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Surface gravity in supergiant stars, by *A. Pannekoek.*

1. The computations on the Stark effect of hydrogen lines in class A stars made at the Amsterdam Astronomical Institute by S. VERWEY [1]) make it possible to derive the gravity acceleration at the surface of a star from the width of the hydrogen lines. These results have been applied by C. SCHALÉN [2]) to his measurements of the line profiles in some A-stars. For the supergiant star α Cygni he derives $\log g = 1{\cdot}2$ (from the width of $H\alpha$ and $H\delta$), assuming the temperature to be 11200°. Because the hydrogen lines are increasing in width for decreasing temperature the result for the gravity must be lower for a lower temperature. Since α Cygni has a spectrum cA2, indicating a greater strength of the metal lines than for A0-stars, its temperature probably is below 10000°. Hence the value 1·2 must be considered as rather too high; for $T = 9100°$ it would be lowered by 0·5.

SCHALÉN makes use of this value of $\log g = 1{\cdot}2$, combined with an absolute magnitude of the star $\mathrm{M} = -3{\cdot}4$, to derive a value of its mass of 0·16 times the sun's mass, entirely at variance with EDDINGTON's mass-luminosity relation. Now it is true that the assumed absolute magnitude is somewhat weakly founded, since it is derived from the parallactic motion of a group of stars, selected as c-stars by their narrow hydrogen lines, in which the main part is formed by O-type stars, with α Cygni as single cA-star added; so it is not certain that they may be treated as a homogeneous group. Nevertheless the discrepancy with EDDINGTON's relation is not dubious. It may be seen at once when for giant stars fitting to the mass-luminosity curve the surface gravity is computed. We have

$$\log g/g_{\odot} = \log M - 0{\cdot}4\ (4{\cdot}8 - \mathrm{M}) + 4 \log T/T_{\odot}$$

where M is the mass, M the absolute magnitude and T the temperature. Taking M and M from EDDINGTON's Table 14 [1]) and considering that a correction factor $-0{\cdot}8 \log T/T_0$ for effective temperature has to be added, we find the results of Table 1 for $T = 10000°$ (and for $T = 3000°$).

TABLE 1.

$\log M$	M	$\log g$ $T = 10000°$	$\log g$ $T = 3000°$
9·11	14·14	7·96	6·29
9·77	6·93	5·73	4·06
0·00	4·64	5·05	3·38
0·75	−1·16	3·48	1·81
1·29	−3·92	2·91	1·24
1·96	−6·71	2·47	0·90

Here it appears that even for the most massive and luminous stars of type A g does not fall below 10^2. The gravity derived by SCHALÉN from the spectrum of α Cygni – which probably is still too large – is much smaller than can be accounted for by any high luminosity, provided that the mass-luminosity relation holds for them.

2. A discrepancy of an analogous character appears in the results derived in 1925 from an investigation of line intensities in Cepheid variables by Dr. J. J. M. REESINCK and the present author [2]). From these line intensities, compared with the intensities in a series of other stars of known gravity and temperature values of gravity and temperature were derived for maximum and minimum of δ Cephei, ζ Geminorum and α Ursae minoris. They led to the result, emphasized in that paper "that a large mass, in accordance with EDDINGTON's relation, is highly improbable for the Cepheids, and that there is some probability that their masses are even smaller than the sun's mass". (p. 56). The values of $\log g/g_{\odot}$, on

1) *Publications Amsterdam* No. 5.
2) A study of hydrogen and silicon line contours in some peculiar stars of type A (*Upsala Univers. Arsskrift* 1936 : 8).

