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Luminosity function and brightness for clusters and galactic clouds,

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Derivation of formulae.

We suppose a mass of stars at distance $10^{0.2\rho}$, distributed according to a luminosity function $\log A(m) = C - r(M - M_0)^2$. As the apparent magnitude is given by $m = M + \rho$, we have also, putting $M_0 + \rho = m_0$ (most numerous magnitude), $\log A = C - r(m - m_0)^2$. The total light of all these stars, expressed in stars of 0.0^m as unity, will be (limits $\pm\infty$)

$$
\begin{aligned}
H &= C \int 10^{-r(m-m_0)^2 - 0.4m}\, dm \\
&= C\, 10^{-0.2(2m_0 - 0.2/r)} \int 10^{-r(m - m_0 + 0.2/r)^2}\, dm \\
&= C \sqrt{\frac{\pi \log e}{r}}\; 10^{-0.2(2m_0 - 0.2/r)}.
\end{aligned}
$$

The contribution of each magnitude class m to the total light follows the same Gaussian curve as the luminosity itself, with maximum for $m_0 - {}^{0.2}/r$. Putting this „most contributing magnitude" $m_0 - {}^{0.2}/r = m_1$ we have

$$H = C \sqrt{\frac{\pi \log e}{r}}\, 10^{-0.4m_1 - 0.04/r}.$$

Now the number of stars of this most contributing magnitude $A_1 = C.\,10^{-r(m_1 - m_0)^2} = C.\,10^{-0.04/r}$ and the brightness of one star of this magnitude $h_1 = 10^{-0.4m_1}$; denoting by F_1 the numerical factor depending on r, we may write

$$H = A_1 F_1 h_1.$$

For the KAPTEYN luminosity curve we have $M_0 = 2.7$, $r = 0.0345$, $m_0 - m_1 = 5.8$; the most contributing absolute magnitude is -3.1; $m_1 = \rho - 3.1$ and the factor F_1 has the value 6.3 ($\log = 0.799$). Thus the formula deduced may be stated in this way: the total light of a mass of stars, following the KAPTEYN luminosity curve, is 6.3 times the light of all stars of the most contributing magnitude. For other luminosity curves of the same type, but with other constants, the factor F_1 decreases with increasing r, i. e. with decreasing spreading of the function. For another magnitude m we have

$$
\begin{aligned}
&\log A = \log A_1 + 0{,}4\,(m - m_1) - r\,(m - m_1)^2 \\
&\log h = \log h_1 - 0{,}4\,(m - m_1)\ ,\ \text{thus} \\
&H = A\, F_1\, h\, 10^{+r(m - m_1)^2}.
\end{aligned}
$$

Putting $\log F_1 + r\,(m - m_1)^2 = \log F$ we have again

$$H = A\,F\,h$$

but now the factor F, denoting how many times the total light surpasses the combined light of all the stars of magnitude m, varies with m. It has a minimum for $m = m_1$ and increases for smaller and greater m. We have computed it for different luminosity curves, i. e. for different values of r being multiples of $r_0 = 0.0345$. The last line contains the mean deviation in magnitude $\sqrt{(1/2\,r)}$.

TABLE 1. *Values of F.*

$\pm(m - m_1)$	$\frac{1}{2} r_0$	r_0	$2 r_0$	$3 r_0$	$4 r_0$	$5 r_0$
0^m	8.9	6.3	4.4	3.6	3.1	2.8
0.5	9.0	6.4	4.6	3.9	3.4	3.1
1	9.3	6.8	5.2	4.6	4.3	4.2
1.5	9.7	7.5	6.4	6.3	6.4	6.9
2	10.4	8.6	8.4	9.4	11.2	13.8
2.5	11.4	10.3	12.0	16.0	23	34
3	12.7	12.9	18.6	31	55	80
3.5	14.4	16.7	31			
4	16.8	22.4	56			
mean dev.	5.4^m	3.8^m	2.7^m	2.2^m	1.9^m	1.7^m

Expressing the total light in magnitudes and putting $-2.5 \log H = m_t$ we have

$$m_t = m - 2.5 \log A - 2.5 \log F$$

which for $r_0 = 0.0345$ becomes

$$
\begin{aligned}
&m_t = m_1 - 2.5 \log A_1 - 2.00 \ \text{and} \\
&m_t = m\ \ - 2.5 \log A\ \ - 2.00 - 0.086\,(m - m_1)^2.
\end{aligned}
$$

If the mass of stars considered extends over a large area of the sky the same formulae may be used, H in this case denoting the surface brightness (total light per square degree in stars 0.0^m) and A denoting the number of stars of magnitude m per square degree.

