig. 19*a* the reaction-curve obtains the shape of fig. 19*b*.

Astronomy. — *"Calculation of Dates in the Babylonian Tables of Planets"*. By Dr. A. PANNEKOEK. (Communicated by E. F. VAN DE SANDE BAKHUYZEN).

(Communicated in the meeting of September 30, 1916)

By the researches of F. X. KUGLER S. J. in Valkenburg we have for some years been acquainted with the methods and results of Babylonian astronomy during the period of its highest development. The material for this was provided by a number of more or less damaged fragments of clay tablets covered with cuneiform writing, which are preserved in the British Museum, and which have been very carefully copied by STRASSMAIER. They contain observations and calculations made in advance of the places of the moon and planets from the 5 centuries before the Christian Era, the complete deciphering and explanation of which is given by KUGLER in his work "Die babylonische Mondrechnung" (1902) and in Vol. I of his larger work "Sternkunde und Sterndienst in Babel" (1907).

The tables of the planets and especially those of Jupiter, which are reproduced and examined in the latter work, show how highly developed the methods of the Babylonian astronomy of those centuries were, with a character of their own differing completely from the contemporaneous Greek and from modern astronomy. The astronomers of Babylon knew not only the regular alternation of direct and retrograde motions of the planets in each synodic period, but they also knew that besides this they do not circulate uniformly in the ecliptic. The calculation of this variable velocity, a consequence of the elliptic motion, was for the Greek astronomers a geometric problem, which they solved by excentric circles and goniometric functions (chords). The Babylonians endeavoured to achieve the same result by purely arithmetical methods. The chief reason of this difference was, undoubtedly, that the Babylonian science, being, as a part of the general religious teaching, the duty of the priest, had no occasion to develop new ideas regarding the position of the celestial bodies in space; its object could, therefore, be no other than by certain mathematical methods to transfer as well as possible to future years, the regular return of periods and variations from the previously observed places in the heavens.

For Jupiter, KUGLER found three kinds of tables. They all contained originally (even though only fragments are left), in five columns beside each other, the heliacic rise, the first station, the opposition, the second station and the heliacic setting for all the successive periods of Jupiter. For each of these phenomena is given: the year (according to the Seleucidian era, which begins 312 B. C.), date (month and day), longitude of the planet, (sign of the zodiac, degrees, minutes). They differ in the way in which the figures in the tables are calculated; the first kind is the roughest, the third the most accurate. If the planet described a circular orbit the construction of such a table were simple enough; each opposition would take place 398,884 days after the previous one and at a longitude 33°8'37"5 larger, and the same interval would hold good for the other special phenomena. In consequence of the elliptical motion the intervals are not always of the same length. Now in the tables of the first kind KUGLER found the following arithmetical process made use of to find the longitude of the planet. In the region of the ecliptic from 240° to 85° longitude (30° ♏ to 25° ♊) 36° is taken as the synodic arc; from 85° to 240° an arc of 30° is taken. If a synodic arc falls partly in one and partly in the other region, a value between these two is taken. If, for instance, one of the phenomena (e.g. the opposition) falls in a certain year on the longitude 215°35′, of the next synodic arc 24°25′ will still fall in the region of the 30°, 5°35′ would project beyond and belong to the region of 36° and must therefore be increased by $^1/_5$ of its value i.e. by 1°7′; the whole arc is then 31°7′ and the following year the same phenomenon takes place at longitude 246°42′. By each time adding a synodic arc calculated in this way, the whole series of values is calculated from the original value. The mean value for the synodic arc which is assumed in this arithmetical process is 33°8′45″, only deviating very slightly from the truth, while as the point of greatest velocity a longitude of 342°30′ was found.

The second kind of tables differ from the first in this, that between the two regions of 30° and 36° transition regions are inserted (from 219° to 272° and from 47° to 99° long.) where the synodic arc is taken = 33°45′. Except for this the calculation is made in the same way. The tables of the third kind, on the other hand, exhibit a more refined method. The velocity, the value, therefore, of the synodic arc, and also the time-interval between two successive oppositions or stationary points (after subtraction of a lunar year of 354 days or 12 lunar months), here increases and decreases gradually: in the Babylonian tables these differences appear in two separate columns. Their rise and fall is not, as in the geometric method, sinusoidal but abrupt; uniform rising up to a certain limiting value and then uniform diminution; which means that the deviation of the accepted positions from a uniform motion is represented by a continuous series of parabolic curves open alternately upwards and downwards. The time-interval between two successive oppositions varies between $50^d 7^I 15^{II}$ (sexagesimal subdivision of the days) and $40^d 20^I 45^{II}$, while after each period it increases or diminishes by $1^d 48^I$; the time of revolution along the ecliptic contains therefore $\dfrac{2 \times 9^d 46^I 30^{II}}{1^d 48^I} = 10\,^{31}/_{36}$ periods, that is $= 11\,^{31}/_{36}$ years. The extreme values for the synodic arc are 38°2′ and 28°15′30″, while here also two successive values differ by 1°48′. The mean value for the synodic arc here as in both the other kinds of tables being 33°8′45″, corresponds to a periodic time of Jupiter of $11\,\dfrac{1370}{1591}$ years, of which the former is an approximate value.

In this manner KUGLER has traced out the rules according to which the longitude of the successive oppositions, stationary points and annual rising and setting of Jupiter was calculated by the Babylonian astronomers. He has, however, paid less attention to the rules for calculating the dates belonging to them in the tables.

