

NOTATIONS

Since in papers on astrophysics different notations are used, we give here the notations used in the present investigation.

<i>A</i>	abundance of an element.	<i>p</i>	total gas pressure.
<i>A_{kj}</i>	transition probability in emission.	<i>P</i>	electron pressure.
<i>B_{jk}</i>	radiation coefficient in transition probability for absorption.	<i>p_n</i>	weight of an atomic state.
<i>B, B'</i>	partition function of atomic states.	<i>r</i>	residual intensity.
<i>c</i>	velocity of light.	<i>s, s₀</i>	diffusion coefficient.
<i>c</i>	= $\frac{1}{4} h\nu/kT$.	<i>t</i>	optical depth.
<i>C</i>	= $\frac{3}{2} c\bar{k}/k$.	<i>T</i>	absolute temperature.
<i>E</i>	intensity of black body radiation.	<i>T₁</i>	effective temperature.
<i>f</i>	oscillator strength.	<i>x</i>	rate of ionization.
<i>g</i>	gravity acceleration.	<i>x_{0, x_{1, x₂}}</i>	fractions neutral, once, doubly ionized.
<i>g</i>	Gaunt correction factor in absorption coefficient.	<i>X</i>	mean ionization of an atmosphere.
<i>H</i>	intensity of net stream of radiation.	<i>y, y_n</i>	rate of excitation (Boltzmann factor).
<i>h</i>	Planck constant.	<i>β</i>	Doppler width constant in frequency.
<i>k</i>	Boltzmann constant.	<i>γ, δ</i>	damping constants.
<i>k, k(ν)</i>	absorption coefficient for unit mass.	<i>ε</i>	electron charge.
<i>\bar{k}</i>	mean absorption coefficient.	<i>θ</i>	angle of radiation with vertical.
<i>K</i>	ionization modulus, temperature function in ionization formula.	<i>κ</i>	= k/P .
<i>I, I'</i>	intensity of radiation streams.	<i>λ</i>	wave-length.
<i>J</i>	intensity of a radiation field.	<i>μ</i>	atomic weight.
<i>l</i>	Doppler width constant of a line.	<i>ν, ν_n</i>	frequency of a line, of an edge.
<i>m</i>	mass of an electron.	<i>ξ</i>	homogeneous depth.
<i>m_H</i>	mass of a hydrogen atom.	<i>ρ</i>	density of matter.
<i>n, n'</i>	concentration of active atoms.	<i>τ</i>	= $5040/T$.
		<i>χ, χ_n</i>	ionization energy from lowest, from <i>n</i> th level.
		<i>ω</i>	damping constant = $4\pi\gamma$.

Introduction.

The formation of Fraunhofer lines in stellar atmospheres has been the object of many investigations. The first fundamental researches of SCHUSTER and of SCHWARZSCHILD on absorption and diffusion in the sun's atmosphere have been used by EDDINGTON and by MILNE to explain the origin and the intensities of Fraunhofer lines. As the problem is complicated by the variability of most of the factors occurring in the formulas, only a first approximation, by means of simplifying assumptions, could be reached in these as well as in many later researches.

It may be asked whether it is not premature to go to further more exact approximations, since there are outstanding discrepancies between theory and observation, indicating that in our fundamental conceptions there remain uncertainties. On the other hand these theories give a good qualitative and often even a quantitative explanation of the wings of the lines, their half width and their equivalent width. An exact quantitative test will be possible only if the theory is worked out to a higher degree of approximation. The test of a theory is not complete until, numerically, its consequences have been worked out to the limit imposed by the accuracy of observations.

Physical theory provides us with a better knowledge of the chief datum, the coefficient of continuous absorption (by the work of KRAMERS, GAUNT, SUGIURA), as well as with a more exact knowledge of the monochromatic absorption and emission values (WEISSKOPF). At the same time the chief astrophysical datum for their application, the composition of the solar and stellar atmospheres, is known in first approximation (by Miss PAYNE, UNSÖLD, RUSSELL).

H. N. RUSSELL recently¹⁾ published an elaborate investigation on the theoretical intensity of Fraunhofer lines, based upon his former researches on the composition of the solar atmosphere and the ionization in stellar atmospheres. My computations, which had started already before the publication of RUSSELL's paper, follow a different way of treating the problem. RUSSELL makes use of an approximative method worked out by MILNE, of deriving a limiting optical depth; the layers below it are neglected, the higher layers are assumed to have constant values of absorption and diffusion. All the variable conditions of ionization and opacity can be put into the assumed limiting optical depth and the constants of the active upper layer. In our work we took account of the gradual variations of the opacity and ionization values in the different layers which contribute in a gradually variable amount to the result. To perform the integrations these gradual variations had then to be systematized in a series of approximate types.

The spectrum of a star is normally determined by a limited number of actual data of nature. First the energy production and the mass of the star which, combined with the radius, determine the stream of energy flowing through the unit surface, hence the effective temperature, and the gravitation at the surface. Secondly the composition of the atmosphere, i.e. the abundance of atoms of each chemical element. Rotation, which modifies the observed appearance of the stellar spectrum, will not be treated here.

We assume an atmosphere with horizontal layers, each of constant density, temperature and composition. For a certain wave length the absorption coefficient is k , the diffusion coefficient is s ; the former varies gradually and slowly with wave length, the latter varies rapidly in the realm of an absorption line. The intensity of a ray of energy with frequency ν , making an angle ϑ with the

¹⁾ Opacity Formulae and Stellar Line Intensities: *Astroph. J.* 78, 239.

outgoing radius, is denoted by $I(\nu, \vartheta)$; i.e. in the directions within an elementary cone $d\omega$ the energy with frequencies between ν and $\nu + d\nu$, streaming per unit time through unit area is $I(\nu, \vartheta)d\nu d\omega/4\pi$. Then the equation of this ray, the fundamental equation of our problem is

$$\cos \vartheta \frac{dI}{d\xi} = (k + s) I - kE - s \int I d\omega/4\pi,$$

where $d\xi = -\rho dh$ is an element of the homogeneous depth. Here $E(\nu)d\omega$ is the black body radiation of the unit mass, evenly distributed over all directions. It depends on the temperature of the layer; the increase of temperature with depth is determined by a mean absorption coefficient \bar{k} in such a way that in first approximation

$$T^4/T_0^4 = 1 + 3/2 \bar{k}\xi.$$

For the solution of the fundamental equation we put with EDDINGTON

$$J = \frac{1}{4\pi} \int I d\omega; \quad H = \frac{1}{4\pi} \int I \cos \vartheta d\omega; \quad K = \frac{1}{4\pi} \int I \cos^2 \vartheta d\omega.$$

Then by integrating over the sphere we get

$$\frac{dH}{d\xi} = k(J - E); \quad \frac{dK}{d\xi} = (k + s)H.$$

In the simple cases of first approximation $K = 1/3 J$, and at the surface of the atmosphere $H = 1/2 J$. By solving the equations

$$\frac{dJ}{d\xi} = 3(k + s)H; \quad \frac{dH}{d\xi} = k(J - E), \quad \text{with } H_0 = 1/2 J_0$$

the stream of energy H , radiated at the surface is found.

This stream $H(\nu)$ for each wave length is the quantity sought. The condition of radiative equilibrium imposed upon the atmosphere, implies that the total stream of energy $\int H(\nu)d\nu$ is constant throughout all the layers and equal to the stream from the interior. This determines the distribution of temperature.

For the solution of the equations the coefficient of continuous absorption $k(\nu)$ and the coefficient of monochromatic absorption, or diffusion coefficient s must be known as functions of the wave length and the physical parameters, as well as the mean absorption coefficient \bar{k} .

The coefficient of continuous absorption.

The absorption formulas.

Two different absorption coefficients appear in the formulas for line intensities; the coefficient $k(\nu)$ for the wave length of the line considered determines the continuous spectrum background, and \bar{k} , the coefficient of absorption for the total radiation, determines the increase of temperature in the deeper layers. The mean absorption \bar{k} can be found as soon as $k(\nu)$ for every wave length is known.

Expressions for the absorption coefficient have been derived by KRAMERS,¹⁾ by means

¹⁾ Philos. Magazine 46. 836.

of the correspondence principle, and afterwards by GAUNT¹⁾ and by SUGIURA²⁾ by means of wave mechanics. The absorption consists of two parts. One part is due to free-free or hyperbolic transitions: a free electron colliding with a positively charged nucleus absorbs radiation of any frequency ν , and increases its translation energy by the amount $h\nu$. The other part is due to ionization: an electron bound to a nucleus is ejected by absorbing a quantum of radiation $h\nu = \chi + \frac{1}{2}mv^2$, where χ is the ionization energy and $\frac{1}{2}mv^2$ its resulting energy of translation.

The absorption in the first case, for one positive nucleus, is given by

$$\alpha_0(\nu) = \frac{4\sqrt{2\pi}}{3\sqrt{3}} \frac{\epsilon^6 g P}{ch(mkT)^{3/2} \nu^3},$$

where ϵ the electron charge, m the electron mass, h and k the Planck and the Boltzmann constants, P the electron pressure have the usual meaning. By g a correction factor is denoted, introduced by GAUNT to express the deviation of the wave mechanics result from KRAMERS' formula. We suppose that all the ionized atoms in the gas may be considered as positive nuclei taking part in this absorption process. If by A we denote the abundance in number for some chemical element (its number of atoms as fraction of the total number of atoms), by x its rate of ionization, by $X = \sum Ax$ the mean ionization of the gas mixture, and by μm_H the mean weight of the atoms, then $X/\mu m_H$ is the number of positive ions per unit mass. Then the absorption coefficient per unit mass from free-free transitions is

$$k_0(\nu) = \frac{4\sqrt{2\pi}}{3\sqrt{3}} \frac{\epsilon^6 gXP}{ch(mkT)^{3/2} \mu m_H \nu^3} \quad 1)$$

The second part of the continuous absorption consists of a series of bands. From each energy level of the atom the electron can be ejected by absorption of $h\nu = h\nu_n + \frac{1}{2}mv^2$. Then ν_n is the frequency of the sharp edge limiting this absorption band at the lower side, and all radiations with larger ν are able to eject the electron from this level, giving it a certain kinetic energy. In each band for larger frequencies the absorption decreases as ν^{-3} . The absorption coefficient for a certain wave length is found by adding together the contributions of all the bands which have their edges at a lower frequency $\nu_n < \nu$. The intensity of each band depends on the relative number of atoms in the corresponding state of excitation. If we represent it by the Boltzmann function, then the absorption for one atom is given by

$$\alpha_1(\nu) = \frac{16\pi^2}{3\sqrt{3}} \frac{\epsilon^6}{ch^3 \nu^3} \sum \frac{p_n}{n^2 B} e^{-h(\nu_1 - \nu_n)/kT} \Delta\nu_n \quad 2)$$

where p_n is the weight of the n th state, ν_1 is the series limit for the lowest state of energy, and the partition function $B = \sum p_n \exp(-h(\nu_1 - \nu_n)/kT)$ for all the states of the atom. For $\Delta\nu_n$, the domain of the n th state, in the case of a simply charged particle (as with a hydrogen atom) we may write

$$\Delta\nu_n = \frac{4\pi^2 m\epsilon^4}{h^3 n^3}, \text{ or } \frac{2\pi^2 m\epsilon^4}{h^3} \left(\frac{1}{(n - \frac{1}{2})^2} - \frac{1}{(n + \frac{1}{2})^2} \right);$$

then the atomic absorption itself becomes

$$\alpha_1(\nu) = \frac{64\pi^4}{3\sqrt{3}} \frac{m\epsilon^{10}}{ch^3 \nu^3} \sum \frac{p_n}{n^5 B} e^{-h(\nu_1 - \nu_n)/kT}. \quad 3)$$

¹⁾ Philos. Transactions A, **229**, 163 (1930).

²⁾ Scientific Papers Tokyo N^o 339, 17. 89.

Here not the ionized but the neutral atoms (or more precisely the ionizable atoms) take part in the absorption process; their number in unit mass can be expressed for each element by $A(1-x)$. Since for the relevant wave lengths ν is much smaller than ν_1 , the exponential BOLTZMANN factor is very small, and the more so as the ionization potential of the element is larger. For this reason we eliminate this factor by means of the SAHA ionization formula

$$\frac{x}{1-x} P = \frac{(2\pi m)^{3/2}}{h^3} (kT)^{3/2} e^{-h\nu_1/kT} \frac{B'}{B}, \text{ or}$$

$$A(1-x) e^{-h(\nu_1 - \nu_n)/kT} = \frac{h^3 A x P}{(2\pi m)^{3/2} (kT)^{3/2}} e^{h\nu_n/kT} \frac{B}{B'}$$

where B' is the partition function for the ionized atom. Then the absorption coefficient per unit mass due to ionization may be written

$$k_1(\nu) = \frac{4\sqrt{2\pi}}{3\sqrt{3}} \frac{\epsilon^8 P}{cm^{3/2} (kT)^{3/2} \mu m_H \nu^3} \sum A x \sum \frac{p_n}{n^2 B'} e^{h\nu_n/kT} \Delta\nu_n \quad (4)$$

where the first summation comprises all the elements, and the second summation for each element extends over all the band edges or levels, for which $\nu_n < \nu$. Substituting for $\Delta\nu_n$ its hydrogen expression

$$k_1(\nu) = \frac{8\pi^2 \sqrt{2\pi}}{3\sqrt{3}} \frac{\epsilon^{10} P}{ch^3 m^{3/2} (kT)^{3/2} \mu m_H \nu^3} \sum A x \sum \frac{p_n}{n^2 B'} \left(\frac{1}{(n-\frac{1}{2})^2} - \frac{1}{(n+\frac{1}{2})^2} \right) e^{h\nu_n/kT}. \quad (5)$$

In these formulas 4) and 5) the different ionization potentials have disappeared. Since the band edges ν_n at the lower (red) side of the relevant wave lengths for different elements are intermixed by chance, the relative contributions of these elements to the absorption coefficient are determined by the factors Ax . Hence all the easily ionizable elements for which x is nearly 1, take part in ratio of their total abundance in the atmosphere. At solar temperature this holds for all the metal atoms with ionization potentials below 7 or 8. The contribution of elements with higher ionization potentials rapidly decreases with their rate of ionization x . It appears that it is only the small neutral fraction of such easily ionizable metal atoms that is responsible for this part of the continuous absorption in the solar atmosphere; because this neutral fraction is itself proportional to the electron pressure, the ionization part of the absorption coefficient k_1 contains the electron pressure as a factor just as well as the free-free transition part k_0 does. In a former paper ¹⁾ an attempt was made to derive the general coefficient of absorption by assuming that these metals constitute a certain fraction (1 % of the mass, $1/40$ % of the number of atoms) in the solar atmosphere. For lower temperatures the number of elements that must be included, decreases, because those with highest ionization potentials get lost one after another. For increasing temperature new elements with higher ionization potentials are included, for which x approaches to 1. Especially the inclusion of hydrogen, as soon as it becomes ionized for the greater part, raises the absorption coefficient considerably.

Some changes have to be made in these formulas for the case, which in the relevant atmospheric layers is nearly realized, that the atoms are affected not by an isotropic field of radiation, but by a nearly homogeneous radiation from below. In this case it can be shown ²⁾ that the BOLTZMANN distribution formula must be replaced by

$$\frac{1}{2} \frac{p_n}{B} e^{-h(\nu_1 - \nu_n)/kT_1}$$

¹⁾ The Theoretical Contours of Absorption Lines, Monthly Not. R.A.S. 91. 139 (1930).

²⁾ B.A.N. 3. 207 (1926).

where T_1 is the effective temperature of the radiation and $1/2$ is the dilution of the radiation. In the derivation of this expression only one higher level above the lowest level was assumed. In the case of more levels each transition from a lower to a higher level contains this dilution factor $1/2$, because it is due to one sided radiation. Hence some rearrangement between all the levels must take place. Since, however, the transitions between the higher levels, in all practical cases, are much less numerous than the transitions from the lowest level to each of them, the latter will dominate over all the other transitions in fixing the new equilibrium. Hence for all the higher levels the same factor $1/2$ may be assumed to hold.

In the same way the ionization formula in the case of a homogeneous radiation field is changed to

$$\frac{x}{1-x} P = 1/2 \left(\frac{T}{T_1} \right)^{3/2} \frac{(2\pi m)^{3/2}}{h^3} (kT_1)^{3/2} e^{-h\nu_1/kT_1} \frac{B}{B'}$$

where T_1 is the effective temperature and T the temperature of the upper layers of the atmosphere. From the relation $(T_1/T)^4 = 2$ we find $(T_1/T)^{3/2} = 1.30 = [0.113]$. Hence the only change to formula 3) for $k_1(\nu)$ is a numerical factor 1.30, if only as the temperature T occurring there we take the effective temperature of the star, and not the temperature of its upper layers. Compared with the case of an isotropic radiation of the effective temperature we have the number of neutral atoms (for strong ionizations) doubled, but the abundance of high levels is halved, so that the factors $1/2$ cancel each other. This implies that for the lowest level a factor 2 should be added.

Composition and ionization of the stellar atmospheres.

To compute the absorption coefficient by means of these formulas, the composition of the atmospheres and the rate of ionization must be known. In a former paper ¹⁾ the electron pressure was computed by assuming the following model composition of the atmosphere, in number of atoms: 5×10^{-5} with $V = 4.3$ (for *K*), 10^{-4} with $V = 5.1$ (for *Na*), 10^{-4} with $V = 6.1$ (for *Ca*, *Al*), 2×10^{-4} with $V = 7.7$ (for *Fe*, *Mg*, *Si*), and 1.0 for *H* with $V = 13.54$. This means that 2 % of the mass consists of metal atoms. Then the mean ionization or electron abundance relative to the atoms, from 10^{-7} at 2000° continually increases, but gradually at a slower rate till a value of nearly $10^{-3.2}$ is reached at 6000° , where the metal ionization becomes complete; then the beginning ionization of hydrogen raises the electron abundance rapidly till at $10,000^\circ$ it becomes nearly 0.5 and at $13,000^\circ$ it reaches 1.

A more exact derivation of the ionization and the absorption coefficient was based by RUSSELL on his results on the composition of the sun's atmosphere ²⁾. The relative abundance of the metallic elements could be determined with fairly great certainty; their absolute abundance i.e. their abundance relative to hydrogen, which (with helium and oxygen) constitutes the main part of the solar atmosphere, remains rather uncertain. In a later paper ³⁾ RUSSELL adopts, for the computation of the ionization, a "representative composition" of the solar atmosphere, consisting of only those elements of lower ionization potential which contribute an appreciable part to the whole, and of hydrogen. The mixture of the first named is:

Element	<i>K</i>	<i>Na</i>	<i>Al</i>	<i>Ca</i>	<i>Cr</i>	<i>Mg</i>	<i>Fe</i>	<i>Si</i>	Total.
Abundance.....	50	126	20	40	10	470	126	158	1000 atoms
Ionization Potential....	4.33	5.09	5.95	6.09	6.75	7.61	7.63	8.14	

¹⁾ Monthly Not. R.A.S. 91. 527 (1931).

²⁾ Astrophys. J. 70. 11 (1929).

³⁾ Mean Ionization in Stellar Atmospheres, Astroph. J. 75. 337 (1932).

The number of hydrogen atoms was taken 61.5 times the total number of these metal atoms. For this model atmosphere the ionization was computed for different temperatures and pressures, separately for hydrogen and for the metal atoms (Tables III and IV of his paper).

These tables of RUSSELL were taken as the basis of my own computations. The qualitative accordance with the results of Miss PAYNE on stellar atmospheres gives a sufficient basis for the assumption of an identical composition of the stellar atmospheres in general.

For the abundance of hydrogen relative to the metals, however, we did not adopt RUSSELL's value 61.5. The number of hydrogen atoms in the second state [3.5], when treated by means of his empirical expression $\frac{5040}{T} (E - C) = \frac{5040}{T} 0.85 E^1$, after correction for a factor 2 due to non-isotropic radiation as derived above, gives [11.8] for the number of hydrogen atoms. The empirical correction $C = 0.15 E$, here applied to the BOLTZMANN distribution is in accordance with the substitution of the effective temperature for the gas temperature in the case of one sided radiation. This result [11.8], compared with [7.2] for the number of *Mg* atoms and [6.7] for the *Ca* atoms shows the number of hydrogen atoms 5000 times more numerous than that of the metal atoms. From the width of the *H α* and the *H* and *K* lines in the solar spectrum, as measured by UNSÖLD, and discussed in our paper already quoted²⁾ a still larger ratio of 10,000 is found. This number is, of course, too large, because the *H α* line in the solar spectrum is certainly broadened by the Stark effect. If we make allowance for a factor 5—10 for this, we come to a ratio 1000. RUSSELL also finds and adopts this value in his last paper³⁾. We will accept this value, thus taking the number of metal atoms 1/1000 of the total number of atoms. Of course it is extremely uncertain and we must keep in mind in our further computations, that it may be changed considerably by later researches. The mean molecular weight of the gas mixture is now 1.029, so little different from 1 that the factor μ in the formulas for k may be omitted.

It is not necessary to take account of oxygen and helium which, perhaps, may contribute some percentages to the whole number of atoms. For oxygen the ionization potential is not much different from that of hydrogen. For helium it is much higher; this means that at 13000° the electron abundance should be only, say 0.98, and then gradually rise to 1. These differences are unimportant. For the same reason second ionizations may be neglected, since they would add an imperceptible number of electrons to the number coming from the totally ionized hydrogen atoms.

We took the ionization of the metal mixture (x_M) and of hydrogen (x_H) from RUSSELL's tables. Then the total ionization

$$x_H + .001 x_M = X$$

is found as a function of temperature and electron pressure (Table 1).

The second table gives $\log p$ for the total pressure $p = P (1 + X)$. The temperatures are chosen by RUSSELL in such a way that $5040/T$ (sometimes here after denoted by τ) takes the values 0.2 0.3 0.4 2.0. This table shows that for low temperatures (and high pressures) only the metals count; when they are nearly ionized and X approaches to 10^{-3} , the ionization of hydrogen begins, and at higher temperatures the contribution of the metal atoms becomes a negligible fraction. Broken lines separate the transition realm from the pure hydrogen and the pure metal values, where in two-figured logarithms the other component is not perceptible. The dotted lines bound the atmospheric layers which are most important in the formation of Fraunhofer lines.

¹⁾ *Astroph. J.* **70**. 53.

²⁾ *Monthly Not. R.A.S.* **91**. 155.

³⁾ *Astroph. J.* **78**. 281.

Table 1. Ionization of a mixture of hydrogen and metal atoms.

T 5040/ T	25200° 0.2	16800° 0.3	12600° 0.4	10080° 0.5	8400° 0.6	7200° 0.7	6300° 0.8	5600° 0.9	5040° 1.0	4200° 1.2	3600° 1.4	3150° 1.6	2520° 2.0
$\log P$													
8	0.69 3.10 0.69	2.38 3.58 2.35	4.06 4.31 3.87	5.66 5.04 4.95	7.20 5.72 5.71	8.73 6.36 6.36	10.22 6.95 6.95	11.71 7.55 7.55					
7	0.14 3.01 0.14	1.40 3.11 1.39	3.06 3.52 2.93	4.66 4.14 4.03	6.20 4.75 4.73	7.73 5.37 5.37	9.22 5.96 5.96	10.71 6.55 6.55	12.17 7.13 7.13				
6	0.02 3.00 0.02	0.53 3.01 0.53	2.06 3.11 2.02	3.66 3.46 3.25	5.20 3.94 3.92	6.73 4.44 4.44	8.22 4.99 4.99	9.71 5.56 5.56	11.17 6.13 6.13	14.08 7.22 7.22			
5	0.00 3.00 0.00	0.10 3.00 0.10	1.11 3.01 1.09	2.66 3.11 2.53	4.20 3.41 3.34	5.73 3.79 3.78	7.22 4.17 4.17	8.71 4.63 4.63	10.17 5.15 5.15	13.08 6.22 6.22	15.96 7.28 7.28		
4	0.00 3.00 0.00	0.01 3.00 0.01	0.33 3.00 0.33	1.67 3.01 1.65	3.20 3.10 2.85	4.73 3.37 3.35	6.22 3.68 3.68	7.70 3.97 3.97	9.17 4.32 4.32	12.08 5.25 5.25	14.96 6.28 6.28	17.80 7.32 7.32	
3	0.00 3.00 0.00	0.00 3.00 0.00	0.05 3.00 0.05	0.74 3.00 0.74	2.20 3.01 2.14	3.73 3.08 2.99	5.22 3.32 3.31	6.71 3.61 3.61	8.17 3.85 3.85	11.08 4.46 4.46	13.96 5.31 5.31	16.80 6.32 6.32	— 8.31 8.31
2		0.00 3.00 0.00	0.00 3.00 0.00	0.16 3.00 0.16	1.23 3.00 1.22	2.73 3.01 2.55	4.22 3.08 3.05	5.71 3.28 3.28	7.17 3.56 3.56	10.08 3.90 3.90	12.96 4.49 4.49	15.80 5.35 5.35	— 7.31 7.31
1			0.00 3.00 0.00	0.02 3.00 0.02	0.41 3.00 0.41	1.74 3.00 1.72	3.22 3.01 2.80	4.71 3.06 3.05	6.17 3.25 3.25	9.08 3.68 3.68	11.96 4.00 4.00	14.80 4.53 4.53	— 6.31 6.31
0				0.00 3.00 0.00	0.06 3.00 0.06	0.80 3.00 0.80	2.22 3.00 2.15	3.71 3.01 2.93	5.17 3.05 3.05	8.08 3.48 3.48	10.96 3.73 3.73	13.80 4.05 4.05	— 5.34 5.34
-1				0.00 3.00 0.00	0.01 3.00 0.01	0.19 3.00 0.19	1.24 3.00 1.23	2.71 3.00 2.53	4.17 3.01 2.98	7.08 3.19 3.19	9.96 3.61 3.61	12.80 3.79 3.79	— 4.56 4.56
-2					0.00 3.00 0.00	0.02 3.00 0.02	0.42 3.00 0.42	1.72 3.00 1.70	3.17 3.00 2.78	6.08 3.04 3.04	8.96 3.40 3.40	11.80 3.66 3.66	— 4.14 4.14
-3						0.00 3.00 0.00	0.07 3.00 0.07	0.79 3.00 0.79	2.17 3.00 2.11	5.08 3.00 3.00	7.96 3.12 3.12	10.80 3.54 3.54	— 3.84 3.84
-4						0.00 3.00 0.00	0.01 3.00 0.01	0.18 3.00 0.18	1.20 3.00 1.19	4.08 3.00 2.97	6.96 3.02 3.02	9.80 3.29 3.29	— 3.72 3.72
-5							0.00 3.00 0.00	0.02 3.00 0.02	0.39 3.00 0.39	3.08 3.00 2.74	5.96 3.00 3.00	8.80 3.08 3.08	— 3.64 3.64
-6								0.00 3.00 0.00	0.06 3.00 0.06	2.08 3.00 2.03	4.96 3.00 3.00	7.80 3.01 3.01	— 3.57 3.57

Table 2. Total pressure of a mixture of hydrogen and metal atoms, Log p .

Log P	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	2.0
8	8.77	10.35	11.87	12.95	13.71	14.36	14.95	15.55					
7	7.38	8.41	9.93	11.03	11.73	12.37	12.96	13.55	14.13				
6	6.31	6.64	8.02	9.25	9.92	10.44	10.99	11.56	12.13	13.22			
5	5.30	5.35	6.12	7.53	8.34	8.78	9.17	9.63	10.15	11.22	12.28		
4	4.30	4.31	4.50	5.66	6.85	7.35	7.68	7.97	8.32	9.25	10.28	11.32	
3	3.30	3.30	3.33	3.81	5.14	5.99	6.31	6.61	6.85	7.46	8.31	9.32	11.31
2	2.30	2.30	2.30	2.39	3.26	4.55	5.05	5.28	5.56	5.90	6.49	7.35	9.31
1	1.30	1.30	1.30	1.31	1.55	2.73	3.80	4.05	4.25	4.68	5.00	5.53	7.31
0		0.30	0.30	0.30	0.33	0.86	2.15	2.93	3.05	3.48	3.73	4.05	5.34
-1			9.30	9.30	9.31	9.41	0.25	1.53	1.98	2.19	2.61	2.79	3.56
-2				8.30	8.30	8.31	8.56	9.71	0.78	1.04	1.40	1.66	2.14
-3					7.30	7.30	7.34	7.86	9.11	0.00	0.12	0.54	0.84
-4						6.30	6.31	6.40	7.22	8.97	9.02	9.29	9.72
-5							5.30	5.31	5.54	7.74	8.00	8.08	8.64
-6								4.30	4.33	6.03	7.00	7.01	7.57

Numerical values.

We write the expression for k_0 in the form

$$k_0(\nu) = C_0 \left(\frac{\nu_0}{\nu}\right)^3 g X P \quad (6)$$

where for ν_0 is taken the frequency corresponding to $\lambda = 4000 \text{ \AA}$, which we shall use as standard wave length. By introducing the numerical values for the physical constants we have

$$C_0 = \frac{4\sqrt{2\pi}}{3\sqrt{3}} \frac{\varepsilon^6}{ch(mkT)^{3/2} m_H \nu^3} = [-1.973] \left(\frac{5040}{T}\right)^{3/2} \quad (7)$$

The other part of the absorption coefficient is taken here in the form 5) holding for hydrogen; we shall see afterwards what has to be changed in it for other atoms. We write it in the form

$$k_1(\nu) = C_1 \left(\frac{\nu_0}{\nu}\right)^3 S P \quad (8)$$

$$S = \sum A x \sum \frac{p_n}{n^2 B'} \left(\frac{1}{(n - \frac{1}{2})^2} - \frac{1}{(n + \frac{1}{2})^2} \right) e^{h\nu_n/kT} \quad (9)$$

$$C_1 = 1.30 \frac{8\pi^2 \sqrt{2\pi}}{3\sqrt{3}} \frac{\varepsilon^{10}}{ch^3 m^{1/2} (kT)^{1/2} m_H \nu_0^3} = [-0.370] \left(\frac{5040}{T}\right)^{5/2} \quad (10)$$

The numerical values of C_0 and C_1 are given in Table 4.

So far the formula for the free-free transition coefficient is easily calculable. There exists, however, a considerable uncertainty as to the correction factor g . GAUNT and SUGIURA have computed by wave mechanics expressions for this factor, but only in limiting cases, by means of approximate formulas. An electron with small translation energy E_2 by absorbing a quantum $h\nu$ acquires a greater translation energy E_1 . The average value of E_2 , the mean translation energy in the electron gas, is $3/2 kT$. Expressed in electron volts it amounts to 3.28, 1.09, 0.66, 0.33 for $T = 25200^\circ$, 8400° , 5040° , 2520° . The difference $E_1 - E_2 = h\nu$ is 3.09 and 2.06 eV for wave lengths 4000 \AA and

6000 \AA . In applying SUGIURA's formula 13) (*l.c.p.* 105), holding for $E_2 \rightarrow 0$ we find incredibly small values for g . If we apply the formula to the separate particles with their different velocities, assuming a Maxwell distribution, the mean g still comes out very small, of the order of 10^{-7} for 2520° , of 10^{-3} for 25200° . Though the reason is not clear a priori, it seems that formula 14) for the limiting case $E_1 - E_2 \rightarrow 0$ is more trustworthy here. This lack of correspondence, of course, throws strong doubts upon the reliability of the values of g in this way derived. SUGIURA gives the formula

$$g = \frac{4\sqrt{3} a_1(a_2 - a_1)}{(a_2 + a_1)(1 - e^{-2\pi(a_2 - a_1)})} \left\{ \log \sqrt{4a_1 a_2(4 + 1/(a_2 - a_1)^2)} + \frac{1}{\sqrt{4a_1 a_2}} \sin(a_2 - a_1) \log \frac{4a_2 a_1}{(a_2 - a_1)^2} \right\}$$

where $a = \sqrt{R\hbar/E}$ and R is the Rydberg constant, hence $R\hbar = 13.54 \text{ eV}$. The second term in parenthesis may be neglected. With the first term the values for $\log g$ in Table 3 were computed.

Table 3. $\log g$ (Correction factor to KRAMERS' formula).

λ / $\frac{5040}{T}$	0.2	0.4	0.6	0.8	1.0	1.2	1.6	2.0
12000	0.466	0.696	0.875	1.000	1.091	1.162	1.264	1.337
6000	0.534	0.800	0.958	1.060	1.133	1.187	1.266	1.321
4000	0.578	0.837	0.977	1.067	1.128	1.175	1.241	1.288
2000	0.632	0.853	0.963	1.030	1.078	1.114	1.165	1.200
1000	0.636	0.812	0.897	0.949	0.986	1.014	1.054	1.083

It is seen from this table that $\log g$ over the wave lengths of practical use (say 7000 to 3000 \AA) is nearly constant; the variations amount to some units of the 2^d place. Though all our computations will be made in two decimal places, there are uncertainties enough in the theory and the numerical values to make these second place figures not always reliable. Hence for $\log g$ a constant average value, holding for the observable region of wave lengths, is inserted in Table 4.

Table 4. Values of some temperature functions.

τ	T	$\log C_0$	$\log C_1$	$\log g$	$\log (\hbar\nu_0/kT)$
0.2	25200°	-3.021	-2.117	0.56	0.455
0.3	16800	-2.757	-1.677	0.71	0.983
0.4	12600	-2.570	-1.365	0.82	1.358
0.5	10080	-2.425	-1.123	0.90	1.649
0.6	8400	-2.306	-0.925	0.96	1.887
0.7	7200	-2.205	-0.757	1.01	2.087
0.8	6300	-2.118	-0.612	1.06	2.261
0.9	5600	-2.042	-0.484	1.10	2.415
1i0	5040	-1.973	-0.370	1.13	2.552
1.2	4200	-1.854	-0.172	1.18	2.795
1.4	3600	-1.754	-0.005	1.23	2.995
1i6	3150	-1.667	+0.140	1.26	3.164
2i0	2520	-1.522	+0.333	1.31	3.455

High temperature stars.

The high temperature stars will be first treated separately, because here hydrogen alone is responsible for the continuous absorption. Data for the medium temperatures, where hydrogen

takes part in the absorption, are then deduced at the same time. In the case of hydrogen, the conditions assumed in the derivation of KRAMERS' formula are realized. Correction factors deduced by wave mechanics are of the order of unity and may be neglected. In the expression for S in formula 9) the first Σ is omitted and $A = 1$. The bandedges are given by $\nu_n = \nu_1/n^2$, the weight of each level $p_n = 2n^2$, and the denominator $B' = 1$. Hence.

$$S = X_H \Sigma \frac{4n}{(n^2 - 1/4)^2} e^{h\nu_1/n^2 kT} = x_H S. \quad (11)$$

The summation extends from $n = \infty$ to n just above the corresponding wave length. The values of the terms for the seven highest edges ($n = 1$ to 7) were computed separately, making use of

$$\log e^{h\nu_1/n^2 kT} = \frac{5040}{T} \times \frac{13.54}{n^2}.$$

They are given in Table 5.

Table 5. Hydrogen bands. Intensities of edges.

ν_1/n^2	25200° 0.2	16800° 0.3	12600° 0.4	10080° 0.5	8400° 0.6	7200° 0.7	6300° 0.8	5600° 0.9	5040° 1.0	4200° 1.2
13.54	3.560	4.914	6.268	7.622	8.976	10.330	11.684	13.038	14.392	17.100
3.385	0.432	0.770	1.109	1.447	1.786	2.125	2.463	2.801	3.140	3.817
1.504	9.497	9.647	9.798	9.948	0.098	0.249	0.399	0.550	0.700	1.001
0.846	8.978	9.063	9.147	9.232	9.317	9.401	9.486	9.570	9.655	9.825
0.542	8.622	8.677	8.731	8.785	8.839	8.893	8.947	9.002	9.056	9.164
0.376	8.349	8.387	8.424	8.462	8.500	8.537	8.575	8.612	8.650	8.725
0.276	8.126	8.154	8.181	8.209	8.237	8.264	8.292	8.319	8.347	8.402

The sum total of all the lower edges ($n = \infty$ to 8) was found by substituting an integral for the sum

$$\Sigma \frac{2n}{(n^2 - 1/4)^2} e^{h\nu_1/n^2 kT} = \int_n^\infty \frac{2}{n^3} e^{h\nu_1/n^2 kT} dn = \frac{kT}{h\nu_1} (e^{h\nu_1/n^2 kT} - 1)$$

where for the limiting $n 7\frac{1}{2}$ was taken. Then in Table 6 the sum total of all the lower bands up to the next edge s_n is given, already multiplied by the factor C_1 .

Table 6. Hydrogen bands. Summarized intensities of intervals.