The most contributing magnitude m_1 is at the same time the central magnitude for the total light, i. e. half of the total light is contributed by brighter, the other half by fainter stars. If the stars down to magnitude m contribute the fraction p of the total light, we have $p = f\left((m - m_1)\sqrt{2.3\, r}\right)$, where f represents the function

$$f(x) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{x} e^{-x^2} dx.$$

If r is known this formula allows to find m_1 from $p(m)$.

These relations may be used to investigate the luminosity curve in distant agglomerations, such as clusters or galactic clouds, when the distance is not known. If the distance is known the luminosity curve may be derived directly as a function of M, as has been done by SHAPLEY for some globular clusters (Studies X. A critical magnitude. *Aph. J.* **49**. 96; *Contrib. M. W.* 155). But as his stars extend over a limited range of magnitude only, the use of the total brightness may also in this case give some information on the number of small stars beyond the limit of the photographs used.

The globular cluster Messier 3.

The total visual brightness has been estimated by HOLETSCHEK (*Annalen Wien* **20** 91); reduced with the Harvard magnitudes of the comparison stars it is found to be 6.56. Photovisual magnitudes have been determined by SHAPLEY, but only for the outer ring. For the inner parts I have constructed a catalogue from the Paris, Bonn and Mt Wilson results, which for some brighter magnitudes may be assumed rather complete (Cf. the next paper „New reduction of VON ZEIPEL's magnitudes"). As these magnitudes are photographic ones, HOLETSCHEK's magnitude cannot be used, since we do not know the mean colour index of the cluster. Fortunately in the case of this cluster we may make use of the valuable measures of its photographic surface brightness by HERTZSPRUNG (Photographische Messung der Lichtverteilung im mittleren Gebiet des Kugelförmigen Sternhaufens Messier 3. *Astron. Nachr.* **207** 89. 1918). In his Table 3 the surface brightness (unit one star 0.0^m spread over a circle of $18''.7$ diameter) is given as a function of the distance to the centre (unit 1 $mm = 16''.98$). As it becomes zero at distance $250''$ it seems that too much has been subtracted for background. By comparing the brightness in the three rings $117''-177''-237''-357''$ $(73, 17, -4 \times 10^{-8})$ with their stardensity after SHAPLEY (41.4, 17.0, 7.9 per circle of $1'$ radius) I found that the values of Table 3 should be increased by 22.10^{-8}. These values rest on magnitudes of comparison stars outside of the cluster determined by himself with the aid of an objective grating. Comparing them with SHAPLEY's magnitudes we find

Nr.	206	1346	1437	1402	1439	9 stars<14
He.	11.04	11.17	12.40	13.39	13.80	14.55
Sh-He.	+0.23	+.63	−.30	+.05	−.19	−.32

The fainter classes alone give a difference of zero point of -0.27. But evidently the scale also is different. As SHAPLEY's magnitudes rest on the scale of the Polar Sequence we must assume them as more correct, and we find for HERTZSPRUNG's magnitudes a correction $-0.20\,(m - 12.8)$. This has a considerable influence on the values of the surface brightness, as they must be corrected by $d \log I = -0.20\,(\log I + 8 - 2.88)$, making the brightest parts fainter, the outer parts brighter. Furthermore instead of his unit the light of one star 15.0^m $(10^{-6} \times$ one star 0.0) per circle of $1''$ radius was adopted. By mechanical integration the total light of each ring, expressed in stars of 15.0^m, was found as follows

$r=0$	$36''$	$54''$	$108''$	$180''$	$240''$	$300''$	$678''$
$L=$	357.6	223.6	473.5	356.4	195.6	157.2	352.2.

The last number has been computed by putting the total light proportional to SHAPLEY's number of stars for the rings $2'-4'-11'.3$; it makes the total light equal to one star 6.69^m. HOLETSCHEK states that the impression of magnitude estimated by him is produced by an image of $4'$ diameter; the light of this disc, being equal to 1130 stars 15.0^m or to one star 7.37^m, gives, combined with HOLETSCHEK's visual magnitude, 0.8 as the mean colour index of this cluster.