With reference to the Jupiter tables of the first kind he says on this head: "Die Regel, welche der Verfasser unserer Tafel bei der Berechnung der Daten befolgte, lasst sich nicht klar erkennen; dagegen ist es nicht schwer, das Bildungsgesetz der Längen festzustellen" [1]. In the tables of the third kind the way to find the dates seems to be indicated by the accompanying column of time-intervals; but here also difficulties arise. For if from the synodic period belonging to the Babylonian values 398.890 the mean value of the time-interval $45^d14^I_, = 45^d.233$ is subtracted, $353^d.657$ results, hence not a lunar year of $354^d.367$ but $0^d.71$ less. The excess of the synodic period beyond the each time tacitly added 12 lunar months is only $44^d.52$, this should, therefore, have been added each time to the previous date, instead of $45^d.23$. Moreover the nature of the Babylonian calendar renders it difficult to calculate the dates in this way. The months have as a rule alternately 29 and 30 days, but occasionally a day must be added (in 30 lunar years 11 days), sometimes, therefore, 2 months of 30 days follow one another; and by this irregularity the whole scheme of calculation, which looks so simple, is upset. Moreover, in the fact that the dates are given in days only, without subdivisions, lies an indication that they were found in a different and simpler way. KUGLER points out that 1 Jupiter period is the same as 13 Babylonian months + 15 days all but $^1/_{123}$ day or also $= 13^1/_2$ Babylonian months $+ 0^d.23$, the same 0.23 which occurs in the mean value of the time-interval $45^d.23$, and that, starting from this principle, a continuous Jupiter calendar might be made, without regard to the varying length of the months. If errors of a single day remained, this was not of consequence for the object of the planet tables [2]. In how far these surmises were true will appear from the following.

II.

It would seem a priori to be improbable that the column containing the time-intervals so carefully worked out should not have been made use of at all in the calculation of the dates. We can moreover put this to the test. The difficulty here lies in the uncertainty as to how long each of the months was which lie between two successive dates. On this account we will leave this point for the present undecided. In the following table, a portion of the Jupiter table of the 3rd kind Sp. II 46, the successive dates (2nd station-

[1]) KUGLER, Sternkunde u. Sterndienst in Babel. I. S. 121.

[2]) KUGLER. l. c. S. 166—169.

ary point) are given (the names of the months are indicated by their numbers in Roman figures, the years are those of the Seleucidian era), preceded by the time-interval according to the table and followed by the difference of the dates. This difference is always a year, increased by one or two months, increased or decreased by some number of days.

Time-interval	Year	Date	Interval between the dates
	190	XII 11	
$41^{d}\ 46^{I}\ 5$			1 year $+ 1^{m} + 11^{d}$
	191	XIII 22	
40 43			„ $+2$ -18
	193	II 4	
42 31			„ $+1$ $+12$
	194	III 16	
44 19			„ $+2$ -15
	195	IV 1	
46 7			„ $+1$ $+16$
	196	V 17	
47 55			„ $+2$ -12
	197	VII 5	
49 43			„ $+1$ $+20$
	198	VII 25	
48 43.5			„ $+2$ -12
	199	IX 13	
46 55.5			„ $+1$ $+17$
	200	IX 30	
45 7.5			„ $+2$ -15
	201	XI 15	
43 19.5			„ $+1$ $+13$
	202	XII 28	
41 31.5			„ $+2$ -18
	204	I 10	
40 58			„ $+1$ $+11$
	205	II 21	

We see here immediately that the varying time-intervals from the first column have certainly been used, as the calculated intervals of the dates rise and fall simultaneously with them. The sum of all the time-intervals from the first column is $579^{d}40^{I}$, the interval between the first and last date is 14 years 2 months and 10 days, which, taking into consideration 5 inserted months (191, 194, 197, 199, 202 were leap-years with 13 months) is equal to 175 months and 10 days. The mean of these 13 time-intervals is $\frac{579^{d}.67}{13} = 44^{d}.59$, the mean difference between 2 successive dates is $\frac{175 \times 29.5306 + 10}{13} =$ $= 398.30$, thus $43^{d}.93$ more than a lunar year; here again, therefore, the tabular values for the time-interval appear to be $0^{d}.7$ greater than the excess of the actual synodic intervals above a lunar year.

If, however, we now take the sum of the intervals in the last column, the riddle is solved; we find 13 years + 19 months + 10 days. The two last terms are precisely equal to the sum of the first column, $579^2/_3$ days, if only for each of the 19 months a value of 30 days is assumed. From this it follows

that in the calculation of the Babylonian planet tables normal months of 30 *days were assumed.*

In this way the difficulty was overcome of not knowing beforehand in the compilation of the tables, which months would have 29 days and which had 30. This of course applied only to the surplus of the Jupiter period beyond the lunar year of 354.37 days. If this surplus was 44 days, 1 month + 14 days was always added in the following year, no matter what the name of the month; therefore either the following month was taken with a date 14 greater or the next but one month with a date 16 smaller. To prevent getting more and more behindhand with the true calendar with its share of shorter months in this way, the number of inserted days had to be taken larger in the same proportion as the normal month of 30 days exceeded the mean length of the true calendar months (29,5 days). The actual mean Jupiter period is according to the data of these tables 398.8895 days; that is $44^d.5224$ more than the mean lunar year 354.3671. If this excess is equal to x real lunar periods, $x \times 30$ must be taken in its place in order on the average to remain equal with the calendar. This $x \times 30 = \frac{44.5224 \times 30}{29.5306} = 45.23$ days, the Babylonian astronomer, to be able to apply his method of calculation, had to add to the previous date each following year. And actually the mean interval of time in the tables that lies half way between the extreme values $50^d 7^I 15^{II}$ and $40^d 20^I 45^{II}$ is exactly $45^d 14^I = 45^d.233$.

