2. Durch die Kombination von zwei vestibulären Reizen ist es bisher nicht gelungen, eine Superposition von zwei Nystagmusformen zu bekommen. Bei einer derartigen Kombination, wenn also bei einem bestehenden vestibulären Nystagmus ein anderer vestibulärer Reiz zugeführt wurde, der ebenfalls Nystagmus auslöst, trat immer nur eine Frequenzerhöhung, resp. Frequenzverminderung auf, auch in den Fällen, in denen die beiden vestibulären Reize je einen Nystagmus von ganz verschiedener Frequenz auslösen.

3. Im Verein mit anderen, oben näher erörterten Tatsachen sprechen die Resultate dieser Versuche dafür, dass eine vestibuläre Genese des Dunkelnystagmus sehr unwahrscheinlich ist.

Astronomy. — *The Stark effect of hydrogen in first type stellar spectra.* By A. PANNEKOEK and S. VERWEY.

(Communicated at the meeting of April 27, 1935).

1. The broad lines of hydrogen in the spectra of class *A* stars were explained by O. STRUVE [1]) as results of a Stark effect in the atmospheres of these stars. By the Stark effect each hydrogen line in a homogeneous electric field is split up into a definite number of components, at distances increasing with the field strength. In a stellar atmosphere there are a number of negatively charged free electrons and positively charged ions, due to the ionization of part of the atoms. They produce electric fields by which the lines, emitted or absorbed by the hydrogen atoms in their vicinity, are split into components. As the distances of the charged particles to the hydrogen atoms are different, and change continually, the electric fields working on an atom are also variable, strong fields occurring less frequently than weak fields. Hence Stark patterns of every scale are superposed, and the result is that in this atom and electron mixture the hydrogen lines are broadened to bands with their intensity decreasing with the wave length difference.

The derivation of the intensity distribution within an absorption line of hydrogen in a stellar spectrum (i.e. the line profile) has to proceed in a number of consecutive steps.

Firstly we want to know the probability of each value of the electric force affecting a hydrogen atom in the atmosphere. This probability for a mixture of atoms and charged particles has been derived by HOLTSMARK. If there are n_e charged particles in unit volume, the mean distance to an atom is $\backsim n_e^{-1/3}$, and the mean electric field strength is $\backsim n_e^{2/3} e$, where e is the electron charge. Hence HOLTSMARK takes a normal electric force

1) Astrophys. J. **69**, 173 (1929).

$F_n = 2.61\ n^{2/3}e$, and the probability of each value of this force is derived by him as a function $W(\beta)d\beta$, where $\beta = F/F_n$. He gives a series development of $W(\beta)$ with increasing powers of β, which is convergent for small β, and another semiconvergent series with increasing powers of β^{-1}; by means of these series the function W can be computed with sufficient accuracy for every β. The shape of the curve is given in Fig. 1. Introducing the electron pressure P, we have for ions and electrons together $n_e = 7.29 \times 10^{15} \times 2P/T$, and $F_n = 46.8\ (2P/T)^{2/3}$.

For each Stark component of a line we denote by I its intensity and by c its displacement in a field $F = 1$. Then the real displacement is $\Delta\lambda = c\beta F_n = 46.8\ c\beta(2P/T)^{2/3}$. Each component, the central one excepted, is spread over all values of β, according to $W(\beta)$, forming a band with

