THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME 84 DECEMBER 1936 NUMBER 5

THE STELLAR TEMPERATURE SCALE[1]

A. PANNEKOEK

ABSTRACT

The methods of determining stellar temperatures from color and energy distribution, from ionization, and from the Boltzmann function are discussed and are shown to depend upon complicated conditions in the atmospheres of the stars which are not sufficiently known at the present time to permit the derivation of reliable temperatures. Direct methods of determining the effective temperatures of stars give 3300° for class cMo and 10,500° for class Ao. These values are regarded as well established, their uncertainty being estimated at no more than 5 per cent. It is suggested that from the degree of accordance between these temperatures and those obtained by other indirect methods information may be obtained concerning the conditions prevailing in the atmosphere of the stars.

1. To find the temperature of a star, the most direct and the most promising method is to make use of the energy distribution in the spectrum. According to Planck's law for black-body radiation, the logarithmic intensity difference for two light sources is a linear function of $1/\lambda$, and the gradient is equal to $0.43 \times 14{,}600/T$, after a correction factor $(1 - e^{-c_2/\lambda T})^{-1}$ has been applied, which is only important for high temperatures and long wave-lengths. Measurements of these gradients—relative to the sun, to a standard star, or to a light source of known temperature—have been made by Wilsing,[2] Rosenberg,[3] H. H. Plaskett,[4] Sampson,[5] Jensen,[6] and by Greaves with his

[1] Part of a symposium on astrophysics, presented at the Harvard Tercentenary Conference on Arts and Sciences, September 3, 1936.

[2] *Publ. Ap. Obs. Potsdam*, **56**, 1909; **74**, 1919.

[3] *Nova Acta K. Leop. Akad.*, **101**, No. 2, 1914.

[4] *Pub. Dom. Ap. Obs. Victoria*, **2**, No. 12, 1923.

[5] *M.N.*, **83**, 174, 1923; **85**, 212, 1925.

[6] *A.N.*, **248**, 242, 1933.

fellow-observers at Greenwich.[7] The first results showed serious discrepancies in the temperature scales. Careful comparisons and reductions of these data by A. Brill[8] have shown that their chief cause was a change of the gradient with wave-length. That the logarithmic intensity difference is not a linear function of $1/\lambda$ is confirmed by the measures of Jensen. So it appears that the radiation of a star cannot be black-body radiation.

Of course, there was no other reason to suppose that the stars radiate as black bodies than that in a first coarse approximation they seemed to do so. This hypothesis was introduced for the sound reason that it was the only means of deriving a temperature from the spectral intensity gradient. This temperature was called "color temperature"; it is of great practical importance because the most easily determinable data on color, such as color index and effective wavelength, depend on the same gradient and, therefore, are strongly correlated to this color temperature.

The ionization theory afforded another method of finding stellar temperatures, by making use of the intensities of absorption lines. The state of ionization of an element depends on temperature and pressure. To eliminate the uncertainties of first appearance, of abundance and of absolute values, Fowler and Milne[9] introduced the method of maxima of absorption lines. Observations show the spectral class for which a line has its maximum intensity. By theory the temperature is computed for which the atoms producing this line have a maximum concentration in the state of ionization and excitation required to produce it. Three assumptions are involved in this deduction: the abundance of the element is constant over the spectral sequence; the absorption coefficient is constant for all spectral classes; and the pressure is the same in all these atmospheres. Because this pressure is unknown, an absolute determination is not possible. By the assumption that the Balmer series has its maximum at 10,000° (A0), the pressure was fixed; and it determined the temperature of other maxima. The application of this method for lower temperatures is not easy, because there are few pronounced maxima. For high temperatures, however, just where the gradient ceases to

[7] *Greenwich Obs.*, 1932; *M.N.*, **94**, 488, 1934.

[8] *A.N.*, **218**, 209, 1923; **219**, 27, 356, 1923. [9] *M.N.*, **83**, 403, 1923; **84**, 499, 1924.

give reliable results, it proved most useful. Especially through the work of Mrs. Cecilia H. Payne-Gaposchkin[10] the temperature scale of the B and O stars could be determined by the maxima of *Si*, *C*, *O*, and *N* in different stages of ionization.

These ionization temperatures may be considered as an extrapolation to high temperatures of a scale based on spectral energy distribution. Hence, the assumption of the stars radiating as black bodies —which we know they do not—is underlying the entire temperature scale.

Now it seems that such an assumption cannot be avoided. Even in the case of the sun we make use of it; the effective temperature is the temperature of a black body that emits the same quantity of energy. For this reason the color temperature is sometimes called another kind of effective temperature. There is, however, an important difference: the effective temperature measures the total stream of energy passing through a unit surface area of the star. This stream is determined by the interior conditions; it determines (for given conditions of composition of the atmosphere and of gravitation at the surface) the state of the stellar atmosphere, the temperature and the ionization in each layer, the spectral distribution of the emitted light, and the line intensities. The amount of this energy stream is the independent datum, and all the other quantities are functions of it. The effective temperature is a number indicating this energy stream. Therefore, the effective temperature is not an uncertain value founded on an incorrect assumption, but it is the exact expression of a fundamental datum of the star. When we speak of the temperature of a star, this effective temperature is meant. Color temperature and ionization temperature have the meaning, that they may be considered as more or less correct substitutes for the effective temperature.

Table V in Brill's discussion of the stellar temperatures in the *Handbuch der Astrophysik*[11] shows the large discrepancies between the different determinations. The differences between the temperature scales resulting from different comprehensive discussions, though smaller, still show their uncertainty, as may be seen from a

[10] *Harvard Circ.*, Nos. 252 and 256, 1924.

[11] *Handbuch d. Astrophysik*, **5**, Part I, 150–151, 1932.

comparison of the scale derived by Mrs. Payne-Gaposchkin and quoted by Eddington,[12] the scale given by Russell, Dugan and Stewart,[13] and the scale given by Brill as the result of his discussions.[14] That these figures do not even mark the limits of uncertainty is shown in the result of the careful determination at Greenwich of the

	B0	A0	F0	G0	K0	M0
C. H. Payne-Gaposchkin	20,000	10,000	7500	5600	4000	3000
Russell, Dugan, and Stewart	23,000	11,000	7400	5600	4200	(3100, M2)
A. Brill	22,000	13,500	8550	5800	4370	3240

color temperature by comparison with laboratory standards. Such absolute measures are difficult, and the elimination of the instrument introduced the uncertain element of atmospheric coloring. Still, the main result cannot be much in error, and it gave 18,000° for the color temperature of A0 stars.

2. The theoretical determination of the absorption coefficients of atoms under the conditions prevailing in stellar atmospheres, by the work of Kramers,[15] Sugiura,[16] Gaunt,[17] and Menzel and Pekeris,[18] provides a new foundation for the determination of stellar temperatures. It must be possible now, at least in the first approximation, to derive, first, the energy distribution in the stellar spectrum as a function of the temperature and, second, the variation of the line intensities with temperature for the real atmosphere, without the assumptions that were necessary ten years ago. Of course, theoretical discussions on the structure of the atmospheres are needed, which can be given as yet only to a certain degree of approximation. For a wave-length in the continuous spectrum, the intensity is found by solving the equations for the integrated light:

$$\frac{dJ}{d\xi} = 3kH \; ; \qquad \frac{dH}{d\xi} = k(J - E) \; ; \qquad J = 2H \text{ for } \xi = 0 \, .$$

[12] *The Internal Constitution of the Stars*, p. 141, 1926. [13] *Astronomy*, **2**, 753, 1927.