In Table 2 the number of stars between limits of 0.2^m is given for each of the rings. The Harvard variables are added in the next column with the mean magnitude they happened to have on the photographs used.

The outer rings show the well known strong maximum between 15.2 and 16.2, corresponding to the critical photovisual magnitude 15.4, and the minimum at 16.4. The inner rings show an incompleteness for increasing limiting magnitude: for C certainly below 16.1, for B below 15.2, for A below 14.5. All rings show a secondary maximum for the brightest stars (14.3 for E, D, C, 14.5 for B, 14.2 for A), followed by a faintly indicated minimum at 14.5—7. The coincidence of these maxima indicates that the scale of

TABLE 2. *Number of stars in M.* 3.

Ring	*A*	*B*	*C*		*D*		*E*	
0	36″	54″	108″		237″		678″	
13.0	1	0	0		0		3	
2	0	0	0		0		0	
4	2	0	0		0		1	
6	3	0	0		0		0	
8	5	0	1		0		1	
14.0	12	4	6		1		1	
2	23	4	5		6		1	
4	18	10	11		5	1	6	
6	18	15	6	1	5		3	
8	19	8	6	1	5		6	
15.0	16	13	10	2	6		8	1
2	10	22	22	0	8	2	10	
4	7	21	28	5	22	7	12	3
6	6	11	36	8	39	14	25	12
8	4	12	30	6	25	19	26	14
16.0	4	7	34	5	40	5	38	9
2	1	5	47		27	4	24	3
4		5	29		35	1	20	2
6			26		25		13	
8			16		28		34	
17.0			6		34		37	
L.	358	224	473		543		518	

magnitudes cannot be systematically much different for the different rings.

As the surface brightness of the outer ring E rests chiefly on extrapolation, we must make use of the next ring D for which the data are most complete. The total light of all stars counted down to 17.0 is 229.7, which is 0.423 × the total light of the ring; thus the central magnitude for the total light lies below 17.0. To the fraction $p = 0.423$ belongs $x = (m - m_1)\sqrt{2.3\, r} = -0.14$. In the case of the KAPTEYN luminosity curve $1/\sqrt{r_0} = 5.4^m$ we have $m - m_1 = -0.5$ and the central magnitude $m_1 = 17.5$. Using SHAPLEY's distance $\rho = 20.7$ the most contributing magnitude for the KAPTEYN curve is $20.7 - 3.1 = 17.6$. This accordance shows that half of the light of the cluster is coming from stars below the same limit of magnitude as also according to the KAPTEYN luminosity curve contribute half the total light. In this globular cluster the abundance of small stars (i. e. smaller than $M = -3.1$, the magnitude of white giants) relative to the brighter ones is the same as in the region surrounding the sun. If it is allowed to assume that this concordance also holds for the smallest dwarfs — the total brightness cannot give any information on them, as they contribute nearly nothing to it — the total number of stars in the cluster must surpass the number of visible stars (down to 17.0) about 160 times. This result corresponds to what has been deduced by H. VON ZEIPEL (Recherches sur la constitution des amas globulaires, *K. Sv. Vet. H.* **51** Nr. 5. 1913) from a wholly different line of research.

In comparing the number of stars of the separate magnitudes with the total light such an accordance cannot be expected. For SHAPLEY has shown (*loc. cit.*) that the luminosity curve for globular clusters, even after omitting the variables, shows a strong excess of stars with M about -5. Computing therefore F from A, the counted number of stars of magnitude m (including the variables) and $I = H/h$, the total brightness expressed in stars of magnitude m (Table 3, „Uncorrected", where m 14.0 means 13.5 — 14.5), we must find a small minimum value for this critical m. The question may be put whether the great number

TABLE 3. *Comparison of F.*

	Uncorrected.			Smoothed.		Corrected.			
m	A	I	F	A	F	A	I	F	F_c
14.0	17	216	13	14	15	16	195	12	18
14.5	28	344	12	46	7.5	27	310	11	14
15.0	76	543	7.1	104	5.2	54	491	9.1	11
15.5	181	861	4.8	136	6.3	97	778	8.0	8.9
16.0	196	1360	6.9	168	8.1	125	1230	9.8	7.7
16.5	154	2160	14			145	1950	13	6.9
						(171)		(11)	