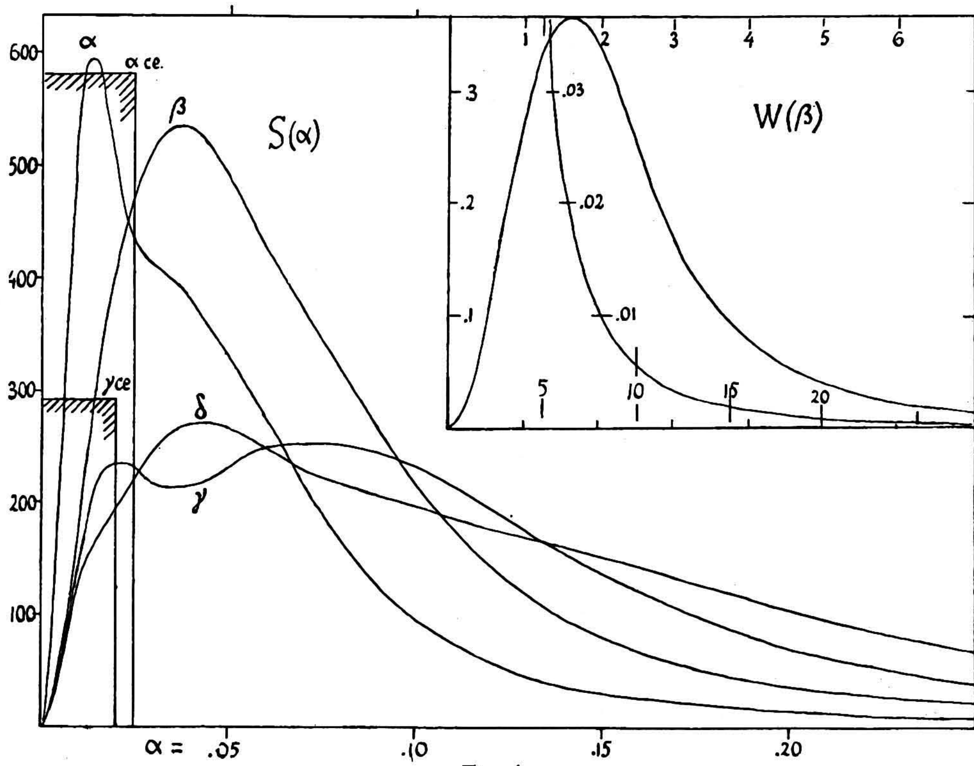

Fig. 1.

intensity distribution W, the scale in wave length being given by c. The bands produced by all the components are superposed. Denoting $c\beta$ by a, the intensity at some $\Delta\lambda$ due to each component is given by $IW(a/c)d(a/c)$ and the total intensity arising from all the components is $\Sigma IW(a/c)da/c$. If we write $S(a)da$ for this expression, then $S(a) = \Sigma I/c$. $W(a/c)$ can be computed from I and c, the data of the Stark components, for every value of a. As we now have $\Delta\lambda = 46.8\ a(2P/T)^{2/3}$ we can say that a represents the wave length difference for some standard conditions. For other values of P and T we can compute the relation of the scale of α to the scale of wave lengths.

The function $S(\alpha)$ determines the intensity distribution in a hydrogen line, spread by the Stark effect into a band, in a gas of determined P and T. It can be computed for each hydrogen line for which the distances c and the intensities I of the components are known. For the four lowest members of the Balmer series $H\alpha$—$H\delta$ the intensities as computed by SCHRÖDINGER by means of wave mechanics, were used. The functions $S(\alpha)$ for these four lines are represented by the curves in Fig. 1. Since the I mean here the intensities in fractions of the total intensity and the unit of W is chosen so as to make $SW(\beta)d\beta$ put $= 1$, the total intensity $\int S(\alpha)\, d\alpha$, after adding the central component, must be $= 1$ too. In the figure the intensities of the central components of $H\alpha$ and $H\gamma$ are represented by the surface of rectangles at $\alpha = 0$; the whole figure of course must be doubled and symmetrically repeated for negative values of α.

Here the hydrogen line without the Stark effect was supposed to have no width. In reality it extends over a finite width owing to resonance damping and Doppler broadening. So the different effects must be combined now. In the absence of Stark effect the diffusion coefficient in the realm of the line consists of two terms, the numerical expressions of which are given by

$$\frac{s}{n} = [6.204]\left(\frac{\lambda}{4000}\right)^2 \frac{\delta}{\gamma}\frac{1}{\Delta\lambda^2} + [11.596]\left(\frac{\lambda}{4000}\right)\left(\frac{5040}{T}\right)^{1/2} e^{-(\Delta\lambda/l)^2},$$

$$l = 0.122\,\frac{\lambda}{4000}\left(\frac{T}{5040}\right)^{1/2}.$$

Here n is the concentration of active atoms, l determines the width of the Doppler core, and δ/γ is the Weisskopf-Wigner correction factor of the damping constant. The numbers in brackets are logarithms, and $\Delta\lambda$ is expressed in A.U.

Each value in this intensity distribution is now spread and broadened by the Stark effect. Since the intensity contained in the Doppler core is 10^4 times greater than the total intensity of the resonance wings, the latter may be entirely neglected, and only the second term in s/n has to be taken into account. This means that each point of the $S(\alpha)$ curve must be broadened to a Doppler curve. Since the width of the Doppler curve for H is of the order of $\frac{1}{2}$ A.U. and the Stark broadening amounts to tens of A.U., it is clear that in its main part the $S(\alpha)$ curve remains unchanged. In the central parts of the line the Doppler broadening of the inner parts of the $S(\alpha)$ curve fills up the zero intensity in the middle of the gap in this curve. Where a central Stark component is present ($H\alpha, H\gamma$) its Doppler curve contributes to produce a strong central peak. Compared with the great intensity of the Stark wings, produced by the dispersion of the high intensity of the Doppler core, the resonance wings are insignificant.