[14] *Handbuch d. Astrophysik*, **5**, Part I, 193, 1932.

[15] *Phil. Mag.*, **46**, 836, 1923.

[16] *Scientific Papers, Inst. Phys. Chem. Res.*, Tokyo, **17**, No. 339, 89, 1931.

[17] *Phil. Trans.*, A, **229**, 163, 1930. [18] *M.N.*, **96**, 77, 1935.

where $\xi = -\int \rho dh$ is the homogeneous depth, $4\pi J$ = the total radiation, $4\pi H$ = the net stream, $4\pi E$ = the black-body radiation, k = the absorption coefficient, all holding for a special λ. The solution for the energy stream emitted at the surface is:

$$H = \frac{1}{2+\sqrt{3}} \int_0^\infty E(\xi) e^{-k\xi\sqrt{3}} k\sqrt{3}\, d\xi \, .$$

Denoting by $\bar{k}$ the mean absorption coefficient (Rosseland mean), we have

$$T^4 = T_0^4\left(1 + \tfrac{3}{2}\bar{k}\xi\right) \qquad (T_0 = \text{the surface temperature}),$$

$$E = \frac{c_1}{\lambda^5}\left(e^{c_2/\lambda T} - 1\right)^{-1} = \frac{c_1}{\lambda^5}\left(e^{c_2/\lambda T_0 (1+\frac{3}{2}\bar{k}\xi)^{-\frac{1}{4}}} - 1\right)^{-1} .$$

In my former computation, a linear function for E was assumed,[19] namely, the first two terms of a Taylor development of this expression. Since the errors of this approximation may be rather large in some cases, we will use here the exact expression, reducing the integral to the form used by Lindblad[20] and Milne.[21] Putting

$$k\sqrt{3}\, d\xi = dt \, , \qquad \frac{c_2}{\lambda T_0} = a \, , \qquad \frac{\sqrt{3}}{2}\frac{\bar{k}}{k} = p \, ,$$

we have

$$4\pi H = \frac{4\pi}{2+\sqrt{3}} \frac{c_1}{\lambda^5} \int_0^\infty \frac{e^{-t}dt}{e^{a(1+pt)^{-\frac{1}{4}}} - 1} = \frac{4\pi c_1}{2+\sqrt{3}} \left(\frac{T_0}{\lambda}\right)^5 f(a, p) \, ,$$

$$f(a, p) = \int_0^\infty \frac{a^5 e^{-t}dt}{e^{a(1+pt)^{-\frac{1}{4}}} - 1} \, .$$

For this function $f(a, p)$ tables are given by Milne (using Lindblad's data); since they, however, extend only from $p = 0$ to $p = 2$, they had to be extended to special large values of p needed here. The results for the net stream $4\pi H$ are compared with the black-body radiation for the effective temperature

$$\pi E(T_1) = \frac{\pi c_1}{\lambda^5}\left(e^{a_1} - 1\right)^{-1} ,$$

[19] *Ibid.*, **95**, 529, 1935.

[20] *Upsala U. Årrskrift*, p. 33, 1920.

[21] *Phil. Trans.*, A, **223**, 201, 1922.

where $\alpha_1 = c_2/\lambda T_1 = \alpha\sqrt[4]{1/2}$. In the tabulated values the factor πc_1 has been omitted in $4\pi H$ as well as in πE. The values for $\bar{k}/k$ were taken from the data of *Amsterdam Publications* No. 4 (for log $g = 4.4$).

In Table I the results are given for effective temperatures $T_1 = 5040/0.2$, etc., corresponding to $T_0 = T_1\sqrt[4]{1/2}$. They afford the differences in the gradient $\varphi = c_2/T(1 - e^{-c_2/\lambda T})^{-1}$ between the stellar and the black-body radiation for the intervals 6000–5000 A and 5000–4000 A. The differences from the former computation are not large, and the theoretical color temperatures show the same character. The high color temperature of 18,000° found at Greenwich for the A0 stars, for which usually an effective temperature of about

TABLE I

$\frac{5040}{T}$	log p			log $4H/c_1$			log E/c_1			$\Delta\varphi$		φ	
	6000	5000	4000	6000	5000	4000	6000	5000	4000	5500	4500	5500	4500
0.2....	−1.07	−0.88	−0.65	0.837	1.103	1.416	0.910	1.179	1.492	+0.02	0.00	0.90	0.79
0.3....	−0.76	− .56	− .31	0.528	0.773	1.062	0.611	0.851	1.118	− .03	− .10	1.05	0.90
0.4....	−0.37	− .16	+ .10	0.296	0.539	0.838	0.356	0.564	0.780	− .24	− .38	1.06	0.86
0.5....	−0.14	+ .07	+ .34	0.101	0.342	0.637	0.122	0.295	0.458	− .47	− .61	1.07	0.87
0.6....	−0.06	+ .17	+ .41	9.900	0.130	0.389	9.899	0.037	0.143	− .63	− .70	1.16	1.05
0.7....	+0.08	+ .20	+ .24	9.733	9.904	0.019	9.684	9.783	9.831	− .50	− .31	1.55	1.71
0.8....	+0.14	+ .08	− .09	9.559	9.606	9.503	9.472	9.533	9.520	+ .08	+ .41	2.39	2.70
0.9....	+0.08	− .05	− .31	9.330	9.290	9.052	9.262	9.284	9.211	+ .43	+ .76	3.01	3.33
1.0....	+0.03	−0.15	−0.47	9.103	8.983	8.635	9.054	9.035	8.901	+0.70	+0.98	3.56	3.83

10,000° is assumed, is not only confirmed but is even surpassed by the theoretical values. Computation shows constancy and even temporary decrease of color temperature, in going from A0 to B0, at variance with observation, which, after allowance has been made for space reddening, indicates a slow but steady increase. Hence it is not possible to derive effective temperatures from the observed gradients in the case of hot stars, so long as the qualitative discrepancy between theory and observation is not removed.

In order to investigate it more closely, the energy distribution over the entire spectrum was derived for the case of an effective temperature of 10,080° ($5040/T = 0.5$). The data for the computation and the results are given in Table II.

The variations in surface radiation depend on $p = 0.87\bar{k}/k$; the mean absorption $\bar{k}$ determines the increase of temperature with depth, and the special absorption $k(\lambda)$ determines from what depth we find radiation of this λ reaching the surface. For these high-temperature stars where the absorption coefficient is determined by hydrogen, it jumps at each band edge to a higher value on the blue side but then decreases continually with λ^3 down to the next edge. The emitted radiation is strong on the longer wave-length side of each edge and falls there abruptly to a smaller value. (We disregard here the smoothing of the highest intensity peak at each edge in consequence of the line-crowding.) This small emission at the short

TABLE II

λ	p	a	log $4H/c_1$	log E/c_1
12,000	0.33	1.41	9.20−10	9.25−10
9000	0.69	1.88	9.54−10	9.64−10
8206	{1.10 0.30}	2.06	9.80−10 9.69−10	....
7000	0.49	2.42	9.91−10	9.95−10
4000	2.19	4.22	0.64	0.46
3646	{2.90 0.15}	4.63	0.75 0.27	....
3000	0.27	5.63	0.35	0.56
2500	0.47	6.76	0.39	0.54
2000	0.91	8.45	0.45	0.41
1500	2.17	11.27	0.62	0.00
1000	7.30	16.90	1.01	8.82−10
911	{9.6 0.00}	18.54	1.09 6.9 −10	8.42−10

wave-length side of each edge is often spoken of as an "absorption band," for which a correction should be applied to get the clear spectrum. Figure 1 shows that this is not a correct statement of the phenomena. The whole spectrum is distorted; and corrections should also be applied, even larger ones, to the bright parts on the long wave-length side of the edges.