The regularly varying time-intervals given in the tables have, therefore, actually been used for forming the dates. But how? It is not probable that values rounded off to days were used for the intervals: as a matter of fact this would not give the results of the table. It is more probable that the time-intervals with their fractions were constantly added to the dates already found and from the list so obtained the fractions were finally omitted. We do not know what fractions were assumed at the starting point of the tables; if we suppose that the first date in the table must be called 190 Adaru 11 12^I, we obtain the results that are brought together in the following table. In the 3rd column "date calculated" the date is calculated in 60th parts of a day, starting from the above mentioned value and adding each time the time-interval of the 1st column. As all months are reckoned as 30 days it is only necessary to add the excess of each time-interval over and above 30, while, every time that we come to more than 30, 30 is subtracted.

TABLE I.

From the Jupiter table of the 3rd kind Sp. II 46 (Kugler p. 152).

D. Second stationary point.

Time-interval	Date	Calc. date	Time-interval	Date	Calc. date
	190 XII 11	11d 12I		211 IX 1	1 10.5
41d 46I 5			46 40.5		
	191*XIII 22	22 58.5*a*		212 X 18	17 51
40 43			44 52.5		
	193 II 4	3 41.5		213* XII 3	2 43.5
42 31			43 4.5		
	194*III 16	16 12.5		214 XII (16)	15 48
44 19			41 16.5		
	195 IV 1	0 31.5		216* I (27)	27 4.5
46 7			41 13		
	196 V 17	16 38.5		217 II (8)	8 17.5
47 55			43 1		
	197*VII 5	4 33.5		218* III 21	21 18.5
49 43			44 49		
	198 VII 25	24 16.5*b*		219 IV 5	6 7.5*a*
48 43.5			46 37		
	199*IX 13	13 0		220 V 23	22 44.5
46 55.5			48 25		
	200 IX 30	29 55.5		221* VII 11	11 9.5
45 7.5			50 1.5		
	201 XI 15	15 3		222 VIII 2	1 11*c*
43 19.5			48 13.5		
	202*XII 28	28 22.5		223 IX 21	19 24.5
41 31.5			46 25.5		
	204 I 10	9 54		224* XI 7	-5 50
40 58			44 37.5		
	205*II 21	20 52		225 XI 22	20 27.5
42 46			42 49.5		
	206 III 4	3 38		227† I 5	3 17
44 34			41 1.5		
	207 IV 18	18 12		228 I 16	14 18.5
46 22			41 28		
	208†VI 4	4 34*a*		229* II 27	25 46.5
48 10			43 16		
	209 VI 23	22 44		230 III 10	9 2.5
49 58			45 4		
	210*VIII 13	12 42		231 IV 25	24 6.5
48 28.5					

The dates of the Babylonian table prove, with little exception, to agree with the dates of the third column when rounded off to the nearest whole number of days. In 4 cases the Babylonian date is 1 different (*a* too small, *b* too large), while from the year 222 (*c*) onwards all dates deviate in the same direction by 1—2 days from

*) Leap years with 2nd Adaru (XII). †) Leap-years with 2nd Ululu (VI).

the calculation. Whereas in the four cases clerical errors of the Babylonian copyists are not impossible, the permanent deviation in the last 9 values points to an error of calculation, as, once an error has been made in the addition, this error is carried on in all the subsequent values. Probably (as may easily occur in the Babylonian method of writing figures) the difference 50 1 30 was read as 51 30, whereby all subsequent dates would become $1^d28.^15$ too large.

III.

We will now proceed to the calculation of dates in the Jupiter tables of the first and the second kind. KUGLER has converted the data of table Sp. II 101, the largest of the first kind, into Julian dates; if we take the successive differences between these, they vary so irregularly between 365 + 37 and + 29 days, that to search for the method of calculation seems indeed hopeless. If, however, guided by what we found in the tables of the third kind, we assume that a normal month of 30 days is used in the calculations all the time, a much greater order and regularity immediately appear in the differences. These differences, which in Table II are placed in the 2^{nd} column, show the same character as the differences of longitude: a number of times a greatest value of 48^d alternates with a smallest one of 42^d, in the same intervals as for the longitude synodic arcs of 36° and 30° alternate, while at the points of transition intermediate values appear (See table II p. 692).

It is natural to assume the same method of calculation for these intermediate values as for those in the longitude, viz. as long as the planet stands in the region of 30°, the time-interval 42^d holds, and as long as it stands in the region of 36°, 48^d holds. Then the number of days of the time-interval must always be exactly 12 more than the number of degrees of the synodic arc. The following list shows that this is not always the case.

