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Studies on line intensities in stellar spectra, II. The variations in the spectrum of some Cepheids,

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1. From the amplitude of the photographic light-curve of η Aquilae, compared with the visual amplitude, SCHWARZSCHILD deduced in 1899 that in maximum the colour of the star was more blue than in minimum. This could be explained either by an increase of temperature at maximum or by tides in an absorbing and colouring atmosphere caused by a companion. The same difference in colour index has since been found for other Cepheids; for δ Cephei the visual and the photographic range, according to ADAMS and SHAPLEY, are 0.76 and 1.25 magnitude, corresponding to a variation of spectral class from *F*2 to *G*4. The corresponding shift of the maximum of the continuous spectrum has been observed by ALBRECHT.

By comparing two photographs of the spectrum of δ Cephei, one near minimum, one near maximum, ADAMS and SHAPLEY (The Spectrum of δ Cephei; *Contrib. Mt. Wilson* N° 22, 1916) detected variations in the intensities of the spectral lines. The hydrogen line $H\gamma$ and enhanced lines of *Fe*, *Ti*, *Sr* and *Cr* are strong at maximum (*Mg*+ λ 4481 even very strong) and much weakened at minimum; low temperature lines of *Ca*, *Fe*, *Ti* and *Cr* are weak at maximum and strengthened at minimum; the strong *Ca* line λ 4227 is also strengthened at minimum. Thus a higher temperature at maximum is indicated, and from the intensities of the hydrogen lines compared with other ones the spectral classes *F*4 at maximum and *G*2 at minimum are found. From the lines fluctuating with the intrinsic luminosities a variation of $1^{m}.1$ in absolute magnitude was derived. In a subsequent investigation (W. S. ADAMS and A. H. JOY; Some spectral Characteristics of Cepheid Variables; *Contrib. Mt. Wilson* N° 53, 1918) these conclusions are somewhat modified: only the hydrogen lines show this great range in spectral class (here *F*2 to *F*9 for δ Cephei, *F*2 to *F*8 for ζ Geminorum), but as to "the more general features of the spectra, in particular the intensities of the arc lines of the various elements, and other characteristics which seem to be primarily a function of general spectral type" there is little or no difference between maximum and minimum (*F*9 to *G*0 for both δ Cephei and ζ Geminorum). Furthermore there is an increase in the intensity of the enhanced lines at maximum, which probably must be ascribed to an increase in absolute brightness. ($0^{m}.9$ for the average of nine Cepheids).

In his study on "The Cepheid Problem" (*Public. Dominion Obs.* Ottawa. IX 1. 1925) F. HENROTEAU gives estimates of the relative intensity of the enhanced *Ti*+ line λ 4534.14 and the *Ti* arc line λ 4534.95. If the latter is taken = 10, then the former changes from 24 (at maximum) to 6 (at minimum), rather parallel to the light variations.

2. Our former investigations show that the character of a stellar spectrum cannot be expressed by noting only the behaviour of the enhanced lines in general. Each spectrum is characterized by two parameters, which may be found from the behaviour of spectral lines that also form a two-dimensional multitude. The classification of the lines used in our research has been performed in *B. A. N.* 79; we will now try by means of this classification to obtain data on the parameters of the stars used, which will be called the temperature and the gravity parameter.

To the negatives mentioned in the preceding article two plates of ζ Geminorum are added (one at maximum and one at minimum), that have not been used in the classification of lines, because the dispersion was somewhat smaller than for the others. Since they did not show considerable differences, they are treated in the same way as the other plates (three estimates for each plate) and are included in our discussion.

The reduction of the estimates of all negatives to the same scale was performed in the following way. Each star was first reduced to the average of its group (the groups are indicated in Table I *B.A.N.* 79), and then the groups were reduced to a common scale; the last named reduction was of course much less certain than the former. The common scale used here is not the same as the scale used in *B. A. N.* **79**; their relation is given by the little table I. The compression of the upper part of the scale gives the advantage of diminishing the weight of the high intensities.

The resulting intensities for all the stars are given in Table II.

TABLE I.

B.A.N. 79	*B.A.N.* 87	*B.A.N.* 79	*B.A.N.* 87	*B.A.N.* 79	*B.A.N.* 87
1	2.4	8	9.8	20	18.2
2	4.1	10	11.2	25	21.7
3	5.5	12	12.6	30	25.2
4	6.7	14	14.0	35	28.7
5	7.7	16	15.4	40	32.2
6	8.4	18	16.8		

TABLE II. Reduced line intensities in stellar spectra.

