## BULLETIN OF THE ASTRONOMICAL INSTITUTES OF THE NETHERLANDS.

1927 April 28

Volume IV.

No. 121.

## COMMUNICATIONS FROM THE ASTRONOMICAL INSTITUTE AT AMSTERDAM.

The determination of absolute line intensities in stellar spectra, by A. Pannekoek.

1. For the development of astrophysics an accurate knowledge of the intensities of the absorption lines in stellar spectra is necessary. Many valuable results have been obtained by simply estimating the relative intensities of stellar lines on an arbitrary scale; so the whole Mount Wilson work on spectroscopic parallaxes has been made by estimates of high precision; the results published in B. A. N. 79 and 87, too, may show the accuracy attainable. Still this method has serious disadvantages. Estimates cannot be repeated ad libitum, in consequence of bias by former results; the accuracy of objective measures, on the other hand, may be increased by repetition. Moreover, estimates must be made in arbitrary or subjective scales, so that a comparison of different stars is only possible by means of rather uncertain scale reductions. Absolute values of the intensities, which could be compared with some theory, cannot be obtained by estimates. Thus methods for deriving absolute line intensities from photographs of stellar spectra are highly desirable.

If by some instrumental means we measure the difference of blackness of the photographic spectrum between the absorption line and the continuous background we get an objective value of the line intensity instead of a subjective estimate. This may be useful in avoiding subjective errors and increasing the accuracy. But also in this case two different plates, with different exposures and different development cannot be compared directly. This is only possible by converting the density on the negative into brightness of light. If we call the line intensities that are obtained in this way "absolute", it must be borne in mind that they relate only to the spectrum as produced by the optical system of the spectrograph, where the lines have a width of 0.5 to 1 A. And not even exactly to this optical image, since, by the extension of the photographic action beyond the limits of the light, the resulting photographic image differs somewhat from the optical image. Thus we suppose an optical image exactly corresponding to the photographic image, only the scale of blackness converted into a scale of brightness; the difference of brightness within and beside a spectral line in this spectrum will be called its absolute intensity. In such a way all the spectra taken with the same instrument, or with nearly identical instruments, become comparable, without regard to exposure time, plate brand and development.

The relation between the brightness of the light Iand the density of the silver deposit s = f(I) is special for each plate, and on the same plate varies with the wavelength. In B.A.N. 41 it was found that s may be expressed by a quadratic exponential function of log I. Exact investigations by HURTER and DRIFFIELD and others show that this "characteristic curve" is nearly straight over a great part of its course, while only for very faint and very bright intensities the slope is less. In order to derive this relation we want a series of continuous comparison spectra on the same plate, produced by a light source, for which the diminution of intensity for each spectrum is exactly known. Exposures of the same source with different exposure times are not equivalent, because the Schwarzschild exponent p in the relation  $f(s) = It^p$ is different for different plates.

In his valuable memoir "The wedge method in stellar spectrophotometry" (Public. Dominion Astrophysical Observatory Victoria, Vol II, N°. 12) H. H. PLASKETT describes a contrivance of placing a neutral glass wedge before the slit, so that he gets a spectrum the intensity of which varies regularly with the height. At my request Mr. PLASKETT kindly has photographed some star spectra and on the same plates impressed the spectrum of an artificial light source (an amyl-alcohol flame) taken through this wedge. By placing a number of small strips at equal distances before the wedge the continuously decreasing spectral band was divided into a number of narrow bands. The midst of each of these bands correspond to regularly increasing light intens-

ities. Thus by measuring, for different wave lengths, the blackness in the midst of each band we get s for a number of regularly increasing values of I. So we are able to derive s=f(I) for each wavelength. Then by measuring s in the star spectrum we may derive I. In order to fix the scale of wavelengths an impression of the mercury lines 4347-4339 was made upon the continuous spectrum.

The wedge is not quite colourless, the wedge constant (per mm) for differen th being (l. c. p. 227—228) for

 $\lambda$  4100 4200 4300 4400 4500 4600 4700 4800 4900  $\sigma = 0.300$  0.284 0.270 0.258 0.248 0.240 0.234 0.228 0.224

Thus the difference of intensity between two consecutive bands varies with the wave length.