	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2
$C_1 s_8$	6.520	6.972	7.296	7.551	7.762	7.942	8.100	8.241	8.368	8.593
$C_1 s_7$.637	7.093	.420	.679	.894	8.078	.240	.385	.516	.749
$C_1 s_6$.781	.243	.577	.842	8.064	.255	.424	.576	.715	.963
$C_1 s_5$.966	.439	.785	8.062	.296	.401	.683	.849	9.002	9.279
$C_1 s_4$	7.218	.714	8.085	.387	.648	.840	9.090	9.284	.467	.806
$C_1 s_3$.608	8.162	.594	.960	9.286	9.579	.867	0.132	0.386	0.868
$C_1 s_2$	8.393	9.141	9.774	0.342	0.872	1.375	1.855	2.320	2.772	3.646
$C_1 s_1$	1.744	3.538	5.204	6.800	8.352	9.874	11.374	12.855	14.323	17.229

To these values the free-free transition coefficients must be added :

$$k(\nu) = (C_1 s + C_0 g) \left(\frac{\nu_0}{\nu} \right)^3 x_H P = \kappa(\nu) P. \quad (12)$$

The region of wave lengths for which k is wanted, is almost entirely situated between the 2^d and the 3^d hydrogen edge (3646 Å and 8203 Å); for this part, and for the wave lengths below the Balmer edge we have

τ	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2
Log C_1s_3	7.61	8.16	8.59	8.96	9.29	9.58	9.88	0.14	0.39	0.87
Log C_1s_2	8.39	9.14	9.78	0.34	0.87	1.37	1.86	2.32	2.77	3.65
Log C_0g	7.54	7.95	8.25	8.47	8.65	8.81	8.94	9.06	9.16	9.33
Log ($C_1s_3 + C_0g$)	7.88	8.37	8.75	9.08	9.38	9.65	9.93	0.17	0.41	0.88
Log ($C_1s_2 + C_0g$)	8.45	9.17	9.79	0.35	0.87	1.37	1.86	2.32	2.77	3.65

For other wave lengths the factor $(\nu_0/\nu)^3$ must be added. In Table 7. the coefficient of $x_n P$ is given for the standard wave length, and the corrections to other wave lengths are added. For wave lengths below the Balmer edge these corrections are variable; for the wave lengths between

Table 7. Continuous absorption by hydrogen atoms. Log κ/xH .

τ	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2
Standard	7.88	8.37	8.75	9.08	9.38	9.65	9.93	0.17	0.41	0.88
λ										
3000	+ .20	+ .43	+ .67	+ .90	+ 1.12	+ 1.35	+ 1.56	+ 1.78	+ 1.99	+ 2.40
3500	+ .40	+ .63	+ .87	+ 1.10	+ 1.32	+ 1.55	+ 1.76	+ 1.98	+ 2.19	+ 2.60
3646	+ .45	+ .68	+ .92	+ 1.15	+ 1.37	+ 1.60	+ 1.81	+ 2.03	+ 2.24	+ 2.65
"										
4000										
5000										
6000										
7000										

this and the Paschen edge they are constant. The same computations have been effected for $g = 1$; for this case the deviations from the above values are

$\lambda > 3646$:	— .18	— .16	— .13	— .10	— .08	— .06	— .06	— .04	— .02	— .01
$\lambda < 3646$:	— .04	— .02	— .01	— .01	0	0	0	0	0	0

It appears that for the highest temperatures the free-free transitions contribute nearly $1/3$ of the whole absorption in the Balmer line region. With decreasing temperature this fraction also decreases; the reason is obvious, because at the highest temperatures the number of electrons is overwhelming in comparison with the ionizable atoms. The influence of the uncertainty of the factor g may be judged after the differences brought about by its omission.

It must be remarked that for hydrogen exact values for the continuous absorption coefficients due to ionizations from the three lowest levels have been computed by means of wave mechanics by SUGIURA. ¹⁾ His expressions for df/dE (formulas 80) 88) and 100) of his paper) are rather complicated. If, however, the numerical values for these expressions are computed for different values

¹⁾ Scient. Papers Tokyo, 1929. Vol 11.

of $E(= (\nu - \nu_n)/\nu_1)$ they vary with ν almost exactly as ν^{-3} . The limiting values at the edges $\nu = \nu_n$ for $n = 1, 2$ and 3 are

$$27.3 \cdot 10^{-4}; \quad 2^{10.5} \cdot 10^{-8}; \quad 27.3^2 \cdot 377 \cdot 10^{-12}.$$

MENZEL, in his elaborate memoir on the Solar Chromosphere¹⁾, represented them by $C \cdot n^{-1.5}$. If this relation holds for the higher levels, we may represent SUGIURA's expressions by $273^{-1} \cdot 10^{-4} \cdot n^{1.5} (\nu_n/\nu)^3$, and the atomic absorption coefficient becomes

$$a(\nu, n) = \frac{27 \cdot \pi^3 \cdot e^6 \cdot 4\pi^2 m e^4}{3e^4 n^2 c h^3 \nu^3 h^3 n^{2.56}}$$

which is MENZEL's formula 4.40 (p. 223 *l.c.*). The difference with the formula used above is the substitution of

$$\frac{27 \cdot \pi^3}{3e^4 n^{2.56}} \text{ for } \frac{16 \pi^3}{3\sqrt{3} n^3}.$$

There is no doubt that SUGIURA's values must be preferred to the more general equation of KRAMERS—GAUNT. The differences in $\log k$, for

$n = 1$	2	3	4	5	6	7	amounting to
-.091	-.053	-.027	-.008	+.006	+.018	+.028	

are so small, especially for the relevant regions, that compared with the outstanding uncertainties in the absorption values, they are without importance. Hence it was not deemed necessary to make any change to the values computed with the GAUNT formula.

The mean absorption coefficient in high temperature stars.

The stream of energy in each layer of the atmosphere of a star for each wave length directly depends on the temperature distribution and the absorption coefficient. The total stream of energy for all the wave lengths is a given quantity, determined by the energy production in the interior of the star. It, therefore, determines the temperature distribution in such a way that the corresponding total stream has just the required value. Besides the continuous absorption of each wave length $k(\nu)$ we have the effect of the Fraunhofer lines, which also absorb part of the energy. This effect will be treated afterwards (*cf.* p. 36); we will restrict ourselves here to the continuous absorption.

With sufficient approximation we may in the interior use for each ν the formula

$$3 kH = dJ/d\xi = dE/d\xi$$

where H , k and E , the black radiation, are variable with wave length, and E depends on the temperature in the layer considered. Hence we find for the total stream

$$H = \int H(\nu) d\nu = \int \frac{1}{3 k(\nu)} \frac{dE(\nu)}{dT} \frac{dT}{d\xi} d\nu = \frac{1}{3} \frac{dT}{d\xi} \int \frac{1}{k(\nu)} \frac{dE(\nu)}{dT}.$$

If we express this total stream in its dependence on the temperature gradient by a mean absorption coefficient \bar{k} , we have

$$H = \frac{1}{3\bar{k}} \frac{dT}{d\xi} \int \frac{dE(\nu)}{dT} d\nu.$$

¹⁾ Publications Lick Observatory, Vol 17.

Hence the mean absorption coefficient to be used here, the ROSSELAND mean, is determined by

$$\frac{1}{k} \int \frac{dE(\nu)}{dT} d\nu = \int \frac{1}{k(\nu)} \frac{dE(\nu)}{dT} d\nu.$$

In the outer layers of an atmosphere, where only radiation streams from below are passing, we have in the same way

$$H(\nu) = 1/2 \left(E(\nu) + \frac{1}{3k(\nu)} \frac{dE(\nu)}{dT} \frac{dT}{d\xi} \right)$$

for each wave length, and for the total stream :

$$H = 1/2 \int E(\nu) d\nu + 1/6 \frac{dT}{d\xi} \int \frac{1}{k(\nu)} \frac{dE(\nu)}{dT} d\nu = 1/2 \left(E + \frac{1}{3k} \frac{dT}{d\xi} \int \frac{dE(\nu)}{dT} d\nu \right).$$

Here the same Rosseland mean appears as the absorption coefficient determining the total stream of radiation. It is found by averaging the reciprocal absorption coefficients, with weights proportional to the fluxion of the temperature of the black body radiation.

Introducing the Planck function this weight is found

$$\frac{dE(\nu)}{dT} = \frac{8\pi h\nu^3}{c^2} \frac{h\nu}{kT^2} \frac{e^{h\nu/kT}}{(e^{h\nu/kT} - 1)^2}$$

The absorption coefficient contains as its main variable factor $(\nu_0/\nu)^3$; hence it is this factor the reciprocal of which must be averaged with the computed weight :

$$\begin{aligned} 1/\left(\frac{\nu_0}{\nu}\right)^3 &= \int \left(\frac{\nu}{\nu_0}\right)^3 \frac{dE(\nu)}{dT} d\nu : \int \frac{dE(\nu)}{dT} d\nu, \text{ or} \\ \left(\frac{\nu_0}{\nu}\right)^3 &= \int \frac{\nu^4 e^{h\nu/kT}}{(e^{h\nu/kT} - 1)^2} d\nu : \int \frac{\nu^7 e^{h\nu/kT}}{\nu_0^3 (e^{h\nu/kT} - 1)^2} d\nu. \end{aligned}$$

A correction factor must still be added, deduced by ROSSELAND¹⁾ on account of the stimulated emission going in the direction of the stimulating radiation, hence representing a part of the radiation that passes unobstructed. It is taken into account by adding a factor $1 - e^{-h\nu/kT}$ to the absorption coefficient, so that

$$\left(\frac{\nu_0}{\nu}\right)^3 = \int \frac{\nu^4 e^{h\nu/kT}}{(e^{h\nu/kT} - 1)^2} d\nu : \int \frac{\nu^7 e^{2h\nu/kT}}{\nu_0^3 (e^{h\nu/kT} - 1)^3} d\nu.$$

By substituting $h\nu/kT = y$ it takes the form

$$\left(\frac{\nu_0}{\nu}\right)^3 = \int \frac{y^4 e^{-y}}{(1 - e^{-y})^2} dy : \left(\frac{kT}{h\nu_0}\right)^3 \int \frac{y^7 e^{-y}}{(1 - e^{-y})^3} dy. \quad (13)$$

The first of these integrals, taken between the limits 0 and ∞ , is equal to $4/15 \pi^4$.

In the formula for the absorption coefficient in high temperature stars

$$k(\nu) = (C_1 s + C_0 g) \left(\frac{\nu_0}{\nu}\right)^3 x_n P = S' \left(\frac{\nu_0}{\nu}\right)^3 x_n P \quad (14)$$

the first factor, called here S' , is constant over large intervals of ν , from one hydrogen bandedge to the next, and then jumps up to a higher value.

¹⁾ The Principles of Quantum Theory, Handbuch der Astrophysik III. 1, S. 456—57.

A. Table of $\varphi(y) = \frac{15}{4\pi^4} \int_0^y \frac{y^2 e^{-y}}{(1-e^{-y})^3} dy$; $y = hv/kT$.

<i>y</i>	Log φ	<i>y</i>	φ	<i>y</i>	φ	<i>y</i>	φ
0.07	2.125 ²⁹¹	0.8	0.0033	4.6	20.24 ¹⁷⁸	8.4	119.15 ²⁵⁴
08	.416 ²⁵⁷	.9	.0062 ²⁹	.7	22.02 ¹⁸⁷	.5	121.69 ²⁴⁹
.09	.673 ²³⁰	1.0	.0107 ⁴⁵	.8	23.89 ¹⁹⁵	.6	124.18 ²⁴⁴
.10	.903 ²⁰⁹	.1	.0176 ¹⁰²	.9	25.84 ²⁰³	.7	126.62 ²⁴⁰
.11	3.112 ¹⁹¹	.2	.0278 ¹⁴⁶	5.0	27.87 ²¹⁰	.8	129.02 ²³⁵
.12	.303 ¹⁷⁶	.3	.0424 ²⁰¹	.1	29.97 ²¹⁸	.9	131.37 ²³⁰
.13	.479 ¹⁶³	.4	.0625 ²⁷²	.2	32.15 ²²⁶	9.0	133.67 ²²⁴
.14	.642 ¹⁵¹	.5	.0897 ³⁶⁰	.3	34.41 ²³³	.1	135.91 ²²⁰
.15	.793 ¹⁴¹	.6	.1257 ⁴⁶⁸	.4	36.74 ²³⁹	.2	138.11 ²¹⁴
.16	.934 ¹³³	.7	.1725 ⁵⁹⁷	.5	39.13 ²⁴⁶	.3	140.25 ²¹⁰
.17	4.067 ¹²⁶	.8	.2322 ⁷⁵¹	.6	41.59 ²⁵¹	.4	142.35 ²⁰⁴
.18	.193 ¹²⁰	.9	.3073 ⁹³²	.7	44.10 ²⁵⁷	.5	144.39 ¹⁹⁸
.19	.313 ¹¹³	2.0	.4005 ¹¹⁴	.8	46.67 ²⁶²	.6	146.37 ¹⁹³
.20	.426 ¹⁰⁷	.1	.515 ¹³⁸	.9	49.29 ²⁶⁷	.7	148.30 ¹⁸⁸
.21	.533 ¹⁰³	.2	.653 ¹⁶⁵	6.0	51.96 ²⁷¹	.8	150.18 ¹⁸³
.22	.636 ⁰⁹⁷	.3	.818 ¹⁹⁷	.1	54.67 ²⁷⁵	.9	152.01 ¹⁷⁷
.23	.733 ⁰⁹⁵	.4	1.015 ²³¹	.2	57.42 ²⁷⁹	10.0	153.78 ¹⁷⁰
.24	.828 ⁰⁹¹	.5	1.246 ²⁶⁹	.3	60.21 ²⁸¹	.5	161.88 ¹⁶⁵
.25	.919 ⁴⁰⁴	.6	1.515 ³¹¹	.4	63.02 ²⁸⁴	11.0	168.73 ¹⁵⁷
.3	5.323 ⁶⁴⁰	.7	1.826 ³⁵⁷	.5	65.86 ²⁸⁶	.5	174.45 ¹⁴⁷
.4	5.963 ⁴⁹⁹	.8	2.183 ⁴⁰⁷	.6	68.72 ²⁸⁸	12.0	179.15 ¹³⁸
.5	6.462 ⁴¹⁰	.9	2.590 ⁴⁶⁰	.7	71.60 ²⁸⁹	.5	182.97 ¹³⁰
.6	6.872 ³⁴⁸	3.0	3.050 ⁵¹⁸	.8	74.49 ²⁸⁹	13.0	186.30 ¹²⁴
.7	7.220 ³⁰²	.1	3.568 ⁵⁷⁹	.9	77.38 ²⁹⁰	.5	188.47 ¹¹⁹
.8	7.522 ²⁶⁸	.2	4.147 ⁶⁴⁴	7.0	80.28 ²⁹⁰	14.0	190.38 ¹¹⁵
.9	7.790 ²³⁹	.3	4.791 ⁷¹²	.1	83.18 ²⁸⁹	.5	191.87 ¹¹⁵
1.0	8.029 ²¹⁷	.4	5.503 ⁷⁸³	.2	86.07 ²⁸⁸	15.0	193.02 ¹¹⁵
.1	8.246 ¹⁹⁹	.5	6.286 ⁸⁵⁸	.3	88.95 ²⁸⁸	16	194.57 ⁸⁹
.2	8.445 ¹⁸²	.6	7.144 ⁹³⁵	.4	91.83 ²⁸⁵	17	195.46 ⁴⁹
.3	8.627 ¹⁸²	.7	8.079 ¹⁰¹⁴	.5	94.68 ²⁸⁴	18	195.95 ²⁷
		.8	9.093 ¹⁰⁹⁵	.6	97.52 ²⁸¹	19	196.22 ¹⁴
		.9	10.188 ¹¹⁷⁹	.7	100.33 ²⁷⁹	20	196.36 ⁷
		4.0	11.367 ¹²⁶	.8	103.12 ²⁷⁶	21	196.43 ⁴
		.1	12.63 ¹³⁵	.9	105.88 ²⁷³	22	196.47 ²
		.2	13.98 ¹⁴⁴	8.0	108.61 ²⁶⁹	23	196.49 ¹
		.3	15.42 ¹⁵²	.1	111.30 ²⁶⁶	24	196.50 ⁰
		.4	16.94 ¹⁶¹	.2	113.96 ²⁶²	25	196.50 ¹
		.5	18.55 ¹⁶⁹	.3	116.58 ²⁵⁷	26	193.51
		.6	20.24	.4	119.15		

Hence the integral

$$\frac{1}{k} = \frac{15}{4\pi^4} \frac{1}{x_H P} \left(\frac{kT}{h\nu_0}\right)^3 \int \frac{1}{S'} \frac{y^2 e^{-y}}{(1-e^{-y})^3} dy$$

may be written as a series of separate terms :

$$\frac{1}{k} = \frac{15}{4\pi^4} \frac{1}{x_H P} \left(\frac{kT}{h\nu_0}\right)^3 \left\{ \frac{1}{S'_1} \int_{y_1}^{\infty} + \frac{1}{S'_2} \int_{y_2}^{y_1} + \dots \right\}$$

In order to facilitate the computation of these terms B. STRÖMGREN has computed a table of the integral ¹⁾

$$\varphi(y) = \frac{15}{4\pi^4} \int_0^y \frac{y^7 e^{-y}}{(1 - e^{-y})^3} dy, \quad (15)$$

so that now

$$\frac{1}{\bar{k}} = \frac{1}{x_n P} \left(\frac{kT}{h\nu_0} \right)^3 \left\{ \frac{1}{S'_1} (\varphi(\infty) - \varphi(y_1)) + \frac{1}{S'_2} (\varphi(y_1) - \varphi(y_2)) + \dots \right\},$$

Denoting the series in parenthesis by I we have

$$I = \sum \frac{1}{S'_n} (\varphi_{n-1} - \varphi_n) \quad \bar{k} = \frac{1}{I} \left(\frac{h\nu_0}{kT} \right)^3 X P = \bar{\kappa} P. \quad (16)$$

STRÖMGREN'S table is computed to one decimal only. Since the divisors S' are small for the higher n , where φ , too, becomes small, it is necessary to have more decimals here. For this reason a table with more decimals is given in Table A p. 18.

We have substituted X for x_H because this computation holds only for those temperatures and pressures where the influence of the metals is imperceptible. The details of the computation are given in Table 8. Values for g were adopted, not exactly equal to those of Table 4, because the wave lengths chiefly determining the mean absorption are below the observable wave lengths for high temperatures, above them for low temperatures. The values adopted are

for	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
log g	0.63	0.75	0.84	0.90	0.94	0.98	1.03	1.07	1.11
log $C_0 g$	7.61	7.99	8.27	8.47	8.63	8.78	8.91	9.03	9.14

They are used, with the data of Table 6, to compute $C_1 s + C_0 g = S'$ in the 4th column of Table 8. The values of $y_n = h\nu_n/kT$ corresponding to the band edges are

$$y = 2.30 \log e^{h\nu/kT} = 2.30 \frac{13.54}{n^2} \frac{5040}{T}.$$

The values of φ corresponding to these edges, and the differences $\varphi_{n-1} - \varphi_n = \Delta\varphi$ are found in columns 2 and 3. The results for $\Delta\varphi/S'$ show that the energy in the case of the hottest stars breaks through chiefly at the lower side of the first (Lyman) edge. For lower temperatures the interval between the 2^d and the 3^d edge, at the lower side of the Balmer edge, begins to contribute, and at 10,000° it contributes half the total transparency. Below 10,000° this interval becomes predominant. Adding together the contributions of all these intervals we find the results of Table 9. A second computation was made with $g = 1$. The separate values are found in the last column of Table 8; the results are included in Table 9.

The differences between the consecutive values of \bar{k} run somewhat irregularly. This is a consequence of the irregular character of the transparency, which is greatest just below each edge. In Figure 1, p. 21 the contributions to I of each interval between two consecutive edges are drawn as a function of $5040/T$ on a logarithmic scale. It is seen there at once how the total transparency, built up as the sum total of these parts, must show considerable irregularities in its course with temperature.

¹⁾ Zeitschrift für Astrophysik, 4, 150 (1932).

Table 8. Mean absorption by hydrogen bands.

τ	y	$\varphi(y)$	$\text{Log } \Delta\varphi$	$\text{Log } S'$	$\text{Log } \frac{\Delta\varphi}{S'}$	Idem $g = 1$	τ	y	$\varphi(y)$	$\text{Log } \Delta\varphi$	$\text{Log } S'$	$\text{Log } \frac{\Delta\varphi}{S'}$	Idem $g = 1$
0.2	∞	196.5	2.14	1.74	0.40	0.40	0.7	21.82	196.5	2.199	1.376	0.823	0.824
	6.24	58.5	1.766	8.459	3.307	3.357		5.46	38.17	1.570	9.643	1.927	1.984
	1.56	0.110	9.04	7.91	1.13	1.34		2.42	1.058	0.002	9.112	0.890	1.124
	0.69	0.00154	7.16	7.76	9.40	9.74		1.36	0.0534	8.68	8.93	9.75	0.18
	0.39	0.00008						0.87	0.00518	7.64	8.89	8.75	9.25
0.3			1.74	3.54	8.20	8.20	0.8						
	9.35	141.25	2.147	9.171	2.976	3.001		24.94	196.5	2.140	1.855	0.285	0.285
	2.34	0.893	9.944	8.385	1.559	1.733		6.24	58.53	1.752	9.912	1.840	1.881
	1.04	0.0131	8.09	8.17	9.92	0.25		2.77	2.071	0.293	9.310	0.983	1.177
	0.58	0.0007						1.56	0.1059	8.98	9.11	9.87	0.23
0.4							0.9						
	12.47	182.5	2.252	9.787	2.465	2.476		1.00	0.0107	7.96	9.03	8.93	9.43
	3.12	3.679	0.559	8.763	1.796	1.936		0.69	0.0015				
	1.39	0.0602	8.76	8.488	0.27	0.59		28.06	196.5	2.063	2.320	9.74	9.74
	0.78	0.00291	7.42	8.39	9.03	9.48		7.02	80.86	1.887	0.165	1.722	1.762
0.5							1.0						
	15.59	193.9	2.264	0.348	1.916	1.921		3.12	3.679	0.541	9.476	1.065	1.236
	3.90	10.19	1.000	9.082	1.918	2.032		1.75	0.2006	9.26	9.25	0.01	0.36
	1.73	0.189	9.255	8.732	0.523	0.806		1.12	0.0198	8.23	9.16	9.07	9.56
	0.97	0.00938	7.93	8.61	9.32	9.75		0.78	0.0029				
0.6							1.0						
	18.71	196	2.241	0.874	1.367	1.369		7.79	102.8	1.97	2.77	9.20	9.20
	4.68	21.66	1.326	9.373	1.953	2.029		3.46	5.964	1.986	0.410	1.576	1.598
	2.08	0.490	9.668	8.940	0.728	0.975		1.95	0.3514	0.749	9.635	1.114	1.266
	1.17	0.0244	8.34	8.79	9.55	9.95		1.25	0.0345	9.50	9.38	0.12	0.45
	0.75	0.0023					0.87	0.0052	8.47	9.28	9.19	9.67	

Table 9. Results for mean absorption by hydrogen.

τ	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$\text{Log } I$	3.311	2.993	2.552	2.227	2.075	1.998	1.912	1.820	1.718
$\text{Log } \left(\frac{h\nu_0}{kT}\right)^3$	0.455	0.983	1.358	1.649	1.887	2.087	2.261	2.415	2.552
$\text{Log } \frac{\kappa}{X}$	7.14	7.99	8.81	9.42	9.82	0.09	0.35	0.60	0.83
$\text{Log } I(g=1)$	3.362	3.025	2.591	2.292	2.148	2.072	1.978	1.886	1.789
Difference \bar{k}	-.05	-.03	-.04	-.06	-.07	-.07	-.07	-.07	-.07

Absorption by a mixture of metal atoms.

For the lower temperatures the absorption is caused at an increasing extent, and at last entirely, by the metal atoms. Here the conditions are very different from the hydrogen atmospheres in that the atom mixture has a large number of absorption edges at small distances, irregularly distributed, instead of the few edges of hydrogen. The Kramers formula is not directly applicable here; by some kind of extrapolation we must try to find a method of working representing the most probable suppositions.

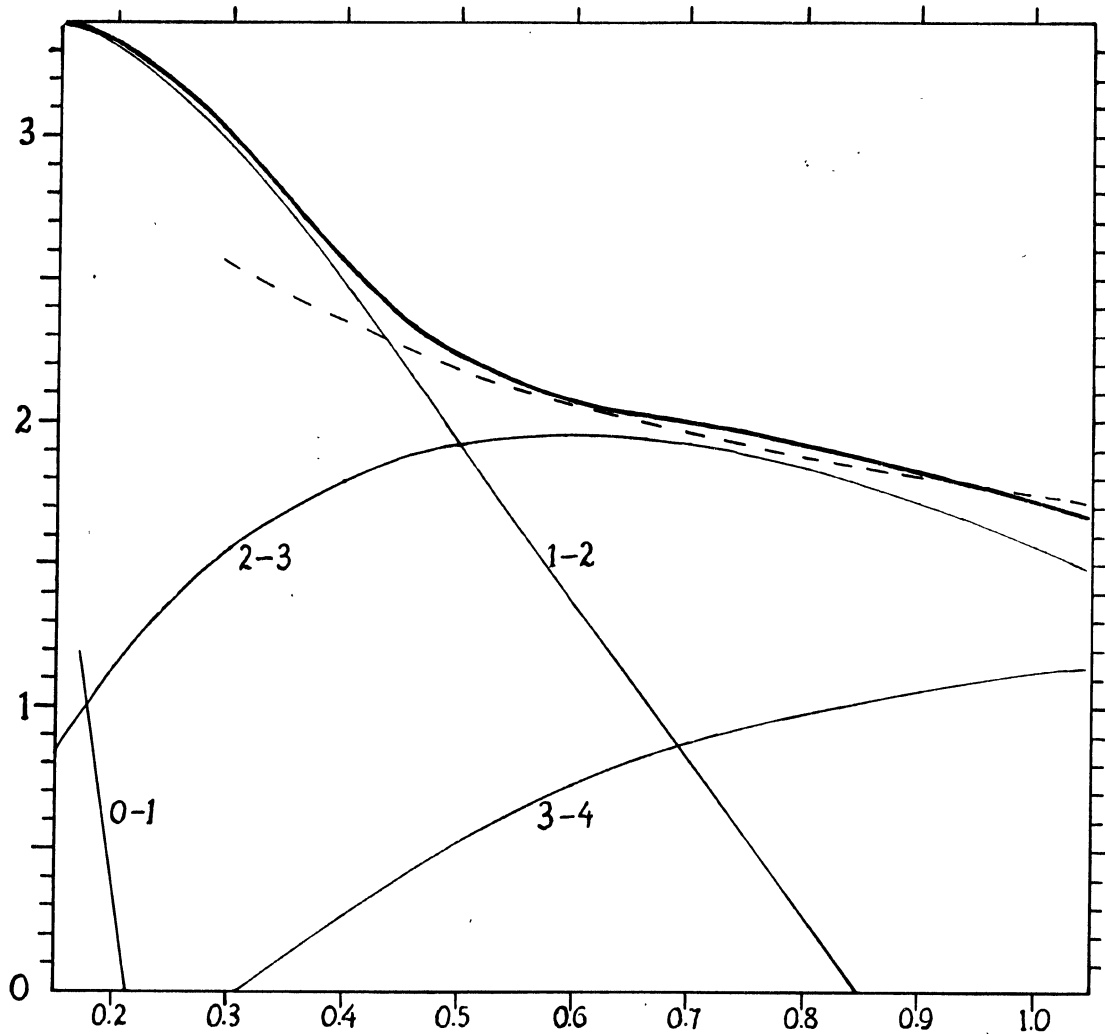


Fig. 1. Transparency of a hydrogen atmosphere.

KRAMERS in his well known paper introduces the apriori probabilities of one electron running freely through space V with velocity v , and the same electron bound to the positive particle in an orbit with weight a :

$$P_i dv : P_n = 4\pi \sqrt{m^3 v^2} dv e^{-hv/kT} : ah^3.$$

Then for equilibrium we have the equality of emissions and absorptions $P_n Q_{n \rightarrow i} = P_i Q_{i \rightarrow n}$, where the Q are transition probabilities between the neutral and the ionized states. $Q_{n \rightarrow i}$ contains the atomic absorption coefficient and the intensity of the incident radiation, $Q_{i \rightarrow n}$ contains the capture factor β , which is derived by means of the correspondence principle. In this way the weight factor a appears in the denominator of the absorption coefficient a , from which our k is derived ; in the case of hydrogen the weight of all the coincident orbits with total quantum number n is n^2 . In the computation of β for each velocity v of the free electron all the

cases $E = \frac{1}{2} mv^2 - h\nu$, where E is comprised between $E(n - \frac{1}{2})$ and $E(n + \frac{1}{2})$ (and so ν is comprised in the corresponding interval $\Delta\nu$) are collected into one atomic level $E(n)$, giving rise to a band that extends from the edge ν (for $v = 0$) to $\nu = \infty$ (for $v = \infty$). In the case of hydrogen, where $E(n) = hR/n^2$ this interval introduces (by dE/dn) the factor $2/n^3$, or $(n - \frac{1}{2})^{-2} - (n + \frac{1}{2})^{-2}$.

A mixture of metal atoms in its absorbing effect upon radiation may be considered as one kind of atoms having all the band edges of the mixture, each weakened in proportion to the fractional abundance of the corresponding constituent in the mixture. We may compare this case with that of hydrogen, if we suppose each band of hydrogen to be broken up into a number of minor bands, nearer together and with correspondingly smaller intensities. The weights p_n/B in the numerator and n^2 in the denominator are cancelled as with hydrogen. One method of working now consists in supposing the number of separate edges, into which each band is broken up, to increase indefinitely. Then the sum is replaced by an integral, and for the atomic absorption instead of 2) we have

$$\frac{16\pi^2}{3\sqrt{3}} \frac{\epsilon^6}{ch^3 \nu^3} \int_0^\nu e^{-h(\nu_1 - \nu_n)/kT} d\nu_n.$$

The integration is extended over all the elementary edges ν_n below the frequency for which the absorption is computed. The result of the integration is

$$\frac{16\pi^2}{3\sqrt{3}} \frac{\epsilon^6 kT}{ch^4 \nu^3} \left(e^{-h(\nu_1 - \nu)/kT} - e^{-h\nu/kT} \right).$$

Multiplying by $A(1-x)/m_H$ to have the mass absorption coefficient, and eliminating, just as p. 8, the factor $1-x$ by the ionization formula (without the factors B , because we consider an abstract element in general), we find

$$k_1(\nu) = 1.30 \frac{4\sqrt{2\pi}}{3\sqrt{3}} \frac{\epsilon^6 x_M P}{chm^{3/2} (kT)^{1/2} m_H \nu^3} \left(e^{h\nu/kT} - 1 \right).$$

This smoothed formula comes instead of 4). It agrees with the formula 1) for $k_0(\nu)$ but for the additional factors $1.30 (e^{h\nu/kT} - 1)$ coming instead of g . Hence the total absorption coefficient may be written

$$k(\nu) = C_0 \left(\frac{\nu_0}{\nu} \right)^3 x_M P \left\{ g + 1.30 \left(e^{h\nu/kT} - 1 \right) \right\}. \quad (17)$$

The differences in $k(\nu)$ between this smoothed formula and the real values, which change by irregular jumps, will not be large (cf p. 28—29). If, however, we want to make use of the smoothed formula to compute the mean absorption coefficient, the matter becomes more critical. The out-streaming energy passes most easily through the most transparent wave length regions just at the red side of the edges. By the smoothing process these peaks of maximum transparency are obliterated, and the total absorption must be found too large.

Among the most abundant constituents of the stellar atmospheres we find such elements as *Mg*, *Na* and *Ca*, which in the region of the spectrum chiefly determining the mean absorption have only a limited number of levels. Hence it is desirable to follow here another method of working, which treats the actual levels of these elements separately.

Whereas with hydrogen all the states with the same n actually coincide in one level, strictly separated from the next n , this is not the case with other atoms. Each state is defined by three quantum numbers $n_1 + n_2 + n_3 = n$, expressing the impulse integrals in three dimensions (with a fourth number due to electron spin). The energies of these states, also if they have the same n ,

are different; the levels are intermingled. They form a two dimensional series (because n_3 does not affect the energy), where the simple sweeping up of all ν into one series of discrete edges, as with hydrogen, is not possible.

So we can extend the KRAMERS formula to other atoms only by some uncertain analogy. Usually it is made in this way. In formula 3) the factor $1/n^5$ occurs; because for hydrogen $n^2 = \nu_1/\nu_n$, this relation is assumed to hold for other atoms, and for $1/n^5$ is substituted $(\nu_1/\nu_n)^{5/2}$, where ν_1 is the frequency corresponding to the lowest level of the atom.

It must be remarked, however, that this factor consists of two parts $1/n^3$ and $1/n^2$, having an entirely different origin and meaning. The former is got by sweeping up an emission value β , extending over the spectrum, into separate band edges. If we now take e.g. the sodium atom, we have a number of series: S, P, D, \dots each extending to $n = \infty$. In each of them $\nu = R/n'^2 = R/(n + \mu)^2$, where μ varies considerably in the lower levels, and for higher n becomes constant. We may consider and determine n' and μ as continuous functions of n . In the sweeping up process, by analogy with hydrogen, we have to take the range $\Delta\nu_n$ from $n - 1/2$ to $n + 1/2$; for the higher members this gives $2R/n'^3 dn'/dn$ (with $dn'/dn \rightarrow 1$ for increasing n); for the lower members of the series it is more correct to take $R((n'_{n-1/2})^{-2} - (n'_{n+1/2})^{-2})$. In the case of the lowest levels this depends, it is true, upon a very uncertain extrapolation for $n - 1/2$; fortunately it will appear that just these lowest levels do not matter very much, because they have only a small influence on the total absorption.

The other factor $1/n^2$ is introduced as the a priori weight of the state with chief quantum number n . In other atoms this state is broken up into a number of states, forming a second dimension series. Now the capture function β extending over the whole spectrum $\nu = 0 - \infty$, can be swept up only once into separate states, each of which fills a domain between $n - 1/2$ to $n + 1/2$. In equalizing the emissions and the absorptions it is not allowed, therefore, to take for a the weight of one special state (defined by n and l). We have first to pile up the S, P, D, \dots series into one series, and though the terms do not coincide, their weights may be added. Each of these states, defined by two quantum numbers n and l , has a weight $p_{nl} = 2j + 1$ where $\sum_l p_{nl} = n^2$. We now assume, as the most rational assumption by analogy, though it is certainly not exactly true, that the portion in this division attributed to, say $3P$, corresponds to the part the weight of $3P$ contributes to the total weight of all the states with $n = 3$. Then we have for the a priori weight a , attributed to the absorptions from the states with a certain n , the total weight $\sum p_{nl} = n^2$. Hence in the denominator of the absorption formula the factor n^2 remains, and it means the total weight of all the states with chief quantum number n .

As to the multiplicity of the terms we have to consider that it depends on the introduction of an additional quantum number from the electron spin, which gives a new impulse integral of a new dimension. So it must be subsumed under the former cases in such a way, that all the multiplet series of the atom are substituted for the one series in the simple case. Hence to the denominator n^2 is added the multiplicity factor M , which for Na, K is 2, for Mg, Ca is $1 + 3 = 4$, for Al, Sc, \dots is $2 + 4 = 6$, for Ti is $1 + 3 + 5 = 9$, etc.

This procedure can be followed only in cases where the levels can be classified in regular series and the total quantum number can be found. In the case of more complicated atoms, where more electrons with different n take part in forming the levels, it is not possible to arrange them into separate series. We cannot coordinate groups of such terms to a Kramers integral over a certain $\Delta\nu$, and the whole process of dividing up the continuous function into band edges loses its meaning. For such atoms, as those of iron, it does not seem possible to derive, by some kind of rational analogy, absorption values for the separate edges.

Continuous absorption for lower temperatures.

The RUSSELL mixture of metal atoms contains some elements for which the series and the quantum numbers are well known and the band edges are not too numerous (*Na, K, Mg, Ca*), and others with an enormous number of overlapping multiplets, to which no chief quantum number can be assigned (*Fe*). Hence we will use both methods, and divide the elements into two groups :

1. For *Na, K, Mg, Ca* we compute the separate band edges and their contribution to the continuous absorption. For the medium temperatures we add *H* to this list ;
2. All the remaining constituents are taken together (*Fe* complex) and considered as one element with a great number of close band edges, which must be treated by the smoothed formula. Its abundance is taken 314×10^{-6} ($\log A = 6.50$).

The absorptions resulting from these two groups must still be increased by the absorption due to free-free transitions.

For the four elements named above, with *H*, a list of all the levels was made for which the ionization potential $V_n > 0.5$, making use of the data published by Miss C. E. MOORE ¹⁾. Multiplets in every case were taken as one entry. They are given in Table 12. For each level the value

$$C_1 A x \frac{p_n}{Mn^2 p'_1} ((n'_{n-1/2})^{-2} - (n'_{n+1/2})^{-2}) e^{h\nu_n/kT}$$

was computed (p'_1 , the weight of the lowest level in the ionized atom, stands for B' , of which it is the chief term), for each of the temperatures from 8400° to 2520° and for two electron pressures *P*. These were chosen in such a way that they included the most efficient part of the atmosphere : viz $\log P = 2$ and 0 for temperatures 8400° — 4200° and $\log P = 0$ and —2 for lower temperatures. The values of *A* used are

$$\log A = 6.100 \text{ (Na), } 5.699 \text{ (K), } 6.672 \text{ (Mg), } 5.602 \text{ (Ca).}$$

The ionization *x* for each element, as computed with RUSSELL's data, is contained in Table 10. The weights p_n/p'_1 for each multiplet and the factor Mn^2 are given in the 4th column of Table 12.

Table 10. Ionization of elements. Log *x*.