λ	Character	δ Cephei		α Urs. min.		ζ Gemin.		α CMi	δ Equ	ξ UMa *br*	ξ UMa *fa*	μ Cnc	ξ Boo	ε Hya	ε Leo	π Cep	15 Lyn	β Her	η Dra
		max.	min.	max.	min.	max.	min.												
4387.0	E H	8.6	8.7	9.6	8.6	8.4	7.6	5.9	5.0	5.2	4.8	4.7	2.7	4.2	9.7	4.0	1.2	4.7	3.6
88.1	AE L	4.5	4.5	4.7	3.5	5.2	4.7	2.6	2.7	2.6	4.2	4.5	3.8	2.1	4.6	2.4	3.3	3.3	2.9
88.6	AE M	4.8	4.9	5.5	4.1	4.2	4.4	4.5	3.2	3.5	4.2	6.3	5.2	0.0	5.0	3.7	0.0	3.0	2.4
89.4	AE LL	1.7	2.4	2.5	2.3	4.0	3.8	1.0	2.9	2.0	4.4	0.7	2.5	0.0	4.6	2.1	3.7	2.8	2.2
90.1	A LL	1.1	3.4	2.4	2.4	4.2	3.7	1.7	3.4	2.6	4.4	8.4	5.0	4.2	4.8	4.5	4.2	6.2	7.2
91.2	AE M	7.3	8.5	8.9	8.5	7.8	10.2	7.7	7.7	5.2	8.4	8.5	7.8	6.1	8.7	5.0	8.2	7.1	7.4
91.9	A LL	3.4	5.1	4.4	4.3	5.6	5.0	3.4	9.0	4.5	10.2	6.9	7.9	8.9	10.2	10.2	7.9	7.7	9.6
93.4	A LL	0.8	1.4	0.4	1.0	1.9	0.7	1.4	2.4	2.0	4.2	4.8	9.5	2.6	4.6	5.0	4.2	5.4	3.8
94.2	E H	8.3	6.9	9.2	8.9	7.2	5.0	7.1	7.3	6.5	4.8	7.5	0.0	5.7	8.5	5.0	5.3	6.4	6.5
95.2	AE M	15.4	13.0	14.8	13.9	15.9	13.4	15.2	14.4	16.3	14.2	14.9	13.0	16.5	14.7	15.0	11.6	13.9	15.5
96.0	E H	7.5	6.2	8.3	8.1	6.5	5.6	6.2	4.7	2.6	4.2	3.5	0.0	1.5	4.6	0.6	4.4	2.9	1.7
97.2	A L	0.0	0.0	0.0	0.0	0.0	0.0	0.2	5.7	1.2	3.5	3.1	0.0	2.6	0.0	0.9	2.3	3.4	3.1
98.3	E M	9.3	10.7	10.3	10.7	11.3	10.0	7.0	7.9	2.0	5.7	7.4	2.5	4.2	10.8	3.2	4.8	6.2	6.3
99.9	AE H	10.1	10.5	9.9	10.2	9.7	7.9	9.8	10.7	10.5	9.1	8.1	7.7	7.2	9.2	4.5	6.9	6.8	5.6
4400.6	E M	10.3	11.9	10.9	11.1	13.4	11.9	9.1	9.1	10.7	8.5	8.6	7.7	9.5	10.9	6.0	7.9	9.0	9.2
01.6	A L	8.2	11.4	9.2	10.2	9.5	11.3	13.0	14.1	16.1	13.9	13.3	13.2	13.6	13.3	10.5	8.2	11.0	11.8
03.4	AE LL	4.4	5.4	5.4	4.8	7.3	6.4	4.2	6.5	6.5	8.9	5.3	2.7	6.1	7.9	4.8	3.5	6.1	5.8
04.9	A LL	15.9	16.2	14.5	14.1	14.9	17.0	19.2	19.2	23.3	25.8	24.3	25.8	25.9	22.4	21.6	30.2	30.4	30.4
06.7	A LL	0.0	0.2	0.5	0.6	0.6	0.5	0.7	2.4	4.5	4.6	4.8	3.4	5.7	2.4	2.8	1.2	5.0	5.0
07.8	A M	6.3	6.7	7.2	6.8	6.2	4.4	8.3	9.5	10.3	9.4	6.9	9.2	7.2	7.3	7.4	5.3	6.0	6.5
08.6	A L	6.1	10.6	8.2	7.9	9.3	11.1	9.0	11.1	10.4	10.2	9.9	9.9	11.6	11.7	10.2	7.9	8.9	12.3
09.5	E M	8.8	11.8	10.4	10.8	10.2	11.1	7.7	8.3	6.3	2.3	4.4	2.5	7.3	10.0	7.5	2.0	7.2	8.0
12.1	E H	6.5	6.7	6.9	6.9	7.7	7.1	4.5	5.1	2.0	0.0	3.1	3.8	6.9	7.8	7.4	8.6	6.3	7.9
13.8	E H	6.2	5.6	5.9	5.3	6.2	5.7	2.6	0.0	3.0	0.0	2.4	1.7	2.9	5.0	3.2	4.2	3.1	1.5
17.0	E HH	10.4	8.2	9.4	8.8	8.2	7.4	8.0	7.9	6.3	5.5	4.0	1.9	8.5	7.0	6.0	8.2	7.1	7.5
17.9	E H	11.3	9.0	10.3	9.7	9.1	7.2	8.8	9.6	7.6	5.5	9.5	5.3	6.9	7.8	6.2	5.9	6.8	6.9
18.5	AE H	5.9	3.5	6.5	5.8	4.3	4.6	6.3	6.9	5.8	3.1	4.4	3.2	4.5	5.6	4.2	5.5	4.0	3.4
19.0	AE L	0.3	1.4	1.0	1.0	1.0	0.7	0.2	0.3	0.7	0.9	1.4	1.9	1.8	2.4	2.8	1.2	2.9	1.6
20.7	E L	3.7	4.0	4.5	4.3	5.6	4.6	0.5	1.9	0.7	0.9	1.4	1.1	3.9	4.6	5.3	5.3	4.4	5.4
22.0	E M	6.1	5.3	6.5	5.0	6.2	3.1	4.4	4.0	2.0	5.2	3.5	8.3	9.8	4.6	8.0	8.2	5.6	7.2
22.8	AE M	6.0	6.5	7.1	6.1	6.6	4.8	6.9	8.1	6.7	5.9	4.7	7.0	6.7	7.3	7.5	7.4	6.0	7.9
23.4	AE LL	2.5	2.4	2.7	2.0	1.7	1.4	2.4	2.2	2.0	4.2	5.6	5.0	7.2	4.6	7.2	6.6	6.2	8.3
25.6	A M	6.9	6.8	7.7	7.0	7.6	9.4	9.6	10.2	10.0	9.9	10.5	10.1	12.3	10.5	13.8	12.0	9.8	11.0
27.4	AE L	7.6	11.6	10.6	10.8	10.4	12.9	10.2	11.9	12.6	12.7	11.6	10.4	12.3	13.0	11.6	12.9	9.8	11.4
28.7	A LL	0.3	0.8	0.5	1.0	2.8	1.7	0.7	0.0	2.3	5.5	4.2	2.5	2.5	3.6	4.2	2.6	3.5	4.3
29.4	E L	1.6	1.5	2.2	2.6	4.3	3.3	0.5	0.3	0.0	1.8	0.0	1.5	2.3	4.6	3.4	2.0	3.2	2.9
31.5	E M	2.2	2.0	3.2	2.8	4.6	1.4	0.2	1.1	0.0	0.4	0.0	0.0	0.9	3.0	0.9	0.0	1.4	2.3
32.2	E L	2.7	2.1	3.6	2.4	3.1	2.9	0.5	3.1	0.0	1.8	1.2	0.6	1.6	2.0	0.9	1.2	2.1	3.1
33.4	A L	3.3	1.6	3.7	3.5	4.2	3.0	5.1	2.9	7.1	6.1	4.0	6.5	5.6	4.0	4.0	3.5	4.7	3.8
34.0	A L	3.3	2.8	3.5	3.5	5.1	4.1	4.2	4.6	4.8	5.9	3.5	5.6	5.8	5.0	5.0	4.6	5.2	5.8
35.2	A L	7.6	8.9	9.0	8.7	9.7	9.4	10.9	12.6	13.3	12.7	11.8	10.4	10.3	8.9	8.9	7.6	8.5	10.1
35.8	AE M	5.8	6.0	7.3	6.7	7.4	6.3	7.7	7.7	8.0	7.2	4.7	4.2	7.3	6.4	4.8	5.3	4.4	6.8
36.5	A LL	0.5	1.6	1.0	0.8	0.9	1.5	1.9	2.2	3.8	0.4	3.5	4.2	5.3	4.2	5.7	4.4	3.1	4.8
37.1	A L	0.5	1.2	1.2	1.3	2.4	1.3	2.4	4.3	3.5	0.4	3.7	2.5	3.5	2.4	2.4	3.7	2.5	3.8
37.8	AE M	0.3	0.9	0.2	0.4	0.4	2.6	1.0	3.2	0.0	0.0	1.4	1.1	5.8	4.4	5.0	4.6	4.3	5.6
38.4	A M	1.0	1.0	0.3	0.9	1.9	2.7	1.4	4.0	1.2	0.9	2.9	1.9	3.1	2.4	3.0	2.6	2.6	2.9
40.6	AE M	2.1	1.3	1.8	1.4	2.6	1.2	0.7	2.2	1.2	1.8	1.4	2.5	3.1	1.6	0.0	2.6	3.0	3.1

TABLE II. (Continued).