The thin end of the wedge and its top are visible as a white line running over the blackest spectrum, and a black line running below the spectra. Their distance was measured 3.815 mm; the distances of the central lines of the bands from the first named line were measured

Since the height of the wedge itself is 5.915 mm the optical system of the spectrograph gives a diminution of 1.551 times, and the central lines of the bands correspond on the slit to distances

from the thin end of the wedge. The distances between these points are

These differences being practically equal, we may attribute to the bands whole numbers I to 7 denoting the intensity of the band in a decreasing logarithmic scale. The value of unity in this scale is given by

$$\log I_n/I_{n+1} = 0.675 \ \sigma = p$$

where, with the above values of  $\sigma$ , the scale value  $\rho$  for  $\lambda$  4100 to 4900 is

By means of these factors the band numbers may be transformed into intensity logarithms.

In this method it is assumed that, if in the flame spectrum the densities  $s_1$  and  $s_2$  (for the same  $\lambda$ ) is caused by illuminations  $L_1$  and  $L_2$ , the same densities  $s_1$  and  $s_2$  in the star spectrum are caused by illuminations  $L_3$  and  $L_4$  proportional to the former:  $L_1$ :  $L_2 = L_3$ :  $L_4$ . This certainly is not strictly true if the exposure time is different for the star and the

flame; but the error may be expected to be of the second order and may be neglected in these first attempts.

## 2. Measures with the Hartmann microphotometer.

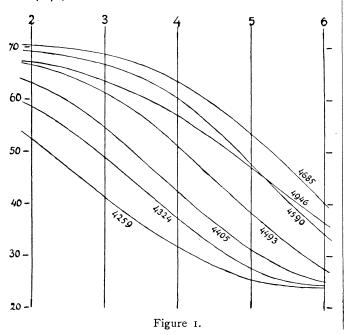
On the plates containing the spectrum of  $\delta$  Cephei Max. and ∂ Cephei Min. the comparison spectra as well as some points of the star spectrum have been measured with the Hartmann microphotometer. For the comparison spectra the circular diaphragm, for the star spectrum the slit diaphragm of the Lummer Brodhun prism was used. Afterwards a systematic difference between these measures was suspected; a series of special measures showed that with the slit — where the measures were more difficult — the dark shades were measured fainter, the faint shades darker than with the hole. A correction, varying regularly from + 1.5 for s = 62 to - 1.5 for s = 36, has been applied to the slit measures. We give here the discussion of the & Cep Max. plate only, the other having been made in the same way. In Table 1 the blackness for spectra 2-6 is given, already reduced to a homogeneous scale according to B. A. N. 44, 20; each line gives the mean of 4 points at intervals of 2 mm. The first column gives the scale reading in mm and the wavelength. The blackness of the non-exposed film was 23.9.

TABLE I.

Scale	, λ	Band	2 3	4	5	6	$n_{\circ}$	r
8.4 24.4 31.4 35.4 39.4 43.4 47.4 51.4 55.4 62.6	4946 4685 4590 4540 4493 4448 4405 4364 4324 4259	67.1 70.3 69.0 68.0 66.8 64.8 63.4 61.2 58.7 52.4	63.2 68.6 66.7 64.2 61.5 57.6 54.7 52.0 48.8 41.5	57.0 63.7 60.3 56.1 50.8 45.8 42.2 39.9 37.4 31.8	46.8 53.6 47.5 42.8 38.3 34.1 31.3 29.6 27.6 25.5	36.4 40.2 34.4 31.2 27.6 25.9 25.1 24.7 24.2	4.72 5.27 4.84 4.47 4.08 3.69 3.36 3.08 2.84 2.34	0.88 1.12 1.16 1.11 1.07 1.03 0.99 0.95 0.93 0.89

The derivation of the intensity, i. e. firstly of the band number n for a measured blackness s, would be a simple process of interpolation between the figures of Table I, if only the intensity intervals of the bands were very small. Since they are, however, rather large, a knowledge of the function s = f(n) is necessary and the interpolation becomes a more complicated process. If we represent the values of Table I in a diagram (see Fig. I) we see that the character of the curves for all wave lengths is the same, having the largest slope for an intermediate intensity. Their maximum slope, as well as their value for a certain intensity, vary with the wavelength; the maximum slope, corresponding to the largest gamma, is found for