<i>P</i>	8400	7200	6300	5600	5040	4200	3600	3150	2520
<i>Na</i>	2	0	0	0	—0.02	—0.29			
	0	0	0	0	0	0	—0.06	—0.50	—2.64
	—2						0	—0.01	—0.73
<i>K</i>	2	0	0	0	0	—0.05			
	0	0	0	0	0	0	—0.01	—0.05	—1.11
	—2						0	0	—0.05
<i>Mg</i>	2	0	—0.01	—0.06	—0.34	—0.99	—2.66		
	0	0	0	0	—0.01	—0.04	—0.75	—2.25	—4.02
	—2						—0.44	—2.02	—5.31
<i>Ca</i>	2	0	0	0	—0.02	—0.10	—0.90		
	0	0	0	0	0	0	—0.03	—0.43	—1.59
	—2						—0.01	—0.14	—2.27
<i>H</i>	2	—1.23	—2.73	—4.22	—5.71	—7.17	—	—	—
	0	—0.06	—0.80	—2.22	—3.71	—5.17	—	—	—

¹⁾ Term Designations for Excitation Potentials. Princeton Observatory, 1934.

In Table 11 the actual quantum limits $n'(2^{1/2}, 3^{1/2}, \dots)$ are derived. From ν_n or V_n for each level the actual quantum numbers $n' = \sqrt{R/\nu} = \sqrt{13.54/V}$, corresponding to the whole numbers n , were found. From curves representing $n' = f(n)$ the values of n' corresponding to halves in n were read. Then the factor $(n'_{n-1/2})^{-2} - (n'_{n+1/2})^{-2}$, denoted in the tables by $2/n'^3$ was computed (Table 12, column 6).

Table 11. Actual quantum limits.

Sodium.						Potassium.									
n	2S		2P		2D		n	2S		2P		2D		2F	
3	1.63	1.13	2.11	1.60	2.98	2.47	3								
4	2.64	2.14	3.13	2.62	3.96	3.47	4	1.77	1.25	2.28	1.72	2.86	2.36		
5	3.64	3.14	4.13	3.63	4.97	4.47	5	2.81	2.29	3.27	2.76			4.00	3.50
6	4.67	4.15		4.63	5.96	5.46	6		3.32	4.28	3.77			5.00	4.50
7	5.63	5.15									4.78			6.00	5.50

Magnesium.																
n	1S		1P		1D		1F		3S		3P		3D		3F	
3	1.33	0.72	2.01	1.54	2.68	2.24					1.66	1.04	2.83	2.32		
4	2.46	1.91	2.98	2.49	3.59	3.13	3.97	3.47	2.32	1.78	2.82	2.24	3.84	3.33	3.97	3.47
5	3.48	2.97	3.97	3.48	4.53	4.06	4.97	4.47	3.35	2.84	3.86	3.34	4.83	4.33	4.97	4.47
6	4.50	3.99		4.47	5.50	5.01	5.98	5.48		3.86	4.87	4.37		5.35		5.47
7	5.49	5.00									5.88	5.38				

Calcium.																
n	1S		1P		1D		1F		3S		3P		3D		3F	
3					2.00	1.50							1.94	1.31		
4	1.49	0.95	2.07	1.09	3.02	2.51	3.97	3.51			1.80	1.22	3.08	2.53	3.92	3.43
5	2.61	2.04	3.77	3.05	4.13	3.58	4.92	4.44	2.48	1.94	2.93	2.37	4.09	3.60	4.90	4.41
6	3.81	3.22	4.50	4.24	5.03	4.60	5.90	5.41	3.52	3.00	4.01	3.48	5.05	4.57	5.88	5.39
7	4.66	4.28	5.30	4.87		5.53		6.40	4.53	4.03		4.53	6.01	5.53		
8		5.07							5.52	5.03						

Then for each temperature the variable factors $\log e^{h\nu_n/kT} = \frac{5040}{T} V_n$ were added. Thus for each of these temperatures two series of band intensities were obtained, for two pressures different by their x . These band intensities had to be added up from the lowest ($V_n = 0$), at every edge adding its intensity to get the total intensity for the next interval. The summation started with $V_n = 0.5$; for all the edges below this limit the sum total was computed by means of the smoothed formula for $\Sigma Ax(Na + K + Mg + Ca)$ and increased by $S(5\frac{1}{2})$ for hydrogen. In this way tables of C_1S (formulas 9) 10)) for all the intervals were obtained. It must be remarked, that in this computation the coefficient 1.30 in C_1 was omitted.

To these values first the corresponding values from free-free transitions C_0gX were added, constant for each table; the total electron pressure X was taken from Table 1. Because these tables

Table 12. List of levels and band edges.

	Level	V_n	$p_n/p_1' : Mn^2$	$\text{Log } \frac{p_n}{p_1' Mn^2}$	$\text{Log } ^2/n^3$		Level	V_n	$p_n/p_1' : Mn^2$	$\text{Log } \frac{p_n}{p_1' Mn^2}$	$\text{Log } ^2/n^3$
H	1	13.54	2 : 1	0.301	0.551	Na	5 ² S	1.02	1 : 25	8.602	8.637
Mg	3 ¹ S	7.71	1 : 36	8.444	0.219	Ca	5 ¹ P	0.95	3 : 100	8.477	8.715
Ca	4 ¹ S	6.09	1 : 64	8.194	9.938	Ca	6 ¹ S	0.93	1 : 144	7.842	8.621
Na	3 ² S	5.12	1 : 9	9.046	9.752	Mg	4 ³ D	0.92	15 : 64	9.370	8.566
Mg	3 ² P	4.91	9 : 36	9.398	9.861	Mg	5 ³ P	0.91	9 : 100	8.954	8.572
K	4 ² S	4.32	1 : 16	8.796	9.652	Ca	4 ³ F	0.88	21 : 64	9.516	8.526
Ca	4 ³ P	4.20	9 : 64	9.148	9.693	Ca	4 ¹ F	0.86	7 : 64	9.039	8.484
Ca	3 ² D	3.58	15 : 36	9.620	9.630	Mg	4 ¹ F	0.86	7 : 64	9.039	8.519
Ca	3 ¹ D	3.39	5 : 36	9.143	9.456	Mg	4 ³ F	0.86	21 : 64	9.516	8.519
H	2	3.38	8 : 4	0.301	9.454	Mg	5 ¹ P	0.86	3 : 100	8.477	8.512
Mg	3 ¹ P	3.28	3 : 36	8.921	9.416	Na	4 ² D	0.86	5 : 16	9.495	8.519
Ca	4 ¹ P	3.17	3 : 64	8.671	9.866	K	4 ² F	0.85	7 : 16	9.641	8.508
Na	3 ² P	3.03	3 : 9	9.523	9.389	H	4	0.85	32 : 16	0.301	8.508
K	4 ² P	2.71	3 : 16	9.273	9.316	Ca	6 ³ P	0.84	9 : 144	8.796	8.529
Mg	4 ² S	2.52	3 : 64	8.671	9.282	Ca	5 ³ D	0.81	15 : 100	9.176	8.466
Mg	4 ¹ S	2.24	1 : 64	8.194	9.206	Ca	5 ¹ D	0.79	5 : 100	8.639	8.488
Ca	5 ² S	2.20	3 : 100	8.477	9.189	Na	5 ² P	0.79	3 : 25	9.079	8.466
Ca	5 ¹ S	1.98	1 : 100	8.000	9.158	K	6 ² P	0.74	3 : 36	8.921	8.425
Na	4 ² S	1.94	1 : 16	8.796	9.068	Ca	6 ¹ P	0.67	3 : 144	8.319	8.128
Mg	5 ¹ D	1.88	5 : 36	9.143	8.988	Mg	6 ¹ S	0.67	1 : 144	7.842	8.358
K	5 ² S	1.72	1 : 25	8.602	9.000	Mg	5 ¹ D	0.66	5 : 100	8.699	8.319
Mg	4 ² P	1.70	9 : 64	9.148	9.040	Ca	7 ³ S	0.66	3 : 196	8.185	8.343
Mg	3 ² D	1.69	15 : 36	9.620	8.980	Ca	7 ¹ S	0.62	1 : 196	7.708	8.196
K	3 ² D	1.66	5 : 9	9.745	8.959	Na	6 ² S	0.62	1 : 36	8.444	8.309
Ca	5 ³ P	1.58	9 : 100	8.954	8.980	Mg	5 ² D	0.58	15 : 100	9.176	8.258
Mg	4 ¹ P	1.52	3 : 64	8.671	8.896	Mg	6 ³ P	0.57	9 : 144	8.796	8.251
Na	3 ² D	1.52	5 : 9	9.745	8.908	Ca	5 ³ F	0.56	21 : 100	9.322	8.230
H	3	1.50	18 : 9	0.301	8.895	Ca	5 ¹ F	0.56	7 : 100	8.845	8.219
Ca	4 ¹ D	1.49	5 : 64	8.893	8.907	Mg	5 ¹ F	0.55	7 : 100	8.845	8.224
Ca	4 ³ D	1.43	15 : 64	9.370	8.898	Mg	5 ³ F	0.55	21 : 100	9.322	8.221
Na	4 ² P	1.38	3 : 16	9.273	8.844	Na	5 ² D	0.55	5 : 25	9.301	8.217
K	5 ² P	1.27	3 : 25	9.079	8.785	K	5 ² F	0.54	7 : 25	9.447	8.213
Mg	5 ³ S	1.21	3 : 100	8.477	8.755	H	5	0.54	50 : 25	0.301	8.213
Mg	5 ¹ S	1.12	1 : 100	8.000	8.704	Ca	6 ¹ D	0.54	5 : 144	8.541	8.163
Ca	6 ³ S	1.09	3 : 144	8.319	8.695	Ca	6 ³ D	0.53	15 : 144	9.018	8.182
Mg	4 ¹ D	1.05	5 : 64	8.893	8.617						

should be used for the mean absorption too, and here the part of the spectrum about $V = 1.5$ dominates the total transparency, values for g somewhat different from Table 3 were adopted, viz

$\tau =$	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	2.0
$\log g$	0.92	0.98	1.03	1.07	1.11	1.18	1.23	1.27	1.33

Then, secondly, the part of the absorption caused by the Fe complex was added by means of the smoothed formula

$$C_0(Ax)_{Fe} \left(e^{h\nu/kT} - 1 \right).$$

Here also the factor 1.30 has to be added afterwards. The ionization x was computed simply

as if the whole complex consisted of iron atoms. Thus the product $\log C_0(Ax)_{Fe}$ was assumed ($\log A = 6.50$)

$\tau =$	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6
$\log P = 2$	4.19	4.28	4.31	4.07	3.46	1.89		
0	4.19	4.29	4.38	4.45	4.49	3.82	2.30	0.71
-2							4.17	2.71

For the sum total of the three parts we write

$$S'' = C_1 S + C_0 g X + C_0 A x_{Fe} (e^{h\nu/kT} - 1) \quad (18)$$

$$k(\nu) = S'' \left(\frac{\nu_0}{\nu}\right)^3 P = \kappa(\nu) P \quad (19)$$

(S'' differs from S' used before in that it contains the ionizations x). Owing to the exponential factor the third term runs up continuously with the frequency. Hence S'' is not constant in each interval between two edges, but runs up towards the large ν values, more slowly or more rapidly according to the relative amount of the terms.

The absorption coefficient κ decreases in each interval with decreasing wave length and then jumps up at each band edge to a higher value. It is represented on a logarithmic scale in Figures 2 en 3 (p. 28 and 29). for the different temperatures $6300^\circ - 2520^\circ$ for the observable wave length interval 7400 to 3000 \AA , after correction $+ 0.11 = \log 1.30$ (the free-free part does not require this correction, but the difference is at most 0.01 or 0.02). We see that for 6300° the hydrogen edge at 3646 \AA is still the most important one. Then it decreases, and first the Mg edges at 3790 \AA and 7310 \AA become visible among the minor ones. But soon they are surpassed in importance by the Na edge at 4086 \AA and the Ca edge at 3454; below 5000° this Na edge and the K edge at 4541 \AA give the most important jumps.

The sudden jumps in the continuous absorption, deduced here by theory, have been ascertained by observation only in the case of the second hydrogen edge, the limit of the Balmer series, in class A stars. The Mg edge at 7310 \AA has been sought for in vain on Utrecht spectrograms by MINNAERT. In the stars of lowest temperature they may be obliterated by the dense bandlike groups of molecular lines. On reproductions of spectra of class K stars no trace of dark bands having their edges at 4086 \AA or 4541 \AA is found. A small hump in the relative intensity curve of a $G8$ and a $A4$ star on Göttingen spectrograms, published by L. BIERMANN,¹⁾ may perhaps be due to the smoothed effect of the Na edge at 4086 \AA . Generally the tails of absorption line series crowding just at the edges in the most transparent parts of the spectrum, must round the sharp edges. There may be at work other smoothing processes in the absorption by metal atoms, taking its origin perhaps from the mutual perturbations of orbits in these complicated atoms. Hence the question may be posed whether it is better to assume for the coefficient of continuous absorption these abruptly varying values, or some smoothed function of the wave length.

In the latter case it is rational to make use of the smoothed formula for the complex of all the metal atoms and to add the absorption due to hydrogen

$$k(\nu) = \left\{ (C_1 s + C_0 g) x_H + 1.30 C_0 \left(e^{h\nu/kT} - 1 + g/1.30 \right) x_M \right\} \left(\frac{\nu_0}{\nu}\right)^3 P.$$

For g we take the values of Table 4 just as with hydrogen. The results for

$$1.30 C_0 (e^{h\nu/kT} - 1) + C_0 g$$

¹⁾ Untersuchungen über Sternatmosphären (Veröff. Göttingen **34**, 308).

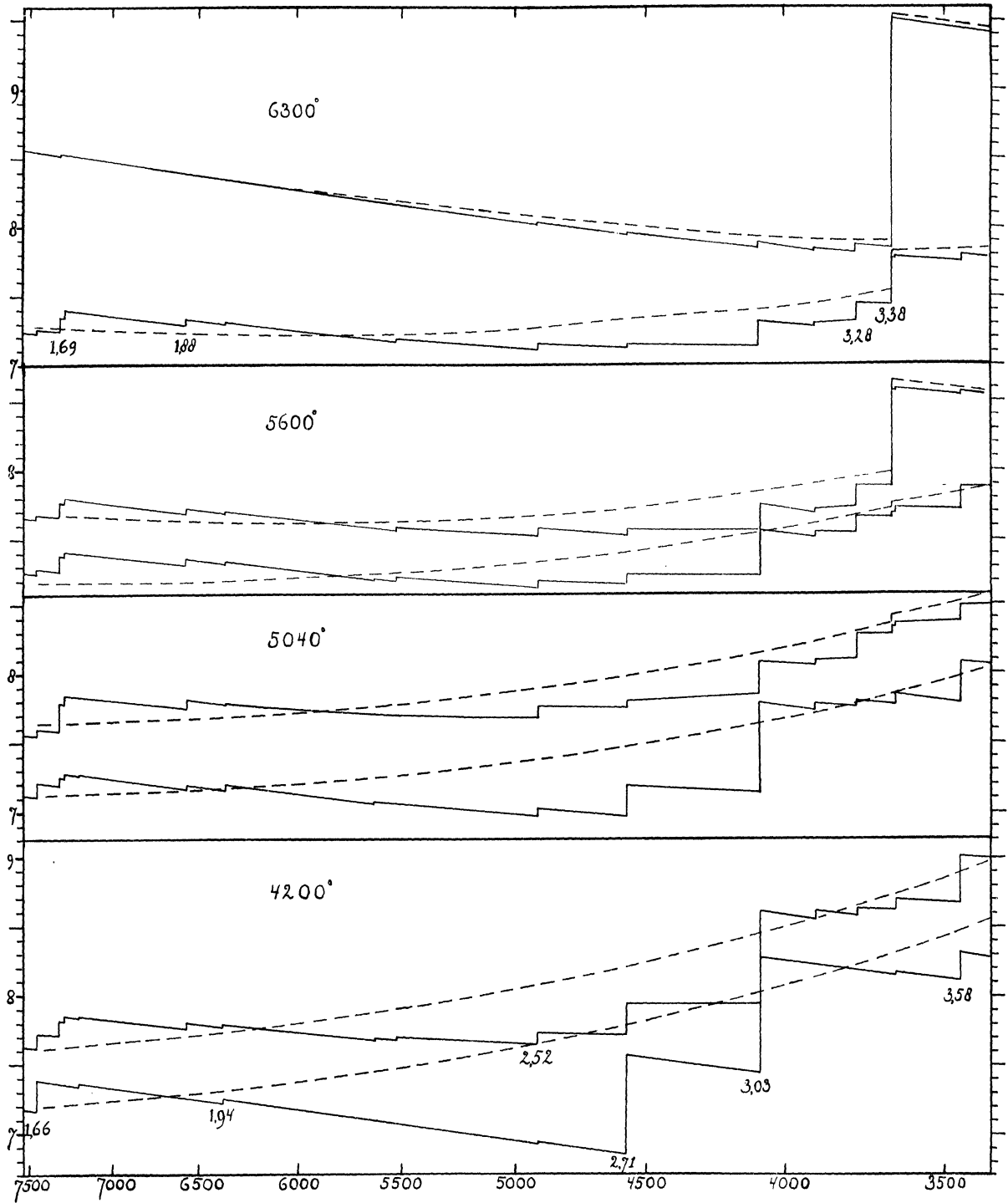


Fig. 2. Variation of absorption coefficient with wave length.

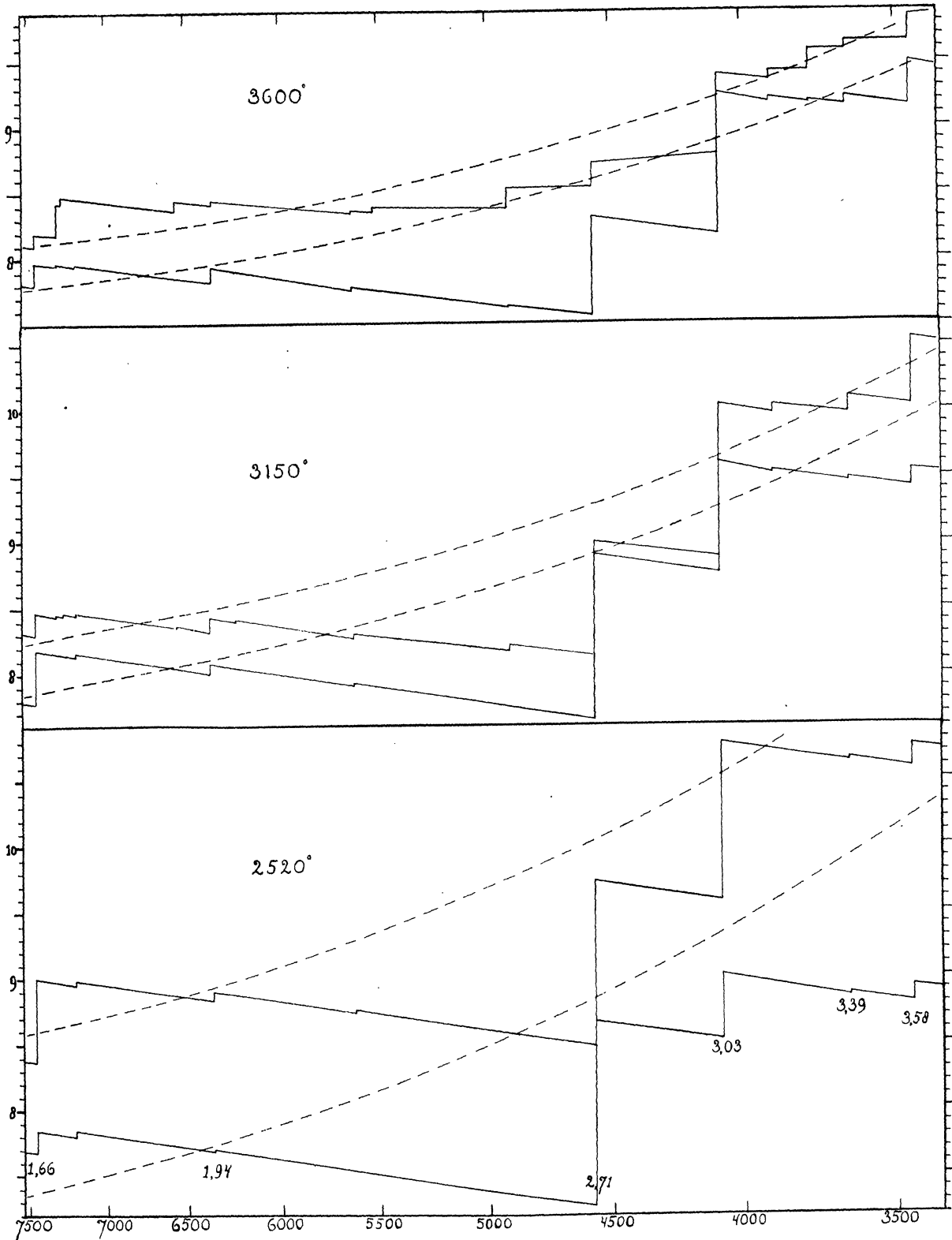


Fig. 3. Variation of absorption coefficient with wave length.

are given in Table 13 in such a form that for $\lambda = 4000 \text{ \AA}$ its value is given as a standard value, whereas for other wave lengths the deviations from these standard values are given separately. The changes in these results, if we take $g = 1$, are added in the lower part of the table.

Table 13. Continuous absorption by metal atoms after the smoothed formula. Coefficients of $x_m P$.

	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	2.0
Standard	8.41	8.87	9.29	9.69	0.09	0.48	0.86	1.23	1.97	2.68	3.39	4.77
λ												
3000	-.13	-.02	+.10	+.22	+.33	+.44	+.55	+.65	+.86	+1.07	+1.27	+1.69
3500	-.07	-.03	+.02	+.07	+.12	+.17	+.22	+.26	+.35	+.44	+.53	+.71
4000	0	0	0	0	0	0	0	0	0	0	0	0
5000	+.16	+.10	+.04	-.03	-.11	-.18	-.25	-.32	-.45	-.57	-.70	-.94
6000	+.32	+.23	+.12	+.01	-.11	-.24	-.35	-.47	-.69	-.91	-1.12	-1.53
7000	+.48	+.37	+.23	+.09	-.07	-.23	-.38	-.53	-.82	-1.10	-1.38	-1.91
Logarithmic deviations for $g = 1$												
3000	-.08	-.04	-.02	-.01	0	0	0	0	0	0	0	0
3500	-.11	-.07	-.04	-.02	-.01	0	0	0	0	0	0	0
4000	-.14	-.10	-.06	-.04	-.02	-.01	-.01	0	0	0	0	0
5000	-.21	-.17	-.12	-.08	-.05	-.03	-.02	-.01	-.01	0	0	0
6000	-.27	-.23	-.18	-.14	-.10	-.07	-.05	-.04	-.02	-.01	0	0
7000	-.31	-.28	-.24	-.19	-.15	-.12	-.09	-.07	-.04	-.02	-.01	0

By means of these tables the values of x were derived for the same electron pressures, for which the computation by separate edges had been made. They form continuous curves with only jumps at the hydrogen edges. These curves are inserted into the drawings of the stepped diagrams Figs. 2 and 3. It appears that generally the continuous curves run somewhat higher than the mean of the discontinuous stepped graph, especially for the lower temperatures. Partly this discrepancy may be due to the spreading of the atom levels belonging to a certain n , which especially for the widely separated lowest levels with high V may cause strong irregularities. For lower V and greater wave lengths, where the levels become closer and more regular, the discrepancy is smaller.

The mean absorption for lower temperatures.

The chief purpose of the construction of the tables of discontinuous absorption values jumping at every bandedge, was the computation of the mean absorption. In this case formula 13) for the Rosseland mean, by the use of 19) takes the form

$$\frac{1}{k} = \frac{15}{4\pi^4} \frac{1}{P} \left(\frac{kT}{h\nu_0} \right)^3 \int \frac{1}{S''} \frac{y^7 e^{-y}}{(1 - e^{-y})^3} dy.$$

Here the use of the function φ is somewhat cumbersome, because the intervals between the separate edges are small, and the limits for y must be computed separately by $y = 2.30 (5040 V/T)$. It is easier in this case to make use of the mean value of the integrand φ' over each interval and to introduce, instead of $y = h\nu/kT$ another argument $y' = 0.43y = \log e y = 5040 V/T = V\tau$, which is already known for each edge and occurs in the tables. We put

$$\frac{15}{4\pi^4} \int \frac{y^7 e^{-y}}{(1 - e^{-y})^3} dy = \frac{15 \times 2.30^8}{4\pi^4} \int \frac{y'^7 [-V\tau]}{\{1 - [-V\tau]\}^3} dy' = \int f(y') dy'.$$

A table of $f(y')$ can be easily constructed and is given as Table B, p. 32. Then for the integral we put a summation:

$$\int \frac{1}{S''} f(y') dy' = \sum \frac{f(y')}{S''} \Delta y' = I'. \quad (21)$$

(I' distinguished from our former I , because the ionization x is included here). In each interval $f(y')$ and S'' both are changing, the latter by the contribution of the Fe absorption. Therefore for each interval the value of $f(y')/S''$ was computed for its limits, and the mean of these limiting values was assumed to hold for the interval. In some cases of large intervals another computation was made after dividing it into a series of smaller intervals; the differences were hardly perceptible.

To illustrate the character of the S'' function and to show how it produces the mean absorption a graphical representation is made for some temperatures. (Fig. 4, p. 33). The ordinate represents $\log S''$ as a broken line, and the curve for $\log f(y')$. The importance of each interval for the mean absorption depends on the vertical distance of the curve above the broken line, this distance being $\log f(y')/S''$.

From the result of the summation I' the mean absorption coefficient is found by

$$\bar{k} = \frac{1}{I'} \left(\frac{h\nu_0}{kT} \right)^3 P = \bar{x}P. \quad (22)$$

The numerical results are

	8400°	7200°	6300°	5600°	5040°	4200°	3600°	3150°	2520°
$\log P$	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	2.0
$\log I'$							4.952	5.247	5.610
$\log \left(\frac{h\nu_0}{kT} \right)^3$									
	2.187	2.833	4.114	4.802	4.873	5.191	5.422	5.654	6.841
	3.341	4.551	5.027	5.166	5.463	5.729			
	1.887	2.087	2.261	2.415	2.552	2.790	2.990	3.164	3.455
							8.04	7.92	7.85
$\log \bar{x}$	0	9.70	9.25	8.15	7.61	7.68	7.60	7.57	6.61
	+2	8.55	7.54	7.23	7.25	7.09	7.06		

It is, of course, not well possible to make the same elaborate computations for each value of P and T . In the case of $k(\nu)$ it was possible to compute the absorptions due to free-free transitions, hydrogen and metal atoms separately and afterwards add the results together. In the case of the mean absorption this cannot be done, because the reciprocals of the total values must be averaged. On the other hand it is an easy matter to perform the computation by means of the smoothed formula; here, however, we have to consider that the mean absorption is found systematically too large, because by the smoothing process just the regions of greatest transparency are cut off. Is it possible to find such systematic corrections to the results deduced with the smoothed formula, by means of our above results of the summation over all the separate edges?

The smoothed formula must now be applied to all the constituents, including hydrogen. Hence

$$k(\nu) = C_0 \left(\frac{\nu_0}{\nu} \right) XP \left\{ g + 1.30 \left(e^{h\nu/kT} - 1 \right) \right\}. \quad (23)$$

Since in the construction of the tables of the absorptions of the separate edges the coefficient 1.30 was omitted, it must be omitted in our comparison also. Then we may write

B. Table of $f(y') = \frac{15}{4\pi^4 \text{Mod}^8} \frac{y'^7 e^{-y'}}{(1 - e^{-y'})^8}$; $y' = \log e^{h\nu/kT} = \text{Mod} \times \frac{h\nu}{kT} = \text{Mod} \times y$.

y'	Log $f(y')$	y'	Log $f(y')$	y'	Log $f(y')$	y'	Log $f(y')$
0.05	5.215	1.6	1.345 ₄₀	4.3	1.618 ₃₁	7.4	0.168 ₅₉
.1	6.444 ₇₂₅	1.65	1.385 ₃₈	4.4	1.587 ₃₁	7.5	0.109 ₆₀
.15	7.169 ₅₂₀	1.7	1.423 ₃₅	4.5	1.556 ₃₄	7.6	0.049 ₆₀
.2	7.689 ₄₀₆	1.75	1.458 ₃₃	4.6	1.522 ₃₄	7.7	9.989 ₆₁
.25	8.095 ₃₃₄	1.8	1.491 ₃₁	4.7	1.488 ₃₆	7.8	9.928 ₆₂
.3	8.429 ₂₈₄	1.85	1.522 ₂₈	4.8	1.452 ₃₈	7.9	9.866 ₆₁
.35	8.713 ₂₄₆	1.9	1.550 ₂₅	4.9	1.414 ₄₀	8.0	9.805 ₆₂
.4	8.959 ₂₁₇	1.95	1.578 ₄₆	5.0	1.376 ₄₂	8.1	9.743 ₆₃
.45	9.176 ₁₉₅	2.0	1.603 ₃₉	5.1	1.336 ₄₃	8.2	9.680 ₆₃
.5	9.371 ₁₇₆	2.1	1.649 ₃₄	5.2	1.295 ₄₄	8.3	9.617 ₆₄
.55	9.547 ₁₆₀	2.2	1.688 ₂₈	5.3	1.253 ₄₆	8.4	9.553 ₆₄
.6	9.707 ₁₄₇	2.3	1.722 ₂₃	5.4	1.210 ₄₈	8.5	9.489 ₆₄
.65	9.854 ₁₃₅	2.4	1.750 ₁₈	5.5	1.166 ₄₉	8.6	9.425 ₆₅
.7	9.989 ₁₂₅	2.5	1.773 ₁₄	5.6	1.120 ₄₇	8.7	9.360 ₆₅
.75	0.114 ₁₁₆	2.6	1.791 ₁₀	5.7	1.074 ₄₈	8.8	9.295 ₆₆
.8	.230 ₁₀₇	2.7	1.805 ₇	5.8	1.027 ₄₉	8.9	9.229 ₆₆
.85	.337 ₁₀₁	2.8	1.815 ₂	5.9	0.979 ₅₀	9.0	9.162 ₆₆
.9	.438 ₉₄	2.9	1.822 ₀	6.0	0.930 ₅₀	9.1	9.096 ₆₆
.95	.532 ₈₈	3.0	1.824 ₄	6.1	0.880 ₅₂	9.2	9.030 ₆₇
1.0	.620 ₈₃	3.1	1.824 ₇	6.2	0.830 ₅₂	9.3	8.963 ₆₈
1.05	.703 ₇₈	3.2	1.820 ₉	6.3	0.778 ₅₄	9.4	8.895 ₆₈
1.1	.781 ₇₃	3.3	1.813 ₁₂	6.4	0.726 ₅₄	9.5	8.827 ₆₈
1.15	.854 ₆₈	3.4	1.804 ₁₄	6.5	0.674 ₅₄	9.6	8.759 ₆₈
1.2	.922 ₆₅	3.5	1.792 ₁₇	6.6	0.620 ₅₅	9.7	8.691 ₆₉
1.25	.987 ₆₁	3.6	1.778 ₁₉	6.7	0.566 ₅₆	9.8	8.622 ₆₉
1.3	1.048 ₅₇	3.7	1.761 ₂₁	6.8	0.511 ₅₆	9.9	8.552 ₆₉
1.35	1.105 ₅₄	3.8	1.742 ₂₃	6.9	0.455 ₅₇	10.0	8.483 ₇₁₀
1.4	1.159 ₅₁	3.9	1.721 ₂₅	7.0	0.399 ₅₇	11.0	7.773 ₇₃₆
1.45	1.210 ₄₈	4.0	1.698 ₂₇	7.1	0.342 ₅₈	12.0	7.037 ₇₅₆
1.5	1.258 ₄₅	4.1	1.673 ₂₈	7.2	0.284 ₅₈	13.0	6.281 ₇₇₅
1.55	1.303 ₄₂	4.2	1.646 ₂₈	7.3	0.226 ₅₈	14.0	5.506 ₇₉₀
1.6	1.345	4.3	1.618 ₂₈	7.4	0.168 ₅₈	15.0	4.716 ₇₉₀

$$I_g = \int \frac{f(y')}{(e^{h\nu/kT} - 1 + g)} dy'; \quad \bar{\epsilon} = \frac{C_0}{I_g} \left(\frac{h\nu_0}{kT}\right)^3 XP. \quad (24)$$

The value of this integral was found by numerical integration: for

g	=	[9.887]	1	11	[1.42]
I_g	=	0.883	0.873	0.655	0.517
$\log I_g$	=	9.946	9.941	9.816	9.714

Interpolating $\log I_g$ for the g used in the computation of the tables, and computing $\bar{\kappa}$ by means of the X in Table 1, we find

τ	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	2.0
$\log I_g$	9.84	9.83	9.82	9.81	9.80	9.78	9.77	9.76	9.74
$\log \bar{\kappa} \left\{ \begin{array}{l} P = -2 \\ 0 \\ +2 \end{array} \right.$	9.68	9.25	8.18	7.63	7.73	7.68	8.07	8.08	8.05
	8.52	7.50	7.27	7.28	7.22	7.26	7.74	7.69	6.85

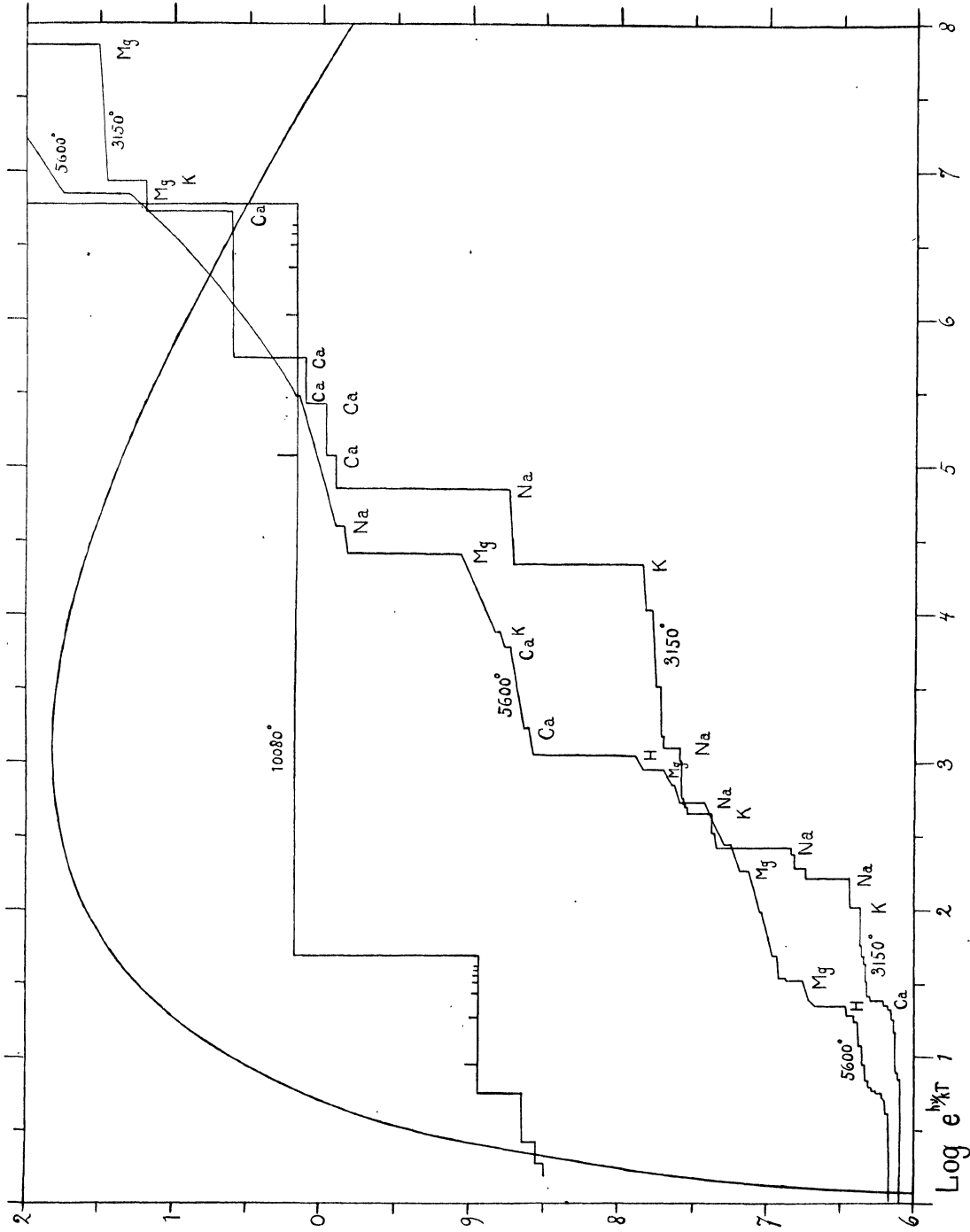


Fig. 4. The function S'' producing the mean absorption coefficient.