of stars of this critical magnitude is reached at the expense of adjacent magnitudes, or is superposed on a regularly increasing mass of stars. In the first case a regular curve must be got by smoothing away the irregularity, in the other case by subtracting the surplus. In the first way (*vide* Table 3, „smoothed") we still get values of F too small to be reconciled with a most contributing magnitude at 17.6, as for $m - m_1 = 2.5$ F cannot be smaller than 10. Proceeding in the other way we subtract, besides the Harvard variables, 73 other stars in rings $D + E$ (according to SHAPLEY's table p. 96 *l. c.* we omit 15 stars b 5—9, 22 stars a 0—5, 9 stars a 5—9, 27 stars f 0—5), making 5 stars 15.0—.5, 32 stars 15.5—16, 4 stars 16.0—.5 for ring D. If the light of the omitted stars (52.8) is subtracted, the total light of the remaining stars, considered as a regular mass, becomes 491, the proportion p for the limit 17.0 becomes 0.36, $x = -0.25$ and m_1 for r_0 is changed into 17.9, still sufficiently accordant.

The values of F (Table 3 „corrected") still show a minimum at 15.5; by increasing somewhat the number of omitted stars these middle values can be raised also to nearly 10. This corresponds to the average of the values F_c computed for a KAPTEYN curve with $m_1 = 17.6$; but they do not show the corresponding regular decrease. Considering only the average we may say that the data are in accordance with the supposition that the stars of the cluster, after excluding SHAPLEY's critical stars, follow a KAPTEYN luminosity curve. They do, it is true, not prove this curve, as curves with different spreading all give the same value of nearly 10 for F at a distance 2^m from the minimum. The deviations from the computed F_c exhibited by the table, the observed F being too great for the faintest and too small for the brightest stars, may perhaps be attributed to special causes. As to the last phenomenon, Table 2 has already shown in all rings a secondary maximum for stars between 14 and 14.5, which may thus be considered as another less important critical group in excess to the regular curve. For the faintest class on the other hand it is probable that the catalogue is not complete: assuming proportionality with the number in ring E we might expect for 16.5 — 17 26 more stars, giving the values in parentheses in Table 3. Perhaps in both rings this faintest class is still more incomplete, though it seems doubtful whether the incompleteness is so great as required by the KAPTEYN curve. By long exposure photographs, reaching one magnitude deeper, with long focus, in order that the fainter stars will not cause an appreciable background blackening, these questions may be settled and the spreading of the luminosity curve may be determined.

For the inner rings the information on the light curves is much more restricted by the incompleteness of the data. By comparing the different rings as to the number of stars and the total light, the relative abundance of bright and faint stars may be found. (Table 4).

TABLE 4. *Comparison of rings.*

	$C:D$			$B:C$			$A:B$		
13.6 — 14.6	30 :	18 =	1.7	33 :	30 =	1.1	76 :	33 =	2.3
14.0 — 15.0	42	28	1.5	50	42	1.2			
14.6 — 15.6	118	103	1.1	75	118	0.6			
15.0 — 16.0	174	181	1 0						
total light	473	543	0.87	224	473	0.47	358	224	1.6

In each inner ring compared with the adjacent outer ring the bright stars are more numerous than they should be if they contributed evenly to the total light. It was known already that in the innermost part, within 40″, the bright stars are relatively more numerous than in the outer parts (VON ZEIPEL *l. c.* p. 6). Table 4 shows that this holds already for ring C (54″ — 108″), and for each inner ring at an increasing rate. Compared with D the number of stars 13.6 — 14.6 in C, B, A is 1.7, 1.8, 4.2 while the total light is 0.87, 0.41, 0.66. It must be remarked that this result depends in a large degree on the scale of magnitudes used; if we had retained the scale of HERTZSPRUNG, only correcting his zero point, so that the surface brightness in the inner parts would have been relatively greater ($A = 540$, $B = 280$, $C = 480$, $D = 386$, $C/D = 1.24$, $B/C = 0.58$, $A/B = 1.92$), the excess of bright stars would have been less striking, though still evident.

Comparing the number of stars itself for each ring with the total light, we have not more than one magnitude at our disposition, for which completeness may be supposed.