The energy-curve of the spectrum emitted by the stellar atmospheres in Figure 1 shows, first, that in the Paschen region (between λ 8206 and λ 3646, where the third hydrogen-level determines the absorption) the gradient is stronger than in the black-body radiation; this is the phenomenon already treated, which appears in the color temperature. In the Balmer region (between λ 3646 and λ 911, where the second hydrogen-level dominates the absorption) the deviation is still more striking; here the emitted radiation increases in a steep ascent for the wave-lengths just above the Lyman edge, to dwindle to nothing below 911 A. These regions are, it is true, not observable; but this strong distortion of the spectrum must produce considerable effects in other phenomena. The radiations of wave-lengths 1215 A, 1024 A, 971 A ($L\alpha$, $L\beta$, $L\gamma$), producing the transi-

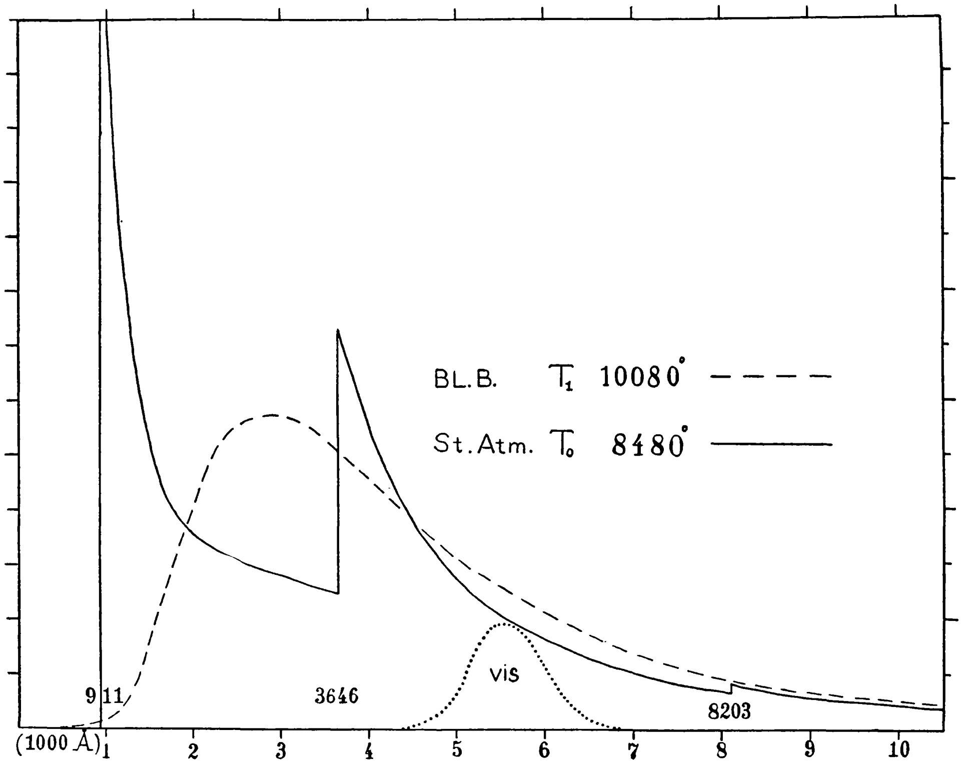

FIG. 1

tions from the lowest to the second, the third, and the fourth level, have here an intensity 22 times, 120 times, and 215 times larger than in the black-body radiation. Hence, in the upper layers of the atmosphere the higher levels must be more strongly occupied than in the case of thermodynamic equilibrium, and the more so the higher the level considered. This must increase the general absorption; moreover, it offers an explanation of the remaining discrepancies in the color temperature of the A stars.

Since the transitions 1–2 and 1–3 chiefly dominate the occupation of these higher levels, we may expect that the number of atoms in the third level is increased 5 to 6 times more than the number of atoms in the second level. Hence, the absorption due to ionizations from the third level is, in the same ratio, stronger than from the second level, relative to the former computations based on the Boltzmann ratios. Since the mean absorption $\bar{k}$ for this temperature[22] is chiefly a combination of second- and third-level absorption, this means that $\bar{k}/k$ and p must be smaller in the Paschen region and larger in the Balmer region. This will produce a smaller difference in intensity gradient between the stellar and the black-body spectrum for the Paschen region than was found in our first computation —a change in the right direction. It will not be easy, however, to determine the exact amount. What we computed here was the spectral-energy distribution in the radiation emitted at the surface of the star. For the deeper layers the composition of the radiation approaches black-body radiation at an increasing rate; and all these deeper layers, with their different occupation of higher levels and their, consequently, different absorption coefficients, determine the real observed spectrum. Accordingly, in a second approximation a more refined treatment will be necessary in which the conditions in each atmospheric layer and its contributions to the total radiation are taken into account.

There is still another point to be considered. We have assumed the surface temperature to be $T_0 = T_1\sqrt[4]{1/2}$ (in this case 8470°), as follows from the Schwarzschild relation for black-body radiation. (The deviation from the linear gradient in the surface layers, giving the exact ratio $T_1/T_0 = 1.23$ as computed by Hopf, does not concern us here, because for us T_0 means only a constant in the linear function we use at greater depth.) For the distorted energy-curve of the real stellar radiation, the same ratio will not hold. Now that it has been found in the first approximation, based on the surface temperature $T_0 = 8470°$, we are able to integrate the radiation over the entire spectrum to find the total amount of emitted energy and to

[22] Cf. Table 8, *Pub. Astro. Inst. Amsterdam*, **4**, 1935.

see to what effective temperature it corresponds. By reading from the curve and by numerical integration, we find

$$\log \int \frac{4H d\lambda}{c_1} = 0.290 ,$$

whereas

$$\log \int \frac{E}{c_1} d\lambda = \log \frac{\pi^4}{15} \left(\frac{T_1}{c_2}\right)^4 = 0.205$$

when computed with $T_1 = 10{,}080^\circ$. The total amount of energy comes out somewhat larger for the stellar atmosphere and corresponds to an effective temperature somewhat higher, $10{,}400^\circ$. This means that for a star with effective temperature $10{,}080^\circ$ the surface temperature is somewhat lower than that computed by the Schwarzschild relation. The difference is not large enough to upset the foregoing results; it has to be taken into account in a second approximation.

