Astronomy. — "*The local starsystem*". By Dr. A. PANNEKOEK. (Communicated by Prof. W. DE SITTER).

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I.

If $A(m)$ denotes the number of stars of magnitude m, Δ the space density of the stars, which for a given line of sight is a function of the distance r, $\varphi(M)$ the luminosity-function, and if for the distance r we introduce $\varrho = 5 \log r$, thus making $M = m - \varrho$, then $\Delta(\varrho)$ can be found from $A(m)$, if both may be represented by quadratic-exponential functions. Thus if we put

$$\log \Delta(\varrho) = h' + k'\varrho - l'\varrho^2; \quad \log \varphi(M) = p + qM - rM^2;$$
$$\log A(m) = a + bm - cm^2$$

we have:

$$h' = a - p + 3.786 - {}^1/_4 \frac{(b-q)^2}{r-c} - \tfrac{1}{2} \log \frac{r-c}{r}$$
$$k' = q - 0{,}6 + (b-q)\frac{r}{r-c}; \qquad l' = \frac{cr}{r-c}.$$

By these formulae (in a somewhat different form) KAPTEYN and VAN RHIJN have deduced the distribution of density in the starsystem surrounding our sun, representing it by a series of flattened surfaces of revolution. [1])

Here the function Δ has been found as a whole from the function A. But the observational data determine this function A for a certain extent of m only. Now the question arises, whether the value $A(m)$ for a given m determines the value $\Delta(\varrho)$ for a certain ϱ. The differential quotients

$$\frac{\partial}{\partial b}(h' + k'\varrho - l'\varrho^2) = -{}^1/_2 \frac{b-q}{r-c} + \frac{r}{r-c}\varrho$$
$$\frac{\partial}{\partial c}(h' + k'\varrho - l'\varrho^2) = -\left\{\tfrac{1}{2}\frac{b-q}{r-c} - \frac{r}{r-c}\varrho\right\}^2 + \frac{\log e}{2(r-c)}$$

show, that for $\varrho_0 = \dfrac{b-q}{2r}$ a variation of b causes not any, a variation of c causes only a slight variation of Δ; so that $\Delta(\varrho_0)$ depends nearly wholly on $a = A(0)$. If we count m and (in order to keep

[1]) J. C. KAPTEYN and P. J. VAN RHIJN. On the distribution of the stars in space...., Astrophysical Journal LII. 289.

the same $\varphi(M)$) also ϱ from the zero point m_0, this means, that in the formula

$$log\, A(m) = a + b(m-m_0) - c(m-m_0)^2$$

the first term $a = log\, A(m_0)$ determines $\Delta(\varrho_0)$ for

$$\varrho_0 = m_0 + \frac{b-q}{2r}.$$

Assuming that the observations determine the values for $A(m)$ of VAN RHIJN from $m = 4$ to $m = 16$ we find, by applying this formula, that they determine the Δ computed from them for $\varrho_0 = 10$ to 17 in the Milky Way (i.e. for $r = 100$ to 2500 parsec) and for $\varrho_0 = 9.5$ to 15 in the polar regions (i.e. for $r = 80$ to 1000 parsec).

As m_0 and ϱ_0 are conjugate values, it is only rational to take ϱ_0 as the zero point in the formula for Δ. If we put

$$log\, \Delta(\varrho) = h + k(\varrho-\varrho_0) - l(\varrho-\varrho_0)^2,$$

we have

$$\varrho_0 = m_0 + \frac{b-q}{2r}; \qquad \frac{1}{l} = \frac{1}{c} - \frac{1}{r}; \qquad k = b - 0{,}6\,;$$

$$h = a - 0{,}6m_0 - p + 3{,}786 - \frac{(q-0{,}6)^2}{4r} + \frac{(b-0{,}6)^2}{4r} + \tfrac{1}{2}\, log\, \frac{r^2}{r-c}.$$