The differences, considered as corrections to the smoothed formula results, are

						— 03	— 16	— 20
+ 02	00	— 03	— 02	— 05	— 08	— 17	— 18	— 24
— 03	+ 04	— 04	— 03	— 13	— 20			

In these values no trace is visible of the steps and the edges of hydrogen which for the higher temperatures should make the absorption by the real edges smaller than by the smoothed formula. This effect of the steps is, however, obliterated by another cause working in opposite direction. In the table of the edges the absorption by hydrogen has a weight 2, whereas in the nameless mixture all the atoms undistinguished have weight 1. Hence to detect the effect of the steps we should compare the smoothed formula with a computation by steps, where the hydrogen edges have the same weight 1. The results of this computation are :

τ	Log $\bar{\kappa}$ for hydrogen weight 1.				Differences between weights 2 and 1.			
	0.6	0.7	0.8	0.9	0.6	0.7	0.8	0.9
$P = 0$	9.47	9.03	7.95	7.57	+ 23	+ 22	+ 20	+ 04
2	8.32	7.40	7.22	7.25	+ 23	+ 14	+ 01	00

Now the new corrections to the smoothed formula are

						— 03	— 16	— 20
— 21	— 22	— 23	— 06	— 05	— 08	— 17	— 18	— 24
— 20	— 10	— 05	— 03	— 13	— 20			

Here the influence of the discontinuous steps is clearly visible, at the left hand side where hydrogen increasingly dominates, and at the right hand side where some few edges of K and Na remain as the only ones. In the middle temperatures where the smoothed contribution of the Fe absorption is greatest, the differences are smallest. This is clearly seen when we compare with the values of $\log X/x(Fe)$:

						0.68	1.96	4.78
3.34	2.73	1.36	0.58	0.50	0.85	2.22	3.57	5.58
2.28	0.99	0.53	0.60	1.01	2.36			

The correlation is evident ; for small values of $\log X/x(Fe)$ the difference is small, and for large increasing values of this quantity the differences at each side tend to a limit.

We first consider the transition region where hydrogen absorption is mixed with the metal absorption. The result of the smoothed formula must be diminished owing to the discontinuous steps, it must be increased in consequence of the double weight of hydrogen, and at last it must be corrected for the factor 1.30, by which the ionization part (not the free-free part) must be multiplied. The first named corrections depend both on the part hydrogen plays in the mixture ; we will assume them to consist of a fraction factor which is simply a function of the relative quantity of hydrogen and metal electrons, and a maximum value holding for a pure hydrogen atmosphere. These maximum values, computed in the way of Table 8 are found to be :

	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2
Step effect	— .72	— .48	— .30	— .25	— .28	— .30	— .29	— .26	— .24
Weight effect	+ .27	+ .27	+ .26	+ .24	+ .24	+ .25	+ .25	+ .25	+ .26
Combined effect	— .45	— .21	— .04	— .01	— .04	— .05	— .04	— .01	+ .02

The irregular course of the combined effect, due to the step effect, is a consequence of the irregular variation of the pure hydrogen absorption with temperature, already mentioned p. 19, and illustrated by Fig. 1 ; there the curve for $\log I$, computed by the smoothed formula and corrected already for the 1.30 factor (these corrections are 0.08 to 0.06) is designated by a dashed line. The quantities called above "new corrections to the smoothed formula" are fractions of this "step effect", and the "differences between weights 2 and 1" are fractions of this "weight effect", both fractions dependent on the hydrogen part in the electron mixture. Assuming them to be the same function of $\log X/x_m$ (as representing the hydrogen importance) we find

for $\log X/x_m =$	0	0.2	0.5	1.0	1.5	2.0	2.5
fraction	0	0.22	0.51	0.79	0.89	0.93	0.97

For all the cases in the transition region the correction to the smoothed formula 23) is now found by multiplying the "combined effect" by this fraction as a function of $\log X/x_m$, and by then adding the correction for the factor 1.30.

The other part of the corrections to the smoothed formula, which holds for the metal atom mixture, increases from -0.04 (for maximum contribution of Fe) to about -0.20 . We can represent them as a function of the values of $\log X/x(Fe)$ given above, or, still better, of $\log X_M/x(Fe)$ which is different only in the transition region. We read from the curve :

$\log x_M/x(Fe)$	0.5	1.0	1.5	2.0	2.5	3.0
Correction	-.04	-.10	-.15	-.18	-.20	-.21

It expresses the decrease of the absorption coefficient in consequence of the discontinuous edges in the metal absorption, especially of K and Na for lower temperatures. It is, however, not certain that these abrupt discontinuities really exist. If the absorption by ionization of the metal atoms is smoothed by some interaction of electrons within these atoms, then this correction must be omitted.

For the computation of the mean absorption $\bar{k} = \bar{\kappa}P$ we have now the following methods of working.

For the cases of pure hydrogen absorption (in Table 1 situated on the left hand side of the first broken line) Table 9 p. 13 gives the values of $\log \bar{\kappa}/X$, to which only $\log X$ has to be added.

For the transition region (on Table 1 between the two broken lines) and for the region of metal absorption (in Table 1 on the right hand side) the smoothed formula is used to compute the first uncorrected values of $\bar{\kappa}/X$ in Table 14. Here the notations of formula 23) are used, but we computed the values by means of formula 22), by making use of $g/1.30$ to interpolate I_g , and then adding 0.113 to the result. The changes by taking $g = 1$ are added at the bottom of the table.

Table 14. Mean absorption coefficients for an atom mixture.

	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	2.0
Log g	0.75	0.84	0.90	0.94	0.98	1.03	1.07	1.11	1.18	1.23	1.27	1.33
Log $I(g/1.30)$	9.889	9.880	9.874	9.869	9.864	9.858	9.853	9.848	9.838	9.832	9.826	9.819
Log $C_0 \left(\frac{h\nu_0}{kT}\right)^3$	8.226	8.788	9.224	9.581	9.882	0.143	0.373	0.579	0.936	1.236	1.497	1.933
Log $\bar{\kappa}/X$	8.45	9.02	9.46	9.82	0.13	0.40	0.63	0.84	1.21	1.52	1.78	2.23
Diff. $g = 1$	-.06	-.07	-.07	-.08	-.08	-.09	-.09	-.10	-.11	-.11	-.12	-.13

By adding $\log X$ we find $\bar{\kappa}$ for every T and P , but still uncorrected. Now the corrections for hydrogen absorption must be added in the transition region; they are computed, as explained above, by multiplying the "combined effect" by the fraction factor, and the results are given in Table 15.

Table 15. Corrections to mean absorption for hydrogen.

P	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2
8	-38	-10	0	0					
7	-41	-12	0	0					
6		-17	-01	0					
5		-19	-02	0	0				
4			-03	0	0				
3				-01	0	0			
2				-01	-02	0			
1					-03	-01	0		
0						-04	0		
-1						-05	-02	0	
-2							-03	0	
-3								-01	
-4								-01	0
-5									+01
-6									+02

Table 16. Corrections to mean absorption for metal edges. (All negative.)

P	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	2.0
8	00	02	07	12	15	17	18					
7	00	01	06	12	15	17	18	20				
6		00	04	10	14	16	18	20				
5		00	01	05	11	15	18	20	21			
4			00	03	06	11	16	19	21	21		
3				01	04	06	11	16	21	21	21	
2				00	02	04	05	10	19	21	21	
1					00	02	04	05	15	21	21	21
0						01	03	04	08	19	21	21
-1						00	02	04	03	13	21	21
-2							00	02	04	07	18	21
-3								01	04	04	11	21
-4								00	04	04	05	21
-5									02	04	04	19
-6									00	04	04	14

The corrections to $\bar{\kappa}$ for the metal edges are computed in Table 16 from $\log x_m/x(Fe)$, by means of the curve with readings given above. For the transition region, where the contribution of the metal atoms is decreasing from 1 to 0 they have been multiplied by corresponding factors. Whether the corrections of Table 16 should be applied is doubtful. In our further discussions for the present we have omitted them.

To the $\bar{\kappa}$ found by means of these tables still a correction for the influence of the Fraunhofer lines must be applied.

General absorption by Fraunhofer lines.

Part of the energy streaming through the atmosphere is absorbed in the Fraunhofer lines and re-emitted, i. e. partly sent back. Since the total radiation stream is a given quantity, this obstruction increases the temperature gradient. The total absorption which determines the temperature gradient is the combined effect of the mean continuous absorption and the line absorption.

In order to make an estimate of the total amount of this line absorption, each line with its special profile is replaced by another having intensity zero over its equivalent width, hence taking away the same amount of energy of the spectrum. Then in our procedure of computing $\bar{\kappa}$ we have to make this change: in the integration of the transparency $1/k(\nu)$ over the spectrum we have to omit all the equivalent widths of the Fraunhofer lines as intervals of transparency zero.

Our empirical data on the amount of absorption of the Fraunhofer lines date from the last few years only. The equivalent width of a large number of such lines has been measured by G. F. W. MULDER¹⁾ at the Heliophysical Institute at Utrecht, and he derived that 8.3 % of the total

¹⁾ Aequivalente breedten van Fraunhoferlijnen in het zonnenspectrum. Diss. 1934.

energy of the spectrum was taken away by them. Strictly speaking we have to derive the fraction constituted by all the Fraunhofer line widths for each part of the spectrum, and then apply these fractions to the values of $(1/S'') f(y') dy'$ (form. 21) for each part. When, however, as is the case with the sun, this function varies little over the relevant realms of the spectrum, we may say that the total absorption is increased by the same fraction as is taken away from the energy of the spectrum. Hence for the sun, where 0.917 is left of the total light ($\log = - .038$) we assume that the logarithm of the mean absorption coefficient has to be increased by $+ .038$.

This holds for the sun only. Is it possible, by some plausible argument, to derive from it values for other stellar atmospheres, i. e. for other temperatures? Of course we can only make a rough estimate, but since the total effect is small this may be sufficient for our purpose. The width of a line is determined, in first approximation, by $V(s/k)$, where s depends on the concentration of the active atoms. The absorption coefficient k , as a function of P and T , is given in Table 18. The mean value of k in the layers which chiefly produce the lines, as will appear from the discussion below (p. . .), from high temperatures going downward, first increases till at $10,000^\circ$ a maximum is reached, then rapidly decreases to nearly $1/100$ of the highest value, at 6000° , and then again slowly increases with decreasing temperature.

For s we first take the metal lines, produced by the metal mixture. From MULDER'S work it appears that the density of Fraunhofer lines is strongest between 3500 and 4000 A, where 38 % of the energy is taken away by them, and gradually decreases for greater wave-length, to 10 % at 5000 A and 2 % about 7000 A; also between 3500 A and 3000 A it is somewhat smaller, 22 %. With increasing temperature neutral atoms gradually are replaced by the same number of ionized atoms, which have their chief terms and lines at smaller wave-lengths. Hence we may expect that the maximum found by MULDER'S will be displaced to smaller wave-length with higher, to greater wave-length with lower temperature. At the same time, however, the intensity curve of the continuous spectrum is displaced to smaller wave-length with increasing temperature. In the absence of quantitative data on these qualitative changes we will assume that, with changing temperature, we keep the same general distribution of Fraunhofer lines, only displaced in the same way as the continuous radiation curve. Then, as to s , the conditions are the same for all temperatures, and the total effect of the metal lines will only depend on the variations of k with temperature. This means that for temperatures from 6000° increasing to $10,000^\circ$ the effect of the Fraunhofer lines must decrease from 0.04 to imperceptible. For temperatures below 6000° the effect should slowly decrease to nearly half its solar amount. This, however, is in contradiction to the observed fact that in the spectral classes *K*, *M*, the absorption lines are stronger than in the solar spectrum. Whether in the general distribution the lines in reality are still increasing in the inaccessible spectral regions below 3000 A, which greater number, for lower temperature proceeds to and appears in the visible wave-lengths, or whether it is due to blending of the metal lines with an increasing number of molecular lines — in any case we have to represent it by slowly increasing values for the correction to the general absorption coefficient.¹⁾

The metalloid atoms in the hottest stars may constitute a higher fraction of the total at-

¹⁾ After the adopted values had been used already in the further computation, a paper was published by G. SHAJN, where the amount of line and band absorption in stellar spectra was investigated (Monthly Not. R A S 94, 642). His results may be summarized in this table:

10000°	6000°	4000°	3000°	2500° (<i>a</i> Her)
0.03	0.06	0.08	0.12	0.28.

They are generally larger than the values adopted in Table 17 below; but for our purpose the differences are hardly material.

mosphere than the metal atoms; but by the high value of k their total absorption is very small. The matter is different for hydrogen, because its abundance is practically 100 %. The Balmer lines absorb an important part of the radiation, the more so because by the STARK effect their width is considerably increased.

The equivalent width of the whole series of Balmer lines has been measured by S. GÜNTHER¹⁾ for a large number of class *B* and *A* stars. The greatest values are shown by such *A0* stars as α Lyrae and γ U Maj, where the equivalent width from 9.4 Å ($H\alpha$) increases to 15.6 Å ($H\delta$) and then decreases to 1.3 Å ($H\nu$). We compute how much the sum of transmission integrals $1/s' \int \varphi'(y) dy$ is diminished when for the $\Delta y' = (y'/\lambda) \times$ equivalent width the transparency $1/s'$ is taken 0. The result is that, for 10.000°, from the integral over the Balmer line region (between 2d and 3d band edge), amounting to [1.886] = 77, the part taken away is 6.6 ($H\alpha$ to $H\delta$ contribute 1.7). The sum total of the integrals is 162; the chief remaining part is contributed by the region between the 1st and 2d edge, which contains the Lyman lines. (They are indicated on the diagram p. 33). We have no empirical data on them; they are certainly stronger than the Balmer lines but are situated in a region with ten times smaller φ . If we assume that they take away the same fraction, we may estimate that 9 % of the passing energy is held back by the hydrogen lines. This holds for stars with the strongest lines; for α Cygni in the same way only 3 % is found.

Then $\log \bar{k}$ for 10.000° has to be increased by .04 at most. For higher temperatures the hydrogen lines decrease continually; if for 5040/ T = 0.4 0.3 0.2 we assume corrections + .03 + .02 + .01, this cannot give errors of any importance. Below 10.000° the hydrogen lines decrease, while at the same time the metal lines are increasing; we may assume their combined effect to remain .04 in this interval. Table 17 contains the adopted corrections to the mean absorption coefficients previously derived.

Table 17. Corrections to the mean absorption coefficient.

5040/ T	.2	.3	.4	.5	.6	.7	.8	.9	1.0	1.2	1.4	1.6	2.0
Log k	0.5	0.7	0.9	0.9	0.2	-0.7	-1.0	-1.0	-1.0	-0.9	-0.8	-0.7	-0.7
$1/\sqrt{k}$	0.6	0.4	0.3	0.3	0.8	2.2	3.1	3.1	3.1	2.8	2.5	2.2	2.2
Corr. log \bar{k}	+ .01	+ .02	+ .03	+ .04	+ .04	+ .04	+ .04	+ .04	+ .04	+ .05	+ .06	+ .07	+ .09

Tables for the continuous absorption coefficient.

The values of k are now computed by means of formula 20) and Tables 7 and 13.

$$k(\nu) = \{ (7) x_n + (13) x_n \} P$$

where x_n , x_m and P are taken from Table 1. This table has been extended to intermediate temperatures, because in the transition region and for lower temperatures the changes are rather rapid. For the temperatures 1.1 1.8 the data of RUSSELL's tables were used; for 0.55 to 0.95 the values of the ionization x and X were computed here; the results are given in Tables 18 and 19. The coefficients of k were interpolated from Tables 7 and 13. In Table 21 the values are given for wavelengths 7000, 6000, 5000, 4000, 3500, 3000. The mean absorption coefficient, as computed by Table 14, with the corrections of Table 15, for atom mixtures, and by Table 9 with formula 16) for hydrogen, both corrected by the values of Table 17, is given in Table 20 for the same range of temperatures and pressures.

¹⁾ Zeitschrift für Astrophysik, 7, p. 106.

Table 18. Ionization of hydrogen and metal atoms (Supplement to Table 1)

T 5040/ T	9164° 0.55	7754° 0.65	6720° 0.75	5929° 0.85	5305° 0.95	4582° 1.1	3877° 1.3	3360° 1.5	2800° 1.8
— 7	5.43 4.43	6.97 5.06	8.48 5.67	9.97 6.26	11.44 6.54	— log x_H ; — log x_M			
6	4.43 3.70	5.97 4.19	7.48 4.71	8.97 5.28	10.44 5.84	12.62 6.68			
5	3.43 3.23	4.97 3.60	6.48 3.97	7.97 4.40	9.44 4.89	11.62 5.68	14.52 6.73		
4	2.43 3.04	3.97 3.21	5.48 3.53	6.97 3.82	8.44 4.14	10.62 4.75	13.52 5.73	16.38 6.81	
3	1.45 3.00	2.97 3.03	4.48 3.18	5.97 3.48	7.44 3.72	9.62 4.09	12.52 4.84	15.38 5.81	
2	0.57 3.00	1.98 3.00	3.48 3.03	4.97 3.15	6.44 3.43	8.62 3.74	11.52 4.17	14.38 4.88	18.64 6.33
1	0.10 3.00	1.01 3.00	2.48 3.00	3.97 3.02	5.44 3.13	7.62 3.53	10.52 3.81	13.38 4.24	17.64 5.35
0	0.01 3.00	0.29 3.00	1.49 3.00	2.97 3.00	4.44 3.02	6.62 3.23	9.52 3.63	12.38 3.86	16.64 4.55
— 1	0.00 3.00	0.04 3.00	0.60 3.00	1.98 3.00	3.44 3.00	5.62 3.04	8.52 3.43	11.38 3.69	15.64 4.10
— 2	"	0.00 3.00	0.12 3.00	1.01 3.00	2.44 3.00	4.62 3.00	7.52 3.14	10.38 3.57	14.64 3.83
— 3			0.01 3.00	0.29 3.00	1.46 3.00	3.62 3.00	6.52 3.02	9.38 3.33	13.64 3.69
— 4			0.00 3.00	0.04 3.00	0.58 3.00	2.62 3.00	5.52 3.00	8.38 3.09	12.64 3.61
— 5				0.00 3.00	0.11 3.00	1.63 3.00	4.52 3.00	7.38 3.02	11.64 3.47

Table 19. Total ionization and pressure (Supplement to Tables 1 and 2)

T 5040/ T	9164° 0.55	7754° 0.65	6720° 0.75	5929° 0.85	5305° 0.95	4582° 1.1	3877° 1.3	3360° 1.5	2800° 1.8
7	4.39 11.39	5.05 12.05	5.67 12.67	6.26 13.26	6.84 13.84	— log X ; log p			
6	3.63 9.63	4.18 10.18	4.71 10.71	5.28 11.28	5.84 11.84	6.68 12.68	6.73 11.73		
5	3.02 8.02	3.58 8.58	3.97 8.97	4.40 9.40	4.89 9.89	5.68 10.68	5.73 9.73	6.81 10.81	
4	2.33 6.33	3.14 7.14	3.53 7.53	3.82 7.82	4.14 8.14	4.75 8.75	4.84 7.84	5.81 8.81	
3	1.44 4.46	2.70 5.70	3.16 6.16	3.48 6.48	3.72 6.72	4.09 7.09	4.84 7.84	5.81 8.81	
2	0.57 2.67	1.94 3.94	2.90 4.90	3.14 5.14	3.43 5.43	3.74 5.74	4.17 6.17	4.88 6.88	6.33 8.33
1	0.10 1.35	1.01 2.05	2.37 3.37	2.97 3.97	3.13 4.13	3.53 4.53	3.81 4.81	4.24 5.24	5.35 6.35
0	0.01 0.31	0.29 0.47	1.48 1.49	2.68 2.68	3.00 3.00	3.23 3.23	3.63 3.63	3.86 3.86	4.55 4.55
— 1	0.00 9.30	0.04 9.32	0.60 9.70	1.94 9.94	2.87 1.87	3.04 2.04	3.43 2.43	3.69 2.69	4.10 3.10
— 2		0.00 8.30	0.12 8.37	1.01 9.05	2.33 0.33	2.99 0.99	3.14 1.14	3.57 1.57	3.83 1.83
— 3		0.00 7.30	0.01 7.31	0.29 7.47	1.45 8.47	2.91 9.91	3.02 0.02	3.33 0.33	3.69 0.69
— 4			0.00 6.30	0.04 6.32	0.58 6.68	2.47 8.47	3.00 9.00	3.09 9.09	3.61 9.61
— 5				0.00 5.30	0.11 5.36	1.61 6.62	2.99 7.99	3.02 8.02	3.47 8.47

Table 20. Logarithm of mean absorption coefficient \bar{k} .

Log P	0.2	0.3	0.4	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.8	2.0		
7	4.01	3.67	3.00	2.47	2.31	2.14	1.98	1.80	1.64	1.48	1.30	1.12	0.94	0.75										
6	3.13	3.48	2.86	2.24	2.07	1.95	1.85	1.73	1.60	1.45	1.28	1.11	0.94	0.75	0.40	0.04								
5	2.15	2.91	2.77	1.95	1.68	1.53	1.45	1.39	1.34	1.27	1.16	1.04	0.89	0.73	0.40	0.04	9.70	9.30						
4	1.15	2.00	2.51	1.82	1.36	1.02	0.89	0.82	0.78	0.76	0.74	0.70	0.64	0.56	0.33	0.01	9.70	9.30	8.91	8.53				
3	0.15	1.01	1.79	1.72	1.24	0.72	0.32	0.18	0.15	0.13	0.08	0.06	0.06	0.03	9.99	9.80	9.59	9.27	8.91	8.53	7.77			
2	9.15	0.01	0.84	1.30	1.11	0.64	0.07	0.60	9.40	9.39	9.42	9.39	9.35	9.32	9.34	9.36	9.26	9.09	8.84	8.50	7.77	7.01		
1		9.01	9.84	0.44	0.58	0.45	9.99	9.42	8.91	8.63	8.59	8.62	8.60	8.63	8.55	8.58	8.62	8.58	8.48	8.32	7.75	7.01		
0				9.46	9.67	9.80	9.71	9.33	8.79	8.25	7.87	7.74	7.78	7.83	7.85	7.78	7.80	7.85	7.86	7.80	7.55	6.98		
— 1				8.46	8.68	8.85	8.96	8.94	8.66	8.16	7.59	7.12	6.89	6.90	7.04	7.07	7.00	6.97	7.03	7.06	7.00	6.76		
— 2					7.68	7.86	8.00	8.11	8.14	7.97	7.51	6.94	6.44	6.10	6.09	6.22	6.29	6.18	6.15	6.19	6.27	6.18		
— 3						6.86	7.00	7.13	7.25	7.32	7.23	6.85	6.31	5.76	5.17	5.26	5.41	5.46	5.39	5.31	5.41	5.48		
— 4								6.13	6.26	6.38	6.48	6.46	6.18	5.68	4.62	4.29	4.43	4.56	4.63	4.56	4.49	4.60		
— 5										5.39	5.52	5.62	5.65	5.48	4.48	3.53	3.44	3.58	3.70	3.77	3.63	3.68		

The monochromatic line-absorption.

Line Intensity.

The diffusion coefficient s appearing in the formulas, is determined by the line absorption and emission, which have large values only in the immediate vicinity of the "monochromatic" line frequency $\nu_{23} = (E_3 - E_2)/h$, due to the transitions between two states indicated here by 2 and 3.

The value of this coefficient depends firstly on the number of atoms capable of producing the line considered. This number is composed by the factors :

A , the abundance (in number of atoms) of the element considered in the stellar atmosphere
 $x_0, x_1, x_2 \dots$ the fraction of these atoms which is neutral, once-or doubly ionized,

y_n , the fraction of this kind of atoms which is excited to the n th level or energy state.

To find the number of transitions $2 \rightarrow 3$, the number of atoms for state 2 must be multiplied by the probability of transitions $2 \rightarrow 3$; in each transition a quantum of frequency ν_{23} is absorbed. The number of emissions of the same ν_{23} is found by multiplying the number of atoms for state 3 by the probability of transition $3 \rightarrow 2$. The first named probability is proportional to the intensity of the radiation $I(\nu_{23})$ producing the transition; the latter is chiefly spontaneous. The probabilities are expressed by the Einstein coefficients A and B :

$$B_{23} I_{23} \text{ and } A_{32} + B_{32} I_{23},$$

obeying the relations

$$B_{23} \frac{8\pi h \nu_{23}^3}{c^2} = \frac{p_3}{p_2} A_{32}; \quad B_{32} \frac{p_3}{p_2} = B_{23}.$$

The absolute value of these coefficients is given by

$$B_{23} = \frac{\pi e^2}{m c h \nu} f_{23} \quad (25)$$

where f_{23} is a factor, the "oscillator strength" belonging to this transition 2—3 and to the line with frequency ν_{23} .

The number of atoms in state 2 contained in a volume element dh (height dh over a surface 1) is $A x y_2 \rho dh / \mu m_H$; multiplying by the transition probability $B_{23} I$ and the quantum $h\nu$ absorbed at each transition, we get the absorbed energy, which formerly was denoted by $s I \rho dh$. Hence

$$s = \frac{A x y \pi e^2}{\mu m_H m c} f = \frac{n \pi e^2}{m_H m c}$$

Since $1/\mu m_H$ is the total number of atoms in unit mass (where μ may be taken 1, cf p. 10), we separate the variable factors $A x y f = n$ as the "concentration" of the atoms active in producing the line ν .

The relation of the probability coefficient of spontaneous transition A_{32} to the oscillator strength f_{23} is now given by

$$A_{32} = \frac{p_2}{p_3} \frac{8\pi h \nu^3}{c^2} \frac{\pi e^2}{m c h \nu} f_{23} = \frac{p_2}{p_3} \frac{8\pi^2 e^2 \nu^2}{m c^3} f_{23}.$$

Denoting $f_{23} p_2/p_3$ by $-f_{32}$ (the emission f , mostly taken negative, can be denoted generally by f_e , while the absorption f , with increasing indices, are denoted by f_a), and introducing the "classical damping constant" ω , we have

$$\omega = 4\pi \nu = \frac{8\pi^2 e^2 \nu^2}{3 m c^3}; \quad A_{32} = -3\omega f_{32}. \quad (26)$$

The damping constant ω contains ν^2 as a variable factor. The f coefficients, which directly determine the different intensities of emission lines and Fraunhofer lines, are known only in some few cases. For hydrogen they have been computed for a number of transitions by SUGIURA (after SLACK had computed the A). The f values for transitions proceeding from one definite state are connected by the "sum rule", holding for each state,

$$\Sigma f_a + \Sigma f_e = z$$

where z is the number of outer electrons taking part in the formation of spectral lines.

Intensity distribution within a line.

The diffusion coefficient s is a function of the distance (difference in frequency or in wave length) to the centre of the line with frequency ν_{23} . For atoms at rest this function is

$$m_H \frac{s}{n} = \frac{\epsilon^2}{mc} \frac{\delta}{\Delta\nu^2 + \delta^2}$$

where $\Delta\nu$ is the difference of frequency with the central value ν , and δ is the "half width" of the line. What was denoted by s in the preceding paragraph is the total diffusion coefficient, the integral of $s d\nu$ over the width of the line (i.e. between $\pm \infty$),

$$\int m_H \frac{s}{n} d\nu = \frac{\pi\epsilon^2}{mc}$$

After WEISSKOPF and WIGNER the half width δ is connected with the damping constant by the relation

$$\delta_{23} = \Sigma 3f_{2n} \gamma_{2n} + \Sigma 3f_{3m} \gamma_{3m}; \quad n < 2; \quad m < 3;$$

where each summation is extended over all the downward transitions possible from level 2 in the first, from level 3 in the second term. Strictly speaking the summations should extend over all transitions, the upward ones included, because their totality determines the mean life time of the state in question. The upward transition probabilities BI , however, are, extreme conditions excepted, far smaller and negligible compared with the downward ones. Hence

$$\delta_{23} = \frac{2\pi\epsilon^2}{mc^3} (\Sigma f_{2n} \nu_{2n}^2 + \Sigma f_{3m} \nu_{3m}^2); \quad n < 2; \quad m < 3.$$

In stellar atmospheres we are always concerned with gases of high temperature; through the high translation velocities the frequency of absorbed and emitted radiation is changed according to the Doppler principle. Then the distribution of s as a function of $\Delta\nu$ is given by

$$m_H \frac{s}{n} = \frac{\epsilon^2}{mc} \left(\frac{\delta}{\Delta\nu^2} + \frac{\sqrt{\pi}}{\beta} e^{-(\Delta\nu/\beta)^2} \right); \quad \beta = \frac{v}{c} \sqrt{\frac{2kT}{m_a}}$$

The second term between the parentheses, forming the "Doppler core", is determined by the Doppler spreading β , which depends on the temperature T and the mass m_a of the atom considered. For small values of $\Delta\nu$, in the central parts of the line, it far exceeds the other term, which may be entirely neglected there (for very small $\Delta\nu \ll \beta$ the first term is replaced by another with factor $\Delta\nu$). For larger $\Delta\nu$ the Doppler term rapidly decreases to insignificance and then the first term, forming the "resonance wings", is the only one to be considered. We introduce the ratio of the real to the "classical" half width δ/γ into the first term; introducing then, for practical purposes, wave lengths instead of frequencies, we have

$$\left. \begin{aligned}
 \Delta\nu/\nu &= -\Delta\lambda/\lambda; & \Delta\lambda/\beta &= -\Delta\lambda/l; & 1/\beta &= \lambda^2/cl; \\
 m_n \frac{s}{n} &= \frac{2\pi \epsilon^4}{3m^2 c^4} \left(\frac{\lambda}{\Delta\lambda}\right)^2 \frac{\delta}{\gamma} + \frac{\epsilon^2 \sqrt{\pi} \lambda^2}{m c^2 l} e^{-(\Delta\lambda/l)^2}; & l &= \frac{\lambda}{c} \sqrt{\frac{2kT}{m_a}} \\
 \frac{\delta}{\gamma} &= \Sigma 3f_{2n} \frac{\nu_{2n}^2}{\nu_{23}^2} + \Sigma 3f_{3m} \frac{\nu_{3m}^2}{\nu_{23}^2}.
 \end{aligned} \right\} 27)$$

The ratio δ/γ , as a rule, is not exactly known, but often it may be considerably > 1 , and thus enhances the s values in the wings¹⁾.

Introducing numerical values for the constants and expressing $\Delta\lambda$ in Angstroms, we find

$$\frac{s}{n} = [6.204] \left(\frac{\lambda}{4000}\right)^2 \frac{1}{\Delta\lambda^2} \frac{\delta}{\gamma} + [11.596] \left(\frac{\lambda}{4000}\right)^2 \frac{0.122}{l} e^{-(\Delta\lambda/l)^2}; \quad l = 0.122 \frac{\lambda}{4000} \left(\frac{T\mu}{5040}\right)^{1/2}. \quad 28)$$

As soon as from observations of line profiles the diffusion coefficient s for a certain $\Delta\lambda$ has been found, this formula will enable us to derive the concentration n of the atoms active in producing the line.

Ionization and excitation.

Whereas two of the factors constituting the concentration, viz. the abundance A and the oscillator strength f do not depend on the physical parameters, the other two factors, x and y are variable with temperature and pressure. In thermodynamical equilibrium we have the well known formulas

$$\frac{x}{1-x} P = K = \frac{(2\pi m)^{3/2} (kT)^{3/2} B'}{h^3 B} e^{-\chi/kT}; \quad y_n = \frac{p_n}{B} e^{-(\chi - \chi_n)/kT}; \quad B = p_1 + p_2 e^{-(\chi - \chi_2)/kT} + \dots, \quad 29)$$

where for the first ionization $x = x_1$, $1 - x = x_0$, χ_n is the ionization energy from the n th level and B' is the same partition function for the ionized atom.

In stellar atmospheres there is no thermodynamical equilibrium. The excitation as well as the ionization are determined by the actual radiation falling upon the atoms (represented by $4\pi J$ in our formulas) and by the transition and recombination probabilities. For the excitation a relation is easily found from the Einstein coefficients; for the ionization a formula has been deduced by WOLTJER²⁾:

$$y_n = (y_n)_{th} \frac{J(\nu)}{E(\nu)}; \quad K = K_{th} \frac{\Sigma \int \psi(\nu) J(\nu) d\nu}{\Sigma \int \psi(\nu) e^{-h\nu/kT} (2h\nu^3/c^2 + J(\nu)) d\nu}.$$

Here $4\pi E$ is the black body radiation of the temperature for which $(y_n)_{th}$ has been computed; T is the gas temperature and $\psi(\nu)J(\nu)$ denotes the probability, after MILNE, that an atom is ionized by the radiation $J(\nu)$. This probability is connected with the absorption coefficient $k(\nu) = \psi(\nu)h\nu$. Each integral extends from the series limit ν_n to ∞ ; the summation sign relates to the different levels from which ionization may take place.

The deviation between J and E consists firstly in that in the upper atmospheric layers radiation is coming only from the lower side, hence is diluted radiation. Moreover the frequency ν lifting the atom to the n th level, is the frequency of an absorption line, for which the intensity J is strongly reduced. It would not be correct, however, to introduce this reduction coefficient into the formula. There are many other ways, through other transitions, for the atom to arrive from the lowest to

¹⁾ It is identical with UNSÖLD's f'/f (Zs. f. Astrophysik 2, 200).

²⁾ Cf. Physica V, 406; B.A.N. III, 207; (N^o 110).

the n th level. The most important is through ionization from the lowest level, followed by capture upon the n th level. So the determination of y_n is a complicated matter.

For the ionization the case is analogous. The ionizing radiation is diluted radiation of effective temperature, whereas the gas temperature of the surface layers is lower. Moreover the frequencies ν which are able to eject the electron out of the atom, are absorbed by just this process and have a reduced intensity in the upper atmospheric layers. So the exact computation of the ionization also is complicated and difficult.

We will not treat these problems here. We restrict ourselves to the second approximation, by assuming that ionization and excitation are determined by diluted radiation of effective temperature. The changes necessary for this assumption have been used already in the computation of absorption coefficients (cf. p. 9). Since the more complicated processes are involved in the intensities of the darkest parts of the lines, we will treat here only the parts of moderate darkness, the wings of the lines.

By the differences of temperature and pressure the ionization and the excitation will be variable within the different layers of a stellar atmosphere. The pressure change is the most important. The temperature does not change very much within the relevant layers of the atmosphere; moreover it is the diluted effective radiation temperature rather than the gas temperature that determines the atomic conditions. Hence we will neglect the variations of temperature, so that y becomes constant. The ionization, however, is strongly dependent on the electron pressure. We will treat it as a variable quantity, just as well as the absorption coefficient, and separate it from the product n . Then by n' the product $A y f$ will be denoted.

The formation of absorption lines.

The relevant atmospheric layers.

The residual intensity at some wave-length within a Fraunhofer line is produced by the absorption, emission and diffusion in the upper layers of the atmosphere. Not all the layers take part in the formation of the line in the same measure; it is necessary to know what layers play the chief part in this process.

To find this out we will make use of a first approximate solution of the equations, first by assuming the coefficients k and s to be constant throughout the atmosphere. Putting $1/4 h\nu/kT = c$ we have the equations of the Introduction

$$\frac{dJ}{d\xi} = 3(k + s)H; \quad \frac{dH}{d\xi} = k(J - E) = k(J - E_0(1 + 3/2 c\bar{k}\xi)); \quad H_0 = 1/2J_0.$$

Putting $kd\xi = dt$, $3(k + s)/k = q^2$, $3/2 c\bar{k}/k = C$, the solution is

$$J/E_0 = 1 + Ct - \frac{q^2 - 2C}{q^2 + 2q} e^{-qt}; \quad H/E_0 = \frac{C}{q^2} + \frac{q^2 - 2C}{q(q^2 + 2q)} e^{-qt}. \quad (30)$$

The surface value of H , compared with the surface value for $s = 0$, which expresses the intensity of the continuous spectrum background, gives the residual intensity at the point of the line considered

$$H_0/E_0 = \frac{q + C}{q^2 + 2q} \quad H_0^0/E_0 = \frac{\sqrt{3} + C}{3 + 2\sqrt{3}} \quad r = \frac{\sqrt{3} q + C}{q} \frac{\sqrt{3} + 2}{\sqrt{3} + C}. \quad (31)$$

The residual intensity is variable with C , i. e. with wave-length; for $C = 2$ it is $\sqrt{3}/q$, and it is always of the order of this quantity, i. e. of the order of $\sqrt{k/(k + s)}$.

For large t J approaches to the black body radiation E , and H approaches to the constant interior stream proportional to $1/q^2$. The decrease of J from the interior to its surface value and the increase of H to its surface value depend on the exponential factor e^{-qt} . The layers in which this factor rises from a small value in the depth to nearly 1 near the surface are the layers which chiefly produce the absorption line. The conditions in these layers determine the character of the absorption line. We will consider as the relevant layers those in which this exponential factor increases from 0.1 to 0.9. They are bounded by $qt = 0.105$ and 2.30 , or $\log qt = -1.0$ and $+0.4$. The optical depth corresponding to these boundary values depends on q , which is different for different parts of the line. If we restrict our attention to the wings, where the residual intensity is nearly $1/2$, we have there $q = 3$ (say 2 to 7). Then, for $\log q$ between 0.3 and 0.8 the limits of $\log t$ are -1.8 and 0.1 . For the central parts of the line, where the intensity is only a few hundredths, q is large; if we take e. g. $r = .04$ and $.02$, $q = 43$ and 86 , we find the limits of $\log t = -2.9$ and -1.2 . By means of the relations $dt = kd\xi$, $dp = gd\xi$ we have $kp = gt$.