 $\lambda = 4640$ , while the maximum value of the blackness



$$r(n-n_0) = -3.5 - 3.0 - 2.5 - 2.0 - 1.5 - 1.0 - 0.5$$
 0.0 + 0.5 + 1.0 + 1.5 + 2.0 + 2.5 + 3.0   
 $s = 70.0$  69.2 68.0 66.2 63.7 60.3 55.6 50.0 44.0 38.0 32.3 28.0 25.2 24.1

for the same band is found at  $\lambda$  4730 (this does not mean that the plate is most sensitive for this wavelength, because it depends also on the varying scale of  $\lambda$  and on the temperature of the light source used). These  $\lambda$  of maximum gamma and maximum intensity are in concordance with the curves given by F. E. Ross in his monograph The Physics of the Developed Photographic Image p. 73. In order to get a more accurate curve s = f(n) we will assume that by displacing the curves over a certain amount horizontally and varying their slope we can bring them into exact coincidence; in this way all separate curves may be reduced to one general curve, which is determined by a great number of points. Also if this supposition does not hold exactly, we still get an average curve, which may be used in interpolating between the figures of Table 1. Thus we assume that a value s standing for n in the separate curve, stands for r  $(n-n_0)$  in the general curve; the quantities  $n_o$  (value of n for s = 50) and r, given in the last column of Table 1, have already been smoothed. The general curve obtained in this way is given by

It must be remarked that the shape of this curve differs from the shape given by Ross; ithas a more continuous curvature and only a small straight part. By means of this curve the values of s corresponding to decimals of n were interpolated between the values of Table 1; then with the scale reading (or  $\lambda$ ), and the blackness s as rectangular coordinates an abacus was constructed showing lines of equal n (interval 0.1).

The measures with the Hartmann microphotometer have been used chiefly to study the density function by means of the comparison spectra. On the star spectrum a number of strong lines and of background points have been measured and reduced, but these measures are difficult and liable to systematic errors. It soon became manifest that for a study of the star spectra the registering microphotometer is better adapted.

## 3. Measures with the MOLL registering microphotometer.

With one of the MOLL registering microphotometers of the Physical Laboratory of the Utrecht University photograms have been made from the Victoria spectra of d Cephei as well as from some other star spectrum negatives. I am much indebted to Prof. S. L. ORN-STEIN, Director of this Laboratory, for the kind permission to use this instrument, and to Mr and Mrs VAN DIJK, assistants of the Laboratory, for valuable aid in making the photograms. The slit of

the thermocouple had a width of 0.3 mm; a 30 times magnified image of the spectrum was thrown on it, thus a width of o.o. mm on the spectrum, corresponding in the middle to 0.1 A, was registered. On the photogram the linear scale of the spectrum is magnified 20 times; the width of the narrowest absorption lines on the photogram was I to 2 mm. The vertical distance on the photogram between absolute blackness and non-exposed film was made 99 mm for  $\delta$  Cep M, 51 mm for  $\delta$  Cep m. On a second sheet photograms of the comparison spectra were made. The curves of these photograms are not simple straight or slowly curved lines but are composed of a series of small sinuosities, caused by irregularities in the film or the glass. The range of these irregularities corresponds to o.I' unit of the band number (0.02 in the logarithm); the uncertainty in drawing a straight curve through the middle of the wavelets may be estimated certainly below half this amount. In our photograms, especially for  $\delta$  Cep m a more important cause of error was found in the curves for the blackest bands creeping together near the zero line, where by their mutual influence the readings could be systematically falsified.

The vertical coordinate h in the photogram (reckoned from the zero point, no light, thus blackness ∞) is proportional to the current and this current is assumed to be proportional to the quantity of light falling upon the thermocouple. The blackness s meas1927BAN....4....1E

ured with the Hartmann microphotometer is the logarithm of the light transmitted. Thus  $\log h$  must be proportional to  $s-s_{\infty}$ . Comparing for these comparison spectr athe results for h and s it was found that indeed the relation between  $\log h$  and s is strictly linear between s=25 and 60; only for the highest values of s, above 60, corresponding to the faintest light and the densest silver deposit, a deviation was found. Here the thermocouple registers a fainter intensity of the long heat waves transmitted, while

the visual rays, used in the Hartmann photometer, are less absorbed by the silver grains.

The curves drawn through the photograms were read with a mm scale every 2 cm.; the averages of every 4 consecutive values are compiled in Table 2 (zero of the longitude scale is the mercury line 4358); the small values in the first bands denote the blackest spectra, while the high values converge to the limit for clear film.

TABLE 2.