If we insert now the values $p = -2{,}394$, $q = +0{,}186$, $r = +0{,}0345$ [1]), we get:

$$\varrho_0 = m_0 + \frac{b-0{,}186}{0{,}069}; \qquad \frac{1}{l} = \frac{1}{c} - 29\,; \qquad k = b - 0{,}6;$$

$$h = a + 4{,}937 - 0{,}6\, m_0 + \frac{k^2}{0{,}138} + \tfrac{1}{2}\, log\,(l + 0{,}0345).$$

If over a limited extent of m the number of stars $A(m)$ may be represented by a quadratic-exponential formula it determines $\Delta(\varrho)$ over a limited extent of ϱ also. An adjacent extent of m affording a formula for A with other constants determines another part of the curve for $\Delta(\varrho)$. In case of an irregularly fluctuating course of $A(m)$ and $\Delta(\varrho)$ we may divide them into separate parts and represent each of them by such formulae, thus using the quadratic-exponential formula in an interpolatory manner. It may be noticed that in this case the coefficients c and l (which become zero together) may be negative, if only $l + r > 0$. Of course this solution of the problem to find Δ from A is not rigid, but only a practical and approximate one. If c approaches r very nearly, small errors in c cause enormous deviations in l, making Δ wholly uncertain; if c has a great negative value the result has no real meaning. If c surpasses the value $^1/_{29}$

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1) KAPTEYN and VAN RHIJN, l.c. p. 297.

the solution becomes impossible; this points to discontinuities in the stardensities, (voids in the star masses, influence of distant starclouds) which we will not consider at this moment.

II.

By applying the above-mentioned formulae, in the following research we have tried to determine the shape of our local starsystem. The galactic zone between ± 20° latitude was divided into 12 sectors of nearly 30° longitude. The only sources giving sufficient data on the numbers of stars are the Durchmusterung Catalogues; we have used the counts of STRATONOFF [1]), giving the density per square degree for fields of 5° square according to the *Bonner Durchmusterung* (to 0° decl.), the *Südliche Durchmusterung* of SCHÖNFELD (to —20° decl.) and the *Cape Photographic Durchmusterung*. As class 9.1—9.5 in all these catalogues is incomplete, only the numbers up to magnitude 6.5, 8.0 and 9.0 were used in our computations. The details of the extensive researches that were necessary to find the relation between these empirical scales and the photometric magnitudes will be given elsewhere; the resulting limiting magnitudes are for the different zones of declination of the N. hemisphere:

Decl.	0°—10°	10°—20°	20°—40°	40°—60°	60°—80°	80°—90°		
6.55 DM =	6.37	6.69	6.72	6.75	6.75	6.62		
8.05 „	7.97	8.15	8.26	8.29	8.23	8.10	all	$-0.005\,(D-15)$
9.05 „	9.38	9.35	9.48	9.47	9.24	9.29	„	$-0.012\,(D-15)$

For the *Südliche Durchmusterung* we found

$6.48-0.014(D-15)$; $8.14-0,018(D-15)$; $9.39-0.025(D-15)$

where D denotes the number of stars per square degree up to 9.5 (not up to 10). To reduce these magnitudes to the scale, adopted in our computations, viz. the scale of *Groningen Public.* 18, corrected by the values, given by VAN RHYN in *Groningen Publications* 27 (G.P. 18 c.), we must still add to our results the values —0,17, —0,08, +0,02 for the three limiting magnitudes. For the C.P.D. for the galactic zone the values

5,76 7,86 and 9,46 (scale G.P. 18 c.),

were adopted; but these are much more uncertain than for the catalogues of Bonn.

From these $N(m)$, the number of stars from the brightest to the limiting magnitude m, the numbers $A(m)$, running nearly parallel

[1]) W. STRATONOFF. Etudes sur la structure de l'univers. Publications de Tachkent. Nr. 2 et 3. (1900, 1901).

with them, may be got by the relation $A(m) = \frac{d}{dm} N(m)$.