λ	Character	δ Cephei		α Urs. min.		ζ Gemin.		α CMi	δ Equ	ξ UMa *br*	ξ UMa *fa*	μ Cnc	ξ Boo	ε Hya	ε Leo	π Cep	15 Lyn	β *Her*	η Dra
		max.	min.	max.	min.	max.	min.												
4441.0	A L	1.7	1.4	0.5	0.7	0.6	1.2	1.0	2.2	1.6	1.8	2.9	4.0	3.5	3.3	2.4	4.6	3.9	3.8
41.9	E M	5.7	4.4	6.8	6.0	4.8	3.8	4.5	3.4	2.6	2.3	3.1	4.6	5.3	5.0	4.2	5.3	4.0	4.8
42.5	A M	5.7	5.2	7.1	6.5	5.9	5.2	8.2	7.9	7.1	8.2	7.8	6.8	8.8	7.8	5.6	7.3	6.1	7.6
43.3	AE L	6.4	8.0	8.4	8.0	8.9	9.5	6.7	7.6	6.5	7.9	7.9	6.6	8.3	8.9	8.1	6.4	6.4	7.5
44.0	E HH	12.5	8.2	10.3	9.2	8.7	7.0	9.6	9.3	7.7	7.5	7.4	6.2	7.3	9.2	6.2	5.7	4.4	5.8
44.7	E M	7.0	7.6	8.4	7.9	9.5	10.3	5.7	6.8	5.1	4.8	3.1	3.8	6.5	8.3	5.0	5.7	6.9	7.5
45.5	A L	0.0	0.6	0.0	0.3	0.4	1.4	0.2	0.0	0.0	0.0	1.4	2.9	4.1	3.0	0.9	0.0	4.0	3.4
46.5	E M	1.0	1.6	1.5	1.4	3.3	1.4	0.2	3.2	0.0	0.0	0.0	0.0	0.5	2.4	0.6	0.0	1.4	1.6
47.1	A L	2.1	3.6	2.4	2.1	5.4	5.5	5.7	5.1	5.8	6.1	8.1	5.9	6.5	7.0	5.7	5.3	5.3	5.6
47.9	A L	6.1	5.8	6.9	6.6	7.2	6.3	8.1	9.8	9.4	8.8	9.8	9.1	7.6	7.8	8.3	7.1	6.6	7.6
49.4	AE L	3.6	5.8	4.1	3.7	6.9	9.7	2.4	8.1	5.5	5.0	5.8	5.6	6.1	8.5	7.5	8.2	6.0	4.7
50.7	E M	10.9	13.1	11.7	11.9	12.7	12.5	10.0	11.2	11.9	9.2	11.6	9.4	10.6	12.0	12.2	11.5	8.9	9.6
51.8	A H	4.8	5.7	4.1	4.1	6.0	6.2	6.9	6.9	5.7	4.6	3.5	6.5	5.7	5.4	7.5	7.4	4.0	6.2
54.9	AE L	13.3	17.2	13.1	14.1	15.0	15.0	14.3	14.4	15.1	16.6	17.6	18.8	16.0	15.0	14.7	14.1	12.1	13.1
56.0	A H	4.5	5.2	4.7	4.1	5.3	4.2	8.0	7.9	8.1	7.4	5.5	7.7	3.8	4.6	3.4	3.7	3.5	4.7
56.7	A M	3.4	3.9	2.9	2.6	4.4	4.8	3.4	4.5	3.5	4.9	2.9	6.6	3.4	4.0	3.7	5.7	3.1	3.9
57.6	A L	3.0	3.8	3.9	3.5	3.9	4.1	5.5	6.1	6.4	7.4	7.0	4.8	6.0	5.4	5.3	6.2	4.3	4.8
58.3	A M	2.7	4.5	3.5	3.1	4.9	5.2	6.3	9.0	4.5	5.5	7.0	7.0	4.5	5.7	4.8	3.5	3.4	4.3
59.3	A M	7.7	8.4	8.9	8.6	7.9	8.3	11.8	10.7	12.0	11.5	11.4	9.9	10.0	9.2	10.8	13.3	7.8	9.6
60.4	E M	4.5	4.0	3.7	3.8	4.0	4.3	1.9	1.1	3.5	5.6	1.0	5.2	6.9	5.0	4.5	4.6	4.9	5.5
63.5	AE L	3.1	3.0	2.8	2.9	4.7	6.9	1.7	0.8	2.0	2.7	3.3	4.6	6.4	7.0	7.6	5.3	5.9	4.8
64.7	AE M	11.3	12.4	11.2	12.2	12.3	12.9	11.1	11.5	12.3	11.6	12.8	10.4	11.2	12.9	12.1	11.5	9.5	10.2
66.8	A L	6.8	8.2	8.4	7.8	8.2	7.4	9.1	10.4	10.9	10.9	11.8	9.8	10.8	9.2	11.2	9.9	7.5	7.8
68.7	E H	12.0	10.2	10.8	11.0	10.3	9.1	10.2	9.7	9.2	8.7	6.5	7.2	8.4	9.0	4.8	7.1	5.5	5.8
69.5	AE H	8.7	9.4	8.9	9.6	9.0	9.9	10.6	10.7	10.1	9.6	10.5	9.5	10.9	10.1	10.9	8.8	8.2	9.8
73.0	AE M	7.6	10.6	8.6	8.8	9.3	9.7	7.8	7.7	9.2	7.9	7.5	7.5	10.6	9.3	10.1	8.5	8.2	9.6
76.2	A M	8.0	8.3	7.8	8.3	7.9	7.4	9.7	9.9	11.3	10.3	10.0	9.5	8.7	9.0	8.7	5.3	6.4	7.5
81.4	AE HHH	14.9	10.2	10.6	10.4	10.0	7.8	15.2	11.0	10.4	7.9	10.8	6.6	7.5	7.3	7.4	10.7	5.3	4.8
82.4	A L	5.7	8.2	8.1	7.4	7.6	7.7	9.2	8.2	10.3	9.1	10.5	8.0	11.0	7.5	8.4	11.0	6.9	8.5
82.9	A M	1.2	0.6	0.6	0.6	1.0	0.8	1.9	0.3	2.6	4.2	2.2	4.2	2.8	3.0	3.6	3.5	2.7	2.9
84.3	A M	5.2	6.4	5.7	5.7	7.2	7.9	7.2	6.3	7.0	8.7	6.5	8.8	6.2	7.0	6.6	8.8	4.9	5.8
85.9	A L	3.6	6.2	4.0	4.3	4.7	5.5	4.4	4.8	4.5	7.0	5.9	6.9	5.3	5.6	6.1	4.8	4.0	5.3
87.1	E L	0.7	2.4	2.4	2.8	3.8	3.2	0.0	1.0	2.6	0.9	1.7	1.7	4.6	3.0	3.7	4.2	2.5	2.9
88.4	AE H	6.0	6.7	6.4	6.1	6.1	6.5	6.7	7.4	5.2	6.6	5.6	4.6	5.3	4.4	4.2	6.2	4.2	5.3
89.3	AE H	7.2	7.0	7.4	7.4	6.4	7.0	8.0	7.4	6.1	7.4	5.8	6.8	7.3	6.4	7.0	7.9	5.8	5.8
90.8	A M	0.0	1.2	1.0	1.3	2.6	2.1	4.5	7.4	2.6	5.5	2.6	5.9	4.8	3.6	6.0	4.8	3.2	3.6
91.6	E HH	7.9	6.0	7.2	7.1	6.2	5.1	7.5	7.2	4.8	4.6	3.1	4.0	3.7	5.4	5.4	3.7	2.5	3.7
92.7	A M	1.6	0.3	0.1	0.2	1.4	1.8	0.7	1.9	1.2	1.8	2.7	3.8	2.1	2.4	3.6	5.3	2.1	3.3
93.7	E M	4.3	3.2	3.5	3.7	4.0	4.5	1.0	3.0	0.7	1.8	2.4	1.1	1.8	3.3	0.9	5.3	2.1	1.7
94.7	A L	6.8	8.5	7.9	8.5	8.7	9.9	8.5	8.6	9.2	8.7	11.1	9.6	8.8	9.5	13.1	11.0	8.8	11.0
95.6	AE M	1.8	1.0	1.4	1.2	2.5	1.1	0.5	1.9	0.7	0.0	1.4	2.5	3.0	3.0	2.1	1.6	1.6	2.3
96.2	A L	1.0	1.1	0.8	0.9	1.6	3.7	1.4	3.0	2.0	2.7	3.1	3.4	5.3	4.0	4.8	4.8	3.1	5.4
97.1	AE L	5.4	6.7	6.8	6.4	6.6	5.8	6.5	7.2	7.6	6.6	5.5	5.9	3.7	5.6	4.8	5.7	3.9	5.6
98.0	AE L	0.5	0.8	1.3	0.9	1.8	2.5	0.5	0.5	1.2	1.8	0.2	1.7	0.9	1.6	3.0	4.2	1.7	2.9
99.1	A LL	1.7	2.8	2.1	2.0	3.8	4.5	3.6	8.0	4.5	6.5	6.0	7.3	6.7	5.2	5.8	7.4	6.2	6.2
4500.5	AE L	1.4	1.8	2.0	1.9	2.5	2.8	1.0	1.1	3.3	3.5	0.5	2.1	2.7	3.0	3.0	3.1	3.1	4.5
01.5	E H	10.2	9.6	10.2	10.2	11.0	10.1	9.6	8.9	8.4	8.4	6.9	9.2	7.3	8.4	8.7	8.4	6.9	6.9
02.4	A LL	0.7	1.4	0.4	0.4	0.7	2.9	1.0	1.6	4.0	2.3	3.7	6.5	5.3	5.2	3.9	5.3	6.1	4.8
05.0	AE L	0.2	1.6	1.1	1.3	3.1	3.2	0.5	0.8	2.6	3.5	1.2	2.1	1.2	2.4	1.2	4.0	2.5	2.9
06.8	E M	2.5	3.0	1.9	2.4	4.7	5.7	0.0	0.0	0.7	0.0	0.5	1.7	1.2	3.0	2.6	5.9	2.5	1.5
08.4	E HH	9.5	8.8	8.8	9.0	7.8	7.9	8.5	7.3	4.5	5.2	4.7	3.6	2.3	6.0	4.0	5.3	2.9	3.5
09.5	A L	0.0	0.2	0.0	0.1	0.6	1.5	1.0	1.1	3.0	3.5	1.0	2.5	1.6	0.4	4.4	4.0	2.7	3.6
12.0	AE L	0.7	0.8	1.1	1.0	0.6	1.8	0.5	0.0	0.0	2.3	1.7	0.6	0.7	2.0	1.8	3.1	1.2	1.4
13.0	A L	1.0	1.4	1.2	1.2	1.3	2.3	1.4	1.1	2.6	4.2	2.9	2.7	1.3	2.7	3.0	4.0	2.2	1.7
14.5	A LL	2.1	2.6	1.5	2.2	2.0	3.3	3.4	6.5	4.8	4.6	4.8	5.9	5.9	4.8	3.4	5.7	5.7	4.6
15.5	E HH	9.0	7.6	8.7	8.8	7.7	7.5	7.7	8.5	5.2	4.2	5.2	1.1	4.3	6.4	6.0	6.6	5.7	4.5
20.4	E H	8.3	8.0	8.0	8.2	7.2	9.8	7.9	5.0	5.8	7.0	6.8	3.8	7.0	7.1	6.6	6.9	6.3	5.4
22.8	E M	11.4	14.6	11.6	12.7	15.2	15.4	9.7	9.5	9.8	9.2	11.6	10.1	11.4	11.0	10.9	11.4	10.7	9.8
25.2	E M	8.5	11.0	9.3	9.5	9.6	10.4	8.0	8.7	7.1	8.5	8.4	8.5	8.9	8.0	8.3	8.2	7.6	7.5
28.8	A L	8.6	9.7	9.6	9.8	9.0	9.6	10.4	10.7	11.4	11.9	13.6	12.6	13.7	10.6	13.4	12.3	11.7	12.2
29.7	AE M	7.4	6.8	7.3	7.5	7.4	7.1	8.3	7.9	8.4	7.4	9.3	7.7	9.1	6.4	9.6	8.4	7.5	7.8
33.3	A LL	3.7	5.0	4.3	4.7	4.2	4.1	6.7	6.5	8.6	8.7	8.4	10.2	8.9	6.2	9.0	7.9	7.0	6.2
34.2	E H	11.9	9.5	12.1	11.7	10.4	8.0	11.1	8.5	9.5	8.5	7.5	9.8	9.6	8.6	10.2	7.9	7.4	7.6
35.0	A LL	0.9	1.4	1.4	1.3	2.7	0.9	3.4	4.7	5.8	5.9	2.6	6.6	3.8	3.0	6.2	6.4	2.6	2.8
3 .	A LL	5.4	10.6	7.8	8.7	8.4	10.9	9.7	10.7	12.1	12.6	15.0	14.7	12.6	11.0	15.9	13.5	12.8	8.3