To find now s/n for a gaslayer of determined P and T, we have for the main part simply to distribute the total intensity of the line $\int (s/n) d\lambda$, i.e. the total intensity of its Doppler term $[10.682]\left(\frac{\lambda}{4000}\right)^2$ over the Stark curve $S(a)d(a)$, transformed to the required scale of wave lengths. As $a/\Delta\lambda = 1/46{\cdot}8\ (2P/T)^{-3/2}$, the distribution in $\Delta\lambda$ can be expressed by $(S\Delta\lambda(a/\Delta\lambda))\ (a/\Delta\lambda)d\Delta\lambda$, the $S(a)$ curve is extended by the ratio of the scales of $\Delta\lambda$ and a, and lowered in the same ratio. Then s/n is found by putting the coefficient $[10.682](\lambda/4000)^2$ before it. For the central parts, however, a combination must be made of the $S(a)$ and the Doppler curve, for each $\Delta\lambda$ integrating the contributions of all the adjacent parts of $S(a)$ and, for $H\alpha$ and $H\gamma$, adding the contribution of the central component.

In the stellar atmosphere layers of decreasing density and electron pressure are situated one above another. In the highest layers P is exceedingly small, the Stark effect broadens the line only over a small extent in $\Delta\lambda$, less perhaps than the resonance wings. In the deeper layers the s/n values extend over ever broader ranges of $\Delta\lambda$, at the same time decreasing in amount. The character of the change may be seen from Table 1, where for $H\gamma$ and $H\delta$ the distribution of s, for a certain constant n

TABLE 1. $s \times 10^{-8}$.

$H\gamma$.

$2P$ \ $\Delta\lambda$	0	0.1	0.2	0.5	1.0	2	5	10	20
0	2890	2220	1010	4.2	.019	.0047			
10	2500	2010	1100	33.3	2.4				
10^2	1295	1215	1005	298	33.0	4.4	.4		
10^3	474	416	312	246	233	65.5	5.0	.85	.19
10^4	346	268	131	36.5	53	49.2	46.9	11.8	1.7
3×10^4	341	263	121	6.9	16.8	25.6	26.0	23.3	6.5

$H\delta$.

$2P$ \ $\Delta\lambda$	0	0.1	0.2	0.5	1.0	2	5	10	20
0	2740	2040	844	1.83	.017	.004			
10	2200	1790	968	41.6	4.0	.9			
10^2	830	832	790	347	49	7.0			
10^3	106	128	175	237	180	90	7.8	1.27	
10^4	3.8	5.6	10.3	27.8	42.2	54.7	37.2	16.8	2.7
3×10^4	0.5	0.8	1.5	5.9	13.5	19.9	25.0	18.3	8.65

over $\triangle\lambda$ for different depths, expressed by $2P$ in the case of $T = 11000°$, is given. Passing, for one determinate $\triangle\lambda$, through the different layers, from the surface downward, we find small values at the top, first increasing with depth and afterwards decreasing. For very small $\triangle\lambda$ the surface values are the largest and decrease for deeper layers. A comparison of the tables for these two lines shows the influence of the central component.

The absolute values of s depend on the concentration n. The total abundance of hydrogen in stellar atmospheres may be put 1.00. The non-ionized fraction is given by $1 - x = P/(K + P)$, or for high temperatures P/K. The fraction y_2 of these atoms in the second state, able to absorb the Balmer lines is given by the weight and the Boltzmann function. The fraction of these atoms producing $H\alpha$ (or $H\beta$ etc.) is given by the "oscillator strength" f, computed by SUGIURA. Combining these factors, we may write $n = P/K \,.\, y_2 f = n'P$, and $s = (s/n)n'P$, where s/n is the value derived above.