∂ Cep Max.									δ Cep Min.									
Scale	λ	Bd I	2	3	4	5	6	Scale	λ	Bd 1	2	3	4	. 5	6	7		
- 8.2 - 0.2 + 7.8 + 15.8 + 23.8 + 31.8 + 39.8	4321 4358 4397 4438 4482 4529 4577	5.3 4.3 3.2 2.3 1.9 1.9	12.9 10.5 7.6 5.8 4.3 3.3 3.0	31.6 26.5 21.2 16.2 11.0 7.7 6.0	66.6 59.7 51.9 41.5 29.5 20.5	93.4 91.1 87.8 80.7 67.6 52.7 39.9	97.1 96.1 94.9 90.9 86.1 77.1	- 4.2 + 3.8 + 11.8 + 19.8 + 27.8 + 35.8 + 43.8	4339 4377 4417 4460 4505 4553 4602	1.7 1.6 1.5 1.3	3.4 3.0 2.8 2.5 2.0 1.3	9.0 7.2 5.7 4.2 2.6 2.1 1.8	21.6 18.3 15.0 11.1 7.2 4.9 3.9	39.7 36.4 32.3 25.8 17.9 12.4	48.7 47.3 43.9 36.6 28.0 21.4	48.5 44.8 40.4		

The construction of interpolation tables was made in the same way as explained for the measures with the Hartmann microphotometer: by the reduction r  $(n-n_o)$  a general curve was deduced, which was used to interpolate between the values of Table 2, and to construct an abacus for each photogram.

We will at first consider the background of the spectra. While with great dispersion the absorption lines appear sharp against a constant or slowly varying background, with the smaller dispersion of these star spectra the background between the relatively broad lines shows different degrees of blackness, continuously merging into the borders of the lines. Even where the eye sees a regular blackness, the photogram shows continuous variations in the height of the top line. Between closely adjacent lines, of course, the photogram does not reach the level of the other tops. But also if we exclude all these cases, where the influence of adjacent lines is manifest, and measure only the highest tops at a sufficient distance from these lines, the results show irregular differences. With the aid of the abacus these measures of the tops have been reduced to logarithmic intensity differences with the comparison spectrum. The result for both spectra is shown in Figure 2, giving  $\log I_0/I$  for the same tops in both spectra (horizontal scale is  $1/\lambda$ ).

Large fluctuations are shown in both spectra. They cannot be caused by errors in reading the curves; from the values of Table 2 and the scale value  $\rho$  we find (the band number of the tops for  $\delta$  Cep M varying from 3.0 to 4.2 and for  $\delta$  Cep m from 2.8 to 4.0, the  $H\gamma$  depression excepted) that 0.1 mm corresponds

for  $\delta$  Cep M. to 0.0005—0.001, and for  $\delta$  Cep m. to 0.001-0.003 in the logarithm; it must be remarked that on the minimum photogram with its smaller scale the curve is better defined so that the accuracy of the points in Figure 2 is hardly different for both spectra. That the fluctuations are a real feature of the stellar spectrum is confirmed by their coincidence in most cases for the two spectra; maxima occurring at 4365, 4400, 4413, 4447, 4476, 4505, 4534, 4572 and broad depressions at 4388, 4417, 4433, 4458, 4492, 4528, 4552, 4584. The amount of the fluctuations is much greater for  $\delta$  Cep min., than for  $\delta$  Cep M; this is in accordance with the increase of intensity of the absorption lines from class F to class G. The depressions show that nearly everywhere the apparently continuous spectrum is weakened by absorptions, and it is not certain whether even the highest tops measured represent the full intensity of the continuous radiation. Whether the source of these absorptions has to be found in the wings and the borders of the strong and the moderate absorption lines, or in the bulk of the faint absorption lines (o to oooo in Rowland's solar spectrum) not separately visible, or in some continuous band absorption, must be decided by further researches. It is not probable that in these fluctuations of blackness the Eberhard effect plays a material role; the whole spectrum is only a narrow strip, and the blackest parts do not occur in the vicinity of the most intense lines.