Another computation has been made for the case of the absorption coefficient k being proportional to the electron pressure P . For this case the formulas have been derived in our former paper¹⁾; from the relations given there $x = s_0 p/g$, $a = (k/s_0^2)g/p$ we have $ax^2 = kp/g$. From the manuscript computations of numerical integration for different cases we find the boundary values of x between which the medium four fifths of the absorption took place:

$$\begin{array}{llll} a = 1/4 & r = 0.5 & x = 0.15 \text{ to } 2.4 & \log ax^2 = -2.3 \text{ to } +0.2 \\ a = 10^{-4} & r = 0.036 & x = 3 \quad ,, \quad 45 & -3.1 \text{ to } -0.7. \end{array}$$

These limits are somewhat wider apart than in the former case, because with increasing depth and pressure the absorption now increases less rapidly.

We will assume the limits for $\log kp/g$ to have the round values of -2.0 and 0.0 for the wings (residual intensity about $1/2$), and of -3.0 and -1.0 for the central parts (with residual intensity about .03). The limits are determined by a relation between absorption coefficient k and pressure p , different for different stars, because the gravity acceleration g is involved. Taking some specimens of stars with their $\log g$:

Sirius B 7.5	Kr. 60 4.7	Sun 4.4	Capella A 2.8	α Orionis 1.3
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we find these limits, for the wings and for the centre

$\log kp = 5.5$ to 7.5	2.7 to 4.7	2.4 to 4.4	0.8 to 2.8	-0.7 to $+1.3$
$\log kp = 4.5$ to 6.5	1.7 to 3.7	1.4 to 3.4	-0.2 to 1.8	-1.7 to $+0.3$

Considering these values we will treat as different "atmospheres" (indicated 0, 1, . . . 6) the cases where the relevant layers are included between the values of $\log kp$:

-1.6 to $+0.4$; -0.6 to $+1.4$; 0.4 to 2.4 ; 1.4 to 3.4 ; 2.4 to 4.4 ; 3.4 to 5.4 ; 4.4 to 6.4 .

For the white dwarfs the higher values 7 and 8 (5.4 to 7.4 , 6.4 to 8.4) have been added afterwards.

Derivation of formulas for these layers.

In an atmosphere consisting of one kind of atoms, these will be almost completely ionized in the uppermost layers; here $p = 2P$, and $k \sim xP \sim p$. In the deeper layers the atoms are nearly all neutral; the ionization formula $x = K/(P + K)$ reduces here to $x = K/P$; since $P = xp$ we have $K = x^2p$ and $k \sim xP$ is constant. Hence with increasing depth and pressure the absorption

¹⁾ Monthly Not. R. A. S. 91, 143.

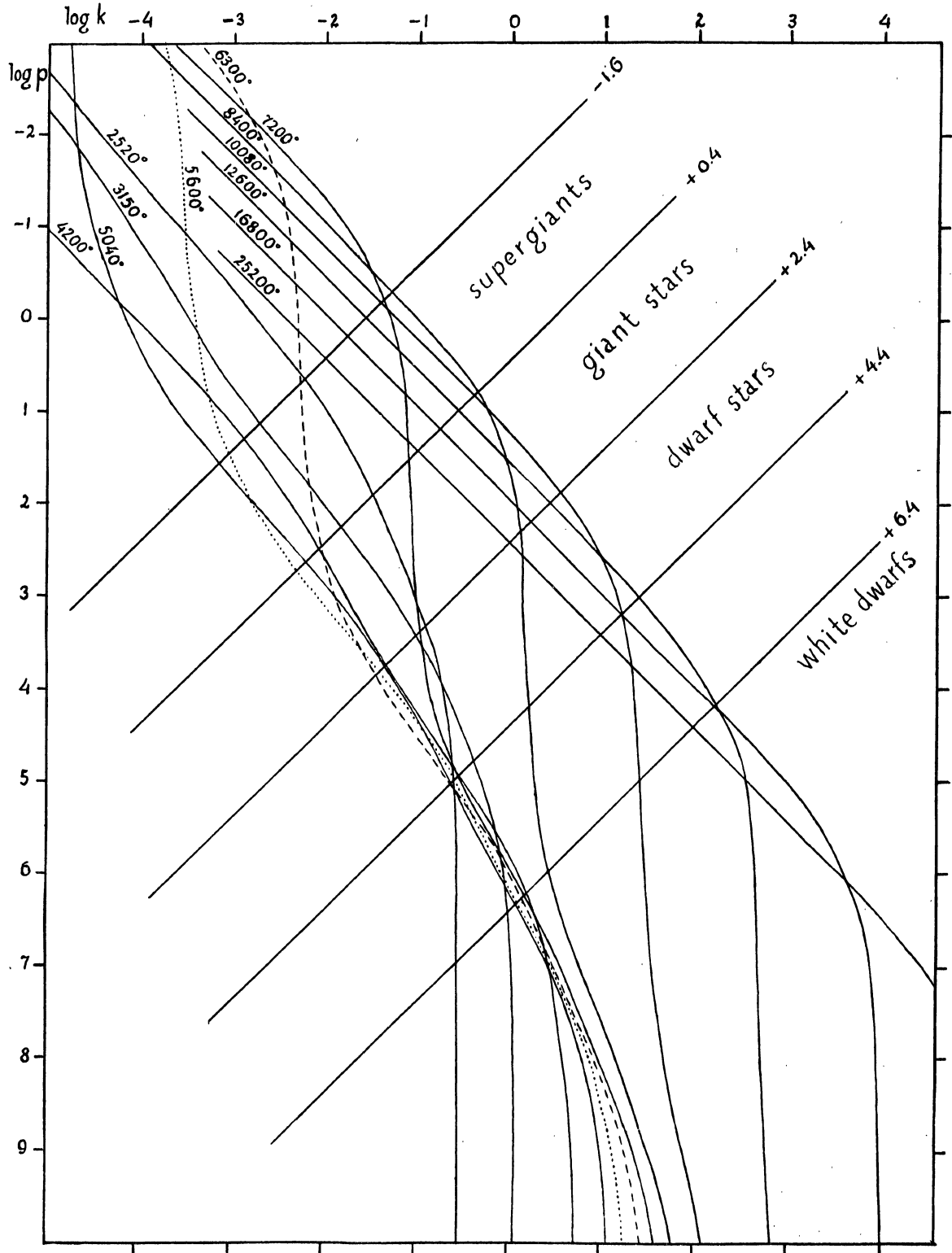


Fig. 5. Variation of absorption with pressure for $\lambda 4000 \text{ \AA}$.

coefficient at first increases proportional to p ; then its increase slackens and in greater depths it remains constant. At low temperature the top layer will be insignificant and in the relevant layers k will be constant; at high temperature the top conditions prevail and k has to be taken proportional to pressure.

In the real atmosphere matters will be more complicated. Because here two factors appear one after another, at low temperature the metal absorption and at higher temperature the hydrogen absorption, the real conditions are here a repetition twice of the simple conditions just described. The exact course of conditions is given by Table 21. It is shown more clearly by the diagram Fig. 5, p. 46, where (for $\lambda 4000 \text{ \AA}$) for each temperature $\log k$ is represented as a function of $\log p$, the total pressure, using the data of Tables 21 and 2. Here we see that for high temperatures (hydrogen absorption) each curve, from the upper layers going inward, runs up $k \sim p$; then for some higher pressure it bends and becomes vertical. For low temperatures (metal mixture) we see the same two parts; here, however, we have not $k \sim p$ in the first part because the mixture of ionized atoms is continually changing; for large p the curves become vertical too. For intermediate temperatures both constituents of the absorption are visible; for the lowest p the curve is inclined $k \sim p$ just as with the hot stars, owing to the hydrogen absorption; then comes a nearly vertical part where the hydrogen absorption would become constant, but soon the metal absorption steps in and produces a k strongly increasing with pressure, till at still greater depth the absorption becomes definitely constant.

It is clear that these irregular variations cannot be treated by a single analytical expression, and that for different conditions of temperature and gravity different formulas have to be used. It is possible, of course, for each special given case to compute the values of k , however irregular, for each depth and then by numerical integration solve the equations. If we wish, however, to find general solutions applicable to different cases afterwards, it is necessary to treat the problem by some kind of analytical expressions, with such parameters as to allow a sufficient approximation to the real cases. This is possible because for each case only a part of the curve, representing the relevant atmospheric layers, has to be used. We found that these layers were bounded by values of $\log kp = \text{constant}$. Hence in the diagram we have only to draw straight lines $\log kp = -1.6, -0.6 \dots +5.4, +6.4$. From each curve they cut off sections which correspond to the active atmospheres in each case. (In the Figure only the atmospheres 0, 2, 4, 6 are given.)

For each of these sections a relation between k and p has to be derived. Since they are mainly straight but differently inclined, expressions of the form $\log k = \log k_0 + u \log p$, or $k = k_0 p^u$ are suggested, where u is a parameter between 0 and 1 as its extreme values. Where the curve in the section considered contains a sharp bend this form is least satisfactory, but even there the deviations between the real k and the formula are small.

For each of the wave-lengths, for which values of k in Table 21 are given, such diagrams on a large scale were constructed. For each point of intersection of the curves and the straight lines the values of k and p were read. Representing them for the two limiting points of each section by $\log k = \log k_0 + u \log p$ the values of $\log k_0$ and of u for each atmosphere were derived.

The diffusion coefficient s too is variable within the atmospheric layers, because of the change of the ionization with pressure. We have $x_0 = P/(K + P)$; $x_1 = K/(K + P)$; the ionization factor depends on the electron pressure, and it is necessary to know for each atmosphere not only k but also P as a function of the total pressure p . Table 2 offers the data for a representation of $P = f(p)$; for the two limiting values of $\log p$ for each atmosphere we read $\log P$ from these curves, and represent them by $\log P = \log a + t \log p$. For the limiting cases of $u = 1$ and $u = 0$ the exponents t are 1 and $1/2$.

Table 22. Data for the different atmospheres.

λ 7000 \AA

π	u and t							Log k_m and log P_m						
	6	5	4	3	2	1	0	6	5	4	3	2	1	0
0.2	00 00	00 00	00 00	00 00	00 00	00 00	00 00	1.85 3.25	1.35 2.75	0.85 2.25	0.35 1.75	9.85 1.25	9.35 0.75	8.85 0.25
.3	00 00	00 00	00 00	00 00	00 00	00 00	00 00	2.10 3.00	1.60 2.50	1.10 2.00	0.60 1.50	0.10 1.00	9.60 0.50	9.10 0.00
.4	83 94	94 00	00 00	00 00	00 00	00 00	00 00	2.26 2.82	1.97 2.31	1.29 1.81	0.79 1.31	0.29 0.81	9.79 0.31	9.29 9.81
.5	21 65	47 78	74 89	89 96	96 00	00 00	00 00	1.97 2.72	1.70 2.20	1.37 1.67	0.91 1.17	0.43 0.67	9.95 0.15	9.45 9.65
.55	05 54	14 61	33 72	63 85	85 94	94 99	00 00	1.49 2.69	1.38 2.17	1.19 1.63	0.88 1.11	0.48 0.58	0.01 0.08	9.52 9.57
.6	03 53	04 53	09 57	25 68	52 81	79 91	92 97	0.92 2.65	0.89 2.14	0.81 1.62	0.65 1.09	0.39 0.55	0.02 0.03	9.57 9.52
.65	09 59	03 54	03 52	07 56	18 63	39 75	67 88	0.38 2.62	0.31 2.09	0.28 1.57	0.22 1.05	0.10 0.53	9.88 9.99	9.55 9.46
.7	29 70	13 62	05 56	03 52	04 53	12 60	26 71	9.98 2.62	9.80 2.07	9.71 1.54	9.66 1.02	9.62 0.50	9.54 9.97	9.38 9.43
.75	50 79	38 75	18 66	07 57	03 53	03 53	08 58	9.73 2.60	9.45 2.06	9.24 1.51	9.12 0.97	9.07 0.45	9.03 9.93	8.97 9.41
.8	55 78	59 81	47 81	27 71	10 60	03 55	03 53	9.64 2.57	9.29 2.05	8.96 1.52	8.71 0.98	8.55 0.42	8.47 9.89	8.43 9.37
.85	48 76	56 80	67 84	60 85	36 77	14 64	05 55	9.57 2.53	9.21 2.01	8.84 1.50	8.47 0.97	8.17 0.44	7.97 9.88	7.86 9.34
.9	52 77	52 77	63 81	72 88	68 88	46 80	21 68	9.51 2.47	9.16 1.96	8.79 1.46	8.40 0.94	7.99 0.43	7.66 9.88	7.41 9.35
.95	55 77	52 75	55 78	67 83	77 90	77 92	56 84	9.45 2.39	9.11 1.90	8.75 1.40	8.37 0.90	7.96 0.39	7.53 9.88	7.16 9.34
1.0	57 76	54 75	54 77	61 81	69 86	77 91	80 95	9.40 2.34	9.05 1.85	8.69 1.36	8.32 0.86	7.93 0.36	7.51 9.85	7.07 9.33
.1	54 77	63 82	63 80	54 78	57 80	67 82	68 86	9.32 2.25	8.96 1.75	8.58 1.25	8.22 0.74	7.84 0.25	7.47 9.74	7.07 9.24
.2	40 71	59 80	71 85	69 84	60 79	55 78	64 83	9.28 2.12	8.96 1.62	8.58 1.12	8.18 0.62	7.78 0.12	7.41 9.62	7.02 9.12
.3	32 66	47 74	64 80	69 86	67 86	67 82	55 78	9.17 2.02	8.89 1.52	8.53 1.03	8.16 0.51	7.74 0.02	7.35 9.51	6.98 9.01
.4	24 61	36 69	47 75	61 80	74 85	77 88	74 86	9.02 1.92	8.80 1.41	8.48 0.91	8.15 0.41	7.74 9.91	7.33 9.41	6.90 8.92
.5	14 58	27 63	40 69	50 76	64 82	74 87	79 88	8.84 1.79	8.67 1.29	8.43 0.80	8.12 0.31	7.75 9.80	7.36 9.30	6.92 8.80
.6	08 53	16 59	29 65	42 71	54 78	68 84	75 89	8.61 1.69	8.50 1.19	8.32 0.70	8.07 0.18	7.74 9.69	7.37 9.18	6.96 8.68
.8	01 51	05 52	12 56	23 62	39 69	49 75	57 78	8.10 1.48	8.07 0.98	7.99 0.48	7.83 9.99	7.60 9.48	7.31 8.98	6.96 8.48
2.0	00 50	01 50	03 51	08 54	19 60	34 67	44 71	7.55 1.28	7.55 0.78	7.53 0.28	7.47 9.78	7.34 9.28	7.15 8.78	6.88 8.28

λ 6000 \AA

0.2	00 00	00 00	00 00	00 00	00 00	00 00	00 00	1.76 3 34	1 26 2 84	0.76 2.34	0.26 1.84	9.76 1.34	9.26 0.84	8.76 0.34
.3	00 00	00 00	00 00	00 00	00 00	00 00	00 00	2.00 3.10	1.50 2.60	1.00 2.10	0.50 1.60	0.00 1.10	9.50 0.60	9.00 0.10
.4	82 94	92 98	98 99	00 00	00 00	00 00	00 00	2.13 2.93	1.67 2.42	1.19 1.92	0.69 1.41	0.19 0.91	9.69 0.41	9.19 9.91
.5	19 62	37 75	65 86	92 96	98 00	00 00	00 00	1.77 2.84	1.57 2.31	1.23 1.80	0.82 1.28	0.34 0.77	9.85 0.25	9.35 9.75
.55	04 53	12 60	30 70	55 81	80 93	94 99	98 00	1.29 2.81	1.20 2.29	1.03 1.76	0.75 1.22	0.36 0.70	9.91 0.19	9.43 9.67
.6	04 54	04 53	07 56	21 65	45 79	74 89	92 96	0.73 2.76	0.69 2.24	0.63 1.72	0.49 1.19	0.26 0.65	9.90 0.14	9.47 9.61
.65	13 60	05 54	03 52	06 54	15 61	34 72	61 85	0.22 2.71	0.14 2.18	0.09 1.67	0.05 1.16	9.94 0.63	9.74 0.10	9.43 9.56
.7	36 71	19 63	09 57	03 53	03 54	10 58	26 67	9.88 2.68	9.67 2.16	9.54 1.64	9.47 1.12	9.42 0.60	9.35 0.08	9.19 9.55
.75	56 79	43 76	25 67	13 58	05 54	02 53	06 56	9.69 2.65	9.39 2.12	9.13 1.60	8.97 1.07	8.88 0.55	8.84 0.03	8.78 9.51
.8	56 78	63 82	54 82	35 71	16 60	06 56	03 53	9.62 2.58	9.26 2.07	8.91 1.55	8.62 1.03	8.42 0.50	8.30 9.98	8.25 9.47
.85	49 76	56 80	69 84	69 85	47 78	23 66	09 57	9.58 2.51	9.22 2.00	8.84 1.50	8.45 0.99	8.11 0.48	7.85 9.95	7.70 9.43
.9	50 77	54 77	64 81	74 88	74 89	56 80	32 69	9.53 2.45	9.18 1.94	8.80 1.44	8.41 0.94	7.98 0.44	7.62 9.92	7.32 9.40
.95	54 76	53 76	57 79	65 83	79 90	85 92	64 85	9.49 2.37	9.14 1.88	8.78 1.38	8.40 0.88	7.97 0.37	7.54 9.87	7.14 9.35
1.0	56 77	56 76	53 77	60 81	71 85	79 90	89 95	9.43 2.32	9.08 1.83	8.72 1.33	8.35 0.83	7.97 0.32	7.53 9.82	7.08 9.31
.1	57 78	64 81	61 81	56 79	55 79	64 81	77 87	9.40 2.19	9.03 1.70	8.65 1.20	8.28 0.70	7.92 0.19	7.53 9.69	7.13 9.19
.2	44 73	63 81	68 84	65 83	60 78	56 77	61 82	9.36 2.06	9.03 1.56	8.64 1.06	8.24 0.56	7.86 0.07	7.49 9.56	7.11 9.07
.3	34 68	49 76	63 81	69 85	69 86	65 82	57 77	9.29 1.94	8.99 1.44	8.65 0.93	8.25 0.43	7.85 9.93	7.44 9.43	7.07 8.93
.4	25 63	38 70	52 76	63 80	72 86	79 88	71 85	9.16 1.83	8.93 1.32	8.62 0.82	8.27 0.32	7.86 9.82	7.44 9.32	7.02 8.82
.5	18 59	30 64	41 70	53 78	67 82	77 88	79 89	9.08 1.68	8.84 1.19	8.59 0.68	8.27 0.19	7.90 9.68	7.49 9.19	7.06 8.69
.6	09 54	19 61	33 67	46 73	57 79	69 85	75 88	8.84 1.57	8.71 1.07	8.51 0.56	8.24 0.06	7.90 9.56	7.51 9.06	7.10 8.56
.8	02 51	06 53	16 57	29 64	43 71	53 76	60 81	8.43 1.32	8.38 0.81	8.28 0.31	8.10 9.82	7.84 9.32	7.53 8.82	7.17 8.32
2.0	01 50	02 51	04 51	11 56	22 63	37 69	50 72	7.93 1.08	7.92 0.59	7.88 0.09	7.80 9.59	7.66 9.08	7.43 8.58	7.15 8.08

λ 5000 \AA

π	u and t							Log k_m and log P_m						
	6	5	4	3	2	1	0	6	5	4	3	2	1	0
0.2	00 00	00 00	00 00	00 00	00 00	00 00	00 00	1.63 3.47	1.13 2.97	0.63 2.47	0.13 1.97	9.63 1.47	9.13 0.97	8.63 0.47
0.3	00 00	00 00	00 00	00 00	00 00	00 00	00 00	1.88 3.22	1.38 2.72	0.88 2.22	0.38 1.72	9.88 1.22	9.38 0.72	8.88 0.22
0.4	77 92	94 00	98 00	00 00	00 00	00 00	00 00	2.00 3.06	1.55 2.56	1.06 2.04	0.57 1.53	0.07 1.03	9.57 0.53	9.07 0.03
0.5	14 59	32 72	63 86	85 93	94 96	00 00	00 00	1.58 2.96	1.39 2.44	1.08 1.91	0.68 1.39	0.21 0.89	9.73 0.37	9.23 9.87
.55	04 53	09 58	24 68	49 80	77 89	92 97	98 00	1.07 2.93	0.99 2.41	0.85 1.88	0.59 1.34	0.22 0.82	9.78 0.31	9.31 9.79
.6	06 54	04 53	06 55	17 62	38 74	67 88	89 97	0.53 2.87	0.47 2.36	0.41 1.85	0.30 1.32	0.09 0.79	9.76 0.26	9.34 9.74
.65	19 61	09 55	05 53	05 52	12 59	29 69	55 81	0.09 2.80	9.94 2.29	9.87 1.78	9.82 1.27	9.72 0.76	9.55 0.23	9.27 9.71
.7	46 72	30 64	14 57	05 54	04 53	09 56	21 65	9.81 2.72	9.57 2.22	9.38 1.72	9.29 1.21	9.23 0.71	9.16 0.19	9.02 9.66
.75	59 79	56 77	38 68	19 60	08 54	05 52	06 54	9.70 2.63	9.36 2.13	9.05 1.64	8.84 1.14	8.69 0.64	8.63 0.14	8.58 9.63
.8	56 77	68 83	64 82	47 72	26 62	12 56	05 53	9.65 2.55	9.29 2.06	8.90 1.56	8.56 1.07	8.30 0.57	8.13 0.08	8.05 9.58
.85	48 77	61 81	74 84	74 86	59 79	35 66	17 57	9.63 2.48	9.26 1.98	8.88 1.48	8.45 0.99	8.08 0.49	7.77 0.00	7.56 9.51
.9	50 77	56 78	65 82	75 88	80 87	67 81	45 71	9.60 2.39	9.24 1.90	8.87 1.39	8.45 0.90	8.02 0.40	7.62 9.92	7.27 9.43
.95	55 77	54 75	57 77	68 84	82 91	85 91	72 85	9.56 2.30	9.21 1.81	8.85 1.32	8.46 0.82	8.03 0.33	7.59 9.83	7.16 9.34
1.0	56 77	54 76	54 78	61 81	74 85	87 92	85 94	9.53 2.25	9.18 1.75	8.82 1.25	8.45 0.74	8.05 0.25	7.61 9.75	7.16 9.25
.1	56 79	65 82	63 79	54 78	57 81	67 82	77 87	9.52 2.10	9.15 1.60	8.77 1.11	8.40 0.60	8.03 0.10	7.65 9.59	7.23 9.10
.2	47 75	67 83	69 85	64 82	59 78	56 77	64 84	9.52 1.95	9.18 1.44	8.79 0.94	8.39 0.44	8.01 9.95	7.64 9.45	7.25 8.95
.3	39 70	54 78	65 84	69 86	71 83	63 80	54 78	9.50 1.80	9.19 1.30	8.83 0.78	8.43 0.28	8.02 9.78	7.63 9.29	7.25 8.79
.4	30 65	44 71	54 78	67 83	77 88	75 88	68 82	9.43 1.65	9.18 1.15	8.85 0.65	8.47 0.15	8.07 9.14	7.64 9.15	7.22 8.65
.5	22 60	33 67	45 72	59 79	71 85	79 89	79 89	9.34 1.50	9.13 1.00	8.85 0.50	8.51 0.00	8.13 9.50	7.70 9.00	7.27 8.50
.6	13 56	25 63	39 69	50 75	61 81	72 86	79 90	9.22 1.36	9.05 0.86	8.82 0.36	8.52 9.86	8.16 9.36	7.76 8.86	7.34 8.36
.8	03 52	10 55	23 60	36 69	45 74	56 77	68 84	8.91 1.07	8.83 0.57	8.69 0.07	8.48 9.57	8.19 9.07	7.85 8.56	7.48 8.06
2.0	01 51	02 52	06 54	20 59	35 67	43 71	49 74	8.51 0.79	8.50 0.29	8.44 9.78	8.32 9.28	8.12 8.79	7.85 8.39	7.53 7.80

λ 4000 \AA

0.2	00 00	00 00	00 00	00 00	00 00	00 00	00 00	1.49 3.61	0.99 3.11	0.49 2.61	9.99 2.11	9.49 1.61	8.99 1.11	8.49 0.61
.3	00 00	00 00	00 00	00 00	00 00	00 00	00 00	1.73 3.37	1.23 2.87	0.73 2.37	0.23 1.87	9.73 1.37	9.23 0.87	8.73 0.37
.4	75 90	92 96	00 00	00 00	00 00	00 00	00 00	1.84 3.20	1.39 2.69	0.92 2.18	0.42 1.68	9.92 1.18	9.42 0.68	8.92 0.18
.5	12 58	27 68	52 82	80 92	94 97	98 00	00 00	1.34 3.12	1.17 2.59	0.89 2.06	0.51 1.55	0.06 1.02	9.59 0.51	9.09 0.01
.55	05 53	07 56	19 65	42 76	68 87	90 97	99 00	0.82 3.07	0.75 2.55	0.62 2.02	0.39 1.48	0.06 0.96	9.62 0.45	9.16 9.94
.6	12 55	06 53	06 53	12 60	32 71	59 84	84 94	0.33 2.98	0.23 2.49	0.16 1.99	0.05 1.47	9.87 0.94	9.58 0.41	9.18 9.89
.65	32 62	19 55	09 53	05 53	10 57	22 66	46 78	9.98 2.86	9.78 2.38	9.65 1.91	9.57 1.41	9.48 0.90	9.36 0.37	9.08 9.84
.7	56 72	43 65	27 58	14 54	06 53	07 55	17 62	9.83 2.71	9.52 2.25	9.26 1.79	9.08 1.32	8.98 0.84	8.90 0.33	8.78 9.81
.75	64 79	63 76	50 69	34 60	20 54	10 52	06 54	9.77 2.58	9.39 2.10	9.04 1.64	8.75 1.18	8.54 0.73	8.41 0.27	8.32 9.77
.8	61 79	72 83	70 81	61 73	41 61	24 56	13 54	9.76 2.47	9.38 1.98	8.97 1.50	8.58 1.05	8.27 0.59	8.01 0.15	7.77 9.68
.85	50 77	64 81	77 85	79 86	70 77	50 66	32 58	9.77 2.37	9.39 1.87	9.00 1.38	8.56 0.90	8.15 0.44	7.78 9.98	7.49 9.55
.9	52 77	56 77	70 83	80 89	82 87	74 79	56 77	9.76 2.27	9.40 1.76	9.01 1.27	8.60 0.78	8.15 0.30	7.72 9.82	7.34 9.37
.95	53 76	53 77	59 80	74 85	87 91	85 92	77 84	9.74 2.18	9.40 1.68	9.03 1.17	8.63 0.68	8.20 0.19	7.74 9.70	7.29 9.22
1.0	54 76	52 76	59 79	67 82	75 88	85 94	91 92	9.75 2.08	9.39 1.58	9.03 1.08	8.65 0.58	8.23 0.08	7.79 9.58	7.33 9.09
.1	63 80	64 81	56 79	54 78	61 81	71 85	82 91	9.77 1.90	9.39 1.41	9.02 0.91	8.65 0.40	8.28 9.90	7.89 9.40	7.45 8.90
.2	59 80	68 86	67 83	63 80	56 77	59 81	70 85	9.83 1.72	9.46 1.21	9.06 0.71	8.67 0.21	8.30 9.72	7.92 9.21	7.53 8.72
.3	47 74	60 81	68 86	70 86	67 83	60 78	56 77	9.86 1.54	9.52 1.04	9.14 0.53	8.73 0.03	8.33 9.52	7.94 9.03	7.57 8.53
.4	38 70	50 75	61 81	71 86	77 88	74 84	64 81	9.87 1.36	9.57 0.86	9.21 0.36	8.82 9.86	8.40 9.36	7.97 8.86	7.57 8.37
.5	30 63	43 71	54 78	65 83	72 88	79 88	77 88	9.85 1.18	9.60 0.68	9.27 0.19	8.91 9.68	8.50 9.17	8.07 8.67	7.64 8.17
.6	20 61	35 68	47 74	59 80	70 86	77 88	82 88	9.81 1.01	9.60 0.51	9.32 0.00	8.97 9.50	8.59 9.00	8.17 8.50	7.73 8.01
.8	09 54	20 59	32 67	45 73	55 78	65 83	74 87	9.68 0.65	9.55 0.16	9.35 9.66	9.07 9.15	8.75 8.65	8.37 8.15	7.97 7.65
2.0	02 51	07 53	18 59	33 67	43 71	49 74	60 80	9.44 0.32	9.40 9.82	9.27 9.32	9.08 8.82	8.82 8.32	8.50 7.82	8.15 7.32

λ 3500 A

π	u and t							Log k^m and log P^m						
	6	5	4	3	2	1	0	6	5	4	3	2	1	0
0.2	00 00	00 00	00 00	00 00	00 00	00 00	00 00	1.69 3.41	1.19 2.91	0.69 2.41	0.19 1.91	9.69 1.41	9.19 0.91	8.69 0.41
.3	00 00	00 00	00 00	00 00	00 00	00 00	00 00	2.05 3.05	1.55 2.55	1.05 2.05	0.55 1.55	0.05 1.05	9.55 0.55	9.05 0.05
.4	85 95	96 00	00 00	00 00	00 00	00 00	00 00	2.32 2.76	1.85 2.25	1.36 1.74	0.86 1.24	0.36 0.74	9.86 0.24	9.36 9.74
.5	27 69	54 83	82 92	92 97	96 00	00 00	00 00	2.24 2.54	1.95 2.01	1.57 1.49	1.12 0.97	0.63 0.47	0.14 9.96	9.64 9.46
.55	08 57	21 67	47 78	75 89	90 97	96 00	00 00	1.93 2.44	1.79 1.91	1.54 1.38	1.18 0.85	0.75 0.34	0.26 9.84	9.77 9.33
.6	02 53	05 55	16 62	39 74	67 88	87 96	98 99	1.48 2.36	1.43 1.84	1.32 1.31	1.10 0.77	0.78 0.24	0.36 9.72	9.89 9.22
.65	02 55	02 52	04 54	14 60	31 71	57 83	83 93	1.00 2.26	0.98 1.72	0.94 1.21	0.85 0.68	0.67 0.15	0.37 9.62	9.98 9.10
.7	06 65	03 57	02 53	03 53	09 58	25 68	50 79	0.54 2.25	0.48 1.66	0.46 1.12	0.44 0.59	0.37 0.07	0.21 9.53	9.95 9.00
.75	22 77	09 68	03 58	01 53	02 53	07 57	20 65	0.20 2.26	0.06 1.65	9.99 1.01	9.97 0.50	9.95 9.97	9.90 9.45	9.76 8.91
.8	43 81	30 81	12 71	03 60	01 55	02 53	06 54	0.00 2.28	9.76 1.68	9.59 1.07	9.50 0.46	9.47 9.89	9.46 9.36	9.41 8.84
.85	48 78	50 82	39 85	18 78	05 64	01 55	02 53	9.94 2.24	9.61 1.70	9.33 1.10	9.12 0.49	9.00 9.87	8.96 9.28	8.95 8.74
.9	49 77	55 79	59 86	47 88	23 80	06 66	02 57	9.91 2.15	9.57 1.63	9.21 1.10	8.89 0.52	8.65 9.91	8.51 9.29	8.15 8.69
.94	53 76	53 76	59 82	67 88	55 91	28 84	09 70	9.91 2.04	9.57 1.54	9.20 1.04	8.82 0.51	8.47 9.93	8.20 9.33	8.03 8.70
1.0	53 76	52 76	59 79	67 84	67 90	61 94	36 87	9.90 1.95	9.56 1.45	9.19 0.96	8.82 0.45	8.42 9.92	8.04 9.36	7.75 8.74
1.	65 81	63 80	55 79	55 80	63 81	69 86	79 92	9.96 1.75	9.57 1.26	0.20 0.75	8.84 0.25	8.47 9.74	8.07 9.23	7.64 8.72
2.	61 82	67 84	67 83	60 79	56 77	64 82	72 87	0.06 1.54	9.68 1.03	9.28 0.53	8.90 0.03	8.52 9.54	8.14 9.04	7.74 8.54
3.	52 78	64 83	72 86	71 85	63 80	56 77	59 79	0.14 1.33	9.78 0.82	9.38 0.32	8.97 9.82	8.57 9.33	8.20 8.83	7.82 8.34
4.	45 72	55 78	67 83	79 88	79 88	68 82	56 80	0.19 1.13	9.87 0.63	9.48 0.14	9.07 9.63	8.64 9.15	8.22 8.65	7.84 8.15
5.	34 66	47 75	60 81	71 85	77 88	77 90	69 86	0.22 0.94	9.93 0.44	9.59 9.93	9.20 9.43	8.78 8.94	8.34 8.43	7.93 7.93
6.	27 65	43 71	56 77	64 84	72 88	79 89	80 88	0.25 0.74	9.99 0.24	9.68 9.74	9.30 9.24	8.90 8.73	8.47 8.23	8.03 7.73
8.	13 58	27 63	43 71	53 76	60 81	71 85	79 88	0.23 0.34	0.05 9.84	9.81 9.84	9.50 8.84	9.14 8.34	8.74 7.84	8.32 7.34
2.0	05 53	15 57	28 64	41 70	48 73	55 78	67 85	0.12 9.96	0.08 9.46	9.85 8.96	9.61 8.46	9.31 7.97	8.97 7.47	8.58 6.97

λ 3000 A

0.2	00 00	00 00	00 00	00 00	00 00	00 00	00 00	1.59 3.51	1.09 3.01	0.59 2.51	0.09 2.01	9.59 1.51	9.09 1.01	8.59 0.51
.3	00 00	00 00	00 00	00 00	00 00	00 00	00 00	1.95 3.15	1.45 2.65	0.95 2.15	0.45 1.65	9.95 1.15	9.45 0.65	8.95 0.15
.4	80 95	96 99	00 00	00 00	00 00	00 00	00 00	2.25 2.86	1.75 2.34	1.26 1.84	0.76 1.34	0.26 0.84	9.76 0.34	9.26 9.84
.5	23 67	52 80	77 89	87 96	96 00	00 00	00 00	2.08 2.64	1.81 2.12	1.46 1.59	1.01 1.08	0.53 0.57	0.04 0.06	9.54 9.56
.55	06 56	18 65	41 75	71 85	89 97	94 00	98 00	1.74 2.55	1.62 2.02	1.39 1.49	1.06 0.97	0.63 0.45	0.16 9.95	9.67 9.43
.6	03 53	05 54	14 60	32 72	63 85	85 94	94 99	1.30 2.45	1.25 1.93	1.15 1.41	0.96 0.88	0.74 0.35	0.25 9.82	9.78 9.32
.65	04 56	02 53	04 53	10 59	27 70	53 81	80 91	0.82 2.36	0.79 1.83	0.76 1.31	0.69 0.79	0.51 0.25	0.24 9.72	9.86 9.19
.7	15 66	05 57	02 54	03 53	08 56	22 65	46 77	0.43 2.31	0.33 1.75	0.28 1.21	0.26 0.69	0.19 0.17	0.05 9.64	9.80 9.08
.75	37 77	20 69	08 60	03 53	02 53	05 56	17 63	0.19 2.27	9.97 1.70	9.85 1.14	9.78 0.60	9.76 0.08	9.71 9.55	9.60 9.02
.8	54 81	48 81	26 73	08 62	03 55	02 53	05 55	0.09 2.21	9.76 1.67	9.53 1.10	9.36 0.55	9.30 9.99	9.27 9.46	9.22 8.94
.85	55 79	63 84	56 85	34 78	14 65	05 57	02 53	0.10 2.11	9.74 1.60	9.38 1.06	9.09 0.50	8.89 9.94	8.79 9.38	8.75 8.84
.9	52 77	63 80	69 87	64 89	43 81	18 67	06 57	0.12 1.99	9.75 1.48	9.37 0.97	8.97 0.45	8.66 9.89	8.43 9.34	8.30 8.78
.95	52 74	55 78	65 84	77 90	74 91	48 84	23 71	0.14 1.87	9.78 1.38	9.41 0.87	8.99 0.37	8.58 9.84	8.22 9.30	7.97 8.73
1.0	54 75	53 78	60 82	75 86	85 92	71 93	53 86	0.16 1.76	9.81 1.26	9.44 0.76	9.03 0.26	8.60 9.76	8.18 9.23	7.79 8.69
1.	67 82	60 79	54 78	59 81	67 83	79 89	90 95	0.24 1.53	9.87 1.03	9.49 0.53	9.13 0.02	8.74 9.52	8.32 9.01	7.87 8.52
2.	63 84	65 85	65 81	56 77	59 80	69 84	79 89	0.37 1.29	9.98 0.78	9.59 0.28	9.22 9.78	8.83 9.28	8.45 8.78	8.02 8.28
3.	60 81	68 86	68 86	68 83	61 78	54 78	63 82	0.49 1.06	0.11 0.55	9.71 0.05	9.30 9.54	8.92 9.05	8.55 8.55	8.16 8.06
4.	52 75	60 80	72 87	80 88	71 85	63 81	59 79	0.61 0.82	0.26 0.32	9.85 9.82	9.43 9.33	9.01 8.83	8.60 8.34	8.23 7.83
5.	42 72	56 79	69 84	77 88	79 89	69 86	63 83	0.72 0.60	0.38 0.10	0.01 9.60	9.59 9.10	9.16 8.60	8.74 8.06	8.33 7.59
6.	37 70	48 76	61 81	71 87	80 90	82 90	75 86	0.80 0.37	0.50 9.88	0.15 9.36	9.76 8.86	9.33 8.36	8.89 7.86	8.45 7.37
8.	26 62	40 71	53 76	61 80	68 84	75 88	85 92	0.95 9.91	0.71 9.42	0.40 8.92	0.04 8.42	9.65 7.92	9.23 7.42	8.78 6.92
2.0	14 57	28 63	41 70	48 73	54 78	65 84	80 90	1.00 9.48	0.84 8.97	0.59 8.48	0.30 7.98	9.95 7.48	9.58 6.97	9.16 6.47

Atm. 8.

	u and t						Log k_m and log P_m					
	7000	6000	5000	4000	3500	3000	7000	6000	5000	4000	3500	3000
0.2	00 00	00 00	00 00	00 00	00 00	00 00	2.85 4.25	2.76 4.35	2.64 4.46	2.49 4.62	2.68 4.42	2.59 4.52
.3	96 98	94 96	94 95	89 96	98 98	94 96	3.08 4.01	2.99 4.11	2.86 4.22	2.70 4.38	3.04 4.05	2.93 4.16
.4	44 77	39 74	32 72	23 68	48 78	42 74	3.02 3.86	2.86 3.98	2.68 4.11	2.46 4.27	3.11 3.80	2.95 3.91
.5	03 54	02 56	03 54	04 54	04 54	03 54	2.15 3.78	1.95 3.90	1.73 4.01	1.48 4.15	2.49 3.60	2.303.70
.55	03 54	06 54	09 55	16 55	02 53	03 54	1.57 3.72	1.38 3.83	1.19 3.95	1.00 4.05	2.02 3.49	1.83 3.59
.6	13 60	17 61	24 62	36 62	03 58	06 54	1.06 3.70	0.91 3.80	0.78 3.87	0.70 3.93	1.53 3.43	1.38 3.52
.65	31 70	36 70	42 70	47 70	11 69	21 69	0.69 3.70	0.60 3.76	0.55 3.80	0.55 3.81	1.11 3.42	1.03 3.47
.7	41 73	44 73	46 73	47 73	25 74	38 75	0.50 3.68	0.46 3.71	0.45 3.71	0.51 3.67	0.80 3.45	0.83 3.43

Atm. 7.