	C. 14.0 — 15.0	C. 15.0 — 16.0	B. 14.0 — 15.0	A. 13.6 — 14.6
$A_m =$	42	174	50	76
$I\ =$	299	751	141	156
$F\ =$	7	4.3	2.8	2.1

This low value of F for the inner parts A and B, indicating that nearly the whole light comes from stars brighter than 16^m, does not appear very probable. With the magnitude scale of HERTZSPRUNG somewhat greater values (3.5 and 3.1) would be found. It is more likely, however, that the cause must be sought for in the catalogue used, in this way that in many instances groups of adjacent stars by their combined images are taken for a single bright star and thus the number of bright stars has been found too great. This is confirmed by the counts of VON ZEIPEL (*loc. cit.*) for bright, moderate and faint stars; for the limiting magnitude of his classes I found 14.87, 15.59 and 16.60 and from the concluded number of 65 stars between 13.87 and 14.87 within 43″.6 the value $F = 3.9$ is deduced. For exact determinations our data are evidently insufficient.

The concentration of bright stars in the centre shows that the most luminous stars are at the same time the most massive stars. In this correlation between luminosity and mass lies the importance of the study of the central parts of the cluster. According to EDDINGTON's theoretical investigations the relation between mass and luminosity for giant stars depends on a parameter, the mean molecular weight, knowledge of which would give information about the inner status of the stars. For these globular clusters the luminosities are immediately found from the parallax; and the mean mass of the different magnitude classes may be deduced from their relative concentration. The construction of a reliable and accurate catalogue of bright stars by means of very large scale photographs with short exposure time does not seem to offer great difficulties. The chief difficulty will be found in the fainter stars, as by their crowding the whole background is blackened; but the surface brightness may be used as a substitute. In this case a careful new determination of the magnitudes of HERTZSPRUNG's comparison stars in order to test the scale of blackness would be very desirable.

The open clusters Messier 11, 37 *and* 35.

For *M* 11 and *M* 35 catalogues with exact measures, based on the Polar Sequence, have been published by F. KÜSTNER (*Veröff. Univ. Sternw. Bonn*, Nr. 18, 1923), while *M* 37 has been the object of a careful study by VON ZEIPEL and LINDGREN (Photometrische Untersuchungen der Sterngruppe Messier 37; *K. Sv. Vet. H.* 61 Nr. 15, 1921). The total light has been estimated by HOLETSCHEK, who gives: for *M* 11 6.8 (diameter at most 5′), for *M* 37 6.5—7 (much dispersed), for *M* 35 5.6 (diameter 20′—30′, at least 15′). Reducing his estimates with the Harvard magnitudes of the comparison stars I find 6.4, 5.9 and 5.4. By comparing extrafocal images in a Zeiss binocular prism-fieldglass I found for *M* 37 and *M* 35 6.2 and 5.4. For photographic magnitudes I have made use of some extrafocal negatives of the Milky Way made by Prof. MAX WOLF at Heidelberg; for the clusters and for some surrounding stars, whose photographic magnitudes were taken from the Draper Catalogue (*Harv. Ann.* 92 and 96), the blackness of the extrafocal discs was measured. For *M* 11 I found thus 6.6; for *M* 37 6.3 and for *M* 35 (at the border of a plate) 5.1; as the stardiscs have a diameter of 40′ these magnitudes give the total light within a circle of 20′ radius. The fields counted have a diameter 5′ for *M* 11 (adopted centre —55″+40″), $6 \times 93'' = 9',3$ for *M* 37 and 20′ for *M* 35 (adopted centre — 150″ + 50″). Thus in the total photographic magnitude a much greater surface is summarized than has been counted out, making it especially for the largely dispersed *M* 35 too bright. Estimating this influence at 0.7^m for *M* 35, and 0.1^m for *M* 37 we will assume the photographic magnitudes 6.6, 6.4 and 5.8 (making the colour index 0.2, 0.3, 0.4); subtracting from *M* 11 the bright star *DM* — 6° 4929 (8.28) we get 6.87 for the total light of the cluster stars. For *M* 37 the photovisual as well as the photographic magnitudes were counted for ring 1—6 and ring 10—15: considering these outer rings as pure background we get the number of cluster stars by subtracting a proportional part

TABLE 5.

Messier 37. photovis. (36.3).

	St.	*Cl. St.*	*L.*	*A.*	*I.*	*F.*	*F_c.*
10							
	1	·1	0.8				
10.5				7	58	8	·7
	7	6	3.0				
11				24	91	3.8	4.2
	19	18	5.7				
11.5				55	145	2.6	3.1
	43	37	7.4				
12				84	229	2.7	2.8
	51	47	5.9				
12.5				83	363	4.4	3.1
	50	36	2.8				
13.				110	575	5.2	4.2
	88	74	3.7				
13.5				155	912	5.9	6.9
	108	81	2.6				
14				160	1450	9	14
	118	79	1.6				
14.5				162	2290	14	34
	150	83	1.1				
15.							
15.5			34.6				
16.							