The hydrogen absorption  $H\gamma$  extends in broad wings in both spectra; in  $\delta$  Cep M it is visible up to 4400, in  $\delta$  Cep m to 4370. On the violet side the wings cannot

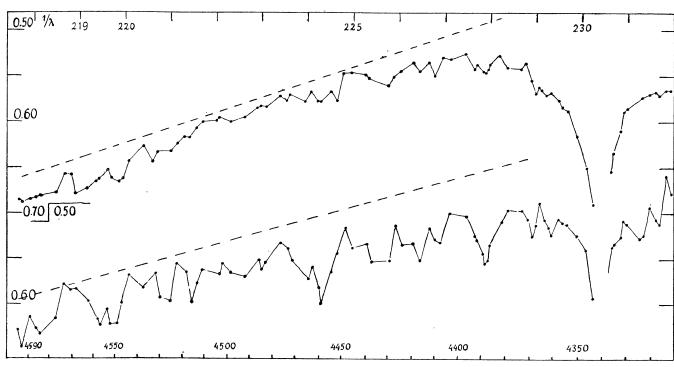


Figure 2.

be followed to the same distance because at 4310 the broad absorption band G begins, which in both spectra considerably lowers the tops of the photogram; here, however, our curves were not continued because the lines of the spectra become less sharp. An attempt to use the highest tops of the photogram as a representation of a continuous black spectrum, and to compute its temperature, failed. If a straight top line is drawn for the larger wave lengths (below 4450 the slope already diminishes somewhat) we find 0.43  $(c_2/T_0 - c_2/T)$  (the coefficient of  $I/\lambda$ ) = 1.80 (max.) and = 1.02 (min.); with the temperature of the comparison light source  $T_o = 2560^{\circ}$  and  $c_2/T_o = 6.12$  this gives  $c_2/T = 1.98$  (max.) and = 3.77 (min.). With the values of  $c_2/T$  in maximum and minimum 2.27 and 2.77 (B. A. N. 87, 54, Fig. 1) the coefficients of  $I/\lambda$  in  $\log L_{\rm o}/L$  should be 1.67 and 1.46; they are represented by the slope of the dotted lines in Fig. 2. For the minimum spectrum the deviations are large; they are, however, in accordance with the deviations from black radiation obtained by A. BRILL in his spectral photometric researches (Astron. Nachr. 219, 353).

It may be remarked that by the shape of the characteristic curve s = f(I), which has a small slope for high intensities, the variations in brightness of the continuous background of the spectrum are considerably diminished in the blackness, thus in the visual appearance of the spectrogram.; they reappear as large variations if we derive this brightness by our method.

4. In the photograms a number of the strongest absorption lines in both spectra have been read. They were reduced in the same way and the results are collected in Table 3, where for each spectrum the first column gives the reading in mm., the second the band number read from the abacus, the third log I relative to the comparison spectrum. The errors of reading the curve are wholly negligible here; according to the figures of Table 2 a difference of 0.01 in band number, or 0.002 in the logarithm, corresponds to 0.2 or 0.3 mm. in the curve. This precision is superfluous, because the errors of the curve itself, caused by irregularities of the silver deposit, will be larger; they may be estimated as soon as we have different photograms made from the same negative. A comparison of several negatives of the same spectrum will then be necessary to determine the real precision attainable by a spectrogram. For the strongest absorption lines the accuracy again diminishes, as shown by the last colum in Table 2. By their great intensity they will appear broad and will thus be estimated strong, but for determining the central intensity accurately strongly exposed negatives will be necessary. This is the case with  $H\gamma$  in  $\delta$  Cep. Max.

The strength of an absorption line is determined by the difference of intensity between the centre of the line and the continuous background. For the background  $\log I_c$  we have adopted a regular curve through the highest tops, defined by the following points

The last column in Table 3 gives the resulting line intensities. For the minimum they are, with the

exception of  $H\gamma$ , considerably larger than for the maximum, in accordance with the more advanced spectrum. If in a diagram the intensities of minimum and maximum are taken as coordinates, it appears that nearly always the high temperature lines, indicated HH and H in B. A. N. 79, are relatively more intense in