Putting

$N(m) = 10^{\alpha+\beta m+\gamma m^2}$ we get $A(m) = {}^1/_{log\, e}\,(\beta + 2\gamma m)\, 10^{\alpha+\beta m+\gamma m^2}$, or

$$log\, A(m) = \alpha + \beta m + \gamma m^2 - log\, log\, e + log\, \beta + log\left(1 + \frac{2\gamma}{\beta} m\right),\text{ thus}$$

$$a = \alpha - log\, log\, e + log\, \beta\,; \qquad b = \beta + \frac{2\gamma}{\beta} log\, e\,; \qquad c = -\gamma + \frac{2\gamma^2}{\beta^2} log\, e.$$

For the mean magnitude m_0 was taken 8.0.

Further data are given by the *Selected Areas* of KAPTEYN; for each of the 6 Northern sectors the mean was taken of all selected areas lying in it. The numbers per half magnitude from 11,0 to 14,5 (for the greater part after the counts of VAN RHYN, kindly communicated to me), were doubled in order to represent the values $A(m)$ for $m = 11{,}25$ to $14{,}25$. They could be represented by linear formulae without perceptible curvature. In these formulae $log\, A(m')$ $= a' + b'(m' - 12{,}75)$ the m' denote photographic magnitudes; as for these faint classes the reduction of photographic to visual magnitude may be represented by $m - m' = -0{,}62 - 0.05\,(m' - 12{,}75)$, we have

$$log\, A(m) = a + b\,(m - 12{,}13), \quad \text{where} \quad a = a', \quad b = {}^{20}/_{19}\, b'.$$

III.

The results of the Durchmusterung catalogues are collected in the next table, where the first column gives the mean galactic longitude of each sector and n the number of fields of STRATONOFF of 23 square degrees on the average.

long.	*n*	*a*	*b*	*c*	ϱ_0	*h*	*k*	*l*
15°	49	0.236	0.478	+0.0086	12.23	9.812	−0.122	+0.011
45	57	351	510	+ 0059	12.70	857	− 090	+ 007
75	48	312	480	+ 0171	12.26	971	− 120	+ 034
105	52	198	475	+ 0121	12.19	810	− 125	+ 018
135	56	147	446	+ 0212	11.77	932	− 154	+ 055
165	49	182	548	− 0126	13.29	540	− 052	− 009
190	38	246	556	− 0246	13.36	548	− 044	− 014
225	68	303	488	− 0017	12.38	793	− 112	− 002
255	48	264	481	+ 0030	12.27	793	− 119	+ 003
285	52	272	503	+ 0032	12.59	9.767	− 097	+ 004
315	68	277	440	+ 0271	11.68	0.204	− 160	+ 127
350	37	102	466	+ 0049	12.06	9.771	− 134	+ 006

For the sectors 315°, 135° and (in a lesser degree) 75° the great positive value of c, approaching r, giving a great value of l that raises also the value of h, causes a strong maximum for Δ, rapidly decreasing on both sides; so we find here a condensation of stars in space. The empirical data upon which it depends, consist in a strong curvature of the A-curve, i.e. a strong increase of the stars of the 8th magnitude, not continuing in the same manner for the 9th magnitude. As in this case small variations in c bring about great variations in Δ, it is necessary to examine the reality of these condensations as to the uncertainty of the starnumbers N, which have by chance distribution a mean error $\mu = \sqrt{N}$. Therefore for these sectors corrections corresponding to this uncertainty were applied in the direction of diminishing c and then the computation has been repeated [1]). Now the results are:

long.	*a*	*b*	*c*	ϱ_0	*h*	*k*	*l*
75°	0.304	0.489	+0.0095	12.39	9.869	−0.111	+0.013
135	0.138	0.455	+ 0134	11.90	9.803	−0.145	+ 022
315	0.266	0.446	+ 0219	11.77	0.062	−0.154	+ 060

The condensation at 135° has wholly disappeared; the great density at 75° joins the dense parts in sector 45°; the condensation at 315°, however, remains evident and does not disappear even by greater corrections to $log\ N$. Unless, perhaps, the scale of magnitudes strongly deviates here from the mean galactic zone, this condensation must be considered as real.