Messier 37. photogr. (27.5).

St.	*Cl. St.*	*L.*	*A.*	*I.*	*F.*	*F_c.*
1	1	0.8				
			2	44	22	14
2	1	0.5				
			6	69	11	7
6	5	1.6				
			35	109	·3.1	4.2
33	30	6.0				
			79	174	2.2	3.1
54	49	6.1				
			114	275	2.4	2.8
73	65	5.1				
			119	436	3.7	3.1
65	54	2.7				
			129	691	5.4	4.2
92	75	2.4				
			132	1090	8.3	6.9
84	57	1.1				
			119	1740	15	14
99	62	0,8				
			98	2750	28	34
89	36	0.3				
			70	4360	62	80
110	34	0.2				
		27.6				

Messier 11. (18.0).

	St.	*Cl. St.*	*L.*	*A.*	*I.*	*F.*	*F_c.*
10							
	1	·1	0.7				
10.5				1	29	29	14
	0	0	—				
11				0	45		
	0	0	—				
11.5				8	72	9	4
	9	8	1.6				
12				38	114	3.0	3.1
	33	30	3.8				
12.5				67	180	2.7	2.8
	41	37	3.0				
13.				83	285	3.4	3.1
	50	46	2.3				
13.5				75	452	6.0	4.2
	32	29	0.9				
14				63	717	11	7
	43	34	0.7				
14.5				76	1140	15	14
	48	42	0.5				
15				84	1800	21	34
	56	42	0.3				
15.5							
			13.8				

Messier 35. (47.9).

	St.	*Cl. St.*	*L.*	*A.*	*I.*	*F.*	*F_c.*
	3	3	13.3				
9.				2	19	10	6
	1	1	2.0				
9.5				4	30	7	5
	3	3	3.8				
10				9	48	5.3	4.6
	6	6	4.8				
10.5				17	76	4.5	4.4
	12	11	5.5				
11				26	120	4.6	4.6
	16	15	4.7				
11.5				29	191	6.6	5.2
	16	14	2.8				
12				32	302	9	6.4
	21	18	2.3				
12.5				41	479	12	8
	28	23	1.8				
13				50	759	15	12
	34	27	1.3				
13.5				59	1200	20	19
	42	32	1.0				
14				74	1910	26	31
	58	42	0.8				
14.5				76	3020	40	56
	55	34	0.4				
15				111	4793	43	
	107	77	0.6				
15.5							
			45.1				

of these stars from the inner region. For *M* 11 the number of background stars to be subtracted was found by counting the ring between 250″ and 350″ in the Bonn Catalogue; for *M* 35 it was computed from the number of stars pro square degree given by VAN RHIJN (*Groningen Public.* 27. Table II) for $b = 5°$.

In Table 5 the results of these counts are given and their comparison with the total light (put after the name, expressed in stars of 10.0^m). The first columns give for intervals of 0.5^m the number of stars counted, the number of cluster stars after subtraction of background stars, and their light in 10.0^m stars; the next columns give A (number per magnitude), I and F for arguments increasing by 0.5^m. At the foot of each column L, the sum total of the light of all magnitude classes, is given; comparing it with the total light we may infer that for *M* 37 the magnitude cannot be anything fainter and the colour index cannot be greater than has been adopted. The same holds for *M* 35; as the stars below 15.5 will not increase the total light more than some few units, the adopted magnitude seems to be a rather good guess. For *M* 11, however, a fainter magnitude seems to fit better; taking 6.8 instead of 6.6 (*C. I.* 0.4) the total light becomes 14.2.