The residual intensity in some point of the absorption line produced by these layers is found by solving the well known equations

$$\frac{dJ}{d\xi} = 3\,(k + s)\,H \;;\; \frac{dH}{d\xi} = k\,(J - E) \;;\; E = E_0\,(1 + {}^3/_2\, c\bar{k}\xi) \;;\; J(0) = 2\,H(0)$$

where $4\pi J$ is the mean radiation intensity, $4\pi H$ the net stream, $4\pi E$ the black body radiation and $d\xi = -\varrho dh$. The absorption coefficient for these high temperature stars where most of the hydrogen atoms are ionized, is proportional to the electron pressure $K = \varkappa P$, and the total pressure $p = 2P$. Then introducing the optical depth $dt = kd\xi = 2\varkappa/g \,.\, PdP$, or $t = (\varkappa/g)P^2$ and $E = E_0(1 + {}^3/_2 ck/\bar{k} \,.\, t)$, we have $s/k = (s/n)n'/k$, and the equations are

$$\frac{dJ}{dt} = 3\left(1 + \frac{n'}{\varkappa}\left(\frac{s}{n}\right)\right)H \;;\; \frac{dH}{dt} = J - E_0\left(1 + \frac{3}{2}\,c\,\frac{\bar{k}}{k}\,t\right); \qquad J(0) = 2H(0).$$

Here $n'/\varkappa$ is constant throughout the layers; (s/n) is given for each $\triangle\lambda$ as a numerically computed table of values, (of which the above table 1 is an extract) depending on P and, by $P = \sqrt{tg/\varkappa}$, on t. The differential equations were solved numerically, in the way indicated in a former paper [1]), for all the values of $\triangle\lambda$. The residual intensity is then found by $r = H_0/H_{0'(s=0)}$, the ratio of the value of the surface intensity H at some $\triangle\lambda$ in the line to the surface intensity in the continuous spectrum.

2. The results of the first computations, made for $T = 11000°$, showed that these four Balmerlines are nearly identical in width and profile, and that the width of the lines for the same temperature increases in a regular way with the gravity. This is shown by the diagram Fig. 2, where the profile of $H\gamma$ is given for $\log g = 1, 2, 3, 4$ and 6. It is clear from the above formulas that for a larger g the same optical depth t corresponds to a larger value of P, with greater density and stronger Stark effect.

[1]) Monthly Notices R. A. S. **91**, 144.

The regularity of this phenomenon suggests a simple relation between hydrogen line width and gravity. Except in the central parts of the line or for very large g the resulting intensity is produced in layers where only the outermost part of the S curve is used, with a considerably larger

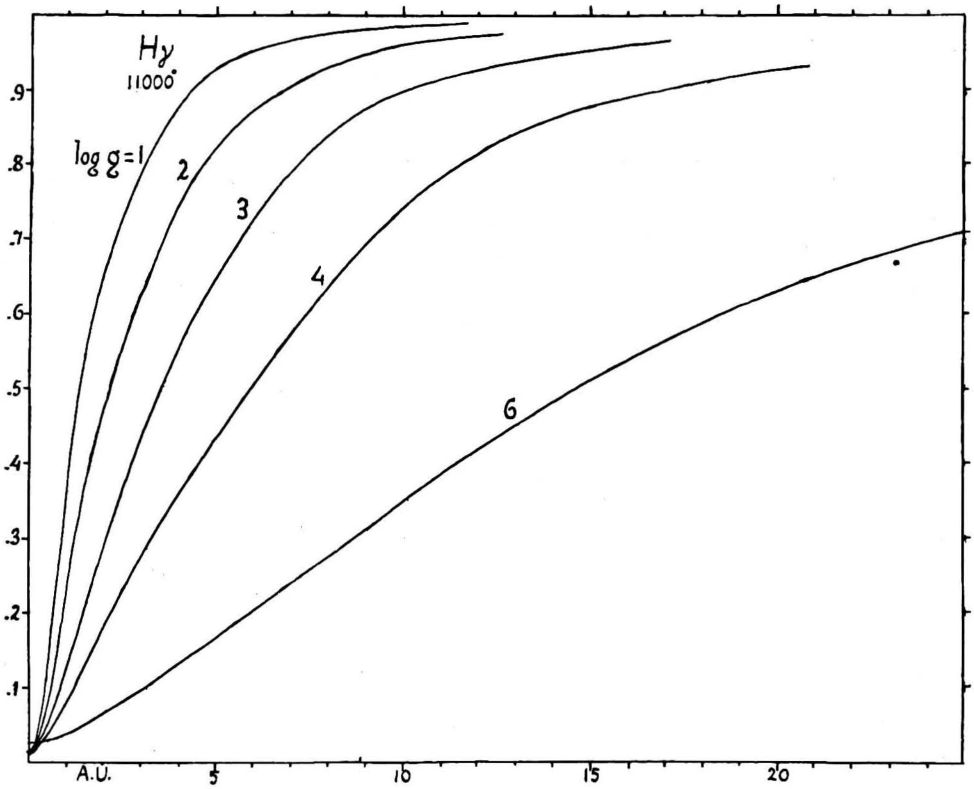

Fig. 2.