3. For the stars of low temperature there are difficulties of another kind. The absorption coefficient is due to the ionization of metal atoms from different levels. Because we do not observe any band edge corresponding to a metal-atom level, we assume that there is some smoothing process at work; and for the absorption coefficient we take the value computed by a smoothed formula:

$$k = \text{Const. } T^{-\frac{3}{2}} \nu^{-3} (e^{h\nu/kT} - 1) .$$

The computations were made for the temperatures 5040° and 3150° ($5040/T = 1.0$ and 1.6). The result is contained in the values of Table III. The absorption coefficient is a minimum for a large infrared wave-length (11,000 A for 5040°, 17,000 A for 3150°), and increases strongly with decreasing wave-length in the visual part of the spectrum (because at low temperatures the lowest levels are strongly occupied relative to the higher levels). The consequence is that the short wave-length part of the spectrum, including the visible region, is much weakened (as is shown in Figure 2 for $T_1 = 3150^\circ$), whereas the maximum in the near infrared is increased and displaced toward the longer wave-length side and the far infrared again is weakened. The slope in the visible part of the spectrum deviates strongly from the corresponding black-body radiation and indicates

TABLE III

	$T_1 = 5040°$				$T_1 = 3150°$			
	a	p	log $4H/c_1$	log E/c_1	a	p	log $4H/c_1$	log E/c_1
50,000	0.68	0.30	....	....	1.08	0.63	6.31	....
25,000	1.35	0.85	7.69	7.68	2.16	1.45	7.36	7.30
20,000	1.69	1.12	8.05	8.00	2.70	1.66	7.64	7.55
15,000	2.25	1.48	8.43	8.37	3.61	1.74	7.94	7.82
12,000	2.82	1.70	8.71	8.62	4.51	1.55	8.08	7.97
9000	3.76	1.70	8.99	8.88	6.01	1.02	8.08	8.04
7000	4.83	1.45	9.12	9.02	7.73	0.51	7.80	7.95
6000	5.63	1.12	9.11	9.05	9.01	0.28	7.48	7.81
5000	6.76	0.76	9.00	9.04	10.82	0.10	6.95	7.55
4000	8.45	0.36	8.64	8.90	13.52	0.02	6.18	7.05
3000	11.27	0.08	7.84	8.49	18.03	0.001	4.85	6.03

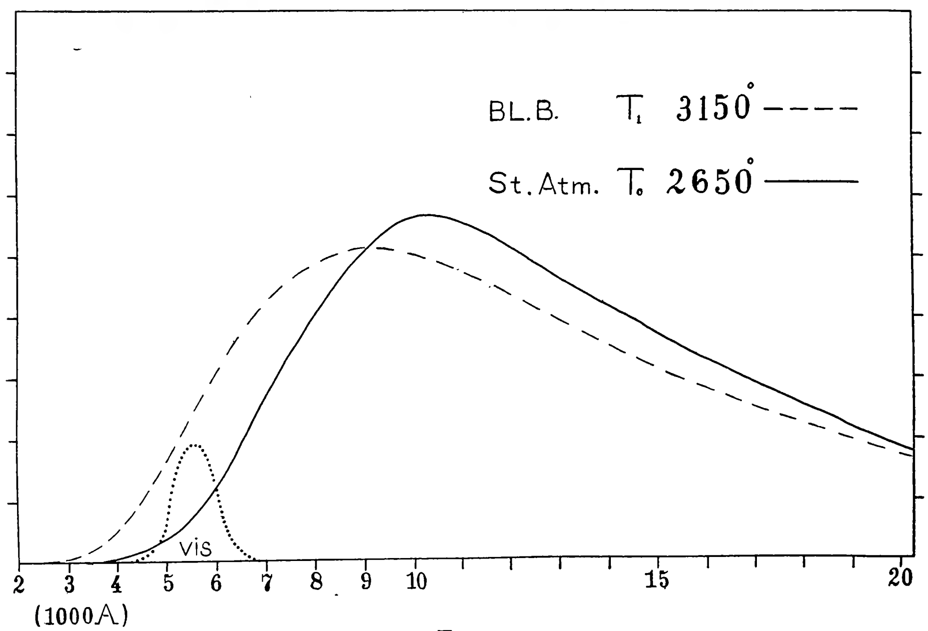

Fig. 2

far too low a temperature. All these deviations are strongest for the lowest temperatures. Integrating the energy values of Table III over the whole spectrum, we find here again a total stream of energy somewhat larger than belongs to the corresponding T_1 (log = 9.032

instead of 9.001 for $T_0 = 4230°$, 8.221 instead of 8.185 for $T_0 = 2650°$) and corresponding to a black-body radiation of 5130° and 3220°, only slightly larger than the values of $T_0 \sqrt[4]{2}$.

We can make use of the measurements of Jensen to derive temperatures from the spectral gradients of low-temperature stars. Jensen measured for a range of wave-lengths the relative intensities of a number of stars relative to α Ursae Minoris. By means of seven stars, which he has in common with the Greenwich observers (η UMa, α Leo, α Lyr, α CrB, α Agl, α UMi, α Aur), we can use the absolute Greenwich gradients to reduce his results for α Orionis to absolute values (i.e., intensities relative to a black body of $T = \infty$) and compare them with H and E multiplied by λ^5. The result of the comparison is shown in Figure 3. (The shorter wave-lengths, from 4000 A downward, are not reliable because here in the class A comparison stars the hydrogen depression makes itself felt.) Compared with the black-body-curve, the slope of the observed intensity-curve indicates a temperature below 3150°. On the contrary, the theoretical curve computed for the stellar atmosphere shows a steeper slope than the star. Hence, T_1 for the star must be above 3150°—or, more exactly, above 3220°—and the theoretical curve belonging to $T_1 = 3600°$ seems to fit better. An exact result is not possible because Jensen's measurements show a variation of the gradient with wave-length which is not present in the theoretical curves.

There are other sources of uncertainty here. It is doubtful whether for these low temperatures we can rely upon the smoothed formula for the absorption coefficient where it is due only to the ionization of a small amount of sodium and potassium atoms. Even if we were able to observe and measure the continuous background of the spectrum, its gradient would give information about the variation of the absorption coefficient rather than about the temperature. But it is well known that, owing to the crowding of absorption lines, chiefly those constituting the molecular absorption bands, the sifting-out of the continuous spectrum is well nigh out of the question. Thus, for such low-temperature stars the gradient of the visible spectrum, observed with present means, cannot give very reliable results for the temperature.

4. The theoretical knowledge of the coefficient of continuous absorption must provide the foundation for the use of line intensities for the derivation of stellar temperature. The intensity of an absorption line depends on the ratio s/k of two quantities: s, the diffusion coefficient in the line (depending on abundance, state of ionization,