	Second stationary point					Heliacic setting			
Time-interval:	42	47	43	46	44	45	44	46	43
Syn. arc + 12:	43.3	45.9	43.1	45.1	43.9	44.9	43.2	45.8	42.4

In the first case the cause of the difference is the synodic arc having been calculated wrong; to the starting point 0°25′ ♏ a synodic arc of 30°5′ belongs, so that the time-interval 42 is correct. But amongst the others there are 4 with deviations of 1 day. This, therefore, requires further elucidation, which was only found after

44*

TABLE II.

From the Jupiter table of the 1st kind. Sp. II 101 (Kugler p. 119).

D. Second stationary point.

Year	Date	Time-interval	Longitude	Synod. arc	Period. number	Date calculated
134*	II 22		0°25′ ♎		21	21
		42		30° 0′		
135	III 4		0 25 ♏		3	3
		42		31 17		
136	IV 16		1 42 ♐		15	16
		48		36 0		
137*	VI 4		7 42 ♑		27	4
		48		36 0		
138	VI 22		13 42 ♒		9	22
		48		36 0		
139	VIII 10		19 42 ♓		21	10
		48		36 0		
140*	IX 28		25 42 ♈		3	28
		48		36 0		
141	X 16		1 42 ♊		15	16
		47		33 53		
142*	XII 3		5 35 ♋		28	3
		42		30 0		
143	XII 15		5 35 ♌		10	15
		42		30 0		
145*	I 27		5 35 ♍		22	27
		42		30 0		
146	II 9		5 35 ♎		4	9
		42		30 0		
147	III 21		5 35 ♏		16	21
		43		31 7		
148*	V 4		6 42 ♐		28	4
		48		36 0		
149	V 22		12 42 ♑		10	22
		48		36 0		
150	VII 10		18 42 ♒		22	10
		48		36 0		
151†	VII 28		24 42 ♓		4	28
		48		36 0		
152	IX 16		0 42 ♉		16	16
		48		36 0		
153*	XI 4		6 42 ♊		28	4
		46		33 3		
154	XI 20		9 45 ♋		11	20
		42		30 0		
156*	I 2		9 45 ♌		23	2
		42		30 0		
157	I 14		9 45 ♍		5	14
		42		30 0		
158	II 26		9 45 ♎		17	26
		42		30 0		
159*	IV 8		9 45 ♏		29	8
		44		31 57		
160	IV 22		11 42 ♐		11	22
		48		36 0		
161	VI 10		17 42 ♑		23	10

TABLE II. (Continued) E. Heliacic setting.

Year	Date	Time-interval	Longitude	Synod. arc	Period. number	Date calculated
135	VII 13		14°40′ ♏		29	13
		45		32°56′		
136	VIII 28		17 36 ♐		11	28
		48		36 0		
137†	X 16		23 36 ♑		23	16
		48		36 0		
138	XI 4		29 36 ♒		5	4
		48		36 0		
139	XII 22		5 36 ♈		17	22
		48		36 0		
141	I 10		11 36 ♉		29	10
		48		36 0		
142*	II 28		17 36 ♊		11	28
		44		31 14		
143	III 12		18 50 ♋		24	12
		42		30 0		
144	IV 24		18 50 ♌		6	24
		42		30 0		
145*	VI 6		18 50 ♍		18	6
		42		30 0		
146	VI 18		18 50 ♎		0	18
		42		30 0		
147	VII 30		18 50 ♏		12	30
		46		33 46		
148*	IX 16		22 36 ♐		24	16
		48		36 0		
149	X 4		28 36 ♑		6	4
		48		36 0		
150	XI 22		4 36 ♓		18	22
		48		36 0		
151†	XII 10		10 36 ♈		0	10
		48		36 0		
153*	I 28		16 36 ♉		12	28
		48		36 0		
154	II 16		22 36 ♊		24	16
		43		30 24		
155	III 29		23 0[1] ♋		6	29
		42		30 0		
156*	IV 11		23 0 ♌		18	11
		42		30 0		
157	IV 23		23 0 ♍		0	23

an examination of the tables of the 2nd kind had shown the way.

From the large Jupiter table of the 2nd kind, which consists of 5 fragments fitting together Sp. II 574, Sp. II 42, Sp. II 107, Sp. II 68 and Sp. II 876, we have put the dates together in table III (the missing ones are represented by numbers in brackets · these numbers were inserted by KUGLER; as will be shown his conjecture usually agrees with the result of our calculations), as well as the longitudes and the synodic arcs [2]).

[1]) Kugler's value 23 36 (difference thus 31 0) is copied erroneously from the cuneiform writing, where 23 0 stands

[2]) Of the longitudes also many are missing owing to damage of the cuneiform texts; but KUGLER was able to reconstruct them with complete certainty.

TABLE III.

From the Jupiter table of the 2nd kind Sp. II 574 etc. (KUGLER p. 128).