than the value for which $S(a)$ has its maximum. These S values on the downward slope have been formed by the $W(\beta)$ function for large β. In this case we may restrict W to the first term of the semiconvergent series, and we have

$$W(\beta)\, d\beta = [.1749]\, \beta^{-5/2}\, d\beta;$$

$$S(a)\, da = \Sigma\, I\, W(a/c)\, da/c = [.1749]\, a^{-5/2}\, da\, \Sigma\, c^{3/2} I = S_0\, a^{-5/2}\, da.$$

$$S\left(\Delta\lambda \frac{a}{\Delta\lambda}\right) \frac{a}{\Delta\lambda}\, d\Delta\lambda = S_0 (\Delta\lambda)^{-5/2} \left(\frac{a}{\Delta\lambda}\right)^{-3/2} d\Delta\lambda = C\, (\Delta\lambda)^{-5/2}\, (P/T)\, d\Delta\lambda.$$

The value of s/n in the outer parts is distributed after $(\Delta\lambda)^{-5/2}$ and proportional to P. Then for the quantity s/k of the differential equation we find

$$s/k = \frac{n'}{\varkappa}\, C P T^{-1}\, (\Delta\lambda)^{-5/2} = n'\, C\, T^{-1}\, (\Delta\lambda)^{-5/2}\, g^{1/2}\, \varkappa^{-3/2}\, t^{1/2} = a\sqrt{t}\,.$$

The equations have now the same form for all $\triangle\lambda$:

$$\frac{dJ}{dt}=3\,(1+a\sqrt{t})\,H\,;\quad \frac{dH}{dt}=J-E_0\left(1+\frac{3}{2}\,c\,\frac{\bar{k}}{k}\,t\right).$$

They cannot be solved analytically but only numerically; the resulting residual intensity is a function of the coefficient a. If for $a=a\,(\frac{1}{2})$ the residual intensity is 0.5, then the corresponding half width $\triangle\lambda$ is determined by

$$n'\,C\,T^{-1}\,\varkappa^{-3/2}\,(\triangle\lambda)^{-5/2}\,g^{1/2}=a\,(^1/_2),\quad\text{or}\quad \triangle\lambda=(Cn'/Ta\,(^1/_2))^{2/5}\,\varkappa^{-3/5}\,g^{1/5}.$$

Analogous formulas are found for other $\triangle\lambda$ corresponding to other values of the residual intensity. This formula gives the dependence of the half width of the line on the physical and stellar parameters. It shows that for g increasing 10 times the half width increases $\sqrt[5]{10}$ times. This approximate computation is confirmed by readings from the curves of the separate numerical solutions. They give

	for $\log g=1$		2		3		4		6
$r=0.3$	$\triangle\lambda=0.82$		1.17		2.0		3.2		8.7
0.5	1.32		2.15		3.5		5.9		14.6
	$\log\triangle\lambda=9.91$	16	0.07	23	0.30	20	0.50	44	0.94
	0.12	21	0.33	22	0.55	22	0.77	39	1.16

The differences in $\log\triangle\lambda$ are nearly 0.20 for a difference 1 in $\log g$.

3. The result that the width of the hydrogen lines increases in such a regular and steady manner with the surface gravity is in the first place important for the recognition of white dwarfs. The white dwarfs as a separate starclass are distinguished by their high mean density — of the order of 10^5 times the sun's mean density — a consequence of normal mass but very small dimensions. This implies a high value of the surface gravity, for Sirius *B* 10^3 times the value for the sun. For Sirius *B* this could be tested by the EINSTEIN displacement of the lines. For white dwarfs which are not members of a binary we have no means to determine the EINSTEIN change of wave length, because we do not know their radial velocity. Here the class *A* spectrum and a large proper motion or a large measured parallax indicating a low luminosity may give a strong presumption that we have to do with a white dwarf. But only the knowledge of its mass gives certainty. For an isolated star this is only possible by means of the surface gravity. The hydrogen lines by their great width allow us to recognize the white dwarf character with certainty.