The central magnitude of total light for *M* 37 visual is 12.1, for *M* 37 photographic 12.4, for *M* 11 13.0 (or with $m_t = 6.8$ 12.7), for *M* 35 10.5. The most contributing magnitude, for which F is minimum, has nearly the same magnitude (12, 12.5, 12.5, 10.5). The value of this minimum F_1 itself is much smaller than for the KAPTEYN luminosity curve, indicating a greater coefficient r and a smaller spreading. For *M* 37 and *M* 11 this spreading of the magnitudes is very small, the coefficient r being nearly $5\,r_0$, giving a mean deviation of 1.7^m only; the most numerous magnitude in this case lies only 1.2^m below m_1. A comparison with the computed F_c in the last columns indicates some systematic deviations, especially for the fainter classes, whose numbers are somewhat too great. Combining the values of log A for *M* 37 vis, *M* 37 photogr. and *M* 11, taking 12.0, 12.5 and 13.0 as identical magnitudes m_1 and plotting the mean values, the summit of the luminosity curve is seen to be flatter than it should be for a parabolic curve

$$2.18 - 0.172\,(m - m_1 - 1.2)^2.$$

	$m_1-1.5$	m_1-1	$m_1-0.5$	m_1	$m_1+0.5$	m_1+1	$m_1+1.5$	m_1+2	$m_1+2.5$
log A	0.84	1.50	1.82	1.97	1.96	1.99	2.06	2.07	2.07
parab.	0.93	1.31	1.68	1.93	2.10	2.17	2.16	2.07	1.89

This excess of small stars is confirmed by the number of w stars (between 15.7 and 16.5) of VON ZEIPEL (*l. c.* p. 119—120). Of course there is no reason to expect that the luminosity curve — especially with such small numbers of stars — exactly fits a Gaussian curve; moreover our local galactic system shows an analogous deviation, as in our surroundings the number of small stars is greater than corresponds to the KAPTEYN luminosity curve. (*cf.* HERTZSPRUNG *B. A. N.* **5**; KIENLE *A. N.* **218**, 119).

The stars in *M* 37 and *M* 11 thus are concentrated within a narrow range of magnitudes. From the distance of *M* 37 $\rho = 15.8$, deduced by VON ZEIPEL by identifying its bright white and yellow stars with our *A—B* stars and *G* giants, we find $m_1 = -3.7$ (vis.) and -3.4 (phot.). These clusters (for *M* 11 we may assume the same condition) therefore contain only giant stars; dwarf stars must be scarce. These open clusters have a constitution not only different from the stellar system as a whole but also from the globular clusters. Their scarcity of dwarf stars is easily understood if we consider such a system in equilibrium with the thinly populated space around, for in such a case a high concentration of massive stars must be accompanied by a very small concentration of small masses.

For *Messier* 35 the minimum F_1 is greater than for *M* 37 and *M* 11, corresponding to $r = 2\,r_0$; here the dwarf stars are much more numerous than in the two other clusters, though less numerous than in the local galactic system. This cluster seems to take an intermediate position; it extends without an appreciable central condensation over a great surface, its figure shows marked deviations from the globular arrangement and at the same time its luminosity curve is more dispersed; in all these properties it approaches to the character of a small galactic cloud.

Small Magelhanic Cloud.

The data for this object are very insufficient. SCHOUTEN has counted on a photographic negative the numbers of stars for magn. 10—15 in an area of 240 sq. min., whose position is not indicated (*Proc. Amsterdam Ac.* 1918). The surface brightness is taken from the *Uranometria Argentina* Atlas, where it is equal to the brightest galactic clouds in Sagittarius and Scutum, thus corresponding to 0.06 stars 0.0^m; the total light of the surface counted is therefore equal to 40 stars 10.0^m. Half this amount is afforded by the (172) stars counted by SCHOUTEN down to $m = 13.4$. The values of F deduced from his A_m are:

m	11.5	12.0	12.5	13.0	13.5	14.0	14.5	15.0
A	18	33	61	107	151	194	270	455
I	159	252	400	630	1000	1590	2520	4000
F	8.8	7.6	6.6	5.9	6.6	8.2	9.4	8.8

They correspond to a Gaussian curve with m_1 about 13^m and $r = r_0$, having the same spreading as a KAPTEYN curve; but the central and most contributing magnitude $m_1 = 13.4$ corresponds (as the distance

$\rho = 21.4$) to an absolute magnitude -8, 5 magnitudes brighter than in our galactic system. So far as these uncertain data may be relied upon the Small Magellanic Cloud consists merely of giants and supergiants, as much dispersed in brightness as the stars in our system, but much brighter. Real measures of the surface brightness will, however, be necessary to get more certain results.

Galactic cloud in Scutum.