1) *The Internal Constitution of the Stars*, p. 137.
2) Studies on line intensities in stellar spectra, II. The variations in the spectrum of some Cepheids. *B.A.N.* No. 87 (Vol. 3, p. 47).

which this conclusion was founded, were: — 3·70, — 3·82 (δ Cep M), — 4·44, — 4·01 (δ Cep m), —4·47, —4·52 (α UMi M), —4·51, —4·59 (α UMi m), — 4·84, — 4·37 (ζ Gem M and m). Hence for these three stars we have log g = 0·4, — 0·1, — 0·2, even smaller than SCHALÉN's value for α Cygni. The theoretical value deduced from M and M satisfying the mass-luminosity relation (e.g. log M = 0·95, M = — 2·3), for T = 6000°, is log g = 2·5. It is true that the spectral values were found by means of a strong extrapolation from a linear relation, so that their exact amount is rather uncertain. Still such a large value as 2·5 cannot be reconciled with the observed line intensities (cf. the diagram Fig. 3, p. 56 in the paper quoted). So there remains a result of the same character as with α Cygni: the surface gravity deduced from the spectrum (there from the Stark broadening of H-lines, here from the relative intensities of arc and spark lines) is much smaller than the value computed from mass and radius.

3. The possibility that perhaps for such supergiants the Eddington relation should not hold, is ruled out for the third case of discrepancy we have now to consider, that of ζ Aurigae. Here mass and radius both are well determined from the elements of the eclipse and the radial velocities. From $M = 3·04 \times 10^{34}$, log $M/M_{\odot}$ = 1·19, R = 1·34 × 10^{13}, log R/R$_{\odot}$ = 2·29, we find log $g/g_{\odot}$ = — 3·39, i.e. log g = 1·0. The decrease of density in the atmosphere has been derived by MENZEL from Miss SWOPE's light-curve. Representing it by exp. $(-\frac{\mu g}{RT} h)$, he finds $\frac{\mu g}{RT} = 8 \times 10^{-13}$, hence for T = 3200°, log R = 7·92, we have log g = — 0·68 — log μ. Thus in the case of an atmosphere chiefly composed of hydrogen the gravity determining the density gradient is 50 times smaller than the dynamical value, and for a larger molecular weight the difference would be still larger.

4. In all these supergiants we have the common feature that the observed surface gravity, deduced from spectral or photometric data, is about a hundred times smaller than the gravity derived from mass and radius.

An explanation by radiation pressure offers itself immediately. Since the pioneer studies of MILNE on the equilibrium of the solar chromosphere we know that radiation pressure is important for Ca +, and in some cases for H and He, but that usually this effect is small, except for very high temperatures. In the case of supergiants the pressure effects on other atoms may be perceptible, because here they find only a small gravity to counteract. Different elements will be differently affected, only some of them experiencing a notable radiation pressure, when they absorb light of average wavelength from the lowest levels in producing strong lines. By their collisions the effects are smoothed, and the atmosphere as a whole experiences a small average radiation pressure, which works against gravity.

A curious circumstance, however, presents itself with this explanation. If we ascribe the difference between the observed and the dynamical gravity to radiation pressure, we have for

α Cygni	dyn. gr. 800 c/s;	obs. gr. 10 c/s;	rad. pr. —790 c/s
Cepheids	„ 320 „ ;	„ 1 „ ;	„ —319 „
ζ Aurigae	„ 10 „ ;	„ 0·2 „ ;	„ — 9·8 „

Radiation pressure depends on temperature and atomic properties, but has nothing to do with the dynamical gravity. Yet we find that in each case it amounts to just 98 or 99 percent of the gravity. We do not see any physical reason why by chance it should be in each case exactly 1 or 2 percent less than the gravity. If, from the general trend of the figures, we are induced to assume for these supergiants mean radiation pressures of the same order of magnitude as their gravity, their exact values may fall as well above as below the gravity values of each separate star. Then there must be other agencies at work, producing an apparent positive residual gravity of only a small fraction of the real gravity. If radiation pressure exceeds gravity, on the whole or in some parts, the atmosphere cannot be in equilibrium. Upward motions of the gases will occur which, perhaps, break down now and then, and produce a different distribution of density, such as belongs to a different apparent gravitation. The most regular case would be that the gases of the atmosphere have a continuous outward motion, and in this moving gas the density will show an outward decrease corresponding to a small positive gravity. It may be of interest to see, whether in this way an explanation may be found.