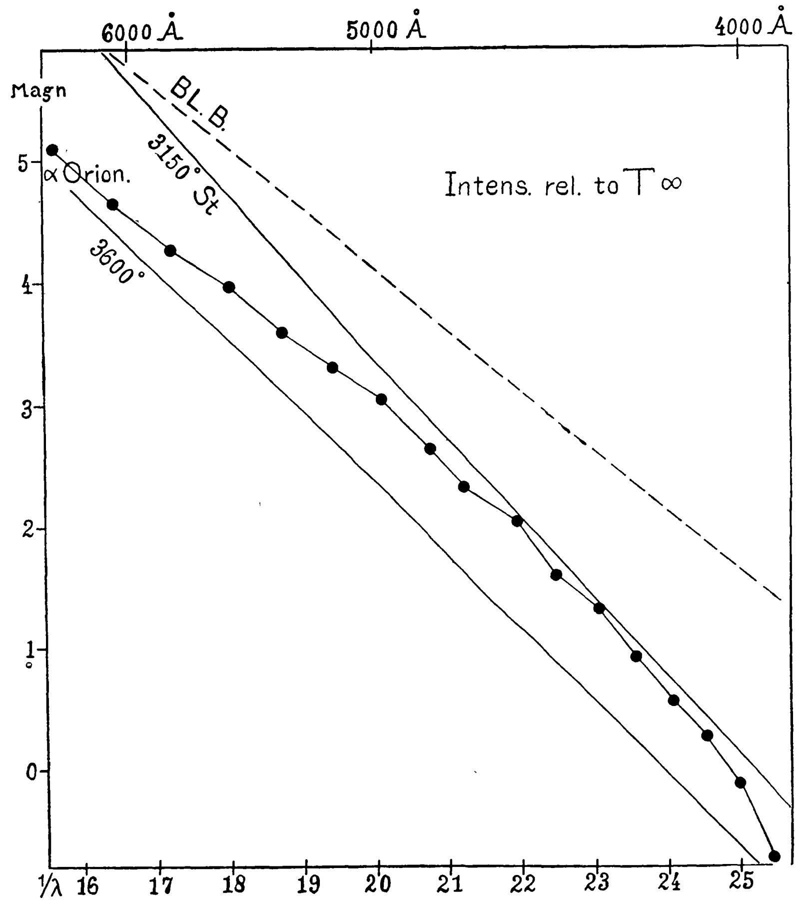

FIG. 3

level, and transition probability of the atoms involved); and k, the continuous absorption coefficient. Because k is variable with temperature in a known way, the maximum intensity of an absorption line is not simply identical with the maximum concentration of the atoms producing it, but one can be derived from the other. Moreover, it is not necessary to assume a pressure; all the layers in an atmosphere with different pressures take part in the formation of the line, and the theoretical treatment of the atmosphere gives the line intensity, where the intensity of the total stream of energy (i.e., the effective temperature T_1) and the surface gravity g appear as the determining parameters. The only assumption we cannot avoid making use of is that of constancy of abundance over the range of spectral classes.

The most conspicuous of the line maxima used in determining stellar temperatures is the maximum of the Balmer series of hydrogen in class A0. Fowler and Milne were able to explain it easily because the hydrogen atoms in the second level of energy have a clearly depicted maximum of concentration, for which 10,000° was assumed. If, however, the variation of k is taken into account, we get an extremely flat and ill-defined maximum at a lower temperature (7000° to 8000° for log $g = 5$ to 2; cf. Figure 6 in *Amsterdam Publications* No. 4). Unsöld has shown[23] that in a pure hydrogen atmosphere the intensity of the Balmer lines must steadily increase for decreasing temperature. The reason is simply that the diffusion coefficient s for these lines depends on the concentration of H in the second level, whereas they are situated in the Paschen region, where the absorption coefficient k depends on the concentration of H in the third level, and that the relative number of the atoms in the second state compared to those in the third state increases with decreasing temperature. Going down below 10,000° the intensity of the Balmer lines must go on increasing because, though N_2 and s are strongly decreasing, N_3 and k are decreasing still more strongly. As soon, however, as k for hydrogen falls below the absorption coefficient due to the metal atoms, the k of the stellar atmosphere begins to increase; and now the intensity of the Balmer lines decreases rapidly. Their maximum, therefore, is not due to themselves but to the transition of the continuous hydrogen to the metal absorption. If the relative abundance of the hydrogen to the metal atoms is assumed to be 50 instead of 1000, the maximum is raised 1000°. It is not likely that in this way we can escape the difficulties. Moreover, the intensity of the Balmer lines is an expression chiefly of their width, which is due to the Stark effect and depends, in the first place, on the surface gravity. Hence, in comparisons with observational data a careful

[23] *Zs. f. Ap.*, **8**, 32, 1934.

arrangement according to the gravity g is first necessary before the variation with T_1 can come out in the right way. We may expect that further approximations in the theoretical treatment of the A stars may clear up the discrepancies; but, for the moment, the maximum of the Balmer lines is not a good indicator of temperature.

This hydrogen case is an example of a more general phenomenon. Between 10,000° and 6000° the continuous absorption coefficient varies irregularly with temperature, because here is the transition from hydrogen to metal absorption. The variations of line intensities in this realm are determined more by these irregularities than by the variations in the element producing the line. The exact course of these irregularities depends on the assumed composition of the atmosphere, especially on the relative abundance of hydrogen and metal atoms. In the maxima of absorption lines within these temperatures, composition and temperature are interrelated.

Below 6000° and above 10,000° the absorption coefficient runs smoothly and slowly with temperature, so that the maxima are related in a simple way to temperature. Lines absorbed by the lowest level of ionized atoms (as H and K of Ca^+) present very flat and uncertain maxima; hence they cannot give exact results for the temperature. The matter is different for lines absorbed by higher levels, where the temperature of the maximum depends chiefly on the excitation potential. Here, however, the crowding and blending of lines make determinations difficult, and exact photometric measures of line intensities are lacking on the whole. Moreover, the variation of k with temperature depends here on the assumed composition of the metal atom mixture.

The case is more favorable for high-temperature stars above 10,000°, where the absorption coefficient depends on hydrogen only and the absorption lines can be separated easily. Thus for the B and O stars the former results of Fowler and Milne and of Mrs. Payne-Gaposchkin from metalloid lines can be put upon better foundations. After the method indicated and used in *Amsterdam Publications* No. 4, the line intensities have been computed for a range of temperature, then plotted, and the temperature of the maximum read. The pressure and the atmospheric parameters s, σ, σ' were taken for two cases of surface gravity, log g = 4.4 (dwarfs) and log $g = 2.4$ (giants). In Table IV the results for the temperatures of the maxima are given, and in the last columns the spectral classes of these maxima are indicated, first according to old results of H. H. Plaskett and Mrs. Payne-Gaposchkin, and then according to the latest results of E. G. Williams.[24] The resulting scale of temperatures is not very different from the former investigations; for B0 the corresponding temperature is nearly 25,000°.

TABLE IV

	Ion. P.	Exc. P.	log g = 4.4	log g = 2.4	H. H. Pl. and C. H. G.	E. G. W.
Si^{+}	16.3	9.8	12,900°	11,200°	A0	....
C^{+}	24.3	18.0	21,000	16,800	....	B3
N^{+}	29.5	18.4	21,000	18,700	B2	B1–2
Si^{++}	33.3	18.9	22,000	19,400	B1–2	B1–2
He	24.5	20.9	18,000	14,400	B2–3	....
O^{+}	34.8	23.3	24,000	21,000	B1	B1
Si^{+++}	44.9	23.9	31,500	27,000	O9	O9
N^{++}	47.4	27.3	35,000	31,500	O7–8	O9
C^{++}	47.7	29.4	30,600	28,000	O9?	O9–B0
O^{++}	54.9	33.7	37,400	33,600	O9–B0	O8
N^{+++}	77.3	39.7	39,000	33,600	....	....
He^{+}	54.2	50.8	44,000	39,000	....	....
C^{+++}	64.2	55.5	50,000:	44,000	....	....

5. Now that the intensity of a line expressed, for example, as the half-width for 50 per cent residual intensity, or as the equivalent width, can be measured as well as computed, it is not necessary to make use of maxima. The intensities themselves may be used, and so a larger quantity of data may be utilized. The number of good intensity measures, it is true, is only small as yet. We can, therefore, make use of a few cases only as instances of the method.

For the line K of Ca^{+} the half-width in the solar spectrum is 6.3 A; this corresponds (according to formulas 28 and 46 in *Amsterdam Publications* No. 4) to log abundance = −4.71, that is, the number of Ca atoms is 20, if all the metals together are taken as 1000. By means of this abundance we compute log $\Delta\lambda^2$ and the half-width $\Delta\lambda$ itself (in the case of disappearing atoms):

$5040/T$ =	0.4	0.5	0.55	0.6	0.65
log $\Delta\lambda^2$ =	−2.58	−1.04	−0.13	+0.63	+1.28
$\Delta\lambda$ =	0.051	0.30	0.86	2.1	4.4 A

[24] *Ap. J.*, **83**, 319, 1936.