TABLE II. (Continued).

λ	Character	δ Cephei		α Urs. min.		ζ Gemin.		α CMi	δ Equ	ξ UMa *br*	ξ UMa *fa*	μ Cnc	ξ Boo	ε Hya	ε Leo	π Cep	15 Lyn	β Her	η Dra
		max.	min.	max.	min.	max.	min.												
4539.0	A L	0.2	1.2	0.4	0.5	0.4	1.6	1.0	1.6	0.7	3.5	0.5	2.1	3.8	2.0	3.4	4.8	3.4	4.0
39.9	E HH	4.4	2.7	3.4	3.6	2.0	2.4	1.9	3.4	1.2	1.8	1.9	1.7	4.3	3.0	3.7	3.1	2.2	2.9
40.7	A M	2.3	1.6	1.4	1.6	1.6	1.8	3.2	5.4	3.5	3.1	2.6	6.8	5.8	3.0	4.5	4.6	4.9	4.8
41.7	E H	6.4	4.8	6.8	6.6	5.7	3.5	4.8	4.5	4.5	3.5	1.2	4.8	4.3	4.4	3.0	4.2	4.6	4.2
42.7	AE M	2.6	1.6	1.3	1.8	1.6	2.3	1.7	1.6	2.0	0.9	0.7	4.2	5.5	3.8	3.4	3.5	5.1	4.5
44.1	E M	4.1	3.6	4.2	4.3	3.4	2.3	2 4	1.1	2.0	0.4	1.2	2.7	4.3	3.0	3.9	3.5	3.3	4.0
45.2	E L	5.4	6.2	6.8	7.2	7.3	7.5	4.7	5.7	5.2	4.6	5.2	8.0	7.4	7.1	7.9	7.3	7.0	6.3
46.1	A M	2.8	2.0	2.1	2.0	0.8	1.9	2.7	2.2	4.1	3.1	1.4	5.2	3.6	3.0	2.4	2.0	2.9	2.9
47.1	A L	2.8	2.6	2.5	2.9	1.5	2.7	3.9	4.5	4.1	4.6	4.4	8.4	5.8	4.4	4.6	5.3	6.1	5.1
48.0	A M	3.2	2.0	2.2	2.5	1.1	2.3	3.6	5.9	4.0	4.2	2.2	5.9	2.9	3.0	2.4	2.3	2.9	2.9
48.9	A L	0.5	1.0	0.4	0.7	0.2	2.0	1.0	3.8	2.6	3.5	1.7	3.2	2.9	2.4	2.4	3.1	2.6	2.3
49.8	E HH	20.0	17.1	18.4	17.8	19.4	16.4	19.2	13.6	13.8	11.4	14.8	11.6	12.5	13.3	14.2	13.0	12.2	11.2
51.0	A M	1.6	0.6	0.2	1.0	1.5	0.4	1.9	3.4	2.6	3.1	0.2	2.7	0.7	0.8	1.5	0.0	0.0	0.4
52.6	AE LL	5.7	7.5	6.8	7.6	8.0	7.5	5.9	6.9	11.8	6.6	8.7	7.3	9.7	7.3	6.4	6.9	8.1	7.8
54.2	E M	9.1	10.2	10.4	10.8	10.5	11.5	9.8	7.7	10.6	8.9	8.1	10.7	10.6	10.0	10.8	8.4	8.4	7.6
55.1	E H	5.1	2.2	2.0	2.3	2.0	2.1	2.4	4.5	0.7	1.3	1.4	2.5	0.7	1.2	1.2	0.8	1.4	1.2
56.1	E H	10.0	9.6	10.4	10.6	9.9	9.8	9.7	6.0	10.1	7.6	9.1	6.6	5.8	7.5	7.9	8.0	5.4	5.6
58.8	E HH	8.6	7.2	8.2	9.0	8.4	6.4	7.2	5.0	4.1	2.7	4.8	4.6	5.3	5.2	3.2	3.3	5.3	3.8
60.4	AE LL	0.9	2.7	1.9	2.1	1.3	4.0	0.7	2.2	0.7	2.3	3.7	4.6	5.0	2.4	1.5	3.7	4.1	3.4
61.3	E L	0.5	1.6	1.2	1.0	0.7	1.2	0.2	0.8	0.7	0.0	0.0	1.5	2.5	0.0	0.3	1.2	1.6	2.4
62.5	E LL	1.4	2.2	2.0	2.6	1.4	2.1	0.0	0.0	0.7	0.0	0.0	2.1	2.9	0.8	0.3	0.8	2.2	2.2
63.9	E M	8.9	8.2	8.8	9.4	9.7	7.3	8.0	7.7	6.5	7.5	7.5	7.0	8.8	7.3	5.0	6.6	6.3	4.5
64.8	AE M	2.7	1.5	1.6	1.9	0.3	0.8	1.9	2.7	0.7	1.3	2.9	3.2	4.5	1.6	2.1	3.5	2.1	1.5
65.8	AE L	5.4	6.0	6 1	6.6	6.1	6.6	5.4	7.4	6.4	7.5	9.2	7.8	11.2	7.3	10.4	7.4	8.8	8.6
66.9	A L	1.5	1.4	0.4	0.3	1.0	0.9	1.0	2.9	1.6	5.0	5.0	5.9	4.5	1.2	4.5	3.1	2.5	3.8
68.6	AE M	3.9	3.4	3.0	2.9	1.9	2.8	1.4	3.0	2.6	1.3	4.8	5.3	5.4	3.8	7.2	5.7	5.2	5.3
71.3	A LL	0.0	2.5	1.9	3.7	1.0	2.6	2.4	5.4	5.0	6.6	5.9	4.0	5.7	3.8	5.8	5.7	5.7	4.7
72.2	E M	10.0	11.8	11.4	11.8	11.6	10.6	9.6	8.4	8.8	8.6	9.0	9.9	10.5	10.4	8.9	8.9	8.7	6.3
74.9	E LL	2.8	4.6	2.7	3.5	4.3	5.1	0.7	2.4	1.6	2.7	1.4	2.3	4.7	4.4	4.4	5.7	4.3	3.5
76.5	E HH	7.2	5.3	6.1	6.7	5.3	2.5	5.4	3.0	3.5	2.7	1.4	1.7	2.4	1.6	0.0	0.8	3.8	2.8
78.8	A M	2.1	1.9	1.7	1.8	2.1	2.3	3.9	3.0	3.3	5.0	2.4	5.9	2.8	1.6	1.5	4.6	3.2	3.0
80.3	E L	6.2	6.7	6.9	6.7	6.4	6.2	4.9	6.0	3.5	5.5	7.3	9.5	9.5	5.6	10.5	11.0	8.5	7.5
81.6	A M	5.3	5.5	5.9	5.4	4.3	3.2	7.8	8.5	8.8	8.4	9.3	10.4	10.9	6.3	10.3	9.8	8.2	8.2
82.9	E H	5.4	5.0	6.3	5.4	5.9	2.5	2.7	0.8	1.6	0.0	2.4	3.8	1.6	1.2	2.8	0.8	1.8	1.0
84.0	E H	11.1	10.2	11.0	10.7	9.6	7.4	8.9	6.0	6.4	5.6	8.2	7.9	8.6	5.6	8.1	4.6	5.9	6.2
85.0	A LL	0.2	1.6	0.6	1.1	0.6	0.4	1.2	1.1	1.6	2.7	3.5	4.8	4.7	1.2	3.6	2.6	2.5	2.9
86.1	A LL	4.0	5.0	4.0	4.3	3.1	3.8	6.0	5.4	7.2	8.1	10.3	11.2	11.0	5.6	10.5	11.0	8.7	8.1
87.2	A L	0.2	0.4	0.5	0.6	0.0	0.4	1.2	0.5	1.2	1.3	1.4	3.2	1.6	1.2	0 0	1.6	2.2	1.8
88.4	E HH	7.4	5.6	6.5	6.5	4.9	3.2	5.5	4.5	3.8	3.5	1.2	4.2	1.6	3.3	0.0	1.2	3.1	1.2
90.3	E HH	7.0	5.9	6.7	6.4	4.9	2.5	5.7	4.5	3.8	4.2	3.1	3.8	2.8	3.3	2.6	2.0	3.0	2.9