In *Monthly Notices R.A.S.* **92** p. 71 YNGVE ÖHMAN published the microphotometer tracing of the spectrum of the star Van Maanen 1166, situated in the h Persei region, of which OOSTERHOFF had suggested that it was a white dwarf by reason of its white colour, its faintness and its large proper motion. The tracing shows only hydrogen lines of an extreme width, between 100 and 200 A.U. for $H\gamma$. It is compared with a tracing of the spectrum of the Companion of o_2 Eridani, which is known to be a white dwarf, and for which the lines have a width of nearly 100 A.U. This width alone is sufficient to establish the white dwarf character of both stars. If the spectra had been standardized, so that we could determine for what residual intensity the width is 100 A.U., the value of the surface gravity could be found fairly exactly. Now we can only make an estimate that the gravity is larger for OOSTERHOFF's star than for o_2 Eridani B. This method is applicable to all single stars suspected to be white dwarfs. A well standardized spectrum suffices to determine their surface gravity — we suppose, of course, that the temperature is known from the spectral class. If moreover, besides magnitudes and spectral class, the parallaxes have been measured, their mass and density can be deduced.

4. This holds not for the white dwarfs only. For all the white stars with strong hydrogen lines the width of these lines — for a given concentration of hydrogen atoms in the second state, which depends on temperature — is directly and strongly variable with gravity. The well known fact that supergiants (α Cygni) have narrow hydrogen lines, is a special instance of this relation. Hence from the width of these lines the surface gravity of white stars can be found.

For this purpose the width of these lines as a function of effective temperature and gravity of the star must be computed. Because we have to do with the wings of the lines only, it is not necessary to make use of the entire complicated S function. As, on the other hand, the use of one single term of the series for W suffices only in the extreme wings, two terms of the HOLTSMARK series were used in these computations

$$W(\beta) = [.1749]\,\beta^{-5/2} + [.88306]\,\beta^{-4}.$$

$$S(a) = S_0\,a^{-5/2} + S'_0\,a^{-4}\ ;\ S_0 = [.1749]\,\Sigma\, c^{3/2} I\ ;\ S'_0 = [.88306]\,\Sigma\, c^3 I.$$

For $\beta > 9$ the error in W is at most 1.6 %, entirely sufficient for the wing intensities.

The absorption coefficient, which for high temperatures was taken $\backsim P$, is variable with some lower power of P for lower temperatures (below 8000° for $\log g = 1$, below 11000° for $\log g = 6$); the values assumed were taken from the computations carried out at the Amsterdam Institute.

The resulting profiles (for $\log g = 4.4$, Sirius, α Lyrae) for the four hydrogen lines are given in Fig. 3 for temperatures 16800°, 12600°, 8400°. We see there that these lines are nearly equal in width; there is a slight increase from $H\alpha$ to $H\delta$. The decreasing number of atoms active in producing these lines is compensated by the increasing width of the Stark pattern. The exponential curve deduced by E. T. ELVEY[1]) from his measurements of $H\gamma$ in α Lyrae is given as a dashed line; the measurements