From some slit-spectra of α Lyrae, taken in August 1935, with a two-prism spectrograph attached to the Wyeth reflector at the Harvard College Observatory, I find an equivalent width of 0.97 A. From theoretical profiles, we found for the relation between half-width and equivalent width:

log E.W.	0.40	0.20	0.00	9.80	9.60	9.40	9.20	9.00	8.80
log H.W.	9.89	9.69	9.48	9.28	9.06	8.95	8.78	8.68	8.54

Thus, an equivalent width of 0.97 A means a half-width of 0.29 A, corresponding to $5040/T = 0.50$. Hence, assuming for α Lyrae the same abundance of *Ca* atoms as for the sun, we find a temperature of 10,100°.

In the same way we can make use of the results of Mrs. E. T. R. Williams-Vyssotsky[25] for the equivalent width of the line K in a number of class A stars. We find (taking $\log g = 4.4$):

	Sirius	Mean A0	A2	A3	A5
E. W.	0.62	1.08	2.94	4.03	4.5
H. W.	0.19	0.33	0.91	1.25	1.39
$5040/T$	0.47	0.50	0.55	0.57	0.58
T	10,700°	10,100°	9200°	8800°	8700°

It must be remarked that the assumed relative abundance of hydrogen and metal atoms goes into this result in its full amount, because it determines the relative values of k for A stars and the sun. If hydrogen atoms are 100 times (instead of 1000 times) more abundant than metal atoms, the temperatures deduced are raised 1200°. Moreover, the assumption of equal abundance of calcium in the sun and the A stars is involved, whereas after the results of W. W. Morgan[26] the abundance of different metals must be different in different A stars. The deviations, however, from constant composition do not seem to amount to large factors; since the variation of the concentration of active atoms with temperature is very rapid, the uncertainties of temperature resulting from these differences will not be large.

There may be a way to escape these uncertainties and to eliminate the differences of abundance and of absorption coefficient at the same time, namely, by comparing the line intensities for ionized and

[25] *Harvard Circ.*, No. 348, 1930.

[26] *Pub. Yerkes Obs.*, 7, No. 3, 1935.

neutral atoms of the same element. The line λ 4227 of *Ca* and the K line λ 3933 of Ca^+ both belong to the lowest level; therefore, no other factors are involved than the variations of k with wave-length, which are small for this small distance in λ and are neglected in the following computation. For α Lyrae the equivalent width of *Ca* 4227 was found from the Victoria spectra to be 0.044 A. According to the curve of growth derived by Struve and Elvey for this star,[27] this value is already situated on the wing part of the curve, where the equivalent width increases as the square root of the concentration. Hence, the ratio of the concentrations of *Ca* and Ca^+ atoms is the square of the ratio of the equivalent widths of λ 4227 and λ 3933, i.e. $(0.044:0.97)^2$, of which the logarithm is -2.70. For the sun we have the half-width of λ 4227, taken from the profile derived by Redman,[28] 0.57 A; compared with the half-width 6.3 A of the K line, we find the logarithm of the ratio of the concentrations log (0.57:6.3) = -2.08. Hence, we find (x = rate of ionization):

$$\log \left[\left(\frac{x}{1-x} \right) \alpha \text{ Lyr} : \left(\frac{x}{1-x} \right) \odot \right] = 0.62 \, .$$

Computing this ratio after the ionization formula $x/(1-x) = K/P$ (taking $\log g = 4.4$), we find:

T	= 10,080°	8400°	7200°	6300°	5800° (☉)
$\log K/P$	= 4.61	3.89	3.31	2.83	2.46

The observational result would give a temperature for α Lyrae nearly 1000° above the solar temperature. In this A0 star both λ 4227 and K have strongly decreased relative to the sun, but nearly in the same ratio, whereas theory demands that λ 4227 should have decreased much more rapidly. Indeed, for a temperature of 10,000° we should expect λ 4227 to be nearly 7 times fainter, so that it would have entirely disappeared in the spectra of A0 stars. Of course, we could assume that the line seen at that wave-length is not the *Ca* line λ 4227 but has some other origin; indeed, a faint Ti^+ line corresponds in wave-length. We shall see, however, that the same

[27] *Ap. J.*, **79**, 409, 1934.

[28] *M.N.*, **95**, 751, 1935.

phenomenon presents itself for other elements; hence, it is not likely that we can so easily explain away the discrepancy.

6. Adams and Russell derived[29] in 1928 the scale of stellar temperatures by comparing the number of atoms in different levels with the Boltzmann function. The factors of abundance and transition probability for each line were eliminated by using for each star relative values compared with the sun. Then the logarithm of the concentration ratio is a linear function of the excitation potential E:

$$\log Y = \text{Const.} - E\left(\frac{5040}{T'} - \frac{5040}{T}\right).$$

The relative concentrations (or numbers) of active atoms producing the lines were found by comparing the stellar with the solar-line intensities and calibrating the Rowland scale of solar intensities by means of multiplets. The *Fe* spectrum bears nearly 90 per cent of the weight, because here the levels extend continuously from $E = 0$ to 4.5 volts. It appeared that for α Orionis and α Scorpii, log Y was not a linear function of E; for large E the variation in log Y slowed down. So it was necessary to replace E in the formula by a quadratic expression $E - 0.083E^2$. This means that the limiting slope of the curve for $E = 0$ was used to derive the difference in $5040/T$. The same quadratic expression was then used for the other stars. In this way temperatures of 9900° for Sirius and 2900° and 3000° for α Ori and α Sco were found.

Here the line intensity, after calibrating, was directly connected with the number or the concentration of atoms. We now know that line intensity is directly connected with s/k, so that numbers expressing line intensities must be multiplied by factors k to find numbers expressing the concentration of atoms. Hence, corrections = $\log (k_{st}/k\odot)$ must be applied to log Y. The values of log k were taken from *Amsterdam Publications* No. 4, using $5040/T = 1.6$, $\log g = 1.4$, for α Ori and α Sco; $5040/T = 0.5$, $\log g = 4.4$, for Sirius; and $5040/T = 0.88$, $\log g = 4.4$, for the sun. The differences Star *minus* Sun vary with wave-length; Dr. Russell kindly provided me with the separate wave-lengths of the lines used, so that for each group the reduction could be computed. In Table V the k correct-

[29] *Ap. J.*, **68**, 9, 1928.

ions and the uncorrected and the corrected values of log Y are given. It appears that by thus taking account of the variations of the absorption coefficient with temperature and wave-length, the constant part of log Y is changed considerably, whereas the slope is

TABLE V

E	α Orionis, α Scorpii						Sirius		
	log Y		log Δk		log Y corr.		log Y	log Δk	log Y corr.
0.06	+2.05	+2.30	−1.30	−1.40	0.75	0.90	−2.66	+2.13	9.47
0.94	+1.28	+1.23	1.54	1.57	9.74	9.66	2.13	2.29	0.16
1.54	+0.54	+0.97	1.33	1.42	9.21	9.55	2.30	2.11	9.81
2.21	+0.30	+0.47	1.44	1.35	8.86	9.12	2.00	2.25	0.25
2.56	−0.06	−0.05	1.67	1.70	8.27	8.25	2.00	2.43	0.43
2.89	−0.19	−0.03	1.40	1.41	8.41	8.56	1.85	2.22	0.37
3.24	−0.33	−0.10	1.41	1.43	8.26	8.47	1.97	2.23	0.26
3.47	−0.54	−0.27	1.52	1.50	7.94	8.23	1.77	2.32	0.55
4.03	−0.76	−0.40	1.57	1.57	7.67	8.03	1.77	2.36	0.59
4.50	−0.87	−0.77	−1.58	−1.60	7.55	7.63	−1.60	+2.38	0.78

changed by only a small amount. Figure 4, where the corrected log Y are represented, shows that the curvature for α Ori and α Sco still exists. Retaining Russell's formula, and hence assuming that in all the stars the occupation of the higher levels follows $E - 0.083\,E^2$, we find the expressions

$$\log Y = 0.88 - 0.82\,(E - 0.083E^2) \text{ for } \alpha \text{ Ori and } \alpha \text{ Sco}\,,$$
$$\log Y = -0.50 + 0.41\,(E - 0.083E^2) \text{ for Sirius}\,.$$

Assuming that the temperature determining the observed distribution over the levels coincides with the effective temperature, we find the values of $5040/T = 1.70$ and 0.47; hence $T = 2960°$ for α Ori and α Sco and $T = 10{,}700°$ for Sirius. We have to consider here that the quadratic expression is only an interpolation formula which certainly does not hold for higher values of E, so that the limiting slope for $E = 0$, on which depends the temperature found, has the uncertainty of an extrapolated value.