A. Heliacic rise.

Year	Date	Time-Int.	Longitude	Syn. arc	Reduc. date	L.—R.D.	Period. numb.	Calc. date
180*	VI 13		10° 0′ ♍		1	9 0	3	13
		42		30° 0′				
181	VI 25		10 0 ♎		1	9 0	15	25
		42		30 7½				
182	VIII 7		10 7½ ♏		1	9 7½	27	7
		45		33 45				
183*	IX 22		13 52½ ♐		4	9 52½	9	22
		47		34 47½				
184	X 9		18 40 ♑		9	9 40	21	9
		48		36 0				
185	XI 27		24 40 ♒		15	9 40	3	27
		48		36 0				
186*	XIII 15		0 40 ♈		21	9 40	15	15
		48		36 0				
188	II 3		6 40 ♉		27	9 40	27	3
		47		35 3¾				
189†	III 20		11 3¾ ♊		2	9 3¾	9	20
		46		33 6¼				
190	IV 6		14 10 ♋		6	8 10	22	6
		41		30 0				
191*	V 17		14 10 ♌		5	9 10	4	18 *a*
				30 0				
192	V (30)		14 10 ♍				16	30
				30 0				
193	VII (12)		14 10 ♎				28	12
				30 38¾				
194*	VIII (25)		14 48¾ ♏				10	24
				33 45				
195	IX (10)		18 33¾ ♐				22	10
				35 6¼				
196	X (27)		23 40 ♑				4	27
				36 0				
197*	XII (15)		29 40 ♒				16	15
				36 0				
199*	I 3		5 40 ♈				28	3
		48		36 0				
200	I 21		11 40 ♉		3	8 40	10	21
		46		34 5				
201	III 7		15 45 ♊		7	8 45	22	7
		45		32 35				
202*	IV 22		18 20 ♋		10	8 20	4	22
		42		30 0				
203	V 4		18 20 ♌		10	8 20	16	4
		43		30 0				
204	VI 17		18 20 ♍		11	7 20	29	17
		42		30 0				
205*	VII 29		18 20 ♎		11	7 20	11	29
		43		31 10				
206	VIII 12		19 30 ♏		12	7 30	23	12
		45		33 45				
207	IX 27		23 15 ♐		15	8 15	5	28 *a*
		48		35 25				
208†	X 15		28 40 ♑		21	7 40	17	15
		48		36 0				
209	XII 3		4 40 ♓		27	7 40	29	3
		48		36 0				
210*	XIII 21		10 40 ♈		3	7 40	11	12

Year	Date	Time-Int.	Longitude	Syn. arc	Reduc. date	L. R.D.	Period. numb.	Calc. date
		48		36°0′	9	7 40	23	9
212	II 9		16°40′ ♉					
		46		33 45¼	13	7 25¼	5	25
213*	III 25		20 25¼ ♊					
		44		32 4¾	15	7 30	17	9
214	IV 9		22 30 ♋					
		42		30 0	15	7 30	29	21
215	V 21		22 30 ♌					
		42		30 0	15	7 30	11	3
216*	VII 3		22 30 ♍					
		42		30 0	15	7 30	23	15
217	VII 15		22 30 ♎					
		45		31 41¼	18	6 11¼	6	30
218*	VIII 30		24 11¼ ♏					
		45		33 45	21	6 56¼	18	15
219	IX 15		27 56¼ ♐					
		48		35 43¾	27	6 40	0	3
220	XI 3		3 40 ♒					
		48		36 0	3	6 40	12	21
221*	XII 21		9 40 ♓					
		48		36 0	9	6 40	24	9
223	I 9		15 40 ♈					
		48		35 42½	15	6 22½	6	27
224*	II 27		21 22½ ♉					
		46		33 45	19	6 7½	18	13
225	III 13		25 7½[1] ♊					
		43		31 32½	20	6 40	0	26
226	IV 26		26 40 ♋					
		42		30 0	20	6 40	12	8
227†	VI 8		26 40 ♌					
		42		30 0	20	6 40	24	20
228	VI 20		26 40 ♍					
		43		30 0	21	5 40	7	3
229*	VIII 3		26 40 ♎					
		44		32 12½	23	5 52½	19	17
230	VIII 17		28 52½ ♏					
		46		33 47½	27	5 40	1	3
231	X 3		2 40 ♑					
		48		36 0	3	5 40	13	21
232*	XI 21		8 40 ♒					
		48		36 0	9	5 40	25	9
233	XII 9		14 40 ♓					
		48		36 0	15	5 40	7	7
235*	I 27		20 40 ♈					
		48		35 23¾	21	5 3¾	19	15
236	II 15		26 3¾ ♉					
		45		33 45	24	5 48¾	1	30
237*	III 30		29 48¾ ♊					
		43		31 1¼	25	5 50	13	13
238	IV 13		0 50 ♌					
		42		30 0	25	5 50	25	25
239	V 25		0 50 ♍					
		42		30 0	25	5 50	7	7
240*	VII 7		0 50 ♎					
		43		30 0	26	4 50	20	20
241	VII 20		0 50 ♏					

[1]) In the cuneiform text the reading is 17 30; the difference shows that this is 10′ too high.

Year	Date	Time-Int.	Longitude	Syn. arc	Reduc. date	L.—R.D.	Period. numb.	Calc. date
242	IX (6)		3°33¾′ ♐	32°43¾′			2	5
243*	X (21)		7 40 ♑	34 6¼			14	21
244	XI 9		13 40 ♒	36 0	9	4 40	26	9
245	XII 27	48	19 40 ♓	36 0	15	4 40	8	27
247	I 15	48	25 40 ♈	36 0	21	4 40	20	15
248*	III 2	47	0 45 ♊	35 5	26	4 45	2	2
249	III 16	44	4 30 ♋	33 45	28	6 30	14	18 *a*
250	IV 1	45	5 0 ♌	30 30	1	4 0	26	1
251*	VI 13	42	5 0 ♍	30 0	1	4 0	8	13