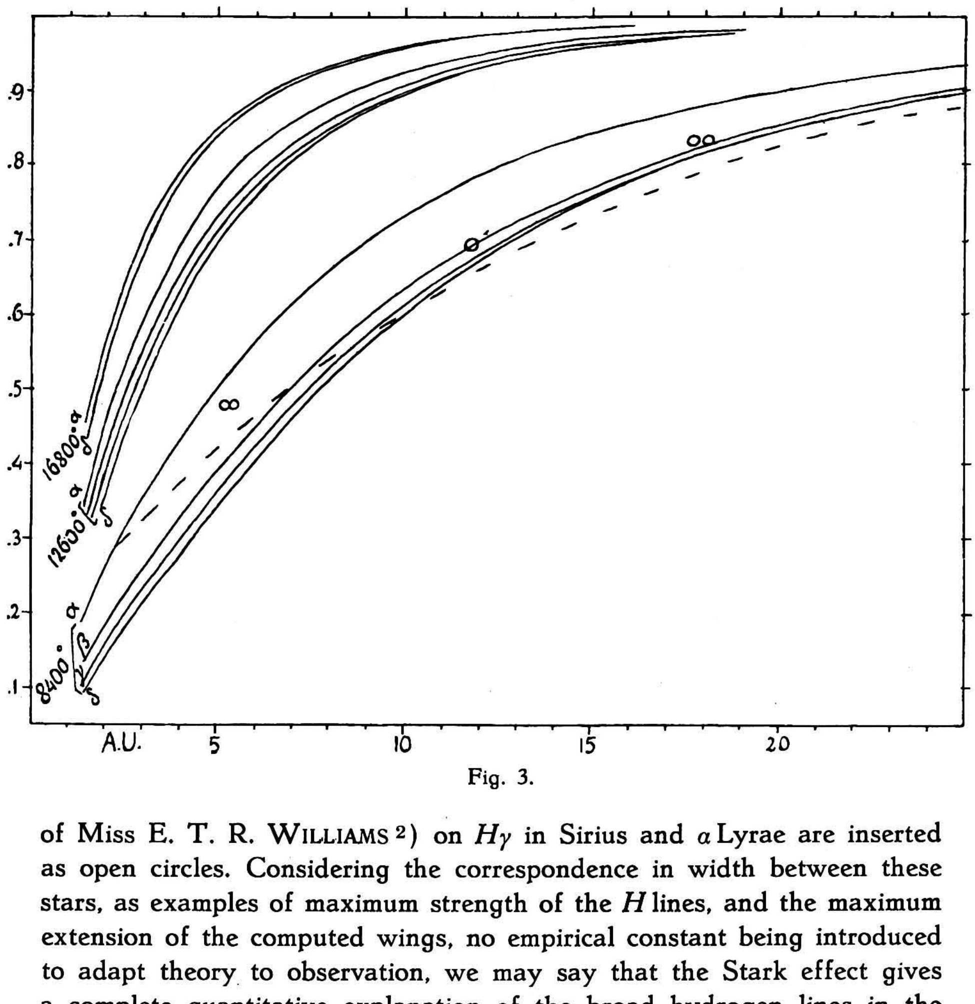

Fig. 3.

of Miss E. T. R. WILLIAMS[2]) on $H\gamma$ in Sirius and α Lyrae are inserted as open circles. Considering the correspondence in width between these stars, as examples of maximum strength of the H lines, and the maximum extension of the computed wings, no empirical constant being introduced to adapt theory to observation, we may say that the Stark effect gives a complete quantitative explanation of the broad hydrogen lines in the high temperature stars. And also that the fundamental constants used as the basis of the computation cannot be much in error. The deviation in the central parts, of course, must be treated separately by taking account of fluorescence phenomena.

The results of the computations yield the half width of the lines in A.U. (for different values of the residual intensity) in its dependence on T and g. A short extract for $H\gamma$ is given in Table 2.
The values for $\log g = 5$ are interpolated. We see here that the width of the Balmer lines attains a maximum at 8000° for $g = 10^4$, at a lower temperature still for giant stars, at 9000°—10000° for the white dwarfs.

1) Astrophys. Journal 72, p. 283.
2) Harvard Circular 348.

TABLE 2.

	r = 0.90						r = 0.50					
	T = 25200°	16800°	12600°	10080°	8400°	7200°	25200	16800	12600	10080	8400	7200
log g = 1	1.0	1.4	2.1	2.9	4.1	6.0	—	—	—	—	1.2	1.8
2	1.55	2.1	3.2	4.6	7.0	9.5	—	—	—	1.35	2.0	2.9
3	2.5	3.4	5.2	7.5	12.5	13.5	—	—	1.45	2.3	3.5	4.3
4	4.0	5.4	8.4	13.3	20.7	16.7	—	1.5	2.3	3.7	6.1	5.8
5	(6.4)	(8.7)	(13.4)	(22.5)	(32)	(21)	—	(2.6)	(4.0)	(6.9)	(9.8)	(7.4)
6	10.6	14.2	22.3	42	45	25	3.2	4.1	6.4	10.8	14.4	8.8
7	17.2	23	39	68	62	30			10.5	19.9	20.8	10.6

This is lower than what is usually assumed. The maximum concentration of neutral H atoms in the second state indeed occurs at a higher temperature, at 10000° for the relevant layers. The difference is due to the rapid decrease of the continuous absorption coefficient k when the temperature goes down below 10000°. This variation of k has the effect to change the scale of stellar temperatures; for class $A0$ 8000° appears to be better than 10000°.

The table shows for each temperature a regular increase of line width with gravity. Hence by means of well standardized spectra the hydrogen lines allow us to find the surface gravity and, if the parallax is known, the mass for each of these stars — supposing the temperature to be known. Here, however, we meet with the difficulty that the temperature scale is not certain. It must be established by the same spectra, chiefly by the state of ionization of different elements. So it is better to say that the hydrogen lines give a relation between temperature and gravity, and that from other lines we have to deduce another relation, so that both may be determined.