For a field of 40′ diameter in the densest part of the bright Scutum cloud magnitude and color index of the stars from 9.5 to 15 have been determined by E. A. KREIKEN (On the colour of the faint stars in the Milky Way and the distance of the Scutum-group, *Dissertation* Groningen, 1923). From his Table 14 I have deduced the number of stars for each half visual magnitude (in class 11.65 vis. are taken together the groups 10.9 — 11.4, *C.I.* — 0.7 to — 0.2; 11.4 — 11.9, *C.I.* — 0.2 to + 0.3; and so on). In Table 6 this number, standing for 0.35 sq. degree, is given under „Ctd", and in the next column under „Red." the number per mag. and per sq. degree. Subtracting the number of stars corresponding to the average galactic circle we get the number of stars A belonging to the cloud proper. The next column L gives the total brightness of each half class expressed in 10^m stars. The surface brightness of this part of the

TABLE 6.

	Ctd.	*Red.*	*A*	*L*	*I*	*F*
9.65	6	34	23	16	254	11
10.15	7	40	23	10	425	18
10.65	11	63	33	9	673	21
11.15	29	166	117	20	1070	9
11.65	48	274	194	21	1690	9
12.15	77	440	312	22	2540	8.1
12.65	169	966	766	33	4250	5.5
13.15	313	1789	1480	41	6730	4.5
				172		

Galaxy, designated 6.0 on chart VII and in the table page 100 in „Die nördliche Milchstrasse" (*Ann. Leiden* XI. 3) has been found by extrapolating from the measures of VAN RHIJN to correspond to 0.058 stars 0.0^m (*Astr. Nachr.* **214**, 392). This value must also be diminished by the light coming from the foreground and background stars in the regular KAPTEYN universe; using the densities given by KAPTEYN and VAN RHIJN we find that the stars up to $\rho = 20$ (10000 parsecs) contribute 0.0212, thus leaving 0.037 for the cloud itself. As the stars down to 13.4 cause a total brightness 0.0172 the central magnitude m_1, lying outside the limits of the table, must be extrapolated at 13.6. The quotient F reaches its minimum for nearly the same magnitude; the minimum value of F indicates a spreading $r = 2r_0$, leaving systematic deviations, whose reality however, on account of as the small number of bright stars, is not certain. The distance of the cloud has been found by KREIKEN 1400 parsecs, making $\rho = 15.8$; this gives $m_1 = 12.7$ for the KAPTEYN curve. Thus the most contributing magnitude is found a magnitude fainter, the spreading somewhat smaller than for a KAPTEYN curve, giving the same m_0 and a frequency of dwarf stars of the same order as in our surroundings. We have a difference for the bright stars; as, however, the zero point of KREIKEN's magnitudes has not been datermined directly (vide p. X of his memoir; if for his region VII the zero point had been adopted in the same way the error would have been 0.9^m), and the surface brightness has been extrapolated, it is doubtful whether this difference is warranted. This dense agglomeration of stars offers a good opportunity for studying directly the luminosity function by extensive star counts, especially its irregularities, that are smoothed and effaced in the process of deriving it from the central parts of the local system.

Galactic clouds in Cygnus and Aquila.

The number of stars below 12^m in these regions show a sudden and strong increase, indicating the appearance of the brightest cloud stars among the nearer system stars, which are soon wholly outweighed by them. Assuming the validity of KAPTEYN's luminosity curve for the remote clouds I deduced a distance beyond $\rho = 20$ from the point of inflexion in the apparent luminosity curve (*M. N.* **79.** 500; *B. A. N.* **11**). It has been shown, however, by Dr. A. KOPFF at Heidelberg that in this case the surface brightness of the Cygnus cloud should be far greater than is given by observation; the KAPTEYN luminosity curve does not hold for this cloud, so that also the distance deduced by it looses its foundation.

I have used the data of *M. N.* **79**, after having changed, according to later results, the limiting magnitude of EPSTEIN to 12.77 and of HERSCHEL to 14.77. For the Aquila-Sagitta region the part on the eastern border of the Milky Way has been combined with the galactic part, as it showed the same peculiarities of structure. The logarithm of the number of stars per magnitude and square degree is given in Table 7 under I and II for Cygnus, bright patch and faint region, and under III and IV for the Aquila-Sagitta region and for the average galaxy there. Subtracting II from I and IV from III we get A for the starclouds alone. The surface brightness in the Cygnus patch and the Aquila-Sagitta region (4.0 and 3.0 on the scale of my tables $S + B + E + P$, *Annalen*