B. First stationary point.

Year	Date	Time-Int.	Longitude	Syn. arc	Period. numb.	Calc. date
180*	X (17)		26°13′ ♍		21	17
181	X (29)		26 13 ♎	30° 0′	3	29
182	XII 13		28 24⅜ ♏	32 11⅜	15	13
183*	XIII 29	46	2 10 ♑	33 45⅝	27	29
185	II 17	48	8 10 ♒	36 0	9	17
186*	IV 5	48	14 10 ♓	36 0	21	5
187	IV 23	48	20 10 ♈	36 0	3	23
188	VI (11)		25 35⅝ ♉	35 25⅝	15	10
189†	VIb 26		29 20⅝ ♊	33 45	27	26
190	VIII 9	43	0 25 ♌	31 4⅜	9	9
191*	IX 21	42	0 25 ♍	30 0	21	21
192	X (3)		0 25 ♎	30 0	3	3
193	XI 16		0 25 ♏	30 0	16	16
194*	XIII 1	45	3 5⅝ ♐	32 40⅝	28	1
196	I (17)		7 10 ♑	34 3⅜	10	17
197*	III 5		13 10 ♒	36 0	22	5

Year	Date	Time-Int.	Longitude	Syn. arc	Period. numb.	Calc. date
198	III (23)		19°10′ ♓	36° 0′	4	23
199*	V (11)		25 10 ♈	36 0	16	11
200	V 28		0 16⅞ ♊	35 6⅞	28	28
201	VII 14	46	4 1⅞ ♋	33 45	10	14
202*	VIII 26	42	4 35 ♌	30 33⅛	22	26
203	IX 8	42	4 35 ♍	30 0	4	8
204	X 21	43	4 35 ♎	30 0	17	21
205*	XII 3	42	4 35 ♏	30 0	29	3
206	XII 17	44	7 36⅞ ♐	33 1⅞	11	18 *a*
208†	II 5	48	12 10 ♑	34 23⅛	23	5
209	II 23	48	18 10 ♒	36 0	5	23
210*	IV 11	48	24 10 ♓	36 0	17	11
211	IV 29	48	0 10 ♉	36 0	29	29
212	VI (16)		4 58⅛ ♊	34 48⅛	11	15
213*	VIII 1		8 43⅛ ♋	33 45	23	1
214	VIII 13	42	8 45 ♌	30 1⅞	5	13
215	IX 25	42	8 45 ♍	30 0	17	25
216*	XI 7	42	8 45 ♎	30 0	29	7
217	XI 20	43	8 45 ♏	30 0	12	20
218*	XIII 6	46	12 28⅛ ♐	33 43⅛	24	6
220	I 23	47	17 10 ♑	34 41⅞	6	23
221*	III 11	48	23 10 ♒	36 0	18	11
222	III 29	48	29 10 ♓	36 0	0	29
223	V 18	49	5 10 ♉	36 0	12	17*a*
224*	VII 3	45	9 39⅜ ♊	34 39⅜	24	3
225	VII 18	45	12 55 ♋	33 15⅝	6	18
226	VIII 30	42	12 55 ♌	30 0	18	30
227†	IX 12	42	12 55 ♍	30 0	0	12
228	X 24	42	12 55 ♎	30 0	12	24

Year	Date	Time-Int.	Longitude	Syn. arc	Period. numb.	Calc. date
		44		30°29$^3/_8$′		
229*	XII 8		13°24$^3/_8$′ ♏		25	8
		46		33 45		
230	XII 24		17 9$^3/_8$ ♐		7	24
		47		35 0$^5/_8$		
232*	II 11		22 10 ♑		19	11
		48		36 0		
233	II 29		28 10 ♒		1	29
		48		36 0		
234	IV 17		4 10 ♈		13	17
		48		36 0		
235*	VI 5		10 10 ♉		25	5
		46		34 10$^5/_8$		
236	VI 21		14 20$^5/_8$ ♊		7	21
		45		32 44$^3/_8$		
237*	VIII 6		17 5 ♋		19	6
		42		30 0		
238	VIII 18		17 5 ♌		1	18
		42		30 0		
239	IX 30		17 5 ♍		13	30
		42		30 0		
240*	XI 12		17 5 ♎		25	12
		43		31 0$^5/_8$		
241	XI 25		18 5$^5/_8$ ♏		7	25
				33 45		
243*	I (11)		21 50$^5/_8$ ♐		20	11
				35 19$^3/_8$		
244	I 29		27 10 ♑		2	29
				36 0		
245	III (17)		3 10 ♓		14	17
				36 0		
246†	V 5		9 10 ♈		26	5
		48		36 0		
247	V 23		15 10 ♉		8	23
		46		33 51$^7/_8$		
248*	VII 9		19 1$^7/_8$ ♊		20	9
		43		32 13$^1/_8$		
249	VII 22		21 15 ♋		2	23 *a*
		43		30 0		
250	IX 5		21 15 ♌		14	5
		42		30 0		
251*	X 17		21 15 ♍		26	17

C. Opposition.

(Only the last part of this table is sufficiently undamaged to be used).