For each line the mean intensity for the whole mass of stars was computed; the deviations from these mean intensities for each of the stars (called x, and expressed in the unit 0.1) were used in deriving the spectral parameters of the stars. We introduce a temperature coordinate p and a gravity coordinate q and we put

$$x = ap + bq + r$$

where the coefficients a and b depend on the character of the line used. For the *HHH*, *HH*, *H*, *M*, *L*, *LL* lines the coefficient a was assumed $+0.5$, $+0.3$, $+0.1$, -0.1, -0.3, -0.5; for the *E*, *AE* and *A* lines the coefficient b was assumed $+0.5$, 0 and -0.5. The constant r is added to neutralize constant errors remaining after applying the scale reduction; such errors might have some influence on account of the different number of lines in the different classes. Each line gives such an equation. Because for all the stars the same groups of lines in each class are used the equations can easily be solved by the method of least squares; the resulting equations are:

$$p = +0.1750\, s_1 - 0.0495\, s_2 + 0.0260\, s_3$$
$$q = -0.0495\, s_1 + 0.0495\, s_2 - 0.0063\, s_3$$

where $s_1 = +0.5\, \Sigma x\, (HHH) + 0.3\, \Sigma x\, (HH) +$
$+0.1\, \Sigma x (H) - 0.1\, \Sigma x (M) - 0{,}3\, \Sigma x (L) - 0.5\, \Sigma x (LL)$
$s_2 = +0.5\, \Sigma x\, (E) - 0.5\, \Sigma x\, (A)$; $s_3 = \Sigma x$

The values of the coordinates p and q computed in this way are contained in Table IV and (for each negative of different phase) in Table VII. They are expressed in an arbitrary scale, while also the zero point, the mean of the stars used, is arbitrary.

For some of these negatives of different quality the probable error of unit weight has been derived from the residuals; the weight of p and q is always 5.7 and 20.2. The results for the probable errors of p and q are contained in Table III.

For ζ Geminorum the residuals are unexpectedly large for negatives that are of a good quality. They may be due partly to the somewhat different dispersion, in consequence of wich the charaeter of the lines in these negatives does not exactly correspond to our classification of lines; perhaps also the overexposure for the greater wavelengths may have caused a deviation in the width of the scale here relative to the other stars.

The chief source of error in these results has its origin in the scale reductions. Since for stars e. g. of extreme temperature the lines of different temperature classes are not distributed haphazard over all intensities, the scale reductions may cause systematic errors that are most perceptible for the extreme stars. Some idea of the order of magnitude of such errors may be gained by comparing the results of different Cepheid negatives of almost the same phase, and also from the following data. The results of *δ Equ* and *ξ_1 UMa* were reduced to one another directly; from their differences, reduced to the general scale, $\Delta p = +13.3$, $\Delta q = +2.5$ was found, while the values of table III show the differences for these stars, each reduced separately, $\Delta p = +16.9$, $\Delta q = +2.2$.

Furthermore we have treated for some stars each of the three series of estimates of a negative separately and deduced p and q from them. The results are also contained in Table III.

TABLE III. Uncertainty of the results for p and q.

Negative	Prob. error		First estimate		2d estimate		3d estimate		Adopted	
	p	q	p	q	p	q	p	q	p	q
δ Cep M	± 2.8	± 1.5	+ 27.9	+ 15.3	+ 42.1	+ 13.7	+ 29.7	+ 15.1	+ 33.9	+ 14.7
δ Cep m	4.0	2.1	— 13.2	+ 14.5	— 18.0	+ 16.3	— 8.8	+ 14.6	— 13.6	+ 15.8
ζ Gem M	5.0	2.6								
ζ Gem m	6.1	3.4	— 24.2	+ 13.6	— 20.6	+ 12.0	— 31.8	+ 13.7	— 27.8	+ 13.5
α CMi	2.2	1.1								
δ Equ	3.3	1.8	+ 5.6	— 24.3	+ 3.6	— 23.7	+ 7.2	— 21.7	+ 7.0	— 23.2
ξ_1 UMa	2.0	1.1								
ε Hya	4.2	2.2								
15 Lyn	5.2	2.8	— 38.2	— 10.4	— 41.4	— 15.6	— 50.1	— 13.2	— 44.7	— 12.1

Since the scale reductions for these separate series have been made wholly independently, the average of the three sets of values does not coincide exactly with the adopted result from the average line intensities of Table II. From the differences between the separate results we find for the probable error of p and q deduced from 3 series the values ± 2.2 and ± 0.6.

3. It may be shown by elementary considerations (vide *B. A. N.* **19**) that the spectrum of a star depends on two quantities characteristic for the star, viz: its effective temperature T, and the quotient of gravity and coefficient of absorption of the surface layers g/k.

This means that two stars with identical values of these quantities have identical spectra (this involves the supposition that there are no real differences in the constituting elements); by increasing T we get the continuous series of spectral classes; by decreasing g/k we also get a continuous series with increasing relative intensity of the enhanced lines. If these quantities are taken as rectangular coordinates, each spectrum may be represented by a point in a plane. Of course instead of these T and g/k we may as well take two functions $f_1(T, g/k)$ and $f_2(T, g/k)$, satisfying certain conditions; then the plane is divided by other, perhaps curved, coordinates. There are some reasons to choose g instead of g/k as a second coordinate.

The absorption k cannot be found from observation of the stars and has no direct relation to astronomical data; only by theoretical researches of EDDINGTON and MILNE we know that it is proportional to $T^{-4.5}$.

The gravity g, on the other hand, may be determined by observational data for a number of stars, and it is just its knowledge that is wanted for other stars.

In the diagram the lines of equal g will be inclined to the lines of equal g/k; in such a way that (taking high temperatures at the left and small g/k at the top) the lines of equal g rise at the right hand side; between temperatures $\log T = 3.7$ and 3.8 (corresponding roughly with *G2* and *F2*) the difference of $\log k$ is 0.45. From the inclination of the ionization curves in the diagram *B. A. N.* **19** we see that by a variation of 0.10 in $\log T$ and 1.6 in $\log g/k$ the ionization remains the same; thus the lines of constant ionization will rise to the right hand side 1.6

relatively to the g/k lines, 1.15 relatively to the g lines.

We cannot expect that our empirical coordinates p and q will coincide exactly with the correct parameters T and g. If we suppose a somewhat simplified case of three stars, a dF and a dG star with equal g, and a c star, with T midway between the other ones, then the difference dF—dG determines the temperature classification and the difference $c - \frac{1}{2}(dF + dG)$ determines the ionization classification (corresponding to the differences a_1 and b_1 in Table II, *B. A. N.* 79). If now our supposition on the character of these three stars is not quite right, the classification of the lines will be somewhat in error, and using them afterwards in deriving the spectrum coordinates they will also yield erroneous results. It is not difficult to see that an error in the first supposition (on the g of dF and dG) will in this way produce only an error in the resulting T of the stars, so that the lines of equal T become inclined to the vertical lines of equal p. On the other hand an error in our supposition on the relative T of the c star will have the result of giving a different g for the two dwarf stars, so that the lines of equal g become inclined to the horizontal lines of equal q. Thus in our real case we may expect that the lines of equal T and those of equal g will be inclined to the axes of the $(p\,.\,q)$ diagram.