0.2	00 00	00 00	00 00	00 00	00 00	00 00	2.35 3.75	2.26 3.85	2.13 3.97	1.98 4.12	2.18 3.92	2.09 4.02
.3	98 99	98 99	96 98	98 99	98 99	98 99	2.60 3.51	2.50 3.62	2.37 3.73	2.23 3.87	2.54 3.56	2.44 3.66
.4	68 87	65 87	58 80	49 81	69 88	65 87	2.68 3.34	2.55 3.45	2.40 3.60	2.20 3.73	2.75 3.28	2.64 3.38
.5	08 56	06 55	05 55	04 54	11 58	09 57	2.09 3.26	1.88 3.37	1.67 3.50	1.42 3.64	2.40 3.07	2.24 3.17
.55	04 52	04 52	05 53	09 53	03 53	03 53	1.54 3.21	1.33 3.32	1.12 3.44	0.89 3.56	1.99 2.97	1.80 3.08
.6	05 56	08 56	13 57	23 58	02 54	02 55	0.97 3.18	0.80 3.27	0.62 3.37	0.48 3.46	1.50 2.89	1.32 2.98
.65	20 66	25 67	34 68	45 69	04 62	10 62	0.50 3.16	0.39 3.23	0.29 3.30	0.25 3.32	1.04 2.83	0.90 2.92
.7	42 76	46 76	49 75	54 75	16 73	29 74	0.21 3.16	0.15 3.21	0.13 3.21	0.17 3.19	0.65 2.85	0.60 2.88

The results for these exponents u and t are given in Table 22 for each atmosphere. Instead of log k_0 and log a we have given here the medium log k_m and log P_m in the atmosphere considered, i.e. the mean of log k and of log P for the two limiting points of the curve section used. The corresponding values of log p_m are not given, because they follow directly from the relations $\log p_m + \log k_m = 5.4 \ 4.4 \ \dots - 0.6$. Seeing the uncertainty in drawing and reading the curves we cannot be sure that the second decimal in the exponents and in the medium values is entirely reliable. To save space the decimal points are omitted for the exponents, and 1.00 is denoted by 00.

The boundary conditions.

Before proceeding to the solution of the equations for these atmospheres, it is desirable to make an inquiry about the boundary conditions at the surface. In first approximation, as is stated in the Introduction, the surface condition $H_0 = \frac{1}{2} J_0$ is taken; but this equation is deduced from an unsatisfactory model. The radiation diagram (giving the radiation intensity as function of the direction) becomes disfigured near the surface, in the manner derived by SCHWARZSCHILD and MILNE for the total and the continuous radiation. Following the same mode of treatment, first from J as a function of the depth we find the intensity I along beams in every direction; by integrating them, for a volume element, over all directions a better value of J and of H is found. This process may be repeated. It is not necessary, for the present purpose of simply deriving boundary conditions, to take account of the variation of the absorption coefficients; hence k , s , \bar{k} will be taken constant here.

Assuming as a first step all outgoing and all inward going radiations to have the constant intensities I and I' , we have the equations

$${}_{1/2} \frac{d}{d\xi} (I + I') = (k + s)(I - I'); \quad {}_{1/2} \frac{d}{d\xi} (I - I') = k(I + I' - 2E).$$

Putting $k d\xi = dt$, $4(k + s)/k = q^2$, $2/q = n$; ${}_{3/2} c\bar{k}/k = C$ the solution is

$$\begin{aligned} I + I' &= 2J = 2E_0 \left(1 + Ct - \frac{1}{1+n} (1 - {}_{1/2} Cn^2) e^{-qt} \right), \\ I - I' &= 2E_0 \left(C + \frac{n}{1+n} (1 - {}_{1/2} Cn^2) e^{-qt} \right). \end{aligned}$$

This value of J is introduced into the first equation of I in the Introduction

$$\cos \vartheta \frac{dI}{d\xi} = (k + s) I - kE - sJ.$$

We put

$$kE + sJ = (k + s)J',$$

$$J' = \frac{k}{k + s} E_0(1 + Ct) + \frac{s}{k + s} J = E_0(1 + Ct) - \frac{s}{k + s} \frac{1 - {}_{1/2} Cn^2}{1 + n} E_0 e^{-qt}.$$

We introduce a new variable x for the optical depth

$$\begin{aligned} dx &= (k + s)d\xi & t &= \frac{k}{k + s} x = n^2 x & qt &= 2nx, \\ J' &= E_0(1 + Cn^2 x) - (1 - n)(1 - {}_{1/2} Cn^2) E_0 e^{-2nx}, \\ \cos \vartheta \frac{dI}{dx} &= I - J'. \end{aligned}$$

The solution of this equation is (writing $180 - \vartheta = \psi$ for $\vartheta > 90^\circ$.)

$$\begin{aligned} I(x, \vartheta) &= \int_x^\infty J' e^{-(z-x)\sec \vartheta} \sec \vartheta dz = E_0 \int_x^\infty \left\{ 1 + Cn^2 z - (1 - n)(1 - {}_{1/2} Cn^2) e^{-2nz} \right\} e^{-(z-x)\sec \vartheta} \sec \vartheta dz, \\ I(x, \psi) &= \int_0^x J' e^{-(z-x)\sec \psi} \sec \psi dz = E_0 \int_0^x \left\{ 1 + Cn^2 z - (1 - n)(1 - {}_{1/2} Cn^2) e^{-2nz} \right\} e^{-(z-x)\sec \psi} \sec \psi dz. \end{aligned}$$

Integrating we have

$$\begin{aligned} I(x, \vartheta) &= E_0 \left\{ 1 + Cn^2 (x + \cos \vartheta) - (1 - n)(1 - {}_{1/2} Cn^2) \frac{e^{-2nx}}{1 + 2n \cos \vartheta} \right\}, \\ I(x, \psi) &= E_0 \left\{ 1 + Cn^2 (x - \cos \psi) - (1 - n)(1 - {}_{1/2} Cn^2) \frac{e^{-2nx} - e^{-x \sec \psi}}{1 - 2n \cos \psi} \right\}. \end{aligned} \quad \left. \vphantom{\begin{aligned} I(x, \vartheta) \\ I(x, \psi) \end{aligned}} \right\} 32$$

These equations give the figure of the radiation diagram for every optical depth, depending on $n = \sqrt{k/(k + s)}$ (increasing with the darkness in the line) as chief parameter. By integration over all directions we get the general radiation integrals

$$\begin{aligned} J &= {}_{1/2} \int_0^1 I(x, \vartheta) d \cos \vartheta + {}_{1/2} \int_0^1 I(x, \psi) d \cos \psi, \\ H &= {}_{1/2} \int_0^1 I(x, \vartheta) \cos \vartheta d \cos \vartheta - {}_{1/2} \int_0^1 I(x, \psi) \cos \psi d \cos \psi, \\ K &= {}_{1/2} \int_0^1 I(x, \vartheta) \cos^2 \vartheta d \cos \vartheta + {}_{1/2} \int_0^1 I(x, \psi) \cos^2 \psi d \cos \psi. \end{aligned}$$

The results are

$$\begin{aligned} J/E_0 &= 1 + Cn^2 x - {}_{1/2} E i_2(x) + {}_{1/2} Cn^2 E i_3(x) - {}_{1/2} (1 - n)(1 - {}_{1/2} Cn^2) \left(\frac{1}{2n} \ln(1 + 2n) + P(x) \right) e^{-2nx}, \\ 2H/E_0 &= {}_{2/3} Cn^2 + E i_3(x) - Cn^2 E i_4(x) - (1 - n)(1 - {}_{1/2} Cn^2) \left(\frac{1}{2n} - \frac{1}{4n^2} \ln(1 + 2n) - Q(x) \right) e^{-2nx}, \\ 3K/E_0 &= 1 + Cn^2 x - {}_{3/2} E i_4(x) + {}_{3/2} Cn^2 E i_5(x) - {}_{3/2} (1 - n)(1 - {}_{1/2} Cn^2) \left(\frac{n-1}{4n^2} + \frac{1}{8n^3} \ln(1 + 2n) + R(x) \right) e^{-2nx}, \end{aligned} \quad \left. \vphantom{\begin{aligned} J/E_0 \\ 2H/E_0 \\ 3K/E_0 \end{aligned}} \right\} 33$$

where

$$Ei_n(x) = \int_1^\infty \frac{e^{-xu}}{u^n} du, \quad \frac{1 - e^{-x(\mu-2n)}}{1 - 2n\mu} = F(\mu, x),$$

$$P(x) = \int_0^1 F(\mu, x) d\mu, \quad Q(x) = \int_0^1 F(\mu, x) \mu d\mu, \quad R(x) = \int_0^1 F(\mu, x) \mu^2 d\mu.$$

The functions $Ei_n(x)$ can be reduced to the exponential integral

$$Ei(x) = \int_x^\infty \frac{e^{-u}}{u} du; \quad Ei_2(x) = e^{-x} - xEi(x); \quad Ei_3(x) = e^{-x} (1/2 - 1/2x) + 1/2x^2 Ei(x);$$

$$Ei_4(x) = e^{-x} (1/3 - 1/6x + 1/6x^2) - 1/6x^3 Ei(x); \quad Ei_5(x) = e^{-x} (1/4 - 1/12x + 1/24x^2 - 1/24x^3) + 1/24x^4 Ei(x).$$

The integrals P, Q, R can be computed numerically for given values of x and n ; though for $\mu = 1/2n$ numerator and denominator in F both become zero, the function itself is continuous. Table 23 gives some numerical values (for $x = 0 P = Q = R = 0$).

Table 23. Functions Ei and P, Q, R .

x	$Ei(x)$	$Ei_2(x)$	$Ei_3(x)$	$Ei_4(x)$	$Ei_5(x)$
1.0	0.2194	0.1485	0.1097	0.08606	0.07044
0.5	.5598	.3266	.2216	.1652	.1309
.2	1.2227	.5742	.3519	.2494	.1922
.1	1.8229	.7225	.4163	.2874	.2190
.0	∞	1.	.5000	.3333	.2500

x	$n = 1.0$			$n = 0.8$			$n = 0.5$			$n = 0.2$			$n = 0.1$			$n = 0.0$		
	P	Q	R	P	Q	R	P	Q	R	P	Q	R	P	Q	R	P	Q	R
1.0	1.756	0.928	0.621	1.478	0.759	0.503	1.172	0.577	0.377	0.959	0.452	0.291	0.903	0.419	0.268	0.852	0.390	0.247
0.5	0.993	.439	.271	0.907	.397	.245	0.806	.345	.211	.722	.302	.184	.697	.289	.176	.673	.278	.168
.2	.488	.178	.102	.482	.171	.0981	.459	.162	.0925	.438	.153	.0873	.432	.150	.0858	.426	.148	.0837
.1	.299	.0912	.0504	.294	.0984	.0494	.288	.0868	.0480	.281	.0844	.0466	.279	.0835	.0461	.277	.0837	.0456

By means of formulas 33) and table 23 the value of $J, 2H$ and $3K$ could be computed for different depths x and different intensities in the lines, dependent on n . For C an average value $3/2$ was assumed. The results for J/E_0 and for $3K/J$ are

x	J/E					$3K/J$				
	$n = 1.0$	0.8	0.5	0.2	0.1	$n = 0.1$	0.8	0.5	0.2	0.1
0.0	0.875	0.709	0.453	0.189	0.095	1.21	1.18	1.15	1.18	1.21
.1	1.101	.895	.574	.239	.120	1.10	1.07	1.04	1.06	1.10
.2	.277	1.033	.662	.277	.139	1.06	1.03	1.01	1.03	1.06
.5	.753	1.388	.881	.377	.193	1.02	1.00	0.98	0.99	1.01
1.0	2.507	1.917	1.182	.521	.275	1.01	1.00	.98	.98	0.99

It appears that in the highest layers $3K/J$ is above 1, and decreases to 1 and somewhat below 1 in the deeper layers. Since the relevant layers extend farther than $x = 1.0$ (exp. $(-2nx) =$

0.1 for $x = 1.15/n$) it does not seem advisable to assume another average value than $K = 1/3J$. For the surface value of $2H/J$ we have, taking $x = 0$, for the same values of n :

for $C = 1$	1.11	1.10	1.10	1.11	1.13
$C = 1.5$	1.14	1.12	1.11	1.12	1.14
$C = 2$	1.17	1.14	1.12	1.13	1.14

Hence this ratio is always greater than 1; it is not greatly variable with C and n , and we may for all these cases take an average value near 1.12. Instead of taking the boundary condition $J_0 = 2H_0$ we will therefore assume $J_0 = 1.8H_0$.

The differential equations.

According to the different variations of k and s through the relevant atmospheric layers we may distinguish different types of atmospheric distribution, involving different types of differential equations for the radiation.

For s , by its dependence on the electron pressure P , we have chiefly three cases. With low temperature, when K is small, the concentration of ionized atoms $x = K/(P + K)$ may be written $x = K/P$; these atoms, which by increase of temperature make their first appearance and grow in concentration, are concentrated in the highest layers and decrease with increasing depth. The neutral atoms, determined by $1 - x = P/(P + K)$ then show a constant concentration. The same is the case for ionized atoms at higher temperatures, with $K \gg P$, when they have reached a constant maximum concentration, before they begin to decrease by second ionization. On the contrary the neutral atoms at that higher temperature are decreasing and disappearing; then $x_0 = 1 - x = P/K$ and their concentration increases with increasing depth. Hence we have these main cases:

- A. Atoms are disappearing; $s = s_0 P/K = \sigma P$
- B. Atoms are constant or at maximum; $s = \text{Const.}$
- C. Atoms are appearing; $s = s_0 K/P = \sigma'/P$.

These types change one into another rather rapidly. For the intermediate cases we keep the same type of variation with depth, but change the constants: $\sigma = s_0/(P + K)$ and $\sigma' = s_0 PK/(P + K)$.

These different distributions of s are combined with different types of distribution of k . According to the formulas representing the different atmospheres we may distinguish them:

1. Proportional absorption $k = k_0 p$, $u = 1$.
2. Less variable absorption $k = k_0 p^u$, $1 > u > 0$.
3. Constant absorption $k = k_0$, constant, $u = 0$.

For these cases we have to transform the fundamental equations

$$\frac{dJ}{d\xi} = 3(k + s)H; \quad \frac{dH}{d\xi} = k(J - E); \quad E = E_0 \left(1 + \frac{1}{2}c \int \bar{k} d\xi\right); \quad J_0 = 1.8H_0,$$

in such a way that they depend on one variable. To avoid complications, we will assume that in the relevant layers $\bar{k} \sim k(v)$; in some special cases this is not exactly true (see p. 61). Then $E = E_0(1 + C \int k d\xi)$.

Case A 1. We have $k = k_0 p$ and $P = ap$ (practically in this case always, with only hydrogen absorption, $a = 1/2$), $s = \sigma P = \sigma ap$. As in all cases A we put $k d\xi = dx$. The equations

$$\begin{aligned} \frac{dJ}{dx} &= 3\left(1 + \frac{s}{k}\right)H; & \frac{dH}{dx} &= J - E_0(1 + Cx), \text{ or} \\ \frac{dJ}{dx} &= 3(1 + a)H; & \frac{dH}{dx} &= J - E_0(1 + Cx); & a = \frac{s}{k} = \frac{\sigma a}{k_0} \end{aligned} \quad 34)$$

have, because α is constant here, an analytical solution

$$\begin{aligned} J &= E_0 \left\{ 1 + Cx - \frac{q^2 - 1.8C}{q^2 + 1.8q} e^{-qx} \right\} \\ H &= E_0 \left\{ \frac{C}{q^2} + \frac{q^2 - 1.8C}{q(q^2 + 1.8q)} e^{-qx} \right\} \\ H_0 &= E_0 \frac{q + C}{q(q + 1.8)} = \frac{E_0}{1.8 + \sqrt{3(1 + \alpha)}} \left(1 + \frac{C}{\sqrt{3(1 + \alpha)}} \right). \end{aligned} \quad 3(1 + \alpha) = q^2 \quad 35)$$

Case A 2. Here we have $k = k_0 p^u$ and $P = ap^t$ ($t > u$), hence $s = \sigma P = \sigma ap^t$.

$$dx = kd\xi = k_0 p^u \frac{dp}{g}; \quad x = \frac{k_0}{(1 + u)g} p^{1+u}; \quad \frac{s}{k} = \frac{\sigma a}{k_0} p^{t-u} = \frac{\sigma a}{k_0} \left(\frac{(1+u)g}{k_0} x \right)^{\frac{t-u}{1+u}}.$$

Putting

$$\frac{t-u}{1+u} = v, \quad \frac{\sigma a((1+u)g)^v}{k_0^{1+v}} = \alpha, \quad \text{we find } \frac{s}{k} = \alpha x^v$$

and the equations are

$$\frac{dJ}{dx} = 3(1 + \alpha x^v)H; \quad \frac{dH}{dx} = J - E_0(1 + Cx). \quad 36)$$

In the limiting cases we have: for A1: $u = 1, t = 1, v = 0$; for A3: $u = 0, t = 1/2, v = 1/2$. When the coefficients of hydrogen and of metal absorption are very different, it may happen in some transition cases that v rises above $1/2$.

Case A 3. This limiting case can be treated entirely as a special case of A 2. We have here $k = \text{Const.}$; $P = ap^{1/2}$; $s = \sigma ap^{1/2}$; $x = kp/g$;

$$\begin{aligned} \frac{s}{k} &= \frac{\sigma a g^{1/2}}{k^{3/2}} x^{1/2}; \quad v = 1/2; \quad \alpha = \sigma a \sqrt{g/k^3}; \\ \frac{dJ}{dx} &= 3(1 + \alpha x^{1/2})H; \quad \frac{dH}{dx} = J - E_0(1 + Cx). \end{aligned} \quad 37)$$

Case B 1. Because in the cases B s is constant, we put $sd\xi = dx$. Then the equations are

$$\frac{dJ}{dx} = 3\left(1 + \frac{k}{s}\right)H; \quad \frac{dH}{dx} = \frac{k}{s} \left\{ J - E_0 \left(1 + C \int \frac{k}{s} dx \right) \right\}$$

Here $k = k_0 p = k_0 \frac{g}{s} x$, or $\frac{k}{s} = \frac{k_0 g}{s^2} x = \beta x$

$$\begin{aligned} \int \frac{k}{s} dx &= \int \beta x dx = 1/2 \beta x^2 \quad \beta = \frac{k_0 g}{s^2} \\ \frac{dJ}{dx} &= 3(1 + \beta x)H; \quad \frac{dH}{dx} = \beta x (J - E_0(1 - 1/2 \beta C x^2)). \end{aligned} \quad 38)$$

Case B 2. In this general case $k = k_0 p^u$; $s = \text{Const.}$ We have

$$\begin{aligned} p &= \frac{g}{s} x; \quad k = k_0 \left(\frac{g}{s} \right)^u x^u; \quad \frac{k}{s} = \frac{k_0}{s} \left(\frac{g}{s} \right)^u x^u = \beta x^u; \quad \int \frac{k}{s} dx = \frac{\beta}{1+u} x^{1+u} \\ \frac{dJ}{dx} &= 3(1 + \beta x^u)H; \quad \frac{dH}{dx} = \beta x^u \left\{ J - E_0 \left(1 + \frac{\beta}{1+u} C x^{1+u} \right) \right\}; \quad \beta = \frac{k_0 g^u}{s^{u+1}}. \end{aligned} \quad 39)$$

The equations of case B 1 are the same as these, with $u = 1/2$.

Case B 3. With k and s both constant we have the simple case, which has always been treated analytically. Keeping $sd\xi = dx$ and $k/s = \beta$, the equations are

$$\frac{dJ}{dx} = 3(1 + \beta)H; \quad \frac{dH}{dx} = \beta \{J - E_0(1 + \beta Cx)\},$$

and the solution

$$H_0 = \frac{\sqrt{\beta} E_0}{1.8\sqrt{\beta} + \sqrt{3}(1 + \beta)} \left(1 + C \sqrt{\frac{\beta}{3(1 + \beta)}}\right)$$

is identical with the solution of case A1, since $\beta = 1/a$.

Case C2. In this general case, where k and s change in opposite directions, we put, as in case B, $sd\xi = dx$. We have

$$k = k_0 p^u; \quad P = ap^t; \quad s = \sigma'/P = \frac{\sigma'}{a} p^{-t}; \quad dx = \frac{\sigma'}{ag} p^{-t} dp; \quad x = \frac{\sigma'}{(1-t)ag} p^{1-t}.$$

$$\frac{k}{s} = \frac{k_0 a}{\sigma'} p^{u+t} = \frac{k_0 a}{\sigma'} \left(\frac{(1+t)ag}{\sigma'}\right)^{\frac{u+t}{1-t}} = \gamma x^w; \quad \int \frac{k}{s} dx = \frac{\gamma}{1+w} x^{1+w}.$$

Hence

$$\left. \begin{aligned} \frac{u+t}{1-t} = w; \quad k_0((1-t)g)^w \left(\frac{a}{\sigma'}\right)^{1+w} = \gamma. \\ \frac{dJ}{dx} = 3(1 + \gamma x^w)H; \quad \frac{dH}{dx} = \gamma x^w \left\{J - E_0 \left(1 + \frac{\gamma}{1+w} Cx^{1+w}\right)\right\}. \end{aligned} \right\} 40$$

The form of the equations is the same as in case B, but here the exponent w is > 1 , whereas u was < 1 . Between the limiting values $u = 0, t = 1/2$ and $u = 1, t = 1$ w runs from 1 to ∞ .

Case C3. In this limiting case the above formulas are directly applicable, with $u = 0, t = 1/2, w = 1$

$$P = ap^{1/2}; \quad s = \frac{\sigma'}{a} p^{-1/2}; \quad x = \frac{2\sigma'}{ag} p^{1/2}; \quad \gamma = 1/2 kg \left(\frac{a^2}{\sigma'}\right).$$

Case C1. This limiting case offers some difficulties; the formulas of the general case C2, for $u = 1, t = 1$, lead to the exponent $w = \infty$. A mathematical solution can be found by integrating from $x = 0$ to $x = 1$ with the first term only, neglecting x^w , and then from $x = 1$ to ∞ by omitting the first term and transforming to a new variable

$$y = \frac{\gamma x^{1+w}}{1+w}; \quad \frac{dJ}{dy} = 3H; \quad \frac{dH}{dy} = J - E(1+y).$$

Then we have for $x = 1$ $J = 4.8 H_0, H = H_0$, and the result is

$$H_0 = \frac{1}{4.8 + \sqrt{3}} E_0 \left(1 + \frac{C}{\sqrt{3}}\right) = 0.1531 E_0 + 0.08838 CE_0.$$

This is a common limiting result for all values of γ . It has only a mathematical importance, because it supposes the value of s to become infinite at the top of the atmosphere.

In the physical problem, however, there is no trace of infinity; for decreasing P the value of s is not $s_0 K/P$, but $s_0 K/(K + P)$, which for $P = 0$ tends to s_0 . And if we consider higher ionizations, we see that for the highest layers s again decreases from s_0 to zero. This latter change has no practical importance; the difficulty is entirely removed, if we use the correct formula for s . This, however, brings complications. It appears that now it is more expedient to introduce other variables.

We put $\xi\sqrt{k_0g} = p\sqrt{k_0/g} = x$; then x is again a numerical quantity without dimensions.

Then

$$k = k_0 p = x \sqrt{k_0 g}; \quad P = 1/2 p = 1/2 x \sqrt{g/k_0}; \quad s = \frac{s_0 K}{K + 1/2 x \sqrt{g/k_0}} = \sqrt{k_0 g} \frac{2s_0 K/g}{x + 2K \sqrt{k_0/g}}; \quad \int k d\xi = 1/2 x^2.$$

Putting $\frac{2s_0 K}{g} = \gamma'$; $2K \sqrt{k_0/g} = G$ we have the equations

$$\frac{dJ}{dx} = 3 \left(x + \frac{\gamma'}{x + G} \right) H; \quad \frac{dH}{dx} = x \{ J + E_0 (1 + 1/2 C x^2) \}. \tag{41}$$

The difficulty of beginning the integrations with an infinite coefficient, is now removed by G . It is necessary to solve the equations for different values of this constant.

The form of the equations chosen in the cases A, B, C, makes it desirable to give attention to the dimensions of the quantities involved. J, H, E are homogeneous, so their dimensions are cancelled. P, p and K are pressures (pres = dyne. cm^{-2}); k, s, s_0 are absorption coefficients (abs = $\text{g}^{-1} \text{cm}^2$); $g/k \sim$ (pres). Then $k_0 \sim$ (abs) (pres) $^{-u}$, $a \sim$ (pres) $^{1-t}$; $\sigma \sim$ (abs) (pres) $^{-1}$ in Case A, $\sigma' \sim$ (abs) (pres) in Case C; and it appears that α, β, γ as well as x are pure numbers. In Case C $1 \sqrt{k_0 g} \sim$ (abs), and γ' and G are pure numbers.

We might have avoided the arbitrary zero point values k_0 and a by introducing medium values for the domain considered:

$$k/k_m = (p/p_m)^u; \quad P/P_m = (p/p_m)^t; \quad s_m = \sigma P_m \text{ or } \sigma'/P_m.$$

Then the ratios of coefficients in the differential equations take the form

$$\left. \begin{aligned} \text{A: } & s/k = (s_m/k_m) ((1 + u)x/x_m)^v; \quad x_m = k_m p_m/g; \quad s_m/k_m = \alpha (x_m/(1 + u))^v; \\ \text{B: } & k/s = (k_m/s) (x/x_m)^u; \quad x_m = s p_m/g; \quad k_m/s = \beta x_m^u; \\ \text{C: } & k/s = (k_m/s_m) ((1 - t)x/x_m)^w; \quad x_m = s_m p_m/g; \quad k_m/s_m = \gamma (x_m/(1 - t))^w. \end{aligned} \right\} \tag{42}$$

Here the quantities introduced are more directly connected with the physical parameters of the atmosphere. Since, however, more quantities are needed here, the mathematical treatment is less easy than with the equations given above A1 — C1.

Integration of the equations.

The differential equations were solved by the method of numerical integration. A certain number of values for the exponents u, v, w were selected, and in each case a series of values for the coefficients α, β, γ were chosen so as to give moderate values for the resulting intensity. The solution of each pair of equations consists of two terms, one containing the factor E_0 , the other the factor $C E_0$ (resulting from the temperature increase in the interior). They are denoted by A I, A II, B I, B II, C I and C II for the three cases.

The method of calculation has been explained in *Monthly Notices* 90, 144. For

$$dJ/dx = 3(1 + \alpha x^v)H, \text{ and } dH/dx = J - E_0, \text{ or } = J - C E_0 x, \text{ we write}$$

$J_{n+1} - J_n = 3(1 + \alpha x^{v_{n+1/2}}) H_{n+1/2} \Delta x; H_{n+1/2} - H_{n-1/2} = (J_n - E_0) \Delta x, \text{ or } = (J_n - C E_0 x_n) \Delta x,$ where Δx is the unit interval in x . Thus the values of J are got for the arguments $n - 1, n, n + 1$, the values of H for $n - 1/2, n + 1/2, n + 1 1/2, \dots$. To start the integration series developments for small x were derived for every case. The integration begins at the surface $x = 0$ with chosen values H_0 and $J_0 = 1.8 H_0$ and proceeds into the deeper layers. The conditions for the right surface value H_0 is that in reaching ever deeper layers $E_0 - J$ and H (for formulas I) must approach to zero asymptotically; or that (for formulas II) $J - C E_0 x$ approaches to zero asymptotically and H takes a small limiting value. If, in the first case, H_0 is too large then $E_0 - J$ will become negative and H reverts before reaching 0; if H_0 is too small, H becomes negative and $E_0 - J$ reverts.

Because the equations in J and H are strictly linear, it is always possible to deduce the value, belonging to a certain H_0 , through linear interpolation between the results belonging to two other H_0 . Hence the computation always begins with two different H_0 , and in the course of the integration new values are interpolated till for the true surface H_0 a sufficient number of decimals has been found.

The integration of the equations C2 in the case of large w offered some difficulties, because for x near and above 1 the variations increased with great leaps. Hence the same transformation was used as with C1. The integration starts with the equations given; but when x approaches to 1 we put

$$\gamma x^w dx = dy; \quad \frac{\gamma}{1+w} x^{1+w} = y; \quad dx = \gamma^{-\frac{1}{1+w}} (1+w)y)^{-\frac{w}{1+w}} dy$$

and the equations are

$$\frac{dJ}{dx} = 3 \left(1 + (1+w)^{-w/(1+w)} \gamma^{-1/(1+w)} y^{-w/(1+w)} \right) H; \quad \frac{dH}{dy} = J - E_0(1 + Cy).$$

For some round value of y the values of J and H must be interpolated from the results of the x integration; then for increasing y the computation goes smoothly.

The results of the numerical integrations are compiled in Table 24. It must be remarked that the last figure given may be one unit or some few units wrong.

Table 24. Results of integrations.

	Coefficients of E_0 .				Coefficients of CE_0 .			
	$\alpha = 0.2$	1	5	25	$\alpha = 0.2$	1	5	25
$v = 0$	0.2705	0.2353	0.1655	0.0941	0.1426	0.0961	0.0390	0.01065
0.1	.2723	.2419	.1778	.1077	.1442	.1006	.0442	.0133
.3	.2754	.2517	.1981	.1304	.1463	.1078	.0537	.0190
.5	.2769	.2584	.2137	.1511	.1482	.1133	.0621	.0251
	$\beta = 0.2$	1	5	20	$\beta = 0.2$	1	5	20
$u = 0$	0.1655	0.2353	0.2705	0.2797	0.03901	0.09606	0.1426	0.1576
0.1	.1606	.2284	.2669	.2780	.03838	.0914	.1379	.1551
.25	.1550	.2192	.2599	.2747	.03806	.0862	.1314	.1514
.5	.1483	.2063	.2485	.2679	.03859	.0805	.1220	.1439
1.	.1408	.1883	.2284	.2553	.04112	.0745	.10775	.1312
	$\gamma = 0.1$	1	10		$\gamma = 0.1$	1	10	
$w = 1$	0.1206	0.1883	0.2417		0.02988	0.0745	0.1212	
2	.1191	.1699	.2143		.03875	.0711	.1058	
5	.1261	.1539	.1794		.05588	.0732	.0910	
10	.1351	.1494	.1642		.06797	.0773	.0869	
∞	.1531	.1531	.1531		.08838	.0884	.0884	
	$\gamma' = 1$	0.2	0.04		$\gamma' = 1$	0.2	0.04	
$G = .001$	0.0470	0.1389	0.2384		0.0164	0.0714	0.1343	
.01	.0674	.1700	.2501		.0238	.0877	.1412	
.1	.1141	.2501	.2660		.1122	.0429	.1504	

Added to this table are the values computed, in the cases A 1 ($v = 0$) and B 3 ($u = 0$), by the analytical solutions given above. Moreover we have for the limiting cases $\alpha = 0$ and $\beta = \infty$, which both correspond to $s = 0$, i.e. to the continuous background of the spectrum

$$H_0 = \frac{1}{1.8 + \sqrt{3}} E_0(1 + C/\sqrt{3}) = 0.2831 E_0 + 0.1635 CE_0.$$

Table 25. Coefficients of E_0 and CE_0 .

v	$\alpha = 0.2$		$\alpha = 1$		$\alpha = 5$		$\alpha = 25$	
0.00	0.2705	0.1426	0.2353	0.0961	0.1655	0.0390	0.0941	0.0106
.05	.2714	.1435	.2387	.0984	.1719	.0417	.1012	.0119
.10	.2723	.1442	.2419	.1006	.1778	.0442	.1077	.0133
.15	.2732	.1448	.2447	.1026	.1834	.0467	.1138	.0147
.20	.2740	.1453	.2473	.1045	.1886	.0491	.1195	.0161
.25	.2747	.1458	.2496	.1062	.1935	.0514	.1250	.0175
.30	.2754	.1463	.2517	.1078	.1981	.0537	.1304	.0190
.35	.2760	.1468	.2536	.1093	.2024	.0559	.1357	.0205
.40	.2764	.1473	.2554	.1107	.2064	.0580	.1409	.0220
.45	.2767	.1477	.2570	.1120	.2102	.0601	.1460	.0236
.50	.2769	.1482	.2584	.1133	.2137	.0621	.1511	.0251
.55	.2770	.1487	.2596	.1145	.2170	.0641	.1562	.0267
.60	.2769	.1491	.2608	.1157	.2202	.0660	.1612	.0282

$\frac{1}{1+u}$	$\beta = 20$		$\beta = 5$		$\beta = 1$		$\beta = 0.2$	
1.0	0.2797	0.1576	0.2704	0.1426	0.2353	0.0961	0.1655	0.0390
0.95	.2789	.1564	.2687	.1401	.2317	.0935	.1628	.0387
.90	.2778	.1550	.2664	.1374	.2278	.0910	.1601	.0384
.85	.2764	.1533	.2634	.1345	.2236	.0886	.1575	.0381
.80	.2746	.1513	.2599	.1314	.2192	.0862	.1550	.0380
.75	.2724	.1489	.2560	.1281	.2145	.0839	.1524	.0381
.70	.2698	.1461	.2516	.1245	.2096	.0818	.1499	.0383
.65	.2669	.1429	.2467	.1207	.2045	.0798	.1475	.0387
.60	.2636	.1394	.2412	.1166	.1993	.0779	.1452	.0392
.55	.2598	.1355	.2351	.1123	.1939	.0761	.1430	.0400
.50	.2553	.1312	.2284	.1078	.1883	.0745	.1408	.0411

$\frac{1}{1+w}$	$\gamma = 10$		$\gamma = 1$		$\gamma = 0.1$		$\frac{1}{1+w}$	$\gamma = 10$		$\gamma = 1$		$\gamma = 0.1$	
0.50	0.2417	0.1212	0.1883	0.0745	0.1206	0.0299	0.20	0.1865	0.0937	0.1567	0.0721	0.1233	0.0514
.45	.2348	.1167	.1827	.0732	.1199	.0320	.18	.1822	.0921	.1550	.0727	.1248	.0540
.40	.2268	.1121	.1772	.0721	.1194	.0345	.16	.1779	.0905	.1534	.0735	.1266	.0567
.35	.2177	.1074	.1717	.0713	.1191	.0376	.14	.1737	.0891	.1520	.0745	.1287	.0596
.30	.2078	.1027	.1664	.0709	.1193	.0414	.12	.1697	.0880	.1508	.0756	.1311	.0628
.25	.1973	.0981	.1614	.0711	.1206	.0459	.10	.1658	.0871	.1500	.0769	.1338	.0663
.20	.1865	.0937	.1567	.0721	.1233	.0514	.08	.1623	.0866	.1496	.0784	.1369	.0700
.15	.1758	.0898	.1527	.0740	.1276	.0581	.06	.1592	.0863	.1497	.0803	.1403	.0741
.10	.1658	.0871	.1500	.0769	.1338	.0663	.04	.1567	.0865	.1502	.0826	.1441	.0786
.05	.1579	.0864	.1499	.0814	.1422	.0763	.02	.1546	.0871	.1513	.0853	.1484	.0833
.00	.1531	.0884	.1531	.0884	.1531	.0884	.00	.1531	.0884	.1531	.0884	.1531	.0884

Derivation of line-widths.

The results of the integration in Table 24 were used to construct tables, where for the values of α , β , γ given, the coefficients AI , AII . . . may be found for every u , v , w . The interpolation was made by means of graphical representations; it was found expedient to take $1/(1 + u)$, $v =$

Table 26. Coefficient $\log C = \log \frac{3}{2} c\bar{k}/k$.