5. In working out the consequences of this hypothesis we cannot assume the effective gravity (gravity minus radiation pressure) to be constant throughout the atmosphere. The radiation pressure is due to radiations of certain λ which are absorbed by the atoms considered. These radiations have intensities smaller than blackbody radiation of the same λ, because the deeper layers of atoms have already absorbed a good deal of their energy. The residual intensity of the net stream (which determines the radial outward pressure) in the surface layers is given (under simplified conditions) by $\sqrt{k/(k+s)}$, where k is the continuous, s the much stronger mono-

chromatic absorption coefficient. In the deepest layers, however, it is given by $k/(s+k)$; the net stream of strongly absorbed radiation is very small in the deepest layers, and rises to a higher limiting value when the surface is approached. So the outward pressure in the deepest layers will be smaller than at the surface. If in the surface layers there is an excess of radiation pressure over gravitation, the reverse will be the case in the deeper layers. Hence instead of a constant value we will have to assume for the effective gravity a more complicated function, positive in the deeper layers and decreasing to a negative value in the higher levels of the atmosphere. We denote by $-g_1$ the effective gravity in the highest, by $-g_1+g_2$ the effective gravity in the deepest levels. Then the function

$$-g_1+g_2/(1+e^{a(r-r_0)})$$

gives a gradual transition between these extreme values, taking place chiefly in the layers where $a(r-r_0)$ varies between $-$ and $+2$ or 3. For a we take $100/r_0$. Instead of a constant g, we take for the outer layers A/r^2. Then the equation of motion for the atmospheric particles under the influence of this effective gravitation and the density gradient is

$$v\frac{dv}{dr}=\frac{A}{r^2}-\frac{g_2}{1+e^{a(r-r_0)}}+\frac{RT}{\mu}\frac{1}{\rho}\frac{d\rho}{dr}, \qquad (1)$$

or

$$\tfrac{1}{2}v^2=-\frac{A}{r}+\frac{g_2}{a}\ln\left(1+e^{a(r-r_0)}\right)+\frac{RT}{\mu}\ln\rho+\text{Const.}$$

By the condition of continuity we have $v\rho r^2=F$, the constant outward stream of matter. Hence

$$\frac{\mu}{RT}\frac{A}{r}-\frac{\mu}{RT}\frac{g_2}{a}\ln\left(1+e^{a(r-r_0)}\right)+\frac{\mu}{2RT}\frac{F^2}{\rho^2r^4}-\ln\rho=C,$$

or, putting $r/r_0=x$ and introducing new constants

$$\frac{\alpha}{x}-\beta\ln\left(1+e^{100(x-1)}\right)+\frac{\gamma}{\rho^2x^4}-\ln\rho=C \quad (2).$$

As an example for numerical computation a star was taken with $r_0=10^{13}$ (nearly as ζ Aurigae), $\mu=1$, $T=6000°$, hence $\log RT/\mu=11{\cdot}7$. For the gravitational terms $\alpha=40$, $\beta=2$ was taken, corresponding to $g_1=2$ c/s, $g_2=10$ c/s. In solving this equation (2) we have to consider that the gravitational potential, expressed by the two first terms, as a function of x ($\varphi(x)$), shows a maximum of $38{\cdot}897$ for $x=1{\cdot}01421$, where $g=0$. The two other terms, considered as a function of ρ (somewhat distorted by the factor $1/x^4$) show a minimum near $\rho^2=2\gamma$. The scales of x and ρ in these two functions must be fitted in such a way that maximum and minimum coincide and their curvatures are neutralized, so that they add everywhere to a constant C. Then for every x the corresponding density ρ is found. The results of this computation (for $\gamma=1$) are given in Table 2. The effective gravity at each level is given by the variation of the gravitational terms $\varphi(x)$ with height; the apparent gravity is given by the decrease of $\ln\rho$ with height:

$$g_{\rm eff.}=10^{-13}\frac{RT}{\mu}\frac{d\varphi}{dx};\quad g_{\rm app.}=10^{-13}\frac{RT}{\mu}\,2{\cdot}30\,\frac{d\log\rho}{dx}.$$

The outward velocity of the atmospheric gas is given by the γ-term:

$$v^2=\frac{2RT}{\mu}\frac{\gamma}{\rho^2x^4};\ 2\frac{RT}{\mu}=10^{12};\ v=10\sqrt{\frac{\gamma}{\rho^2x^4}}\,\text{km/s}.$$