The quadratic deviation from the Boltzmann distribution, the so-called "Adams-Russell phenomenon," has not found an adequate explanation as yet. Dr. Russell, in a letter, suggested the possibility that for these two Mo stars the value of log Y for the lowest E may be systematically wrong, because the lines belonging to the lowest levels are extremely strong in these stars and so cannot be exactly compared with the solar lines. If the result for $E = 0.06$ is excluded, the remaining values, as Figure 4 shows, may be represented by a

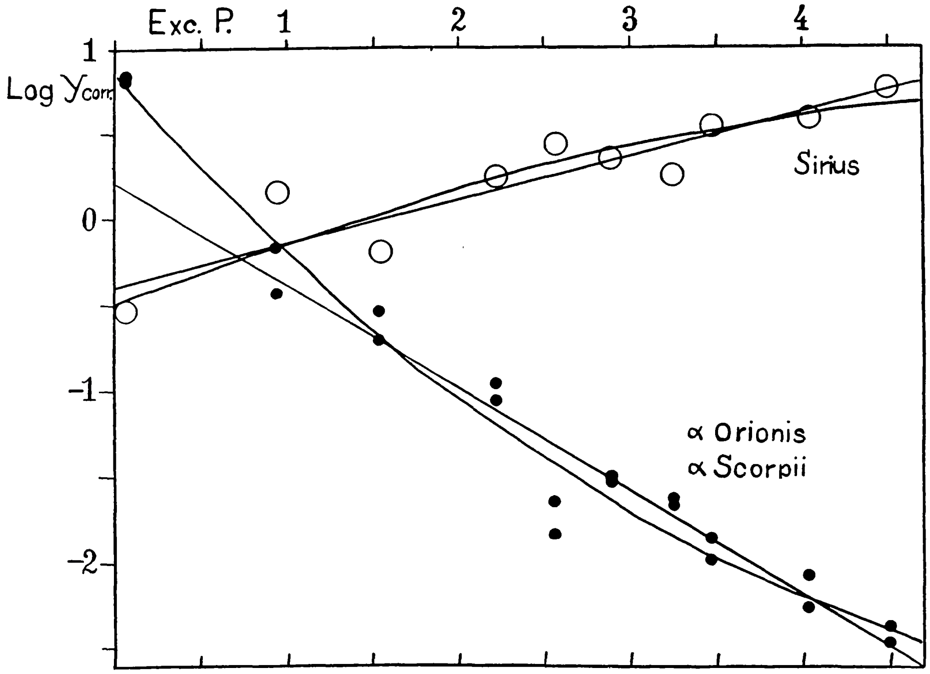

FIG. 4

straight line, which, of course, has a smaller slope. For Sirius, log Y as a function of E is as well represented by a straight line as by the curve of Adams and Russell. The resulting linear expressions are

α Ori, α Sco — Sun		Sirius — Sun	
log Y uncorr.	$= +1.60 \; -0.57E$	log Y uncorr.	$= -2.55 \; +0.21E$
log $k/k_{\odot}$	$= -1.40 \; -0.03E$	log $k/k_{\odot}$	$= +2.15 \; +0.05E$
log Y corr.	$= +0.20 \; -0.60E$	log Y corr.	$= -0.40 \; +0.26E$

Then for $5040/T$ we have 1.48 and 0.62; hence $T = 3400°$ for α Ori and α Sco, and $T = 8130°$ for Sirius. Because the mean slope used here is smaller than the limiting slope in the quadratic formula, the scale of temperatures is narrower.

The data of Adams and Russell used here can give still another result. The constant in the foregoing formulas represents the relative number of atoms for the star and the sun for $E = 0$, i.e., for the lowest level; it depends only on abundance and state of ionization. Comparing these values for *Fe* and Fe^+, the abundance of the element is eliminated in their ratio; and the same holds for the values of k, in so far as we may assume that the average wave-length for the *Fe* and the Fe^+ lines was the same. Hence, in the same way as in the case of *Ca* and Ca^+, this ratio is equal to the ratio of $x/(1 - x)$ for the star and sun. The values of log Y in the case of Sirius given by Adams and Russell for Fe^+, *Ti*, and Ti^+ were first corrected for k and then reduced to $E = 0$ by means of the slope derived from *Fe*.

TABLE VI

E	log Y	log Y corr.	$-0.26E$	log Y_0	E	log Y	log Y corr.	$-0.26E$	log Y_0
	Fe^+					*Ti*			
2.83	−0.18	+2.02	−0.73	+1.29	0.03	−2.00	+0.20	−0.01	+0.19
3.56	−0.54	+1.66	−0.95	+0.71	0.83	2.11	0.09	.22	− .13
					1.02	1.07	1.13	.27	+ .86
	Ti^+				1.44	1.13	1.07	.37	+ .80
					1.82	1.82	0.38	.47	− .09
					2.09	1.00	1.20	.54	+ .66
					2.26	−0.90	+1.30	−0.59	+0.71
1.16	−1.05	+1.15	−0.30	+0.85					
1.57	−0.78	1.42	.41	1.01					
2.00	+0.08	2.28	.52	1.76					
2.93	−0.75	+1.45	−0.76	+0.69					

Taking averages for log Y_0, the ratio log $Y_0^+ -$ log $Y_0^n =$ log $[x/(1 - x)$ St. : $x/(1 - x)\odot]$. We have

log Y_0 for *Fe* -0.40, for Fe^+ $+1.00$ log $Y_0^+/Y_0^n = +1.40$;

log Y_0 for *Ti* $+0.43$, for Ti^+ $+1.08$ log $Y_0^+/Y_0^n = +0.65$.

Computing log $x/(1 - x) =$ log K/P for different temperatures (log $g = 4.4$), we find:

	10,080°	8400°	7200°	6300°	5800°(☉)
Fe	3.68	2.78	2.03	1.37	0.87
Ti	4.11	3.29	2.63	2.10	1.64

The corresponding temperature for *Fe* is 7500°; for *Ti*, 6600°. Hence we meet with the same phenomenon as in the case of *Ca:* the decrease of the lines of the neutral atoms relative to the enhanced lines from the sun to Sirius is far too small for their large difference in temperature. Assuming a temperature of 10,000° for Sirius, all those arc lines which we see in the Sirius spectrum should be invisible after the ionization formula, provided the right curve of growth has been used. But these faint lines probably are situated on that part of the curve of growth where a large variation of atom concentration corresponds to a small change in line intensity. So it is probable, here as well as in the case of *Ca*, that pecularities of the curve of growth, depending upon turbulence and collisions in the star and upon the Weisskopf-Wigner corrections of the atoms, play a rôle and make the exact determination of temperatures by this method difficult. These computations of Y_0 and of their ratio have also been made with the quadratic formula; they give so nearly the same results that it is unnecessary to give them in detail. For the low temperature Mo stars the ratio log $Y_0^+/Y_0^n = -0.55$ for *Fe* and -1.32 for *Ti*. It is not well possible to use these results for the derivation of temperatures because, owing to the chromospheric character of their atmospheres, we do not know what pressure to use in the computation of the ionization. On the other hand, in so far as these relations are trustworthy, they may better be used to derive some information about the pressure in the relevant atmospheric layers, once their temperature is known.