A deviation in opposite sense is shown by the sectors 165°, 190° and (in a smaller degree) 225°, where c and l are negative. Because the number of stars increases from the 8th to the 9th magnitude at an unusual rate — it is known that on this side of the sky the stars of magnitude 9—10 are strongly condensed over a fourth of the galaxy — we find that the space density at first, in the vicinity of the sun, decreases rapidly, but at a greater distance ceases to do so; whether it increases afterwards, cannot be decided by these data.

The results of the *Selected Areas* for the 6 Northern sectors are collected in the next table, where n denotes the number of the areas used (each being $^1/_9$ square degree).

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[1]) The coefficient c will be diminished, if N(6,5) and N(9,0) are increased, N(8,0) is diminished. A small calculation showed that we get an even chance if for the value of this increase or diminution we take 0,6 μ.

long.	*n*	*a*	*b*	ϱ_0	*h*	*k*
15°	7	2.045	0.360	14.65	9.391	—0.240
45	7	2.195	354	14.56	562	— 246
75	6	1.991	301	13.80	567	— 299
105	7	1.981	387	15.04	238	— 213
135	5	1.845	358	14.62	197	— 242
165	7	2.094	352	14.54	468	— 248

While the Durchmusterung Catalogues determine Δ on the average for ϱ from 11,5 to 13,5 ($r = 200$— 500 parsecs) the *Selected Areas* determine it between 13,5 and 16 ($r = 500$—1600 parsecs). In the main the values for Δ from the *S.A.* fit close to the values from the *D.M.* An exception is made by sector 165°; here the Δ curves intersect for $\varrho = 14{,}4$, $log\ \Delta = 9{,}5$, but they show a different course: according to the *S.A.* the density is strongly, according to the *D.M.* it is hardly decreasing. In this sector the number of stars shows a discontinuity, as the strong increase for the magnitude 9—10 does not continue in the lower magnitudes of the *S.A.* Presumably this is caused by the influence of a remote galactic stream, which must be studied in another way.

IV.

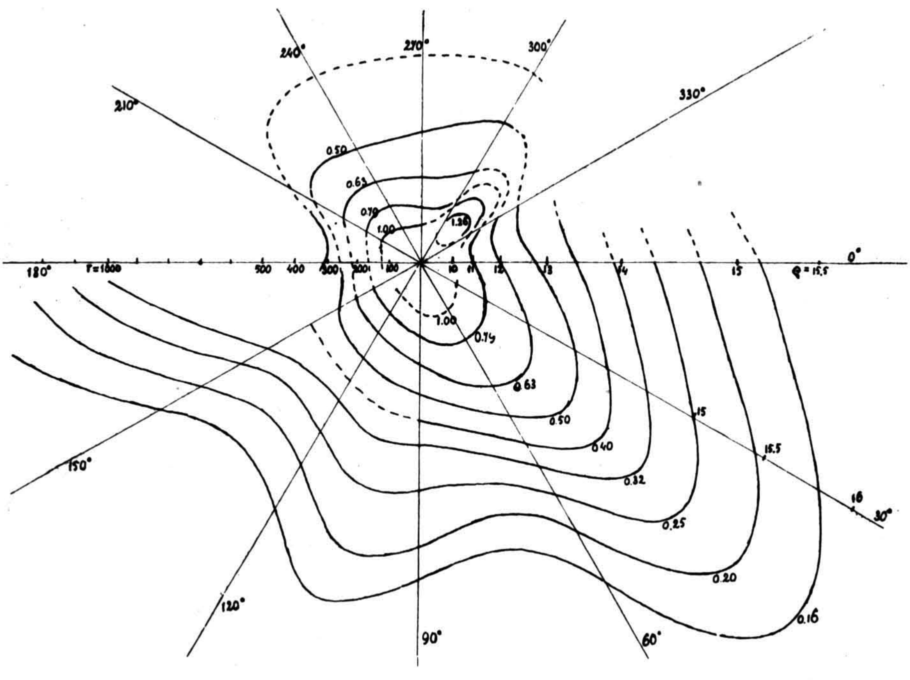
In order to find from these results the distribution of stardensity along the galactic section of the local system, values of ϱ were deduced from our formulae, for which $log\ \Delta$ gets the values 0,0 9,9 9,8 etc. In order to have no more irregularities in the resulting figure than are probably real, the results of the second computation were used for the sectors 75°, 135° and 315°. The values falling beyond the limits of the formula are placed in parentheses.