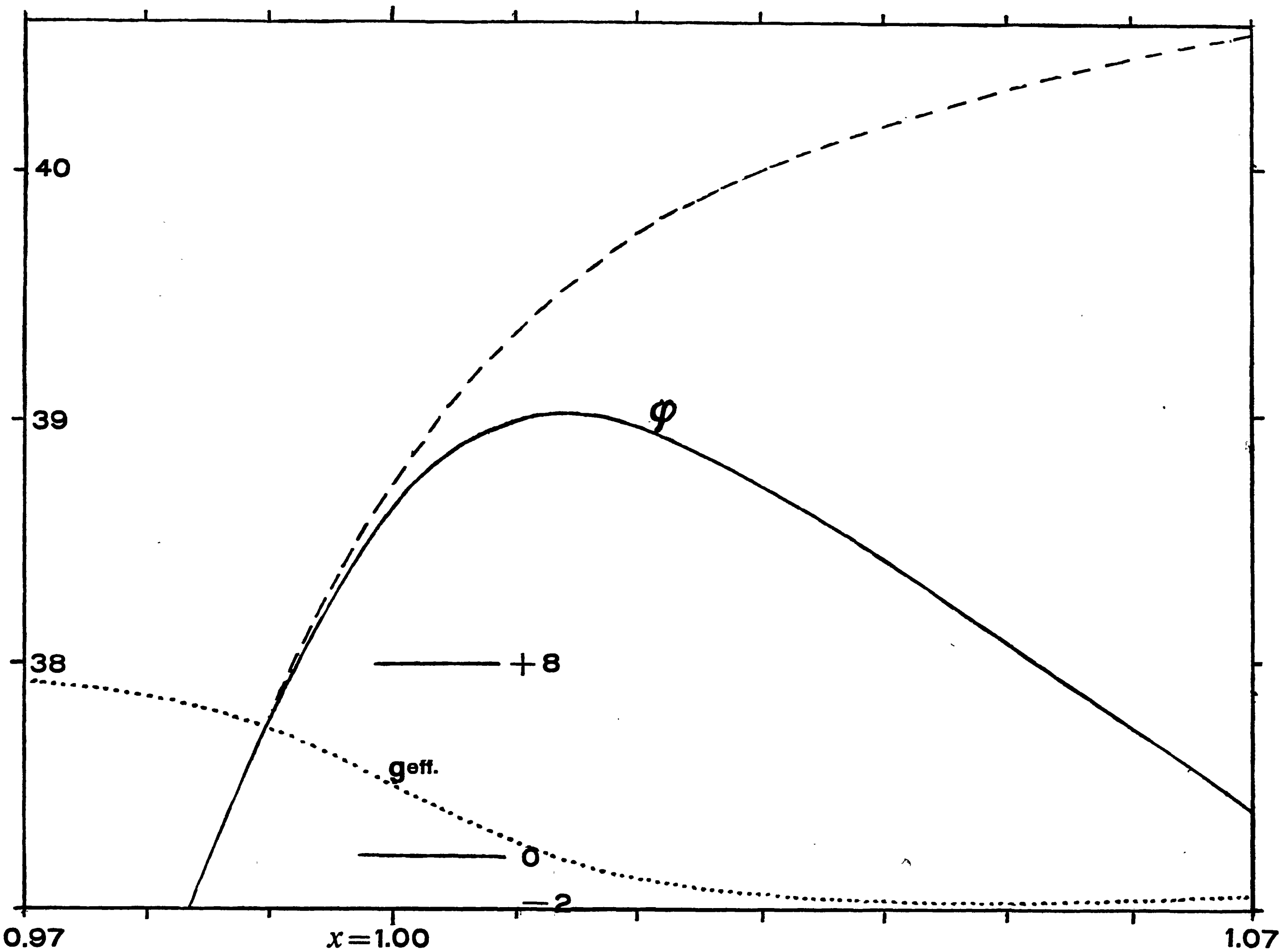
In the diagram Fig. 1 the full line represents the gravitational terms $\varphi(x)$; the dashed line represents the sum total when the γ-term is added to them, so that their vertical distance gives the influence of the stream motion in changing the negative effective into a positive apparent gravity. The latter is indicated by the slope of the dashed line that gives $-\ln\rho$ as a function of the height.