λ 7000 — 3000 \AA								λ 7000 — 3000 \AA							
0.2	8.28	8.50	8.78	9.12	8.75	8.99	1.3	0.36	0.26	0.05	9.64	9.31	8.80		
.3	8.72	8.96	9.24	9.60	9.01	9.26	.4	.33	.21	9.95	9.47	9.09	8.53		
.4	9.24	9.48	9.78	0.14	9.32	9.58	.5	.28	.13	9.82	9.28	8.86	8.24		
.5	9.59	9.84	0.14	0.52	9.47	9.73	.6	.23	.03	9.69	9.09	8.62	7.95		
1.1	—	—	—	—	—	—	.8	.10	9.85	9.41	8.69	8.13	7.34		
.2	0.37	0.31	0.15	9.79	9.50	9.06	2.0	9.94	9.63	9.12	8.28	7.62	6.71		

λ 7000 \AA							λ 6000 \AA							λ 5000 \AA							
6	5	4	3	2	1	0	6	5	4	3	2	1	0	6	5	4	3	2	1	0	
0.55	9.68	9.68	9.67	9.67	9.67	9.67	9.93	9.92	9.92	9.92	9.92	9.92	9.92	all 0.24							
.6	9.76	9.75	9.74	9.73	9.73	9.73	0.01	0.00	0.00	0.00	0.00	0.00	0.00	all .30							
.65	9.87	9.82	9.79	9.77	9.77	9.77	.10	.07	.06	.05	.05	.05	.05	all .34							
.7	0.03	9.92	9.85	9.81	9.80	9.79	.27	.23	.16	.11	.08	.07	.06	all .37							
.75	.19	0.09	9.98	9.90	9.86	9.83	.32	.28	.22	.15	.11	.08	.07	0.38	0.38	0.39	0.39	0.39	0.39	0.39	0.39
.8	.28	.26	0.19	0.08	9.97	9.89	.35	.34	.31	.25	.18	.13	.11	.38	.38	.39	.40	.41	.41	.41	.41
.85	.32	.32	.31	.27	0.18	0.05	.38	.38	.37	.34	.29	.23	.18	.36	.36	.36	.37	.38	.39	.39	.40
.9	.34	.34	.34	.33	.29	.22	.37	.37	.37	.37	.35	.32	.26	.35	.35	.35	.35	.36	.38	.38	.40
.95	.36	.36	.36	.35	.34	.32	.37	.37	.37	.37	.37	.36	.34	.33	.33	.33	.33	.33	.34	.34	.36
1.0	.37	.37	.37	.37	.36	.34	all 0.37							all 0.30							

λ 4000 \AA							λ 3500 \AA							λ 3000 \AA									
6	5	4	3	2	1	0	6	5	4	3	2	1	0	6	5	4	3	2	1	0			
0.55	0.59	0.61	0.62	0.62	0.62	0.62	all 9.47							9.73	9.72	9.72	9.72	9.72	9.72	9.72	9.72	9.72	9.72
.6	.62	.66	.68	.69	.69	.69	9.44	9.43	9.42	9.42	9.42	9.42	9.42	9.71	9.70	9.69	9.69	9.69	9.69	9.69	9.69		
.65	.56	.62	.67	.70	.72	.73	9.46	9.39	9.36	9.35	9.35	9.35	9.35	9.69	9.64	9.62	9.61	9.61	9.61	9.61	9.61		
.7	.45	.52	.60	.66	.71	.74	9.54	9.36	9.30	9.27	9.26	9.26	9.26	9.77	9.62	9.56	9.54	9.53	9.53	9.53	9.53		
.75	.36	.40	.48	.56	.64	.71	9.83	9.52	9.31	9.24	9.19	9.18	9.17	9.89	9.77	9.57	9.49	9.46	9.45	9.45	9.45		
.8	.30	.31	.35	.42	.52	.63	0.03	9.85	9.53	9.26	9.15	9.11	9.10	9.92	9.84	9.69	9.52	9.41	9.37	9.36	9.36		
.85	.24	.24	.26	.29	.37	.47	.04	9.98	9.87	9.51	9.22	9.07	9.03	9.86	9.82	9.77	9.67	9.47	9.35	9.30	9.30		
.9	.19	.19	.19	.20	.23	.27	.02	9.99	9.95	9.83	9.56	9.21	9.03	9.76	9.75	9.73	9.69	9.59	9.42	9.28	9.28		
.95	.13	.13	.13	.13	.13	.15	9.95	9.94	9.92	9.89	9.83	9.57	9.21	9.67	9.67	9.65	9.63	9.61	9.53	9.35	9.35		
1.0	all 0.08						9.87	9.87	9.86	9.85	9.82	9.74	9.51	9.55	9.55	9.55	9.54	9.53	9.51	9.43	9.43	9.43	

Atm. 8 λ 7000—3000 \AA							Atm. 7 λ 7000—3000 \AA						
0.5	9.60	9.85	0.14	0.52	9.48	9.73	9.59	9.84	0.14	0.52	9.47	9.73	
.55	9.74	9.99	.24	.56	9.50	9.75	9.71	9.95	.24	.58	9.49	9.74	
.6	9.86	0.09	.30	.51	9.54	9.82	9.80	0.04	.30	.57	9.46	9.73	
.65	9.99	0.19	.34	.43	9.73	9.97	9.92	.14	.34	.50	9.52	9.80	
.7	0.16	0.28	.37	.37	0.05	0.05	0.12	.27	.37	.40	9.82	9.94	

$(t-u)/(1+u)$ and $1/(1+w) = (1-t)/(1+u)$ as arguments of the table. An extract is given in Table 25, p. 59.

For each wave-length, temperature and atmosphere the values of these three arguments were computed from u and t . Then the values of AI, AII, BI, \dots could be taken from Table 25.

To compute the coefficient $C = \sqrt[3]{2} \bar{c}k/k$ first a table was constructed giving $\sqrt[3]{2} c$ for each wave-length and temperature, and then the ratio \bar{k}/k was taken from the Tables 20 and 21. In most cases this ratio was constant over the range of pressures occurring in the relevant part of the atmosphere used. When, however, the transition from hydrogen to metal absorption just took place in these layers, then the ratio \bar{k}/k varied with the pressure. Then for each atmosphere, taking its mean pressure, a different value of \bar{k}/k was derived. Strictly speaking \bar{k}/k in these cases is variable over the different layers of the atmosphere and should be expressed as some power of p ; this variation, however, was neglected, because it would introduce still another parameter into the formulas, and its effect is probably only small. The adopted values of $C = \sqrt[3]{2} \bar{c}k/k$ are given by Table 26.

The residual intensity

$$r(A) = \frac{AI + AII C}{0.2831 + 0.1635 C} \text{ (or the same with } BI, BII, CI, CII)$$

was now computed, for each temperature, wave-length and atmosphere, for each of the values of α , of β , of γ of our tables. We represent $r(A)$ as a function of $\log \alpha$; this forms a kind of line profile, with the difference that not $\Delta\lambda$ but $\log \alpha$ is the absciss. From this profile we can read the value of $\log \alpha$ belonging to $r = 0.50$, i.e. corresponding to the half width of the line. Other residual intensities, from 0.80 to 0.30 generally, could also be read; but to cover the entire profile and to derive in this way equivalent widths, the numerical integrations should have been extended to a wider range of α values. It appeared advisable not to derive $\log \alpha$ for $r = 1/2$ through graphs, but to compare with some analogous normal curve (computed with constant s and k) and to read to each r from a table representing this curve the corresponding argument; these arguments run nearly linearly with $\log \alpha$ and can be interpolated with second differences.¹⁾ The same was done in the cases B and C to find $\log \beta$ and $\log \gamma$, corresponding to $r = 1/2$.

In Case C1 the variable parameter G depends on the ionization modulus K , which for every element is different. So no general result can be given for each atmosphere. Since the coefficient C is the only other variable on which the result depends, results were derived for regular increasing values of $\log G$ and a number of values of C , from 0 to 3.5. For each $\log G$ the coefficients of E_0 and CE_0 were interpolated from Table 24 (simply assuming them to be quadratic functions of $\log G$), and then for different values of C the residual intensity r was computed for each γ' . On the way described above $\log \gamma' (1/2)$ corresponding to $r = 0.50$ was derived. For the computation of $G = 2K\sqrt{k_0/g}$ Table 27 gives $2\sqrt{k_0/g}$ for the cases where it is needed.

These results have to be used where in Table 22 at the top of each part $u = 1.0$ $t = 1.0$ is found. It must be remarked, that also for u and t approaching to 1 the same difficulty appears in the practical result that then for the three values of γ in our tables three nearly equal values for r are found, not including 0.5. Hence the introduction of $P + K$ is also necessary

¹⁾ The table gives $\log(1/r^2 - 1) - \log 3$, so that for $r = 0.50$ the argument is 0. If now for three values $\log \alpha = -1, 0$ and $+1$ the computed r have corresponding arguments in the table $-(p-a), a, p+q$, so that the first differences are p and $p+q$ and the second difference is q , then the value of $\log \alpha$ to which corresponds argument 0 and $r = 0.50$, is approximately given by

$$-a/p + \frac{1}{2} \frac{a(p-a)}{p^2} \frac{q}{p+q}$$

and exactly by $\sqrt{(p_m/q)^2 - 2a/q - p_m/q}$, where $p_m = p + 1/2 q$.

Table 27. Values of $\log 2 \sqrt{k_0/g} = \log G/K$.

log g	5040/T	7000	6000	5000	4000	3500	3000	log g	5040/T	7000	6000	5000	4000	3500	3000
8.4	0.2	-4.75	-4.84	-4.97	-5.11	-4.92	-5.00	2.4	0.2	-1.75	-1.84	-1.97	-2.11	-1.92	-2.00
	.3	4.44	4.48	4.61	4.64	4.52	4.54		.3	1.50	1.60	1.72	1.86	1.55	1.65
7.4	0.2	-4.25	-4.34	-4.47	-4.61	-4.42	-4.50	.4	.4	1.30	1.40	1.53	1.68	1.24	1.34
	.3	3.96	4.06	4.15	4.34	4.02	4.12		.5	1.14	1.24	1.36	1.49	0.96	1.06
6.4	0.2	-3.75	-3.84	-3.97	-4.11	-3.92	-4.00	1.4	.55	1.05	1.14	1.24	1.33	0.82	0.92
	.3	3.50	3.60	3.72	3.86	3.55	3.65		0.2	-1.25	-1.34	-1.47	-1.61	-1.42	-1.50
5.4	0.2	-3.25	-3.34	-3.47	-3.61	-3.42	-3.50	.3	.3	1.00	1.10	1.22	1.36	1.05	1.15
	.3	3.00	3.10	3.22	3.36	3.05	3.15		.4	0.80	0.90	1.03	1.18	0.74	0.84
4.4	0.2	-2.75	-2.84	-2.97	-3.11	-2.92	-3.00	.5	.5	0.65	0.74	0.86	1.01	0.46	0.56
	.3	2.50	2.60	2.72	2.86	2.55	2.65		.55	0.57	0.67	0.79	0.94	0.32	0.42
3.4	0.2	-2.25	-2.34	-2.47	-2.61	-2.42	-2.50	.6	.6	0.54	0.64	0.74	0.86	0.24	0.34
	.3	2.00	2.10	2.22	2.36	2.05	2.15		0.41	0.2	-0.75	-0.84	-0.97	-1.11	-0.92
	.4	1.80	1.90	2.03	2.18	1.74	1.84	.3	.3	0.50	0.60	0.72	0.86	0.55	0.65
	.5	1.60	1.72	1.79	1.90	1.43	1.50		.4	0.30	0.40	0.53	0.68	-0.24	0.34
								.5	.5	0.15	0.24	0.36	0.51	+0.04	+0.06
									.55	0.07	0.17	0.29	0.44	0.18	+0.08
								.6	0.04	0.14	0.25	0.40	+0.28	+0.16	

in these cases. From the relations between k , p and P it follows that then $u = 1 - 2z$, $t = 1 - z$, for z small. By putting then, somewhat different from case C1

$$\left(\frac{k_0 g}{1-z}\right)^{1/2} g^{-z} \xi^{1-z} = x,$$

$$\gamma'' = \frac{s_0 K}{a g (1-z)} \left(\frac{g(1-z)}{k_0}\right)^{z/2z-2}; \quad G' = \frac{K}{a(1-z)} \left(\frac{k_0}{g}\right)^{1/2} \tag{43}$$

we find

$$k d\xi = x dx; \quad s d\xi = \gamma'' \frac{x^{z/1-z}}{x + G'} \tag{44}$$

The coefficients are slightly altered (α is nearly $1/2$), and the differential equations show only this difference with 41) that in the numerator of the s term a factor $x^{z/1-z}$ appears. Its influence at the start of the integrations, for x increasing from 0.001 to 0.01, is for $z = 0.02$ only a factor 0.87 to 0.91.

By means of the formulas p. 55, 56 α , β , γ are connected with the physical data. It is better to keep g , the gravity, separated from the other quantities, because for other r somewhat different values of $\log g$ correspond to atmospheres 0 . . . 6. So we put

$$a k_0^{-(1+v)} (1+u)^v = V; \quad k_0^{1/(1+w)} = U; \quad a k_0^{1/(1+w)} (1-t)^{w/(1+w)} = W \tag{45}$$

or in terms of medium quantities

$$V = P_m p_m^{-v} k_m^{-(1+v)} (1+u)^v;$$

$$U = p_m^{-u/(1+u)} k_m^{1/(1+u)};$$

$$W = P_m p_m^{-w/(1+w)} k_m^{1/(1+w)} (1-t)^{w/(1+w)},$$

and we find

$$\left. \begin{aligned} \text{Case A: } \log s_0/K &= \log \sigma = \log a - \log V - v \log g \\ \text{Case B: } \log s &= -\frac{1}{1+u} \log \beta + \log U + \frac{u}{1+u} \log g \\ \text{Case C: } \log s_0 K &= \log \sigma' = -\frac{1}{1+w} \log \gamma + \log W + \frac{w}{1+w} \log g. \\ \text{Case C1: } \log s_0 K &= \log \gamma' + \log \frac{1}{2} g \end{aligned} \right\} 46)$$

The temperature function K of the ionization formula must be computed for each element separately.

This is the result of the computations. For each of the differently constituted atmospheres of different effective temperature we find (for different wave-lengths) what value of the diffusion coefficient s_0 or s belongs to the residual intensity $1/2$ and to the half-width of the line defined by that intensity.

Numerical results.

The numerical results of the computations are contained in the Tables 28, 29, 30 for s , σ and σ' . They are arranged in such a way that for each case of T and g the results for the different wave lengths stand beside one another, so that interpolation for other wave lengths is possible. For high temperatures there is a jump at λ 3646 \AA ; hence between 4000 \AA and 3646 \AA the values must be found by extrapolation from the larger wave lengths, whereas below 3646 \AA the values for 3500 \AA and 3000 \AA must be used. For lower temperatures, where the hydrogen absorption is insensible, there is a continuous variation with wave length.

The values of s_0 for Case C1 are given in Table 31 as a function of $\log G$ for a series of regular values of $C = \frac{3}{2} c \bar{k} / k$. Then for any wave length and temperature, for which $\log C$ has been interpolated from Table 24, the value of s_0 can be taken from this table.

It must be remarked that the table values, though they are given in two decimals, have not an accuracy of .01. All the computations of exponents and coefficients, u , t , v , $\log U$, $\log V$, $\log W$, $\log a$, $\log \beta$, $\log \gamma$, have been carried out with two decimals; since, however, they are sometimes multiplied by large values of $\log g$ and $\log k_0$, the results may be several units of the second decimal in error. This is shown at once by the differences between adjacent columns and lines. A certainty of .01 in the logarithm, i.e. of 2 percent in the values, is far beyond the accuracy reached in present day measures of spectral line intensities; in many cases we may even be content with one decimal or a factor $5/4$. So these tables may be made use of and interpolated in two decimals, leaving the last figure some few units uncertain.

Table 28. Log s.

	Log g = 6.4						Log g = 5.4						Log g = 4.4					
	7000	6000	5000	4000	3500	3000	7000	6000	5000	4000	3500	3000	7000	6000	5000	4000	3500	3000
0.2	2.69	2.60	2.46	2.30	2.52	2.41	2.19	2.09	1.96	1.80	2.01	1.91	1.69	1.60	1.46	1.30	1.52	1.41
.3	2.93	2.82	2.62	2.48	2.87	2.75	2.43	2.32	2.18	1.99	2.37	2.25	1.93	1.82	1.68	1.48	1.87	1.75
.4	3.03	2.89	2.70	2.44	3.10	2.92	2.55	2.44	2.23	2.02	2.64	2.51	2.10	1.94	1.76	1.52	2.16	2.02
.5	2.65	2.44	2.10	1.73	2.98	2.79	2.44	2.23	1.92	1.55	2.70	2.51	2.14	1.89	1.70	1.38	2.33	2.20
.55	2.21	1.93	1.57	1.20	2.71	2.45	2.08	1.84	1.47	1.12	2.53	2.31	1.90	1.67	1.34	1.00	2.31	2.08
.6	1.63	1.34	0.98	0.68	2.26	2.02	1.59	1.30	0.95	0.56	2.23	1.97	1.47	1.24	0.88	0.50	2.10	1.86
.65	1.02	0.81	0.56	0.37	1.77	1.55	0.98	0.71	0.39	0.15	1.78	1.53	0.94	0.66	0.34	0.98	1.75	1.49
.7	0.56	0.38	0.36	0.33	1.31	1.11	0.46	0.21	0.03	9.97	1.30	1.06	0.36	0.07	9.81	9.65	1.28	1.05
.75	0.26	0.23	0.21	0.31	0.86	0.84	0.08	9.92	9.88	9.96	0.81	0.67	9.84	9.67	9.57	9.48	0.82	0.59
.8	0.14	0.14	0.17	0.31	0.61	0.74	9.84	9.84	9.78	9.95	0.40	0.41	9.52	9.44	9.42	9.52	0.35	0.27
.85	0.05	0.10	0.09	0.28	0.51	0.73	9.74	9.73	9.79	9.97	0.22	0.46	9.38	9.39	9.46	9.61	9.97	0.08
.9	0.02	0.01	0.08	0.34	0.54	0.81	9.67	9.73	9.76	9.97	0.17	0.47	9.36	9.33	9.38	9.60	9.86	0.09
.95	9.93	0.02	0.05	0.37	0.60	0.85	9.62	9.69	9.74	0.02	0.25	0.46	9.24	9.27	9.35	9.63	9.85	0.11
1.0	9.88	9.97	0.07	0.36	0.60	0.90	9.56	9.60	9.71	9.99	0.21	0.58	9.21	9.27	9.36	9.64	9.87	0.22
1.1	9.84	9.91	0.09	0.46	0.63	1.01	9.52	9.57	9.69	0.06	0.32	0.69	9.14	9.19	9.37	9.67	9.88	0.28
1.2	9.81	9.91	0.10	0.52	0.82	1.22	9.47	9.61	9.76	0.11	0.42	0.76	9.16	9.16	9.39	9.75	0.02	0.38
1.3	9.63	9.78	0.09	0.59	0.92	1.35	9.39	9.54	9.80	0.23	0.57	0.90	9.07	9.24	9.42	9.83	0.18	0.50
1.4	9.47	9.69	0.04	0.62	1.00	1.44	9.26	9.51	9.84	0.29	0.64	1.12	9.03	9.17	9.48	9.97	0.29	0.69
1.5	9.29	9.56	0.00	0.63	1.02	1.59	9.14	9.39	9.80	0.39	0.75	1.23	8.98	9.16	9.52	0.06	0.44	0.86
1.6	9.09	9.40	9.96	0.63	1.08	1.65	9.01	9.30	9.76	0.42	0.83	1.32	8.81	9.12	9.52	0.14	0.52	1.00
1.8	8.65	9.08	9.70	0.50	1.14	1.82	8.63	9.06	9.61	0.44	0.91	1.58	8.55	8.96	9.48	0.17	0.66	1.26
2.0	8.17	8.65	9.36	0.32	1.04	1.88	8.17	8.66	9.34	0.43	0.92	1.72	8.15	8.62	9.29	0.14	0.71	1.44

	Log g = 3.4						Log g = 2.4						Log g = 1.4					
	7000	6000	5000	4000	3500	3000	7000	6000	5000	4000	3500	3000	7000	6000	5000	4000	3500	3000
0.2	1.19	1.10	0.96	0.80	1.02	0.91	0.69	0.59	0.46	0.30	0.52	0.41	0.19	0.09	9.96	9.80	0.02	9.91
.3	1.43	1.32	1.18	0.99	1.37	1.25	0.93	0.82	0.68	0.49	0.87	0.75	0.43	0.32	0.18	9.99	0.37	0.25
.4	1.59	1.47	1.29	1.06	1.66	1.52	1.09	0.97	0.79	0.56	1.16	1.02	0.59	0.47	0.29	0.06	0.66	0.52
.5	1.66	1.53	1.30	1.00	1.89	1.74	1.19	1.04	0.83	0.58	1.40	1.25	0.70	0.56	0.38	0.12	0.90	0.77
.55	1.61	1.40	1.14	0.83	1.95	1.78	1.22	1.02	0.81	0.52	1.51	1.35	0.74	0.59	0.38	0.13	1.05	0.88
.6	1.36	1.08	0.79	0.38	1.86	1.66	1.09	0.87	0.61	0.24	1.55	1.37	0.74	0.55	0.32	0.03	1.13	0.98
.65	0.90	0.63	0.28	9.89	1.65	1.42	0.78	0.51	0.18	9.80	1.45	1.23	0.56	0.31	0.02	9.69	1.14	0.97
.7	0.33	0.03	9.72	9.41	1.27	1.03	0.29	9.98	9.65	9.30	1.19	0.94	0.20	9.93	9.57	9.21	1.01	0.80
.75	9.79	9.53	9.28	9.15	0.81	0.57	9.77	9.43	9.12	8.94	0.79	0.54	9.69	9.40	9.06	8.73	0.73	0.50
.8	9.28	9.15	9.06	9.10	0.34	0.12	9.17	8.95	8.76	8.72	0.31	0.10	9.11	8.84	8.56	8.39	0.30	0.07
.85	9.05	9.00	9.04	9.13	9.86	9.79	8.70	8.63	8.60	8.69	9.86	9.65	8.54	8.38	8.24	8.25	9.82	9.61
.9	8.96	8.99	9.01	9.16	9.56	9.70	8.53	8.57	8.55	8.74	9.42	9.39	8.24	8.15	8.15	8.32	9.34	9.21
.95	8.90	8.90	8.96	9.27	9.48	9.75	8.55	8.52	8.60	8.85	9.14	9.34	8.12	8.11	8.16	8.35	8.94	8.96
1.0	8.85	8.90	9.00	9.27	9.49	9.78	8.47	8.54	8.64	8.86	9.11	9.36	8.09	8.08	8.21	8.43	8.74	8.96
1.1	8.72	8.81	8.96	9.30	9.52	9.92	8.32	8.42	8.56	8.93	9.20	9.52	8.00	8.07	8.22	8.57	8.79	9.10
1.2	8.72	8.77	8.98	9.40	9.74	0.03	8.34	8.44	8.60	8.99	9.27	9.64	7.91	8.02	8.22	8.61	8.90	9.26
1.3	8.71	8.81	9.07	9.46	9.79	0.11	8.27	8.42	8.67	9.05	9.39	9.75	7.89	7.98	8.28	8.66	8.98	9.37
1.4	8.70	8.87	9.11	9.61	9.88	0.24	8.34	8.46	8.76	9.18	9.45	9.87	7.93	8.03	8.30	8.76	9.02	9.46
1.5	8.63	8.88	9.19	9.67	0.04	0.46	8.30	8.50	8.84	9.29	9.62	0.00	7.95	8.12	8.39	8.86	9.19	9.58
1.6	8.64	8.88	9.20	9.79	0.15	0.62	8.29	8.49	8.87	9.40	9.75	0.15	7.91	8.14	8.49	9.00	9.30	9.74
1.8	8.41	8.73	9.21	9.90	0.36	0.88	8.18	8.51	8.96	9.55	0.00	0.47	7.91	8.21	8.62	9.19	9.59	0.07
2.0	8.08	8.52	9.15	9.93	0.45	1.14	7.96	8.38	8.94	9.67	0.14	0.80	7.76	8.15	8.65	9.34	9.81	0.42

Table 28. Log s. (Continued).

Log g = 0.4							Log g = 8.4						
	7000	6000	5000	4000	3500	3000							
0.2	9.69	9.59	9.46	9.30	9.51	9.41	0.2	3.70	3.60	3.46	3.30	3.50	3.41
.3	9.93	9.82	9.68	9.49	9.87	9.75	0.3	3.91	3.76	3.61	3.44	3.82	3.69
.4	0.09	9.97	9.79	9.56	0.16	0.02	.4	3.82	3.61	3.35	3.02	3.87	3.70
.5	0.20	0.06	9.88	9.63	0.41	0.27	5.5	2.90	2.61	2.26	1.87	3.27	3.02
.55	0.27	0.11	9.92	9.68	0.54	0.40	.55	2.28	2.01	1.67	1.41	2.77	2.53
.6	0.28	0.13	9.92	9.67	0.67	0.51	.6	1.78	1.54	1.26	1.10	2.30	2.06
.65	0.24	0.04	9.79	9.49	0.77	0.60	.65	1.35	1.12	1.10	1.05	1.82	1.66
.7	0.04	9.78	9.46	9.12	0.73	0.55	.7	1.07	1.03	0.99	1.01	1.39	1.47
.75	9.63	9.35	9.01	8.62	0.58	0.37							
.8	9.08	8.78	8.47	8.22	0.26	0.02							
.85	8.49	8.22	8.02	7.90	9.80	9.58							
.9	7.98	7.83	7.75	7.85	9.30	9.12	0.2	3.20	3.09	2.96	2.80	3.00	2.91
.95	7.71	7.68	7.71	7.92	8.85	8.76	.3	3.39	3.28	3.18	2.94	3.33	3.21
1.0	7.62	7.65	7.74	7.95	8.50	8.58	4.	3.44	3.28	3.10	2.78	3.55	3.36
1.1	7.58	7.71	7.84	8.12	8.34	8.64	5.5	2.81	2.56	2.22	1.79	3.17	2.93
1.2	7.56	7.66	7.84	8.23	8.50	8.82	.55	2.26	1.97	1.64	1.24	2.77	2.51
1.3	7.50	7.61	7.86	8.29	8.61	9.00	.6	1.67	1.37	1.15	0.91	2.26	2.04
1.4	7.49	7.64	7.86	8.33	8.64	9.07	.65	1.17	0.95	0.75	0.71	1.83	1.57
1.5	7.48	7.68	7.96	8.45	8.76	9.20	7.	0.83	0.73	0.64	0.66	1.33	1.23
1.6	7.55	7.74	8.07	8.54	8.86	9.29							
1.8	7.54	7.86	8.23	8.76	9.16	9.63							
2.0	7.51	7.86	8.34	8.99	9.42	0.00							

Table 29. Log σ.

	Log g = 6.4						Log g = 5.4						Log g = 4.4					
	7000	6000	5000	4000	3500	3000	7000	6000	5000	4000	4500	4000	7000	6000	5000	4000	3500	3000
0.2	9.52	9.30	9.02	8.70	9.17	8.95	9.51	9.31	9.05	8.75	9.17	8.95	9.52	9.30	9.04	8.73	9.17	8.95
.3	0.00	9.81	9.51	9.12	9.88	9.65	9.99	9.83	9.50	9.12	9.87	9.63	9.99	9.75	9.50	9.13	9.87	9.63
.4	0.27	0.01	9.64	9.10	0.40	0.11	0.29	9.99	9.69	9.21	0.41	0.12	0.32	0.04	9.74	9.28	0.44	0.18
.5	0.02	9.58	9.05	8.47	0.49	0.09	0.24	9.88	9.40	8.82	0.70	0.40	0.53	0.06	9.66	9.12	0.84	0.55
.55	9.50	9.07	8.54	7.96	0.29	9.90	0.03	9.51	8.98	8.37	0.66	0.28	0.25	9.87	9.34	8.77	0.95	0.59
.6	8.98	8.50	7.99	7.55	9.96	9.59	9.44	9.00	8.47	7.89	0.44	0.01	9.88	9.45	8.93	8.37	0.81	0.47
.65	8.38	8.02	7.64	7.37	9.57	9.20	8.88	8.45	8.00	7.65	0.11	9.70	9.38	8.91	8.41	7.93	0.58	0.21
.7	7.89	7.61	7.44	7.50	9.10	8.80	8.34	7.94	7.70	7.61	9.70	9.34	8.81	8.35	7.96	7.76	0.23	9.86
.75	7.61	7.43	7.41	7.61	8.57	8.53	7.88	7.69	7.65	7.70	9.20	8.94	8.25	7.92	7.74	7.69	9.84	9.49
.8	7.45	7.42	7.52	7.71	8.21	8.44	7.65	7.56	7.61	7.86	8.70	8.75	7.89	7.77	7.73	7.94	9.28	9.15
.85	7.41	7.43	7.47	7.83	8.25	8.58	7.61	7.62	7.69	8.00	8.51	8.77	7.78	7.70	7.80	8.03	8.83	8.95
.9	7.47	7.44	7.58	8.03	8.27	8.85	7.63	7.60	7.73	8.14	8.47	8.97	7.75	7.78	7.90	8.21	8.69	9.07
.95	7.44	7.51	7.66	8.07	8.45	8.94	7.60	7.63	7.76	8.20	8.62	9.08	7.71	7.76	7.91	8.38	8.78	9.22
1.0	7.44	7.54	7.74	8.23	8.58	9.12	7.54	7.62	7.85	8.34	8.73	9.32	7.70	7.75	7.96	8.45	8.84	9.41
1.1	7.43	7.65	7.84	8.51	8.88	9.52	7.54	7.78	8.03	8.62	9.06	9.66	7.75	7.87	8.12	8.66	9.12	9.76
1.2	7.51	7.69	8.05	8.81	9.28	9.94	7.75	7.90	8.19	8.89	9.43	0.06	7.85	7.96	8.38	9.01	9.49	0.16
1.3	7.48	7.70	8.25	9.07	9.64	0.31	7.77	7.97	8.39	9.20	9.73	0.43	7.88	8.16	8.53	9.31	9.89	0.53
1.4	7.44	7.80	8.34	9.31	9.89	0.71	7.79	8.09	8.61	9.43	0.09	0.88	7.97	8.24	8.75	9.63	0.20	0.90
1.5	7.35	7.80	8.49	9.55	0.17	1.07	7.72	8.13	8.74	9.73	0.38	1.17	8.02	8.38	8.97	9.90	0.54	1.31
1.6	7.30	7.83	8.58	9.73	0.42	1.37	7.71	8.18	8.91	9.98	0.62	1.53	8.02	8.44	9.14	0.21	0.88	1.73
1.8	7.06	7.75	8.64	9.96	0.83	2.00	7.56	8.23	9.07	0.32	1.18	2.24	8.00	8.63	9.42	0.56	1.38	2.40
2.0	6.86	7.58	8.60	0.10	1.11	2.49	7.33	8.10	9.13	0.56	1.52	2.83	7.80	8.56	9.56	0.87	1.83	3.04

Table 29. Log σ . (Continued).

	Log $g = 3.4$						Log $g = 2.4$						Log $g = 1.4$					
	7000	6000	5000	4000	3500	3000	7000	6000	5000	4000	3500	3000	7000	6000	5000	4000	3500	3000
0.2	9.52	9.31	9.05	8.73	9.17	8.95	9.52	9.30	9.05	8.73	9.17	8.95	9.52	9.31	9.05	8.74	9.17	8.95
.3	9.99	9.76	9.50	9.13	9.87	9.63	9.99	9.77	9.50	9.11	9.87	9.63	0.00	9.78	9.50	9.13	9.87	9.63
.4	0.32	0.04	9.72	9.28	0.44	0.18	0.32	0.07	9.73	9.28	0.44	0.18	0.32	0.07	9.73	9.28	0.44	0.18
.5	0.48	0.23	9.82	9.27	0.92	0.63	0.51	0.27	9.86	9.40	0.93	0.65	0.57	0.27	0.90	9.43	0.97	0.69
.55	0.49	0.12	9.67	9.13	1.09	0.79	0.61	0.27	9.84	9.39	1.18	0.90	0.64	0.35	9.97	9.48	1.22	0.90
.6	0.26	9.83	9.34	8.70	1.12	0.78	0.52	0.16	9.67	9.11	1.33	1.00	0.68	0.32	9.90	9.39	1.44	1.15
.65	9.85	9.43	8.88	8.28	1.03	0.62	0.25	9.81	9.29	8.70	1.34	0.99	0.54	0.15	9.67	9.12	1.54	1.26
.7	9.30	8.86	8.36	7.95	0.73	0.39	9.78	9.32	8.79	8.27	1.17	0.80	0.23	9.77	9.24	8.67	1.54	1.18
.75	8.78	8.36	8.00	7.86	0.38	0.02	9.25	8.80	8.36	8.06	0.89	0.52	9.75	9.32	8.78	8.29	1.36	0.97
.8	8.20	7.97	7.84	7.93	9.94	9.61	8.67	8.35	8.08	8.01	0.51	0.15	9.20	8.77	8.33	8.12	1.02	0.68
.85	7.90	7.89	7.86	8.11	9.41	9.29	8.17	8.04	7.94	8.13	0.04	9.75	8.59	8.29	8.12	8.13	0.62	0.28
.9	7.89	7.87	7.97	8.30	8.99	9.24	7.96	7.93	8.01	8.32	9.53	9.48	8.23	8.11	8.11	8.36	0.11	9.89
.95	7.86	7.91	8.01	8.50	8.89	9.37	7.99	8.00	8.13	8.54	9.19	9.48	8.11	8.05	8.20	8.53	9.60	9.68
1.0	7.86	7.90	8.14	8.61	9.02	9.56	7.96	8.05	8.25	8.70	9.13	9.60	8.06	8.13	8.28	8.74	9.39	9.72
1.1	7.81	7.95	8.21	8.82	9.29	9.92	7.91	8.07	8.37	8.98	9.44	0.03	8.12	8.26	8.51	9.10	9.56	0.12
1.2	7.95	8.11	8.44	9.17	9.64	0.33	8.03	8.23	8.54	9.28	9.76	0.42	8.16	8.35	8.71	9.37	9.87	0.52
1.3	8.05	8.28	8.70	9.48	9.98	0.65	8.13	8.37	8.77	9.53	0.10	0.75	8.25	8.45	8.93	9.65	0.20	0.89
1.4	8.13	8.46	8.88	9.72	0.30	1.02	8.30	8.51	9.05	9.84	0.35	1.09	8.36	8.61	9.10	9.89	0.45	1.19
1.5	8.23	8.59	9.15	0.05	0.64	1.42	8.32	8.73	9.29	0.16	0.72	1.45	8.52	8.82	9.33	0.25	0.79	1.57
1.6	8.35	8.75	9.34	0.35	0.97	1.82	8.45	8.87	9.52	0.49	1.07	1.86	8.61	9.03	9.62	0.53	1.14	1.95
1.8	8.32	8.92	9.71	0.85	1.58	2.53	8.61	9.13	9.92	0.98	1.73	2.62	8.84	9.35	0.09	1.11	1.81	2.72
2.0	8.26	8.96	9.94	1.15	2.08	3.23	8.66	9.31	0.18	1.34	2.26	3.39	8.93	9.60	0.39	1.57	2.43	3.51

Log $g = 0.4$							Log $g = 8.4$						
0.2	9.52	9.31	9.06	8.74	9.17	8.95	0.2	9.52	9.32	9.06	8.74	9.14	8.95
.3	9.99	9.78	9.50	9.13	9.87	9.63	.3	9.96	9.76	9.47	9.02	9.86	9.63
.4	0.32	0.07	9.73	9.28	0.44	0.18	.4	9.96	9.65	9.23	8.60	0.16	9.74
.5	0.57	0.29	9.90	9.44	0.97	0.69	.5	9.10	8.69	8.14	7.57	9.69	9.27
.55	0.67	0.38	0.01	9.57	1.24	0.95	.55	8.52	8.12	7.66	7.16	9.33	8.90
.6	0.74	0.43	0.04	9.58	1.47	1.17	.6	7.97	7.60	7.28	7.11	8.92	8.49
.65	0.75	0.39	9.93	9.42	1.70	1.39	.65	7.54	7.31	7.09	7.07	8.40	8.09
.7	0.59	0.16	9.65	9.10	1.76	1.44	.7	7.27	7.17	7.15	7.17	7.86	7.90
.75	0.22	9.77	9.25	8.67	1.72	1.38							
.8	9.70	9.24	8.75	8.37	1.49	1.13							
.85	9.11	8.70	8.36	8.20	1.14	0.80							
.9	8.54	8.32	8.19	8.31	0.68	0.42	0.2	9.52	9.32	9.06	8.74	9.14	8.95
.95	8.23	8.17	8.20	8.56	0.21	0.05	.3	9.98	9.76	9.48	9.10	9.86	9.63
1.0	8.17	8.19	8.35	8.79	9.76	9.92	.4	0.18	9.89	9.46	8.98	0.30	0.04
1.1	8.20	8.32	8.57	9.17	9.62	0.15	.5	9.59	9.19	8.62	8.00	0.16	9.76
1.2	8.25	8.44	8.77	9.48	9.96	0.60	.55	9.05	8.60	8.08	7.60	9.82	9.35
1.3	8.38	8.57	8.96	9.79	0.29	0.97	.6	8.45	8.09	7.60	7.30	9.43	9.04
1.4	8.39	8.68	9.19	0.00	0.56	1.30	.65	7.97	7.61	7.40	7.21	9.01	8.64
1.5	8.58	8.86	9.42	0.32	0.88	1.66	.7	7.53	7.32	7.30	7.27	8.42	8.28
1.6	8.72	9.11	9.71	0.59	1.21	2.01							
1.8	8.99	9.47	0.20	1.21	1.89	2.75							
2.0	9.17	9.76	0.55	1.72	2.53	3.60							

Table 30 Log σ' .