The direction of the true T and g axes in the diagram can only be determined by empirical data of stars for which T and g are known. At the same time we may determine scale value and zero point to reduce the arbitrary scale of p and q to T and g. We do not know what functions of g and of T will have a linear relation to p and q; for the first we will assume *log g*, because this function, but for the mass factor, runs parallel to the absolute magnitude; and we will assume for the other c_2/T, which occurs in PLANCK's formula, for the practical reason that this function is given by HERTZSPRUNG as colour-equivalent; moreover for the moderate extent in temperature (from $F3$ to $G5$) we are using, the precise function which we may choose in this first approximation is rather irrelevant.

Thus for the temperature we assume the relation $c_2/T = Ap + Bq + C$. The values c_2/T are taken from HERTZSPRUNG, Mean Colour Aequivalents (*Annalen Leiden* XIV. 1.), and are used with the weights there given, because the accidental errors of p and q are smaller than those of c_2/T. The values for the Cepheids (with exception of α Ursae minoris) are not used, because the phase for the colour determinations is not known; μ Cancri, not included in HERTZSPRUNG's catalogue, was added from the sources O_4 and P *). For ξ Ursae the values p and q of the two components were combined with weights equal to their luminosity. In this way the formula

$$c_2/T = 2.57 - 0.0105\,p + 0.0036\,q$$

was obtained. In Table IV the computed values are given in the 6th column.

There are some cases of appreciable differences.

TABLE IV. Reduction of spectrum coordinates.

Star	p	q	C_2/T	Wt	comp.	log g	Wt	comp.	Spectrum		M
									H	W	
α UMi	+ 7.4	+ 19.0	2.53	149	2.56				F8	F9	— 3.0
α CMi	+ 35.7	— 18.6	2.21	116	2.13	— 0.74	2	— 0.60	F5	F3	+ 3.2
δ Equ	+ 7.0	— 23.2	2.54	82	2.42	— 0.31	2	— 0.46	F5	F6	4.0
ξ UMa *br*	— 9.9	— 25.4	2.40	125	2.58	— 0.47	2	— 0.45	G0	F9	4.9
ξ UMa *ft*	— 30.9	— 27.4			2.80	— 0.36	2	— 0.46		G2	5.3
μ Cnc	— 31.9	— 22.4	2.43	29	2.82					G3	3.7
ξ Boo	— 40.6	— 29.4	3.02	82	2.89	— 0.48	2	— 0.37	G5	G6	5.8
ε Hya	— 15.0	— 21.8	3.01	148	3.05	— 2.17	2	— 2.25	F8	F9	2.1
ε Leo	— 36.8	+ 0.4	2.92	114	2.95				G0	G0	— 0.9
π Cep	— 46.5	— 13.5	3.16	64	3.01	— 1.80	1	— 1.97	G5	G2	— 0.2
15 Lyn	— 44.7	— 12.1	3.15	69	2.99	— 2.27	1	— 2.08	G0	G4	+ 0.1
β Her	— 60.0	— 4.3	3.17	146	3.19				K0	G5	— 1.0
η Dra	— 55.3	— 11.5	3.19	121	3.11	— 2.33	1	— 2.25	G5	G6	+ 0.7

Comparing δ Equulei and ξ Ursae the spectrum makes the latter star more advanced in colour, while the colour catalogue gives just the reverse. That there cannot be an error of this amount in our result is shown by comparing it with the spectral class given by Mt Wilson and Harvard (columns 10 and 11), which also rest upon line intensities; both ascribe unmistakebly to δ Equulei a less advanced spectrum than to ξ Ursae. On the other hand among the observers of star colours, who observed both δ Equulei and ξ Ursae,

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*) OSTHOFF's colour 3.3 for $5^m.4$, corrected to maximum colour according to HERTZSPRUNG's diagram in *B. A. N.* 37, becomes 3.9, which reduced to *W* gives 2.39 (*wt* 22); MÜLLER and KEMPF *WG* gives 2.55 (*wt* 7); mean 2.43 (*wt* 29).

three (OSTHOFF, KRÜGER, LAU) make the first named star redder, only Potsdam makes it less red.

The conclusion that colour estimate and spectrum from the line intensities do not correspond in these stars must, at the moment, be judged too hazardous, because we do not know what systematic errors may occur in estimating the colour of double stars *). An investigation of the intensity distribution in these spectra either directly or by determination of effective wavelength or colour index seems very desirable.

4. Among the stars used several double stars were included, because values of the gravity may be found for them from the parallax and the orbital elements. For the gravity we have (all values expressed in the sun as unity)

$$g = \frac{\mu_1}{R^2}; \quad \mu_1 = \left(\frac{a}{p}\right)^3 \frac{1}{P^2} \frac{\mu_1}{\mu_1 + \mu_2} = 2 \left(\frac{p_d}{p}\right)^3 \frac{\mu_1}{\mu_1 + \mu_2}$$

where μ_1 and μ_2 are the masses of the components, R is the radius of the star considered, p the parallax, a the semi-axis, P the period, p_d the dynamical parallax computed by the method of JACKSON and FURNER with total mass $2 \times \odot$.

The radius is computed from the absolute magnitude relative to the sun $m + 5 \log p - 0.5 = - 2.5 \log (\sigma R^2)$ where σ is the surface intensity of the star. Combining these formulae we have

$$\log g = 3 \log a - 2 \log P + \log \frac{\mu_1}{\mu_1 + \mu_2} + \\ + \log \sigma + 0.4\, m - 0.2 - \log p$$

$$\text{or } \log g = \quad 3 \log p_d \quad + \log \frac{\mu_1}{\mu_1 + \mu_2} + \\ + \log \sigma + 0.4\, m + 0.101 - \log p.$$

In Table V the data for all the stars are collected. The mass ratio $\mu_1/(\mu_1 + \mu_2)$ for the three last named stars has been deduced from the difference of magnitude by means of the table of BERNEWITZ (*Astr. Nachr.* **213**. 1); from the same source are taken the values of $\log \sigma$ as a function of c_2/T, while the argument c_2/T is assumed according to our discussion above. It must be remarked that the values of the mass and the radius vary strongly with the parallax adopted, but in $\log g$ these variations are neutralized for the greater part, so that g depends only on the first power of the parallax.

For the three last named stars of the table, where not the whole orbit has been observed, no values of a and T are given, but only p_d, deduced by JACKSON and FURNER from moderate arcs. The parallaxes of these stars have been computed from the dynamical parallaxes by assuming the mass of the primary $2.4 \odot$, according to SEARES (*Aph. Journ.* **55**, 194, *Mt. Wilson Contrib.* 226). For δ Equulei, where the combined spectrum of the nearly equal components has been used, the value of $\mu_1/(\mu_1 + \mu_2)$ must be taken 1; the radius in the table denotes here the radius of a single star having the combined light. If a star is really a binary with equal components it will show the same spectrum, because a combined star of twice the luminosity and twice the mass of each component will have the same gravity at its surface as they have. If the components are unequal the larger component, which determines the spectrum, will have the greatest part of the luminosity but only little more than half the mass; thus the spectrum will show a smaller g than is computed from the supposition of one combined star.

These values of $\log g$ are inserted in Table IV and used to derive a formula of the form $\log g = D + Ep + Fq$. In the solution by least squares the stars π Cephei, 15 Lyncis, η Draconis received half weight, because orbit and parallax are both insufficiently known. In this way we found.

$$\log g = - 2.77 + 0.0107\, p - 0.0964\, q.$$

TABLE V. Computation of the gravity for binaries.

Star	B.G.C.	a	P	p_d	$\mu_1/(\mu_1+\mu_2)$	p	m	$\log \sigma$	μ_1	R	$\log g$
α CMi	4187	4″05	39.0	0.280	0.75	0.305	0.5	+ 0.23	1.17	2.78	— 0.74
ε Hya	4771	0.23	15.3	0.030	0.53	0.025 1)	3.7	— 0.51	1.81	12.3	— 2.17
ξ UMa *br*	5734	2.51	59.8	0.130	0.45	0.158	4.4	— 0.13	0.50	1.33	— 0.47
ξ UMa *ft*	»				0.55		4.9	— 0.30	0.61	1.31	— 0.36
ξ Boo	7034	4.97	159.5	0.134	0.56	0.178	4.8	— 0.38	0.48	1.31	— 0.48
δ Equ	10829	0.27	57.0	0.067	1.00	0.053 1)	4.6	— 0.01	4.00	3.10	— 0.31
π Cep	12196			0.026	0.56	0.020 2)	4.7	— 0.47	(2.4)	21.9	— 1.80
15 Lyn	3678			0.014	0.53	0.011 2)	4.9	— 0.46	(2.4)	13.0	— 2.27
η Dra	7634			0.034	0.64	0.028 2)	2.9	— 0.55	(2.4)	23.8	— 2.33

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*) In *B.A.N.* 35 HERTZSPRUNG finds from colour indices for δ Equ. 2.67, making this star still redder.