5. In the central parts of a hydrogen line for ordinary stars a residual intensity of nearly zero is found, because in the upper layers, which determine this intensity, s has a large value. In the case of white dwarfs, for high values of g, the profile of the absorption line is determined chiefly by deeper atmospheric layers of great density. In these layers, as may be seen from the lower lines of Table 1, the value of S from a minimum at the centre increases gradually with $\Delta\lambda$ and, owing to the large scale of $S(a)$, the Doppler curve hardly changes anything in it. In the case of $H\alpha$ and $H\gamma$ the central component, having only the width of the Doppler core, is superposed and stands out against a surrounding small s. So $H\alpha$ and $H\gamma$ in the case of high values of the surface gravity must show a narrow black line in the centre, on a somewhat brighter background.

In $H\beta$ and $H\delta$ which have no undisplaced central Stark component, the centre of the line appears as a bright reversal against the adjacent inner parts of the wings. The computed profiles for the central parts of $H\gamma$ and $H\delta$ are represented in Fig. 4 and Fig. 5 for $\log g = 7$ and 8. These special

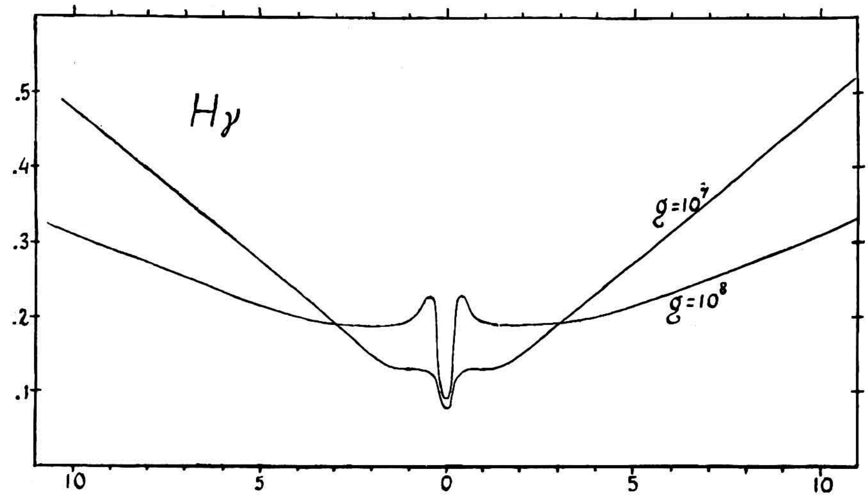

Fig. 4.

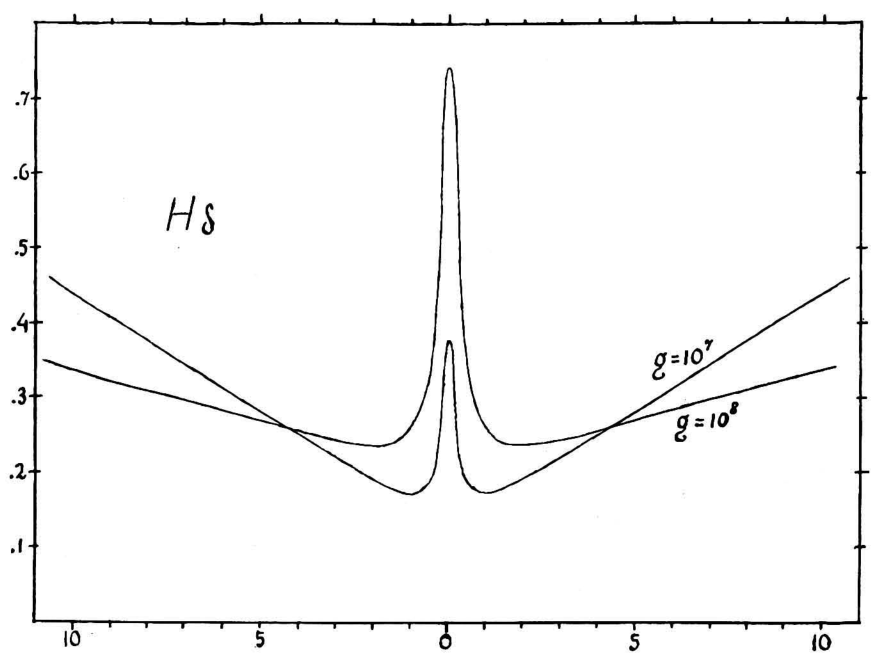

Fig. 5.

central phenomena may serve as another characteristic of stars of very high density. It will, however, be difficult to apply it, because, by the faintness of these stars, spectra with sufficient dispersion can only be obtained by very long exposures.