230	I 9		8°55′ ♏		1	9
		46		33°44$^3/_8$′		
231	II 25		12 39$^3/_8$ ♐		13	25
		47		34 42$^5/_8$		
232*	IV 12		17 22 ♑		25	12
		49		36 0		
233	IV 31		23 22 ♒		7	30 *a*
		47		36 0		
234	VI 18		29 22 ♓		19	18
		48		36 0		
235*	VIII 6		5 22 ♉		1	6

Year	Date	Time-Int.	Longitude	Synod. arc	Period. numb.	Calc. date
		46		34°28 5/8′		
236	VIII 22		9°50 5/8′ ♊		13	22
		46		33 14 3/8		
237*	X 8		13 5 ♋		25	8
		42		30 0		
238	X 20		13 5 ♌		7	20
		42		30 0		
239	XII 2		13 5 ♍		19	2
		42		30 0		
240*	XIII 14		13 5 ♎		1	14
		42		30 30 5/8		
242	I 26		13 35 5/8 ♏		13	26
		47		33 45		
243*	III 13		17 20 5/8 ♐		26	13
		47		35 1 3/8		
244	III 30		22 22 ♑		8	30
		48		36 0		
245	V 18		28 22 ♒		20	18
		48		36 0		
246†	VIb 6		4 22 ♈		2	6
		49		36 0		
247	VII 25		10 22 ♉		14	24 *a*
		45		34 9 7/8		
248*	IX 10		14 31 7/8 ♊		26	10

The time-intervals, derived in the same way using 30^d for each month, show the same character as the synodic arcs: the 48^d and 42^d occur repeatedly several times in succession, just as the synodic arcs are 36° and 30°. The number of intermediate values is in these tables greater than in those of the first kind. Here also it is natural to assume that the intermediate values of the time-interval are calculated in the same way as those of the synodic arc, but the deviations between the first and 12^d + the last are here even more numerous and larger than in the tables of the first kind Even in the constant extreme values deviations occur; now and then 43 (once 41) and 49 stand in place of 42 and 48.

In order to be able to see, if at least on the average the values for the time-interval increase in the same way as the synodic arcs, they were combined into groups of full degrees and the mean was taken. This showed that

with	30°22′	corresponded a mean of	$42.^d7$	(6 values)
„	31 15	„ „	43. 3	(6 „)
„	32 22	„ „	44. 2	(5 „)
„	33 10	„ „	45. 2	(4 „)
„	33 45	„ „	45. 6	(15 „)
„	34 32	„ „	46. 6	(10 „)
„	35 23	„ „	47. 7	(6 „)

This indicates, that with great approximation the synodic arcs

30° 31° 32° 33° 34° 35° 36°

correspond to time-intervals, of

42 43 44 45 46 47 48 days.

That this, however, cannot be quite accurate is shown by the following consideration.

From the length of the synodic arc the time-interval to be added may be calculated, according to the formulae

$$\frac{\text{syn. arc} + 360}{360} \times \text{sidereal year} = \text{synodic period}$$

$$\{\text{synodic period} - \text{lunar year}\} \frac{30}{29.5306} = \text{time-interval}$$

This gives for

synod. arc $= 30°$ synod. period $= 395.698$

$$\text{time-interval} = \frac{30}{29.5306} \times 41.331 = 41.99$$

synod. arc $= 36°$ synod. period $= 401.786$

$$\text{time-interval} = \frac{30}{29.5306} \times 47.419 = 48.18$$

whereas for the mean value there was already found:

syn. arc $= 33°8'45''$ time-int. $45^d.23$.

For the extreme values, therefore, without a great error 42 and 48 may be taken, provided care is taken that the mean value comes out correct. If we take all the time-intervals = the syn. arc + 12 days, the mean value of the time-intervals becomes $45^d 8^I 45^{II} = 45^d.146$, therefore $0^d.08$ less than it should be. In 12 periods this difference must rise to a day.

The longitudes of Jupiter in the table have resulted from successive summation of all the synodic arcs. If the time-intervals are obtained by adding 12 to the number of degrees of the synodic arc, the dates that result from successive summation of the time-intervals must each time get ahead of the longitude by 12 and thus successively differ with it by v, $v + 12$, $v + 24$, $v + 36$ etc. As the degrees of longitude only go to 30, and similarly the dates, the dates must be deduceable from the longitudes by adding

v, $v + 12$, $v + 24$, $v + 6$, $v + 18$, v, $v + 12$ etc.

the same five differences constantly recurring.

This, however, as already said, cannot come out exactly; in order, to find the character and origin of the remainders, we subtract from the successive dates the series of numbers

12 24 6 18 0 12 etc.

and compare the results with the longitudes. We then find the numbers which in the table III on page 694 stand in the column "reduced date", beside which the values of the difference "longitude — reduced date" (L.— R. D.) are placed. These values become gradually smaller, altogether 5 days in the course of the whole table. This is exactly as much as it should be to account for the difference between 45.23, the actual mean time-interval, and 45.146, the mean syn. arc + 12. We now see that a correction for this difference is not introduced gradually, but suddenly, by shifting one day each time after 10—13 numbers; this is done at the places where the horizontal lines are put (the first line is uncertain, as there is some error here).