Log $g = 6.4$							Log $g = 5.4$						Log $g = 4.4$					
	7000	6000	5000	4000	3500	3000	7000	6000	5000	4000	3500	3000	7000	6000	5000	4000	3500	3000
.5	5.14	5.08	4.97	4.83	5.23	5.16	4.23	4.24	4.20	4.11	4.20	4.22	3.09	3.16	3.12	3.14	3.02	3.13
.55	4.71	4.62	4.45	4.26	4.92	4.85	4.06	3.95	3.81	3.67	4.17	4.11	3.23	3.18	3.10	2.99	3.25	3.24
.6	4.10	4.00	3.85	3.69	4.43	4.30	3.59	3.46	3.28	3.11	3.85	3.72	2.93	2.85	2.70	2.54	3.14	3.08
.65	3.47	3.41	3.29	3.20	3.84	3.74	2.93	2.79	2.66	2.53	3.32	3.21	2.38	2.23	2.07	1.95	2.75	2.65
.7	3.01	2.95	2.89	2.89	3.27	3.16	2.39	2.26	2.15	2.16	2.72	2.62	1.74	1.62	1.50	1.45	2.24	2.09
.75	2.62	2.64	2.60	2.69	2.78	2.76	1.86	1.79	1.76	1.77	2.18	2.08	1.18	1.15	1.07	1.02	1.62	1.54
.8	2.50	2.48	2.47	2.50	2.55	2.59	1.58	1.55	1.55	1.55	1.69	1.68	0.71	0.66	0.68	0.73	1.08	1.05
.85	2.40	2.41	2.36	2.48	2.46	2.47	1.46	1.45	1.46	1.48	1.54	1.52	0.49	0.56	0.54	0.58	0.58	0.59
.9	2.28	2.30	2.30	2.36	2.42	2.46	1.42	1.40	1.41	1.43	1.45	1.54	0.46	0.45	0.47	0.50	0.46	0.45
.95	2.13	2.13	2.14	2.22	2.27	2.40	1.37	1.30	1.36	1.38	1.41	1.49	0.43	0.45	0.43	0.44	0.50	0.46
1.0	2.08	2.02	2.06	2.12	2.18	2.32	1.24	1.23	1.21	1.28	1.34	1.42	0.33	0.34	0.36	0.41	0.46	0.48
1.1	1.86	1.87	1.94	1.96	1.94	2.03	0.94	0.92	0.97	1.02	1.14	1.23	0.11	0.07	0.15	0.25	0.24	0.37
1.2	1.72	1.72	1.80	1.79	1.88	1.88	0.78	0.79	0.86	0.87	0.90	0.97	9.83	9.85	9.88	0.03	0.07	0.13
1.3	1.54	1.60	1.64	1.76	1.77	1.84	0.70	0.75	0.80	0.83	0.88	0.89	9.82	9.80	9.77	9.85	9.89	9.97
1.4	1.29	1.37	1.50	1.65	1.78	1.87	0.55	0.59	0.72	0.77	0.84	0.91	9.72	9.74	9.81	9.85	9.88	9.80
1.5	1.05	1.14	1.31	1.59	1.61	1.74	0.37	0.44	0.56	0.73	0.77	0.85	9.62	9.68	9.75	9.82	9.82	9.85
1.6	0.72	0.90	1.09	1.35	1.49	1.63	0.15	0.23	0.37	0.58	0.72	0.79	9.43	9.47	9.64	9.71	9.77	9.81
1.8	0.03	0.29	0.54	0.96	1.18	1.43	9.51	9.75	9.98	0.29	0.45	0.56	8.95	9.10	9.31	9.50	9.62	9.69
2.0	9.36	9.55	9.92	0.32	0.73	1.09	8.82	9.10	9.42	9.87	0.11	0.34	8.31	8.56	8.87	9.18	9.36	9.56

Log $g = 3.4$							Log $g = 2.4$						Log $g = 1.4$					
.55	2.18	2.24	2.19	2.15	2.13	2.20	0.91	0.95	1.10	1.11								
.6	2.19	2.09	2.03	1.88	2.25	2.24	1.20	1.20	1.17	1.13	1.14	1.19	0.04	0.10	0.10	0.15		
.65	1.80	1.69	1.51	1.34	2.08	1.99	1.11	1.01	0.91	0.73	1.25	1.18	0.25	0.21	0.13	0.03	0.24	0.24
.7	1.21	1.05	0.90	0.80	1.64	1.53	0.66	0.50	0.35	0.20	1.00	0.92	9.99	9.90	9.76	9.60	0.22	0.16
.75	0.61	0.49	0.37	0.32	1.10	0.97	0.05	9.90	9.75	9.67	0.54	0.44	9.48	9.35	9.20	9.03	9.92	9.83
.8	0.05	9.94	9.96	9.95	0.52	0.44	9.47	9.37	9.28	9.23	9.97	9.87	8.86	8.76	8.62	8.53	9.45	9.35
.85	9.62	9.58	9.58	9.58	9.93	9.95	8.91	8.86	8.85	8.85	9.41	9.32	8.27	8.20	8.15	8.13	8.87	8.76
.9	9.42	9.41	9.42	9.44	9.52	9.47	8.52	8.50	8.55	8.57	8.86	8.83	7.79	7.78	7.79	7.83	8.20	8.24
9.5	9.48	9.49	9.43	9.44	9.44	9.38	8.34	8.34	8.32	8.34	8.38	8.42	7.36	7.36	7.38	7.38	7.76	7.73
1.0	9.41	9.41	9.41	9.46	9.44	9.46	8.44	8.43	8.43	8.39	8.38	8.32	7.32	7.35	7.30	7.22	7.20	7.31
1.1	9.24	9.27	9.29	9.37	9.37	9.43	8.32	8.34	8.38	8.43	8.46	8.52	7.43	7.44	7.47	7.44	7.48	7.42
1.2	9.00	9.02	9.06	9.19	9.27	9.34	8.18	8.19	8.24	8.34	8.42	8.41	7.31	7.34	7.38	7.41	7.45	7.47
1.3	8.83	8.82	8.89	8.93	8.98	9.11	7.93	7.94	8.02	8.10	8.22	8.31	7.06	7.10	7.20	7.30	7.39	7.45
1.4	8.84	8.85	8.86	8.85	8.80	8.83	7.80	7.83	7.77	7.85	7.89	8.04	6.81	6.83	6.87	7.07	7.13	7.23
1.5	8.74	8.77	8.82	8.84	8.82	8.79	7.79	7.80	7.78	7.79	7.82	7.84	6.79	6.74	6.78	6.85	6.82	7.03
1.6	8.63	8.69	8.72	8.73	8.77	8.76	7.73	7.77	7.79	7.77	7.75	7.72	6.70	6.73	6.80	6.78	6.80	6.79
1.8	8.25	8.38	8.47	8.64	8.71	8.78	7.48	7.59	7.66	7.75	7.77	7.78	6.60	6.67	6.75	6.78	6.77	6.76
2.0	7.75	7.92	8.17	8.40	8.56	8.71	7.07	7.25	7.42	7.61	7.70	7.78	6.34	6.44	6.61	6.76	6.78	6.77

Log $g = 0.4$							Log $g = 8.4$						
.6		8.74	8.62	8.89			0.4	7.23	7.21	7.22	7.11	7.23	7.27
.65	9.12	9.16	9.21	9.17	8.98	9.04	.5	6.47	6.37	6.17	6.03	6.67	6.53
.7	9.19	9.15	9.03	8.95	9.28	9.27	.55	5.82	5.76	5.60	5.43	6.10	5.94
.75	8.88	8.75	8.61	8.46	9.16	9.13	.6	5.29	5.19	5.05	5.01	5.50	5.51
.8	8.33	8.19	8.03	7.91	8.86	8.75	.65	4.79	4.75	4.73	4.75	4.95	4.89
.85	7.71	7.58	7.47	7.42	8.33	8.21	.7	4.53	4.47	4.53	4.54	4.57	4.61
.9	7.14	7.08	7.05	7.05	7.76	7.68							
.95	6.69	6.65	6.64	6.71	7.20	7.15							
1.0	6.15	6.13	6.25	6.35	6.63	6.67							
1.1	6.46	6.43	6.45	6.33	6.33	6.09							
1.2	6.36	6.41	6.39	6.41	6.41	6.40							
1.3	6.26	6.29	6.35	6.43	6.49	6.50							
1.4	5.97	6.00	6.10	6.21	6.27	6.39	0.4	6.14	6.13	6.25	6.15	6.15	6.13
1.5	5.79	5.82	5.85	5.92	6.05	6.20	5	5.90	5.80	5.62	5.44	6.00	5.94
1.6	5.73	5.75	5.72	5.84	5.88	6.01	55	5.34	5.19	5.00	4.81	5.54	5.42
1.8	5.73	5.76	5.74	5.78	5.77	5.67	6	4.68	4.55	4.44	4.33	4.97	4.85
2.0	5.56	5.61	5.73	5.78	5.75	5.68	65	4.13	4.05	3.95	3.96	4.43	4.28
							7	3.68	3.70	3.64	3.71	3.88	3.85

Table 31. Values of $\log \gamma^{(1/2)}$ (Case C 1).

Log g	$C = 0$	0.2	0.5	1.0	1.5	2.0	2.5	3.0	3.5
— 1.0	— 0.78	— 0.76	— 0.72	— 0.68	— 0.66	— 0.63	— 0.61	— 0.60	— 0.59
1.2	.66	.63	.60	.56	.54	.52	.50	.49	.48
1.4	.55	.53	.49	.46	.43	.41	.39	.38	.37
1.6	.44	.42	.39	.35	.33	.31	.30	.29	.28
1.8	.35	.33	.30	.27	.25	.23	.22	.21	.21
— 2.0	.26	.24	.22	.20	.18	.16	.15	.14	.14
2.2	.19	.17	.15	.13	.11	.10	.09	.08	.08
2.4	.13	.11	.09	.07	— .05	— .04	— .03	— .02	— .02
2.6	.07	.05	— .03	— .01	.00	+ .01	+ .02	+ .02	+ .03
2.8	— .02	— .01	+ .01	+ .03	+ .04	.05	.06	.06	.07
— 3.0	+ .03	+ .04	+ .05	+ .07	+ .08	+ .08	+ .09	+ .09	+ .10

Derivation of line intensities.

General remarks.

By means of these tables we are now able to compute the line intensity, here expressed by the half width $\Delta\lambda$ for which the residual intensity is $r = 1/2$, as a function of the stellar parameters for the line considered. This half width is connected with s (which will be written here for $s^{(1/2)}$) by the formulas 27) 28). Assuming $\Delta\lambda$ to be situated on the resonance wings (in other cases the complete formula has to be used) we have.

$$(\Delta\lambda)^2 = [6.204] \left(\frac{\lambda}{4000}\right)^2 \frac{\delta n'}{\gamma s}; \quad n' = A\gamma f.$$

Hence we may consider n'/s as a measure of line intensity. We will compute the variations of this quantity with temperature and gravity. The factors A (abundance of the element) and f (oscillator strength) are constant for the line and affect only the absolute value of n'/s and $\Delta\lambda$. The Boltzmann factor γ varies with temperature. The variation of line intensity then depends only on γ/s . Larger values of s , i.e. larger values of the diffusion coefficient needed for $r = 1/2$, mean that a larger concentration of atoms is necessary to give the same line intensity.

For a constant or a maximum number of atoms in the state of ionization considered, we make use of s of Table 28. For a decreasing line, disappearing by increase of temperature, Table 29 gives $\sigma = s/K$, hence the intensity depends on $\gamma/\sigma K$. For an increasing line, appearing and growing by increase of temperature, Table 30 gives $\sigma' = sK$, hence the intensity depends on $\gamma K/\sigma'$. The ionization modulus K must be computed for each element as a function of temperature.

The electron pressure P (being no stellar parameter) does not occur in these expressions, as also the relevant layers of the atmosphere so far play no rôle here. They are, however, needed to separate these three chief cases, and play a part in the intermediate transition cases. The values of P that we want for this purpose, can be taken from Table 22. Then the fractions in the state of ionization considered, are

$$\begin{array}{lll} \text{increasing atoms} & \text{constant or maximum} & \text{decreasing atoms} \\ \frac{K}{K+P} \quad (P > K) & \frac{K}{K+P} \quad (P < K) & \frac{P}{K+P} \quad (P > K) \\ & & \frac{P}{K+P} \quad (P < K) \end{array}$$

As an instance of transition we take Ca between 5000° and 3000° ($\log g = 4.4$)

$\frac{5040}{T}$	=	1.0	1.1	1.2	1.3	1.4	1.5	1.6
$\log K$	=	2.58	1.86	1.16	0.46	9.77	9.09	8.42
$\log P_m$	=	1.12	0.96	0.76	0.59	0.43	0.26	0.08
$\log \frac{P}{P+K}$	=	-1.47	-0.95	-0.55	-0.24	-0.09	-0.03	-0.01
$\log \frac{K}{P+K}$	=	-0.01	-0.05	-0.15	-0.37	-0.75	-1.20	-1.67

The separation between the increasing and the constant case is made between 1.2 and 1.3, but only above 5040° $P/(P+K)$ may be taken P/K , and only below $1.5 (= 3360^\circ)$ it is constant. For the intermediate temperatures for y/s we take $y/s(K/(K+P))$ at the increasing, $y/s(P/(K+P))$ at the decreasing side; for $y/\sigma K$ we take $y/\sigma(K+P)$, and for yK/σ' we take $y/\sigma'(KP/(K+P))$.

Of course the results will only be approximations then. In our integration we assumed, for the three pure cases, s to be $\sim P$, to be constant, and to be $\sim 1/P$. In the intermediate cases s , in the relevant layers, which extend nearly over a range of $\log P_m \pm 0.5$, varies nearly as some intermediate power of P , which changes from 1 to 0 and from 0 to -1 . Such cases could be treated in the same way; as here, however, a second independent exponent occurs, special integrations would be necessary for every separate case. We will content ourselves, therefore, with the assumption that the true result lies between the results of the two pure cases, and is gradually passing over from one case to the other.

In the first investigations on variations and maxima of absorption lines only the variation in the number of active atoms, expressed by ionization fraction and Boltzmann function, was considered. Now it appears that the irregular variations of $s^{(1/2)}$ with temperature have an important bearing upon the spectral phenomena. These variations are chiefly the variations of the absorption coefficient k with temperature. In the simple theory of constant k and s the residual intensity $r = \sqrt{k/(k+s)}$ for $C=2$, hence $r = 1/2$ takes place for $s = 3k$; so we may say that $s^{(1/2)}$ replaces $3k$ of the simple theory. Varying with the different conditions in the atmosphere the absorption coefficient is contained in $s^{(1/2)}$ with powers between 1 and $1/2$ (and complementarily the gravity is included with powers between 0 and $1/2$); but the chief variation of s with temperature is due to the absorption coefficient. This variation has for every λ and g the same qualitative character. For low temperatures $s^{(1/2)}$ and k decrease slowly with increasing temperature, till at about 6000° the metals are ionized. Then by the ionization of hydrogen k and $s^{(1/2)}$ begin to rise rapidly, till at 10.000° a hundred times larger value is reached. Then, after the complete ionization of hydrogen, they slowly decrease for still higher temperatures.

The rapid rise of k and $s^{(1/2)}$ between 6000° and 10.000° explains the strong decrease in intensity of all the metal lines if we go from spectral class G to class A . A more precise computation by means of our tables is needed, however, to investigate the details in the intensity variations. Some few cases will be treated here.

Variation of line intensity for some elements.

The computation of intensity variations has been carried out for the following lines:

Element	<i>H</i>	<i>Ca</i>	<i>Ca +</i>	<i>Mg</i>	<i>Mg +</i>	<i>Fe</i>	<i>Fe +</i>	<i>He</i>
λ	4341	4455	3934	5184	4481	4383	4233	4471
Ion. Pot.	13.54	6.09	11.82	7.61	14.97	7.83	16.16	24.47
Exc. Pot.	10.16	1.89	0	2.70	8.83	2.39	2.57	20.87

The constant factors A and f , as well as the weights in the Boltzmann function are neglected, because only the variation with temperature is required.

The values of s , σ and σ' were interpolated from the tables for the wave length of each line, for each of the gravity values 6.4, 4.4, 2.4 and 0.4 (which roughly may be said to correspond to white dwarfs, dwarfs, giants, supergiants). The results for $\log n/s$ are represented in the diagrams, Figs. 6—9, where for the temperature also a logarithmic scale has been chosen (one vertical division denotes a factor 10).

The curves are composed of two or three partial curves, the σ , the s , and the σ' part; in the transition parts they do not coincide though they are nearly parallel. Between the s and the σ part the differences are small, but between the s and the σ' part they are sometimes large, indicating that here the atmospheric structure has a large influence upon the result. The transition parts of the curves, where they are passing over from one partial curve to another, are given by dashes, and the further course of the separate curves is indicated by short extensions.

In Fig. 6 the curves for Ca , $Ca +$ and H are given. It must be remarked that the zero point of the vertical logarithmic scale for different lines is arbitrarily different; they are drawn at such heights as give the least confusion.

The four curves for the blue Ca triplet show a maximum at temperatures 3150°, 3600°, 4100, 4700° (for $\log g = 0.4, 2.4, 4.4, 6.4$); the maximum intensity is largest for the lowest g , but during the descent, at temperatures above 5000°, the intensity for large g is largest.

For the lines H and K of $Ca +$ also we see that during increase and maximum the giant stars have stronger lines than the dwarfs, and that the maxima fall at lower temperatures. The four curves show all the same character: after a rapid increase comes a broad maximum (beginning at about 3200°, 3500°, 4200, 4800° for $\log g = 0.4, 2.4, 4.4, 6.4$); during this maximum the intensity still slowly increases and attains a highest top just before the steep decrease begins at 5700° (for 0.4), at 5900° (2.4), 6600° (4.4), whereas for $\log g = 6.4$ the intensity is constant from 5000° to 7000°. Then a rapid and steep fall in intensity takes place, beginning at 6300°, 7000° 7700°, 8400°.

In order to show the origin of these different features in the curves, for one of them (viz. for $\log g = 4.4$) the curve of the number of atoms is given, denoted by crosses, as well as the curve of s , denoted by dots (the curve in Figure 6 holds for λ 4341; but the difference is very small). The number of atoms of $Ca +$ in this case is constant from 4200° to 7300°; the slow increase of line intensity is due to the slow decrease of the s curve, and the highest top even corresponds to a minor depression in the s curve. It is the rapid rise of s from 7000° to 10000° to which the steep descent of the line intensity curve is due.

For hydrogen in the same way the curve of the number of atoms is given (with its constituent parts: above 9000°, where the number of neutral atoms begins to decrease the Boltzmann curve is continued by dashes). It reaches a maximum for 10500° (for $\log g = 0.4, 2.4, 6.4$ the maximum is at 8200°, 9400°, 12000°). The intensity of the Balmer lines, however, does not decrease for lower temperatures, but still increases very slowly, because the s curve decreases at the same rate or even somewhat more than the number of atoms. The intensity of the lines remains nearly the same over a large range of temperature, till at last after reaching a maximum it rapidly falls. For $\log g = 0.4$ this maximum is found at 6800°, for 2.4 at 7200°, for 4.4 at 7900°, for 6.4 at 8700°; and between these temperatures and 10,000° the change in intensity is very small. Figure 6 shows that the maximum for 4.4 occurs close above the temperature where the s curve begins its steep ascent; and so it is for the other gravity cases.

In Fig. 7 the computed intensities for Fe (λ 4383 A) and $Fe +$ (λ 4233 A) are given for

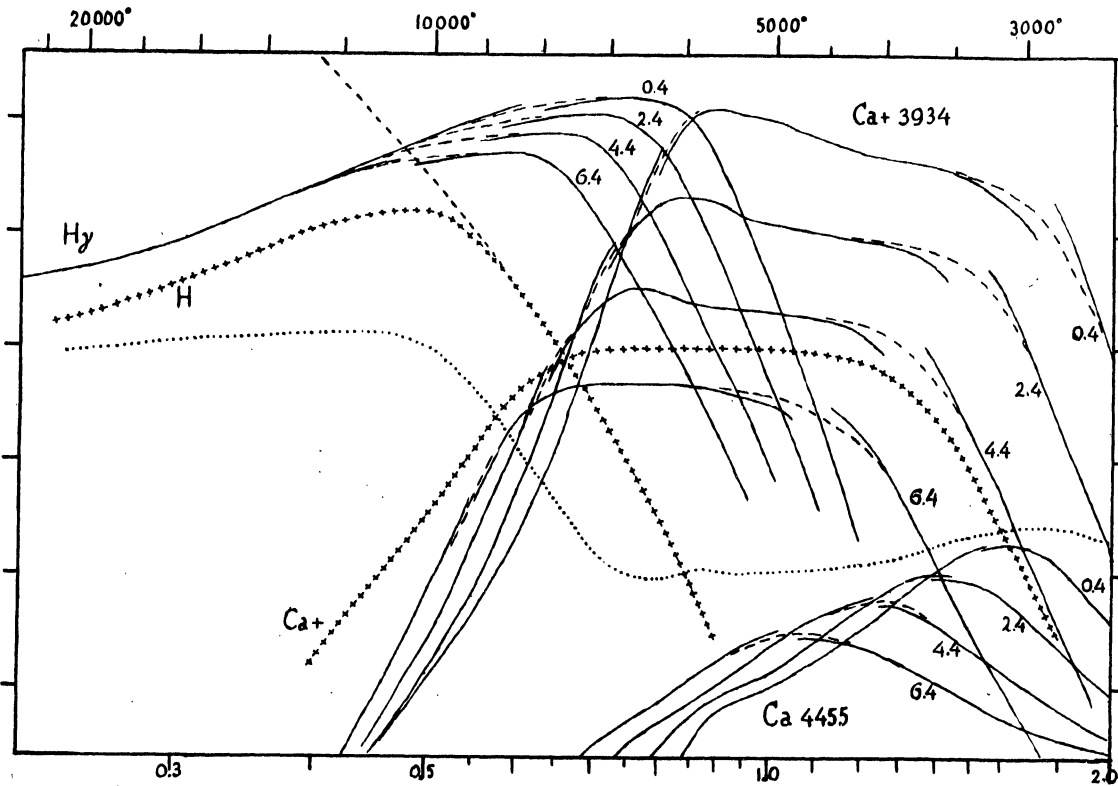


Fig. 6. Variations of Ca , $Ca+$ and H with temperature and gravity.

the same four gravity values. The maxima of the neutral Fe line : 3900° , 4400° , 5000° , 5800° , depend chiefly on the excitation potential, and is different for different atomic levels. The decrease after the maximum becomes steeper when the s curve begins to rise rapidly.

For the $Fe+$ lines the curves show many irregularities. The number of atoms, indicated by crosses (only for $\log g = 4.4$), with the Boltzmann curve continued above and below the range where all the iron atoms are once ionized, reaches a maximum at 8400° , 9200° , 10400° , 12000° . The line intensity, however, has its maximum at 5900° , 6400° , 6900° , 7800° for the same four cases of gravity 0.4, 2.4, 4.4, 6.4. At these latter temperatures a sharp decrease sets in, due to the beginning rapid rise of the s curve. This confirms the remarkable result, derived by RUSSELL in his analytical treatment of the line intensities, that the maximum of $Fe+$ is not due to its own behaviour, but to the behaviour of hydrogen, on which the absorption coefficient depends. In this range of temperatures, between 5600° and 10000° (for $\log g = 4.4$), the number of once ionized iron atoms is constant, and the number in the relevant higher level is only slowly and regularly changing. The dependence of the maximal temperature on gravity is due entirely to the change of the s curve with gravity. The irregularities, which the intensity curves show in the decrease for higher temperatures are due to the same cause. Up to the temperature where the steep increase of s ceases, there is a steep descent; then with nearly constant s the descent becomes slower, but it steepens again when by second ionization the number of atoms begins to decrease more rapidly. At the lower temperature side of the maximum there is some uncertainty caused by the passing over of the curve from Case B to Case C.

Upon these phenomena RUSSELL has based an ingenious method to derive the relative con-

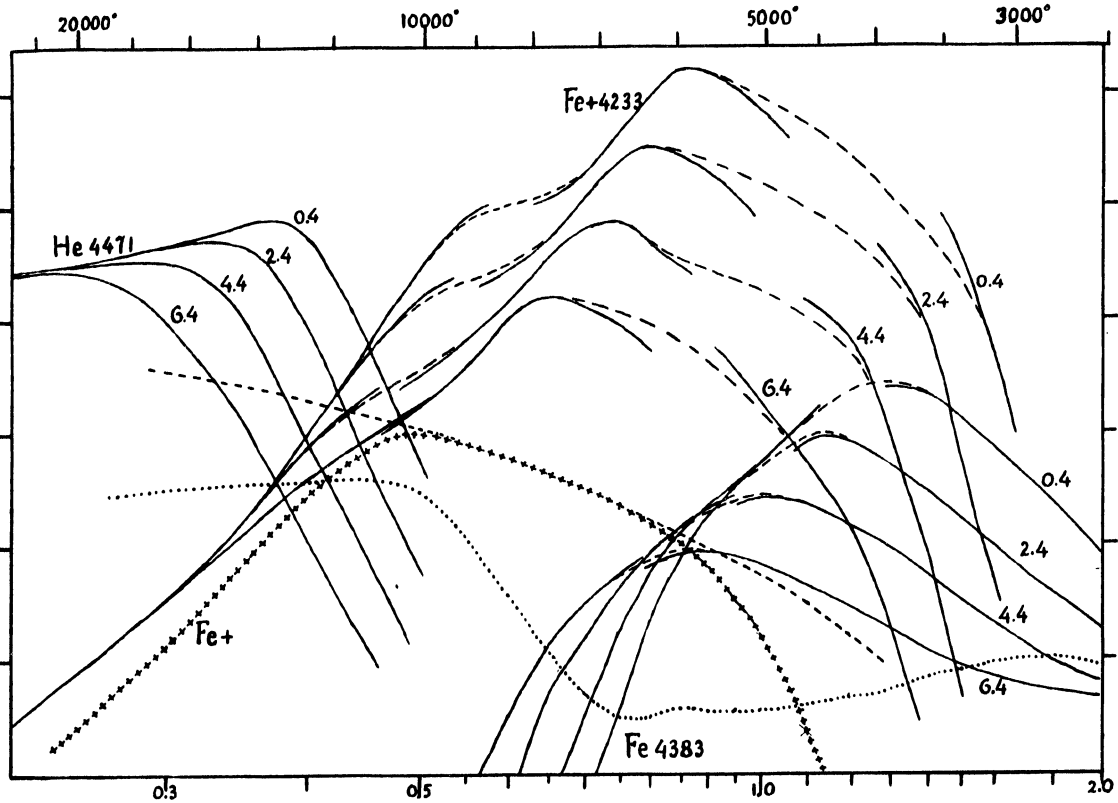


Fig. 7. Variations of Fe , $Fe +$ and He with temperature and gravity.

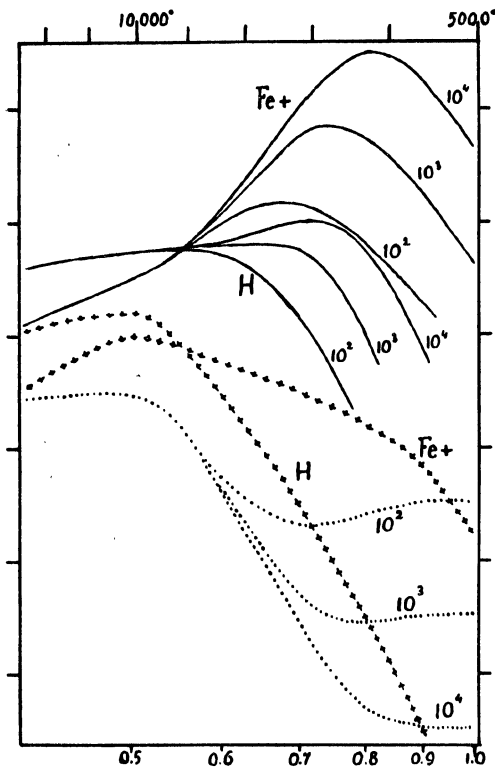


Fig. 8. Influence of metal abundance.

centration of hydrogen and metal atoms in the atmosphere from the maximum of $Fe +$ lines. The variations brought about in the above results by an error in the adopted ratio of $10^3 : 1$, may be derived by varying these assumptions, e.g. by taking this ratio $10^2 : 1$ and $10^4 : 1$. Then the temperature region of the rapid increase of the absorption coefficient changes. For the first case, 10 times more metal atoms, k is 10 times higher for low temperatures, the rapid rise begins at a higher temperature and has a smaller amount.

For 10 times fewer metal atoms the rise begins lower and lasts longer. The maximum value of k for completely ionized hydrogen is the same in all these cases. Because the intensity maximum of the $Fe +$ lines coincides with the beginning of the sharp rise of the absorption coefficient, it must fall at a lower temperature as the relative number of metal atoms is smaller. In Fig. 8 the curves for the variation of $\log k_m$ (for atmosphere 4, i.e. $\log g = 4.4$, and $\lambda 4000 \text{ \AA}$) are given for the three cases: ratio $10^2, 10^3, 10^4$; the second curve is of course only little different from the s curve in the other Figures. The maxima occur at $7500^\circ, 6800^\circ, 6200^\circ$, the second practically coinciding with the previous results. We have added the same

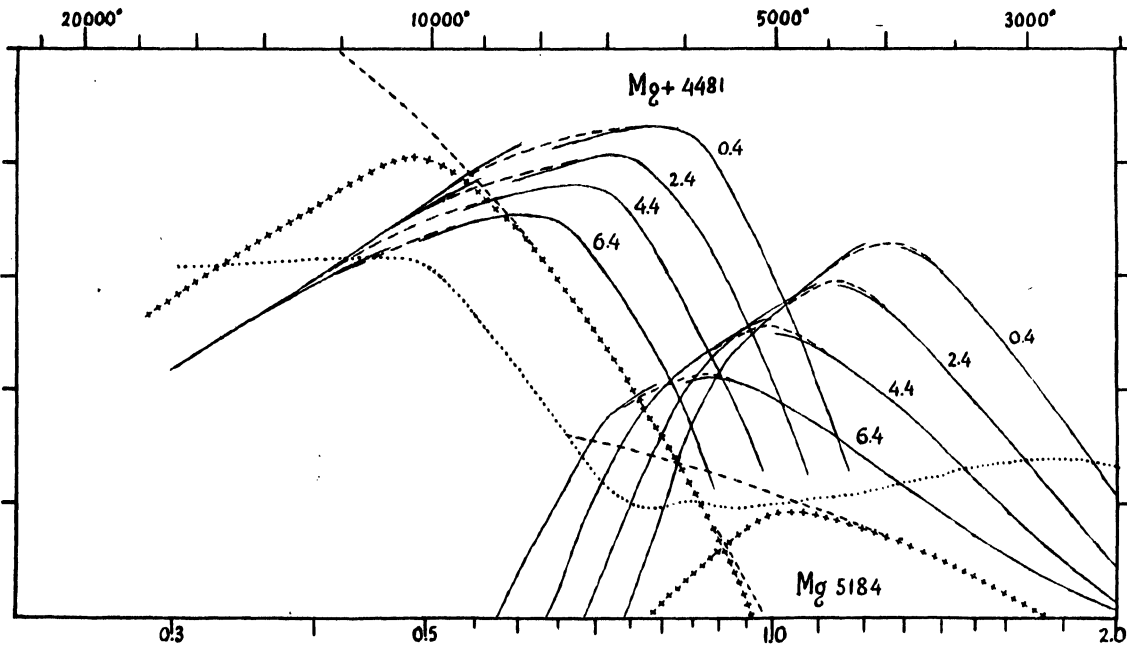


Fig. 9. Variations of Mg and $Mg+$ with temperature and gravity.

curves for hydrogen; here also a variation in the same direction shows itself, though by the flatness of the hydrogen maximum the figures are less exact: the maxima read are 9000° , 7800° , 7000° . In so far as the spectral class here rests on the basis of hydrogen intensities, it means that the variation of the $Fe+$ maximum in terms of spectral class is less than would correspond to the variation in temperature.

In Fig. 7 the curves for He (λ 4771) also are inserted. They fall in the region of complete ionization of hydrogen. The maxima read from the curves are 13800° , 15400° , 17500° and 21000° .

In Fig. 9 the curves for Mg (λ 5184) and $Mg+$ (λ 4481) are given. For each line it shows the intensity curves for the same four gravity values, as well as (for 4.4 only) a curve for the number of atoms, continued by the Boltzmann curve, and the s curve (for λ 4500). The maxima for the green Mg triplet are read 4000° , 4400° , 5100° , 5800° , not much deviating from the maxima of the number of atoms.

For the $Mg+$ line the number of atoms has a maximum at 8200° , 9000° , 10300° , 11800° (Figure 8 contains only the 3d value). The line intensities show a very flat maximum immediately after the steep ascent from lower temperatures, occurring at 6300° , 6900° , 7500° , 8400° , after which the high value remains over a large range of temperatures, only slowly decreasing. There is a strong resemblance to the hydrogen curves, in the shape as well as in the situation, in complete accordance with their common behaviour in stellar spectra. There is a marked difference, however, with the sharp maxima exhibited by the $Fe+$ lines. The reason is found in the different values of the excitation potential. For the Balmer lines and the $Mg+$ line λ 4481 the high excitation potential causes the curve of the number of atoms to run up nearly parallel to the s curve; hence the flat maxima extending over the whole range of temperature in which the absorption coefficient rises rapidly. For the $Fe+$ lines the low excitation potential produces a small slope in the curve for the number of atoms, so that a pronounced maximum occurs near the lowest point of the s curve, where its slope is equal to that of the curve of the number of atoms.

So it appears that the observational data : equality of behaviour of the Balmer lines and the $Mg +$ line λ 4481, but the maximum of the $Fe +$ lines at a more advanced type of spectrum, are in accordance with theory. It is possible, however, that the temperature scale of the spectral classes needs some revision, in such a way that it is lowered for class A and extends over a wider range for this class.

Reduction of observed line intensities.

The preceding computations of the variations with temperature and gravity, to be expected in the line intensities, give only a qualitative survey, in which the maxima are the sole exact quantities which may be used for comparison with observational data. It must be possible, however, to derive data about the stellar atmosphere or about the spectral line from every determination of line intensities in a spectrum. To serve this purpose our computations were made and our tables constructed. We suppose here that the half width of a line has been measured; for the case that equivalent widths have been measured it will be possible to connect them in such a way with the half width, that one quantity can be deduced from the other.

If the measured half width is situated on the resonance wings, we have according to formula 28:

$$n' = [-6.204] \left(\frac{\lambda}{4000} \right)^2 \frac{\gamma}{\delta} s A \lambda^2;$$

in the more general case, when we represent the second part of form. 28) by $F(\Delta\lambda)$, we have $n' = s/F(\Delta\lambda)$. In the case of appearing or disappearing atoms s is replaced by σK or σ'/K or, in the intermediate cases, by the more exact functions of K and P discussed p. 68.

The concentration $n' = Afy$ can thus be found from the measured half width, if only the factor δ/λ is known, for which, on the reverse, these line intensities may give some data. The variations of n' will fix the scale of temperatures and give indications about gravity; if these parameters are known, which is directly the case for the sun, the abundance A of the element may be deduced from the knowledge of f . The abundance of the metals in total, i.e. the relative number of hydrogen and metal atoms in stellar atmospheres may be deduced from the line intensities of hydrogen in low temperature stars (taking account of the Stark effect), where the absorption coefficient is determined by the metal atoms; as well as from the metal line intensities in high temperature stars, where hydrogen determines the absorption coefficient.

It must be borne in mind that the results obtained in this way for concentration and abundance of atoms depend on the absorption coefficient used. It is the ratio of the monochromatic diffusion coefficient to the absorption coefficient that determines a line intensity. This implies that every correction to be applied to the adopted k brings a correction to s and n . In the absorption coefficient the assumed abundance of the active atoms is one of the factors. Hence the concentration n or n' derived from a spectral line is always a relative concentration, relative to the atoms producing the absorption coefficient. This means that for lower temperatures the line intensities reveal the concentration of the element considered as a fraction of the total number of the metal atoms only. For high temperatures, on the other hand, with dominant hydrogen absorption, the line intensities reveal the concentration of that element relative to hydrogen, i.e. to the atmospheric atoms in total. It is clear, moreover, that any change in the coefficients of the formulas used for the absorption, which should be demanded by physical theory, or in the value of free-free transitions or other sources of absorption, will have a direct influence upon the results for the atom concentration derived. By means of the tables of the different quantities used in the derivation of the absorption coefficient it will be possible to apply such corrections to the results as may be needed.