TABLE 2.

$x=r/r_0$	$\log\rho$	grav. terms φ	γ/ρ^2x^4	$g_{\rm eff.}$	$g_{\rm app.}$	v km/s
1·09	−0·376	36·6972	3·994			20·0
1·08	− ·344	37·0364	3·581	−1·70	+0·37	18·9
1·07	− ·308	37·3814	3·154	−1·73	0·41	17·8
1·06	− ·267	37·7308	2·709	−1·75	0·47	16·5
1·05	− ·218	38·0818	2·243	−1·75	0·56	15·0
1·04	− ·157	38·4254	1·762	−1·72	0·70	13·3
1·03	− ·077	38·7376	1·265	−1·56	0·92	11·2
1·02	+ ·037	38·9622	0·778	−1·12	1·31	8·8
1·018	·066	38·9868	·687	−0·62	1·63	8·3
1·016	·097	39·0022	·600	−0·39	1·78	7·7
1·014	·131	39·0070	·517	−0·12	1·95	7·2
1·012	·169	38·9991	·438	+0·20	2·18	6·6
1·010	·210	38·9774	·366	+0·54	2·33	6·1
1·008	·254	38·9408	·301	+0·92	2·53	5·5
1·006	·303	38·8864	·242	+1·36	2·84	4·9
1·004	·356	38·8148	·191	+1·79	3·07	4·4
1·002	·415	38·7240	·147	+2·27	3·36	3·8
1·000	·479	38·6136	·110	+2·76	3·68	3·3
0·99	0·883	37·7774	·0165	+4·09	4·64	1·3
·98	1·417	36·5628	·0013	+6·07	6·15	0·4
·97	2·035	35·1398	·0001	+7·11	7·11	0·0
·96	2·691	33·6306	·0000	+7·54	7·54	

It is not necessary to repeat the computation with another value of γ. If γ is changed by a certain factor, we have to change ρ^2 by the same factor, $\ln\rho$ is changed by a constant, which simply changes C. This means that then for the density of the atmosphere and for the stream intensity a multiple of the former value is taken, but the velocity at each level remains the same.

6. So we see that in this way with a resultant outward acceleration it is possible to find a positive



apparent gravitation. But the velocity of the outward stream of matter is rather high, of the order of magnitude of 10 km/s. In order to see how it depends on the arbitrary assumptions of our model, we put the equation of motion in this form

$$v\frac{dv}{dr} = \frac{1}{2}\frac{d}{dr}(v^2) = \Delta g = g_{\text{app.}} - g_{\text{eff.}}.$$

Hence v^2 is determined solely by the difference in g we want to explain, and by the height of the atmosphere over which the integration of Δg has to be extended. In our assumed case the velocity 10^6c/s is determined by $\Delta g \backsim 2$ and a dimension of the atmosphere $10^{11} - 10^{12}$. The high value of 10 km/s thus is directly connected with the size of these supergiants and the large extent of their atmospheres.

In the bright line stars of the Wolf-Rayet type we have matter streaming radially outward with much higher velocities; it forms a transparent outer atmosphere producing bright lines symmetrically widened by the Doppler effect. In the case of ordinary non-transparent stars we have only the Doppler effect of the visible hemisphere of the star, which should produce a small broadening of the absorption lines and a decrease of the apparent radial velocity of the star by half the amount of the outward velocity. For single stars it will be difficult to find out whether the observed radial velocity is too much negative. It could be decided, if such a supergiant belongs to a group with common motion, or is a member of a spectroscopic binary.

In the case of ζ Aurigae we have empirical data, which, however, are not favourable for this explanation. At the beginning and the end of the eclipse sharp absorption lines of Ca+ and H are produced by the light of the B-star shining tangentially through the lateral parts of the atmosphere. These lines show the radial velocity of those lateral parts, produced by the rotation of the star; but they are free from an outward radial velocity of the atmospheric gases, which should appear in the mean radial velocity derived from other absorption lines of the K-star. The measures made by W. H. Christie and O. C. Wilson indicate for the K-star (from the other