If we leave out these regularly recurring jumps, the differences L—R. D. everywhere show variations up and down. On the other hand they show a great constancy, if we only pay attention to the whole numbers and not to the fractions. If we may consider a few cases in which this does not come out as erroneous, we find this rule: *the Babylonian calculator found the dates by taking the numbers of the degrees from the calculated longitudes, increasing them successively by the periodic series of numbers* $v, v+12, v+24$ $v+6, v+18, v$, *etc., each time after* 10—13 *periods taking the number* v *one higher.*

As a final test, in all the sections of the great Jupiter table of the second kind [1]) the dates were calculated according to the above rule by means of the periodic series of numbers v, $v+12$ etc. The few cases, indicated by *a*, where there is still a day's difference, are not such as to throw a doubt on the correctness of the rule for the calculation that we have found; these are probably due to copying errors or errors of calculation in the cuneiform texts. The first error in the 3[rd] section, where Duzu 31 stands instead of 30, is undoubtedly of that kind In the first error of the first section there was a doubt as to where the periodic number had shifted so that either Duzu 6 or Abu 17 must be one day wrong; we have chosen the transition so, that the latter date, the number of which lies at the edge of the illegible damaged part and has therefore perhaps been misread, was taken to be erroneous. The 3[rd] erroneous number of the 2[nd] section also lies at the edge of a damaged portion.

If we now return to the Jupiter tables of the first kind, we find that our rule applies there also. In the table II on p. 692 which contains the dates and places for the second stationary point and the

1) The columns "reduced date" and "L—R D" have only been computed for the first section, the heliacic rises; the system of calculation having been discovered from these it was not necessary to compute them for the other sections.

heliacic setting from this table, the periodic number and the date calculated are placed in the last two columns. The agreement is everywhere complete, except in the first two dates; but here it can be shown, that there is a copying error in the cuneiform text. We found above (p. 691) that the 2nd synodic arc has been calculated wrong: to a starting point of 0°25′ ♏ an arc of 30°5′ belongs. How could this error have arisen? If we assume that the first two longitudes have been copied wrong and should be 1°25′, the synodic arc belonging to these would be 30°17′, therefore the 3rd longitude 1°42′ as it stands in the table. And then the dates calculated become at the same time one higher: Airu 22 and Simannu 4 as the table gives them. In this way all the dates agree with the calculation.

The fact that here the method appears of using the whole number of the degrees of the longitude and not the nearest number rounded off upwards or downwards, indicates that this may have been done in the tables of the third kind also. We cannot settle this, because it is of no consequence; for in that case the first number only, from which the summation started, needs to be taken 30^{I} greater.

It appears, thus, that the Babylonian astronomers made use of a very simple arithmetical system in order to deduce at the same time the longitude and the date of particular phenomena of Jupiter. By the use of normal months of 30 days and corresponding enlargement of the mean value to be added, they made themselves independent of the unequal lengths of the calendar months. Having noticed that the periodic alternation in the time-interval between two successive oppositions contained about the same number of days as the alternation in the synodic arc degrees, they were able by a very simple process of reckoning to find the date from the longitude. They might have done the same in the tables of the third kind; then the column of time-intervals would not have been necessary and practically the same result would have been arrived at with less trouble. Theoretically, it is true, the periodic variations in the synodic arc and in the time-interval should differ by the influence of the varying velocity of the sun: this inequality has practically no influence upon the periodicity in the synodic arc, while it increases the phase of the periodicity of the time-intervals by about 20°. The Babylonians were indeed acquainted with this inequality in the velocity of the sun; but in the Jupiter tables they have not taken it into account. Although KUGLER finds an indication in the didactic text S H 279 (81 . 7 . 6) that in the tables of the 2nd kind the unequal velocity of the sun was taken into account (p. 149—150), nothing of this appears in the tables. In the tables of the 3rd kind also, where it would have been quite possible to apply a different periodicity to the time-intervals and to the synodic arcs, this has not been done; they run practically parallel, differing only by an unimportant computational quantity; and the method of calculation which is used in the tables of the first and second kind excludes any possibility of taking into account the varying velocity of the sun.

Chemistry. — *"On the System Mercury-Iodide."* By Prof. A. SMITS. (Communicated by Prof. P. ZEEMAN.)

(Communicated in the meeting of September 27, 1916.)

As was already discussed more at length before mercury iodide exhibits a very peculiar phenomenon on being heated, which phenomenon consists in this that after the red phase has been converted to the yellow phase at 127°, the substance remains yellow up to about 188°, but then gradually assumes a more and more pronounced red colour, and finally melts to a dark red liquid at ± 255°,5. This, combined with the fact that the vapour is colourless or light yellow, tells us that as far as the composition is concerned the solid phase lies between the vapour and the liquid at the three-phase-equilibrium solid-liquid-gas. In virtue of these data a pseudo figure was derived that took these above facts into account, and gave, moreover, an exceedingly simple explanation of the fact that on sudden cooling of the vapour the yellow modification always makes its appearance first.

Yet this figure had a drawback, which was felt by me and also by others, and which gave an indication that the view would still have to be modified somewhat. This drawback consisted in this that it was assumed that above the point of transition the yellow rhombic mixed crystals would continuously pass into red tetragonal ones.

As was communicated in the last paper on this subject, Dr. A. L. W. E. VAN DER VEEN had at my request undertaken the crystallographic study of mercury iodide in the hope that this research would bring the problem nearer its solution. This has actually been the case. By making use of a special sublimation arrangement Dr. v. D. VEEN [1]) has succeeded in making crystals of yellow mercury iodide, 2 cm. long above 127°.2, and in studying them accurately microscopically at different higher temperatures. It then appeared that the originally yellow crystal begins to gradually assume an

[1]) Verslag van de gewone vergaderingen der wis- en nat. afd. Kon Akademie, Vol. XXIV (1916) p. 1557.