1) from visual orbit and radial velocity.

2) computed from adopted mass.

Figure 1. Spectrum coordinates.

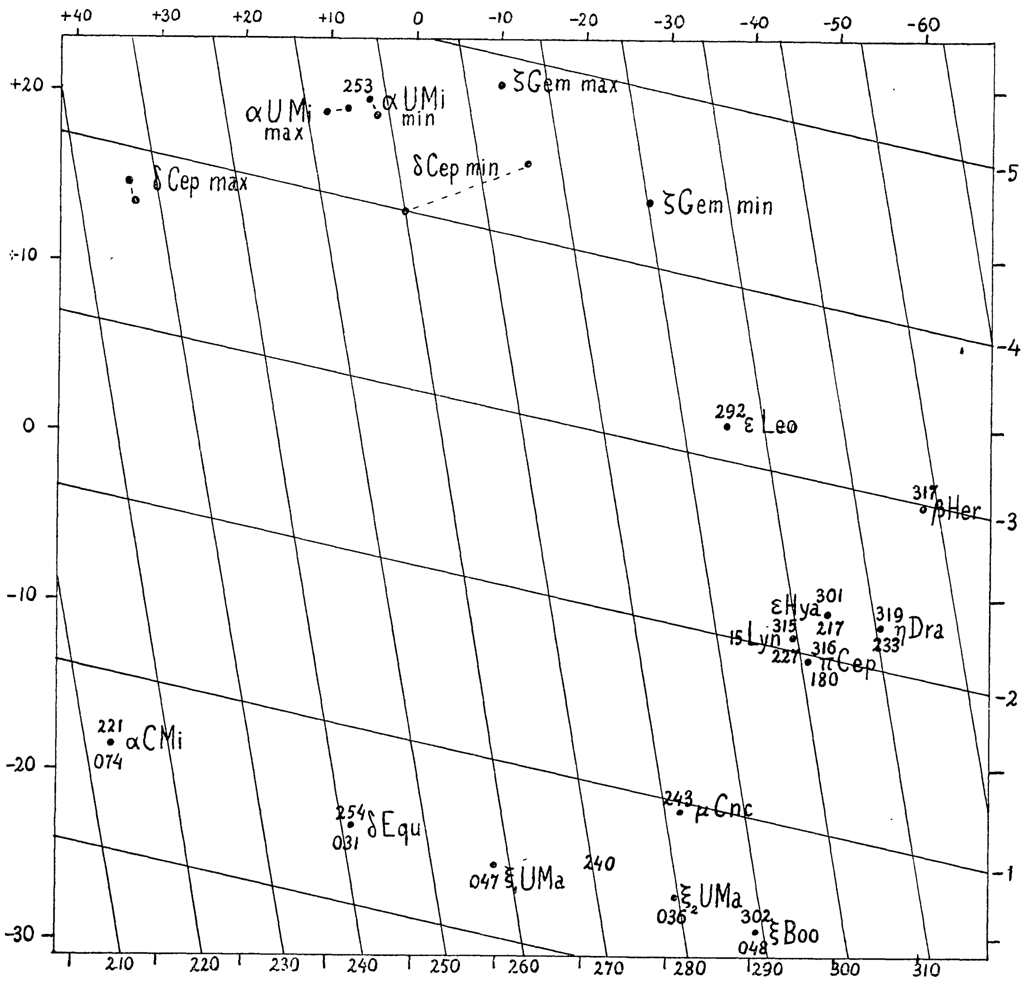
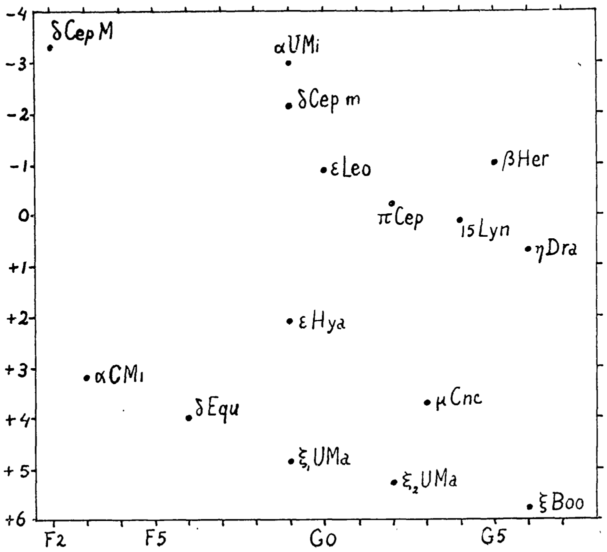


Figure 2. Mount Wilson spectrum coordinates.



The values computed by this formula are contained in column 9 of Table IV.

The remaining differences are small.

5. In our diagram representing each spectrum by its two coordinates p (at the top) and q (at the left hand side), inclined lines of equal log g (at the right hand side) and equal c_2/T (at the bottom) have been drawn after the formulae just derived. Above each point the value of c_2/T after HERTZSPRUNG, below each point the value of -log g computed from orbital motion and parallax has been inserted. For sake of comparison a diagram is added in which the Mount Wilson coordinates (viz: spectral class and absolute magnitude) of the same stars are represented. Since these too are derived from line intensities, only using other combinations of lines, the relative situation of the points should show the same aspect, but on a coordinate framework that may be somewhat inclined and distorted relatively to ours. The chief difference of both representations consists in a distortion, the ordinary giants of class G being situated in our diagram far more to the right, relatively to the c stars and the dwarfs. This corresponds with the wellk nown result that for the same Mt Wilson spectral class these giants have a more advanced colour than the dwarfs (c_2/T for *gG*0 2.88, for *dG*0 2.64, by smoothing HERTZSPRUNG's table, p. 15, *l. c.*), while our p coordinates do not show such a difference. Thus it apppears possible to determine spectral class by line intensities in such a way that for dwarfs and giants it has the same relation to colour and temperature.

Our diagram shows that the direction of the T axis depends chiefly on α Ursae minoris compared with the F and G dwarfs, while the direction of the g axis is only fixed by the series of dwarf stars. The question may be put whether it is permitted to assume the other lines of constant T or g to be parallel to them. Or, in other words, whether an identical change in ionization and line aspect for lower and for higher temperature corresponds to the same change of $\log g$, and whether an identical change in the relative intensity of high temperature and low temperature lines for lower and for stronger ionization corresponds to the same change in T. According to the ionization formula $pxx_0/(1-x)(1+x_0) = f(T)$ (*cf. B. A. N.* **19**, 107) a change in $\log p$ causes an equal opposite change in the value of $\log xx_0/(1-x)(1+x_0) = \xi$, whatever the temperature may be; the curves of constant ionization in the diagram of *B. A. N.* **19** are obtained by vertical displacement of the ionization curves $\xi = 0$; and a vertical displacement $\log g/k$ of the atmospheric curve produces the same change of ionization for each temperature. Thus the first question may be answered in the affirmative. Since the relative frequency of electron orbits of different energy depends solely on temperature and not on pressure the same would hold for the second question, if on a line of constant g the ionization were constant. For constant g, however, the ionization increases with temperature and the intensity differences used to indicate temperature are partly due to increasing ionization. Since the slope of the ionization curves decreases for increasing temperature the same change in ionization corresponds to a smaller change in temperature for high ionizations and small g. For this reason the lines of constant T may converge somewhat at the upper side of the diagram.

6. The variations in the spectrum of the Cepheids can be made manifest by computing the average variations of intensities for lines of the same character. This has been done in a preliminary research, where the scale of *B. A. N.* **79** was still used. The results are collected in the following Table VI. For six classes of lines the average intensity of the lines of high and moderate intensity (the very highest were excluded because here the scale reductions are uncertain) are given there for the F dwarfs, the G dwarfs, the Cepheids, and for maximum and minimum of δ Cephei and ζ Geminorum. The differences F—G and c—(FG) indicate the character of each class of lines.

TABLE VI. Comparison of mean line intensities.