(*See table following page*).

These values have been used for the construction of the figure showing curves of equal density, decreasing with 0,1 for log Δ (at the curves the values of Δ itself are put down); where ϱ is not determined by the data, but has been extrapolated, the curves are dotted.

On the Southern hemisphere, where the density could not be found at greater distances than 500 parsecs, we perceive in sector 315° (Scorpio) the condensation, mentioned above, at a distance 100—200 parsecs, where the density surpasses 1,25. For the rest the central mass with density 1 extends farthest in the direction $l = 60°$; at greater distances the maximum density lies in the sector 45° (Cygnus), where a starstream seems to issue from the system.

long.	log △ =	0.1	0.0	9.9	9.8	9.7	9.6	9.5	9.4	9.3	9.2	9.1
15°	ϱ =		(10.4)	11.5	12.4	13.1 / 13.4	13.8	14.2	14.6	15.0	15.4	15.8
45			(10.7)	12.2	13.4	14.0	14.4	14.8	15.2	15.6	16.0	(16.4)
75			(11.0)	12.1	13.0	13.4	13.7	14.0	14.3	14.6	15.0	(15.3)
105			(10.0)	11.4	12.3	12.9	(13.4)	13.8	14.3	14.7	15.2	15.7
135			(10.0)	11.2	12.0	12.6	(13.1) / (13.0)	13.4	13.8	14.2	14.6	15.0
165						(11.1)	12.2 / 14.0	14.4 / 14.4	14.8	15.2	15.6	(16.0)
190						(11.4)	12.4					
225		(9.8)	10.6	11.5	12.4	13.2	(14.1)					
255		(9.6)	10.4	11.4	12.2	13.0	(13.8)					
285			(10.0)	11.1	12.2	13.3	(14.3)					
315		11.4	12.1	(12.6)								
350			(10.2)	11.1	11.9	12.6	(13.3)					



Whether the remote fluctuations between 75°—150° are real, is not certain; perhaps the inward bending of the curves in sector 135° and 350° is caused by the absorbing nebulae in Taurus and Ophiuchus. Also in the direction 180° the starsystem extends to a great distance; but here the discontinuity already spoken of, and the lack of counts for fainter stars makes the explanation of the data uncertain.

Of course the results of this first investigation have only a provisional character, and that for two reasons. In the first place by the incompleteness of the data: while up to the 9th magnitude the Durchmusterung Catalogues afford a rather complete though coarse material (by the uncertainty of the reduction to the photometric scale), we have hardly any data for the 10th and 11th magnitude. For the fainter classes the Selected Areas form an excellent but very limited material, while it is uncertain in what degree the local irregularities vary the average values for greater regions. Thus we do not know the whole course of $A(m)$ from the 6th to the 14th magnitude, which would be necessary to remove all doubts on the course of $\Delta\ (\varrho)$.

In the second place it must be emphasized that by taking together extended space-sectors with artificial boundaries the real irregularities in the distribution of the stars, with perhaps wholly different boundaries may be partly effaced, partly changed in their character. Moreover by regarding the influence of near absorbing nebulae and of remote galactic objets on the number of stars the results for space density may still be modified.