TABLE 3

_	Maximum Minimum								Maximum					Minimum			
λ	Reading	Band nr.	$-\log I$	$-\log I/I_c$	Reading	Band nr.	$-\log I$	$-\log I/I_c$	λ	Reading	Band nr.	$-\log I$	$-\log I/I_c$	Reading	Band nr.	$-\log I$	$-\log I/I_e$
4340.6 4369.9 71.5 76.1 77.0 83.7 85.5 87.0 91.2 94.2 95.2 94.2 95.2 94.3 99.9 4400.6 01.6 04.9 07.8 08.6 09.5 11.2 12.1 15.3 17.0 17.9 22.0 22.8 27.4 35.2 43.3 44.0 50.7 50.7 50.7 50.7 50.9 50.9 50.9		>> 6 4.25 4.01 4.60 3.96 4.47 4.26 4.16 4.18 4.28 4.49 3.98 4.25 4.02 4.39 3.82 3.84 4.06 3.82 3.74 4.40 4.18 4.30 3.95 3.91 4.16 4.28 4.46 4.58 4.46 4.58	> 1.08 0.751 708 810 697 785 748 783 748 783 694 738 740 698 763 664 673 704 663 743 681 723 743 681 776 714 733 790 760 780 734	> 0.50 208 167 276 165 257 201 205 222 257 168 212 213 171 236 136 136 145 175 134 118 230 191 211 147 139 178 171 185 241 208	46.3 25.8 20.5 35.0 17.1 16.9 18.8 18.5 31.7 19.1 27.0 19.9 29.2 16.2 17.9 13.3 12.1 34.1 19.0 24.7 13.2 16.9 19.4 27.0 19.4 27.0 19.4 27.0 18.1	5.52 4.39 4.08 4.91 4.83 4.54 4.09 4.84 4.16 4.62 4.77 4.16 4.16 4.16 4.16 4.16 4.16 4.16 4.16	0.994 775 720 868 688 848 796 692 716 714 844 725 786 829 696 722 718 650 881 737 796 659 718 750 785 769 841 805 776	0.46 285 230 379 199 358 306 201 224 221 351 231 291 309 240 332 197 2238 164 149 234 293 154 212 242 273 254 325 286 316 297 218 319 219 219 219 219 219 219 219 2	4466.8 68.7 69.5 71.1 73.0 76.2 81.4 90.0 91.6 94.7 97.1 4501.5 08.4 22.8 25.2 28.8 25.2 34.2 41.7 49.8 54.2 56.1 58.8 63.9 72.2 76.5 84.0 88.4	41.2 62.3 49.1 44.7 40.4 70.6 44.4 44.8 46.1 41.6 37.3 58.4 45.0 62.8 42.6 39.0 74.2 42.7 55.6 64.8 37.5 55.6 64.8 37.5 55.6 64.8 37.5 55.6 64.8	4.21 4.73 4.42 4.33 4.25 5.06 4.41 4.53 4.44 4.53 4.48 4.53 4.85 5.21 4.77 4.69 5.60 4.86 6.28 5.34 5.34 5.62 4.94 5.78 5.62	0.714 802 749 733 718 721 853 743 752 761 746 731 802 806 754 806 865 782 828 789 776 927 802 1033 878 878 878 878 876 919 805 940 837	0.151 238 184 166 150 277 166 170 177 159 143 234 202 200 145 195 251 167 209 155 304 171 396 237 24 209 142 209 155 251 167 209 169 169 169 169 169 169 169 169 169 16	15.1 25.3 19.0 15.6 12.7 20.5 14.8 15.1 13.4 13.9 12.1 21.0 15.0 14.4 9.9 13.5 21.3 16.1 12.2 12.8 24.1 11.7 31.2 17.8 12.7 17.8 12.7 17.8 12.7 17.8 12.7 17.8 17.8 17.8 17.8 17.8 17.8 17.8 17	4.41 4.66 4.79 4.49 4.32 4.96 4.64 4.64 4.64 4.64 4.81 5.38 4.71 6.10 5.57 4.86 5.57 6.86 6.16 6.16 6.18	0.748 858 797 761 725 730 827 768 780 763 774 750 864 750 893 797 845 794 803 933 799 1013 906 893 831 877 947 786 922 793	0.221 330 268 232 195 198 292 233 241 223 231 206 318 256 282 231 239 367 229 439 329 358 293 358 293 358 293 358 293 293 358

maximum, the lines M and L more intense in minimum. There are deviations also; though partly caused by accidental errors they are chiefly due to systematic differences between the estimates from which these characteristics have been deduced and the measures. Such differences become also visible in directly comparing estimates and measures of the same lines, and find their origin probably in differences of width, which are included in the estimates and not in the measures.

The chief difficulty in this method of finding absolute line intensities lies in the continuous background. In deriving the intensities of the strongest lines a regular curve through the highest tops has been assumed. This is founded on the assumption that the

lower tops next to the lines are affected by the borders of the lines themselves and thus do not give the true background. For the weakest lines, which present themselves as small depressions of the curve, the immediately adjacent tops and not the highest topline must of course be used. How a medium course shall be followed for lines of medium intensity remains uncertain. This question can only be answered when by special researches the intensity distribution in a line, up to its extreme borders, thus the shape of the intensity curve, has been investigated. Then also the mutual influence of close lines down to doubles and blends has to be studied. These studies are now taken up at this Institute,