Character	Nbr	F	G	c	δ Cephei		ζ Gemin.		F—G	c—(FG)	δ Cep. m—M	ζ Gem. m—M
					max.	min.	max.	min.				
ALL	6	3.1	6.0	1.8	1.1	2.4	1.7	2.5	— 2.9	— 2.7	+ 1.3	+ 0.8
AL	8	8.0	9.5	5.8	4.3	6.7	6.3	7.0	— 1.5	— 3.0	+ 2.4	+ 0.7
AM	6	7.2	7.5	4.7	3.9	4.1	3.9	3.7	— 0.3	— 2.6	+ 0.2	— 0.2
EM	12	5.6	5.0	8.1	6.6	8.3	8.9	8.0	+ 0.6	+ 2.8	+ 1.4	— 0.9
EH	12	5.7	3.8	7.9	7.3	5.9	6.3	4.8	+ 1.9	+ 3.1	— 1.4	— 1.5
EHH	9	4.7	2.1	6.3	6.7	4.3	4.2	3.0	+ 2.6	+ 2.9	— 2.4	— 1.2

The differences between minimum and maximum for both Cepheids show without any doubt that the temperature at minimum is lower than at maximum. This is in accordance with the result deduced by ADAMS and SHAPLEY. It is in variance with the later negative result of ADAMS and JOY, for it shows that the variation of temperature does not only appear in the hydrogen lines, but also in the other lines, the arc lines as well as the enhanced lines. That an increase of temperature at maximum is indicated by the intensity of the spectral lines disproves all theories where the increase of light and the change of colour at maximum is attributed to the disappearance or thinning out of strong absorbing and red-colouring nebulosity in the stellar atmospheres. By a comparison of the AM and EM lines we see, with less certainty, that lines not varying in going from G to F dwarfs, show a small increase of ionization at minimum for δ Cephei, at maximum for ζ Geminorum. Since, however, the ionization from G to F dwarfs increases by temperature, this means that also for the Cepheids the ionization is stronger at maximum than at minimum.

From the whole of the line intensities of Table II the coordinates p and q have been computed, and this has been done also for each of the separate negatives. In Table VII the second column contains the date and time of exposure, and the third column the phase (expressed in periods) counted from maximum: for δ Cephei after HERTZSPRUNG's formula (*Astr. Nachr.* **210**, 17, 1919) (minimum at 0.7), for ζ Geminorum after RABE (*Astr. Nachr.* **219**, 130, 1923) (minimum at 0.5), for α Ursae minoris after PANNEKOEK (*Astr. Nachr.* **194**, 359, 1913). The other columns contain the coordinates p and q, and the values of the parameters c_2/T and $\log g$, computed by means of the formulae given above. They are also inserted in the diagram.

The variation of temperature between maximum and

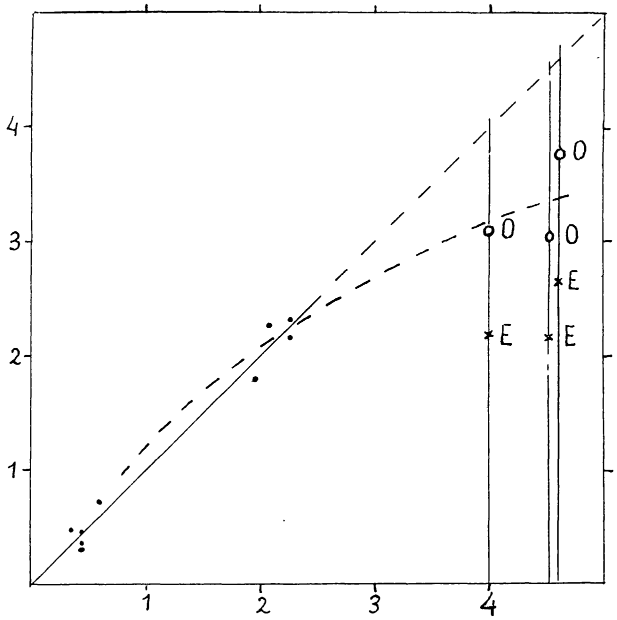
TABLE VII. Results for the Cepheids.

Star	Time	Phase	p	q	c_2/T	log g
δ Cep.	1907 Oct. 7 17^h 11^m	0.974	+ 32.9	+ 13.3	2.28	— 3.70
»	1907 Oct. 7 18 40	985	+ 33.9	+ 14.7	2.27	— 3.82
»	1907 Sept. 25 17 38	741	— 13.6	+ 15.8	2.77	— 4.44
»	1907 Oct. 6 19 36	806	+ 1.2	+ 13.0	2.60	— 4.01
α UMi	1919 Nov. 28 14 15	0.062	+ 10.3	+ 18.8	2.53	— 4.47
»	1919 Dec. 6 18 46	125	+ 8.0	+ 19.0	2.55	— 4.52
»	1919 Dec. 16 0 46	457	+ 4.3	+ 18.6	2.59	— 4.51
»	1919 Dec. 8 13 52	578	+ 5.6	+ 19.5	2.58	— 4.59
ζ Gem.	1899 Dec. 27 23 6	0.108	— 10.0	+ 20.4	2.75	— 4.84
»	1900 Jan. 10 19 25	473	— 27.8	+ 13.5	2.90	— 4.37

minimum for δ Cephei is found to be from 2.28 to 2.77, if the difference of the two minima is real, and from 2.28 to 2.68, if we take the mean for each phase. The latter values correspond to temperatures of 6400° at maximum and of 5400° at minimum. For α Ursae minoris the variation is found to be from 2.53 to 2.58 (temperature 5770° to 5660°), for ζ Geminorum from 2.75 to 2.90 (temperature 5310° to 5030°). (If the lines of constant T converge at the top of the diagram the variations in temperature become somewhat larger.) Thus for ζ Geminorum unmistakably a lower temperature is found than for δ Cephei, again at variance with the result of ADAMS and JOY, but in accordance with the rule established by SHAPLEY that the spectrum becomes more advanced with increasing period.

Results on the other coordinate, the gravity, are much more important, but they are also much more uncertain and difficult. The knowledge of log g would allow to find the mass of the star, because the absolute brightness is known as a function of the period by Miss LEAVITT's relation. But the values of log g, computed by the formula or read from the diagram, rest on a strong extrapolation, and we do not know how much the assumed linear relation between estimated line intensities and log g deviates from the truth. Still some conclusions of interest may be drawn. From the absolute magnitude of δ Cephei — 2.19, or log $I = 3.08$ we find (with log $\sigma = -0.06$ for $c_2/T = 2.48$) log $R = 1.57$, and log $g' =$ log μ — 3.14. If this star has a mass log $\mu = 0.94$ according to EDDINGTON's relation between the masses and the luminosities of the stars (*Monthly Notices* **84**, Plate 8) the gravity at its surface must be log $g = -2.20$. Such a large value cannot be reconciled with the results for q. In order to see within what limits the line intensities of the spectrum determine gravity and mass a diagram is given in fig. 3 where the spectral values of — log g, computed from q by the formula are taken as abscissae, and the orbital values of — log g are taken as ordinates. The value computed from EDDINGTON's relation is denoted by E, the value for log $\mu = 0$ by O. A possible curve cannot be laid through the first named points. If the curve takes its course through the zero points O it means that the line intensities will show a large variation already for an insignificant increase of — log g. Thus we may say that *a large mass, in accordance with* EDDINGTON's *relation, is highly improbable for the Cepheids*, and that *there is some probability that their masses are even smaller than the sun's mass.* *)

Figure 3.



This contradiction to EDDINGTON's relation may raise the question whether the assumptions underlying our formulae can be fulfilled in such pulsating stars; especially whether radiative equilibrium may be assumed and the gradient of the outer layers is determined by the gravity g. While it must be admitted that only an exact theoretical investigation of the conditions in the outer layers of a pulsating star can settle the question, still some general considerations may indicate that an explanation of the discordance in this way is not probable. We deal with the outer layers only, and they will certainly adjust themselves to the intensity of the outward stream of energy going through them very rapidly. The variation of temperature and pressure with depth may perhaps deviate appreciably from a quiet star with the same g; but since the variations are to and fro there will be some turning points, where for a moment ordinary conditions prevail. If we may take it for true (though our few data cannot show it) that also in the other phases the spectrum will afford values for p and q not much different from those found

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*) If WILSON's increase of 30 % to SHAPLEY's parallaxes of the Cepheids is right (*Astron. Journ.* 35. 44), the luminosity of these stars becomes smaller, the computed gravity larger and the difference with EDDINGTON becomes still larger.