7. The effective temperature itself is derived for the sun by measuring the total radiation, the solar constant, and reducing it by means of the angular semi-diameter of the sun to radiation per unit surface. For the stars this method can be applied in a direct way only, where the angular diameter can be determined by interferometer measures, i.e., for red giants and supergiants. The total radiation, the bolometric magnitude, is then derived from the visual magnitude by means of the "reduction to bolometric magnitude," such as is given by Eddington:[30] for 3600°, −0.95 mag.; for 3000°, −1.71 mag.; for 2540° −2.59 mag. Then for α Sco we find from the angular semidiameter of 0".021 and from the visual magnitude 1.2 the

[30] *Op. cit.*, p. 138, 1926.

temperature 2980° (red. to bol. mag. = −1.74; bol. mag. = −0.54). Eddington's table of reduction from visual to bolometric magnitude has, however, been computed for black-body radiation. Hence, this derivation does not materially differ from the computation made in the textbook of Russell, Dugan, and Stewart, where the black-body radiation for λ 5290 is compared with the visual magnitude of the star. A better approximation will be reached if, for the reduction from visual to bolometric magnitude, we make use of the spectral intensity-curve computed here for the stellar radiation. The curve for 3150° in Figure 3 gives a reduction of −2.70 mag., i.e., 1.2 mag. larger than the black-body-curve gives for the same temperature. Making use of the interpolated values: for 3360°, −2.02 mag.; for 3600°, −1.46 mag., we find now from the same data on α Sco the temperature 3310° (red. to bol. mag. = −2.18).

Though in this way the systematic error of former derivations is avoided, the method remains somewhat unsatisfactory because the total radiation is derived from the measurement of a minute part of it at the extreme border of the spectrum. To find the effective temperature directly, bolometric or radiometric measurements of the total radiation are needed. Pettit and Nicholson have measured stellar radiations with vacuum thermocouples,[31] and by means of absolute standardization of their instrument in various ways they could derive stellar temperatures from the interferometer diameters. They found 3270° for α Ori and α Sco.[32] The measures had first to be reduced to "no atmosphere," i.e., corrected for the atmospheric absorption bands in the red and infrared; this correction was derived from measures on solar energy-curves, of course, by supposing the stellar radiation to be black-body radiation of a certain low temperature. This involves the possibility of a certain error; the radiation of the star has a different energy distribution from a black body, and for its derivation even the knowledge of k may not be sufficient because of the strong molecular absorption bands in the stellar spectrum itself. Still, it is not likely that appreciable errors will arise, because the atmospheric bands are well distributed over the most contributing parts of the spectrum. Hence this determination deserves a large weight. We might suppose that a certain limb-darkening of these stars, now theoretically determinable from the knowledge of k, could

[31] *Ap. J.*, **68**, 279, 1928.

[32] *Ibid.*, p. 301.

produce an important correction to the angular diameters derived from interferometer measures. The figures of Table III, however, show that p and $\bar{k}/k$ for visual rays is very small; hence there is no appreciable limb-darkening here.

The accordance of the two results based upon interferometer measures is very satisfactory now, so that 3300° may be assumed as the effective temperature of these cMo stars.

8. The same data may be found in an indirect way from eclipsing variables. If all the data about the eclipses, and hence the ratios of the stellar diameters to the orbit, are well known, the radial velocity gives the diameters in kilometers. Then we want the parallax to derive angular diameters, which, when combined with the observed radiation, give the temperatures.

This method has been applied by S. Gaposchkin[33] to a large number of eclipsing variables. Because most of the parallaxes were highly uncertain, only a rough accordance with the scale usually adopted, with many large deviations, could be ascertained. Lately Pilowsky[34] made another statistical application of this method. A number of eclipsing variables with known radial velocities gave stellar diameters, which were plotted as a function of the spectral class. From a number of binaries the Russell diagram was derived, giving absolute magnitude, i.e., total radiations, as a function of spectral class. Combining both curves, we find the temperature as a function of spectral class. The uncertainty here arises from the individual deviations from the statistical curves; in the first-named curve a deviation of 0.3 in log R occurs, which means a factor of 1.4 in T. In order to strengthen the correlation, the statistical relations of radius, absolute magnitude, and spectral class to mass were determined; but here the deviations from the curve are still larger. Though a general concordance with the usual scale of temperatures is arrived at, it is not likely that this statistical treatment will allow great accuracy. The physical correlation between spectral class and temperature is much narrower and more exact than the statistical correlation between either of them and such data as mass, diameter, and absolute magnitude. So it will be better to look for separate stars for which all the data are well known.

We will consider here only spectral class A, as a well-marked

[33] *A.N.*, **248**, 213, 1933.

[34] *Zs. f. Ap.*, **11**, 265, 1936.

point in the scale of stellar temperatures. Among them we have only three stars with sufficiently reliable trigonometric parallaxes: β Persei, $0''.031 \pm 4$; β Aurigae, $0''.037 \pm 4$; and α Coronae, $0''.053 \pm 10$. For the first star the partial eclipse leaves the ratio of stellar to orbital radius rather indeterminate. For the first and the third stars the velocity of the companion has not been observed; so the relative orbit can be derived only from uncertain estimates of mass. Only β Aur remains, for which the computation has been given already in Russell, Dugan, and Stewart's textbook. For the two extreme suppositions of uniform disk and of completely darkened limb, Shapley found $r/a = 0.147$ and 0.159. With m for each component, $2.07 + 0.75 = 2.82$, with the reduction to bolometric magnitude -0.24 (for $10,000^\circ$, following Eddington), with the radial velocity of 220 km/sec, and with the period of 3.960 days we find for T the values 9630° and $10,130^\circ$. From the energy distribution of Table II and of Figure 2, however, the reduction to bolometric magnitude is found to be 0.23 mag. larger than for black-body radiation; taking this difference into account, the values for T are $10,300^\circ$ and $10,740^\circ$. Hence, $10,500^\circ$ may be assumed as the temperature of this A0 star.

9. Thus, two points in the scale of stellar temperatures may be considered as well established: $10,500^\circ$ for A0 and 3300° for cM0; we may estimate the limit of uncertainty of these values to be nearly 5 per cent of their value. They have been derived by direct methods of determining the effective temperature itself. The results from other methods which gave the first results for temperatures, from color and energy distribution, from ionization, and from the Boltzmann function are so strongly dependent on complicated conditions in stellar atmospheres that it is, as yet, not possible to derive reliable temperatures from them. On the other hand, the temperatures now established may enable us to make a decision between different assumptions made in the discussion of energy gradients and line intensities. With the progress of the theoretical treatment of stellar atmospheres and their absorption coefficients, an increasing accordance may be expected. But this degree of accordance will serve rather as a test of the theories and as a source of knowledge concerning stellar atmospheric conditions than as a determination of temperature.