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The Planetary Theory of Newton

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I

The title under which Newton's chief work became famous in the history of human knowledge, "Philosophiae naturalis principia mathematica," suggesting as it does nothing astronomical, finds its explanation in the beginning of his third book. Having dealt with "the motion of bodies" in the first and second books, in the third he turns to astronomy in treating "the system of the world." Here he tells how first he had written it in a popular way in order that many should be able to read it; but afterwards, considering that those not sufficiently acquainted with the first principles, would not be able to understand the strength of the conclusions and would not give up their prejudices, and, wishing to prevent needless disputes, he chose to give it the mathematical form of propositions. Thus it can be read only by those who first have mastered the mathematical principles laid down in the first book. Not all the propositions demonstrated there will be needed, but only the first sections, the definitions and laws of motion, and such propositions as will be quoted afterwards.

Thus mathematics will be the foundation; mathematics applied to the forces and motions of the bodies. This is what we call the science of mechanics. In his preface to the first edition Newton himself introduces this name of rational mechanics, and he makes a highly instructive comparison with geometry. Practical mechanics, in the manual arts, works more or less imperfectly, just as practical geometry in drawing the lines is not perfect, whereas from olden times the use of geometry was to demonstrate and solve propositions. "Therefore geometry is founded in mechanical practice, and is nothing but that part of universal mechanics which accurately proposes and demonstrates the art of measuring. But since the manual arts are chiefly employed in the moving of bodies, it happens that geometry is commonly referred to their magnitude, and mechanics to their motion. In this sense rational mechanics will be the science of motions resulting from any forces whatsoever, and of the forces required to produce any motions, accurately proposed and demonstrated. . . I consider philosophy rather than arts and write not concerning manual but natural powers, and consider chiefly those things which relate to gravity, levity, elastic force, the resistance of fluids, and the like forces, whether attractive or repulsive; and therefore I offer this work as the mathematical principles of philosophy, for the whole burden of philosophy seems to consist in this—from the phenomena of motions to investigate the forces of nature, and then from these forces to demonstrate the other phenomena. . ."[1]

Rational mechanics was the discipline needed to unite earthly and celestial motions into one system. Earthly motions were ruled by Galileo's laws of falling and gravity, celestial motions were ruled by Kepler's laws of planetary orbits; there was no connection, no link between them. Gradually in the 17th century opinions were expressed that orbital motion was a kind of equilibrium between a centrifugal and a centripetal force. But such ideas could be fruitful only when cleared up by a rational system of fundamental concepts and rules. This was the first thing Newton had to do in his introductory "definitions" and "axioms, or laws of motion." Here he states the principles, since then appearing in every textbook, that every body, if not impelled by a force, continues in its state of rest or of uniform motion; that the change of motion, the acceleration, is proportional to the motive force; and that the mutual actions of two bodies upon each other are equal and contrarily directed. Whereas the first named principles easily proceed from what Galileo and his successors had found, the third one is explained at length to show how it is involved in all practical mechanics. Mass is introduced as quantity of matter "arising from its density and bulk conjointly," sometimes also simply called "the body"; "the quantity of motion arises from the celerity multiplied by the quantity of matter; and the motive force arises from the accelerative force multiplied by the same quantity of matter." Mass and weight are clearly distinguished. "Hence it is that near the surface of the earth where the accelerative gravity, or force productive of gravity, in all bodies is the same, the motive gravity or the weight is as the body; but if we should ascend to higher regions where the accelerative gravity is less, the weight would be equally diminished, and would always be as the product of the body, by the accelerative gravity."

Because the chief aim is the treatment of the freely moving heavenly bodies, here in the introduction centripetal forces are introduced "by which bodies are drawn or impelled, or any way bend, towards a point as to a centre." "Of this sort is gravity . . . and that force, whatever it is, by which the planets are continually drawn aside from the rectilinear motions, which otherwise they would pursue, and made to revolve in curvilinear orbits. . . They all endeavor to recede from the centres of their orbits; and were it not for the opposition of a contrary force which restrains them to, and detains them in their orbits, which I thereforce call centripetal, would fly off in right lines, with a uniform

[1] All the quotations are from the translation of F. Cajori, published Berkeley, 1934.

motion." Then, after mentioning a projectile shot from a mountain horizontally with sufficient velocity, which would go round the earth in an orbit, he proceeds: "the moon also, either by force of gravity, if it is endued with gravity, or by any other force that impels it towards the earth, may be continually drawn aside towards the earth, out of the rectilinear way which by its innate force it would pursue; and would be made to revolve in the orbit which it now describes; nor could the moon without some such force be retained in its orbit . . . it belongs to the mathematicians to find the force that may serve exactly to retain a body in a given orbit with a given velocity . . ." That he himself was this mathematician will appear later on.

Thus the edifice of rational mechanics was erected as the basis for an explanation of the heavenly motions. But this could not be done at the time without the simplifying notions of absolute time and absolute space. So he puts forward the theses: "Absolute, true, and mathematical time, of itself, and from its own nature, flows equally without relation to anything external. . . Absolute space, in its own nature, without relation to anything external, remains always similar and immovable. . . Place is a part of space which a body takes up, and is according to the space, either absolute or relative. . . Absolute motion is the translation of a body from one absolute place into another. . ." Newton was quite well aware that in common life (and in physical experiments as well) we always have to deal with relative places and relative motions. "It may be that there is no such thing as an equable motion, whereby time may be accurately measured. All motions may be accelerated and retarded, but the flowing of absolute time is not liable to any change." "But because the parts of space cannot be seen, or distinguished from one another by our senses, therefore in their stead we use sensible measures of them. . . And so instead of absolute places and motions, we use relative ones; and that without any inconvenience in common affairs; but in philosophical disquisitions, we ought to abstract from our senses, and consider things themselves, distinct from what are only sensible measures of them. For it may be that there is no body really at rest, to which the places and motions of others may be referred."

So absolute time, space, and motion are philosophical conceptions; they are abstractions, nowhere to be found in experience, but nevertheless deduced from the facts of experience by the abstracting power of human mind. Newton considers them as the true reality behind the phenomena; modern natural philosophy considering "observables" as the only reality, has abandoned them entirely. But it must not be lost from view that in the 17th century, in the gradual elaboration of mechanics as the science of motion, this conception of "absolutes" was necessary in order to subject the phenomena to simple mathematics; only by expressing absolute motions in simple formulas the observed relative motions could be deduced. The necessity of Newton's principles cannot be more stringently demonstrated than by the fact that only as late as two centuries afterwards—though relative place could be treated earlier already—the theory of relativity with its complicated algebra was able to handle relative time and relative motion in a satisfactory way. In astronomy, moreover, we have frames of reference the irregularities or the motions of which are so slight—the fixed stars, the rotation of the earth—that for practical use place, time, and motion relative to them could be considered as a sufficient approximation to absolute place, time, and motion, to suggest the conception of such absolutes.

Newton was able to lay the foundations of astronomy not only because, first, he was a renovator of mechanics, by elaborating its principles, but, secondly, because he was a renovator of mathematics too. The mathematics needed in these problems of curvilinear motion is the theory of fluxions, developed by himself at an earlier time already. In the *Principia,* it is true, no fluxions are found; it is all geometry. In some of the biographies it is said that he had worked out the problems and found their solutions by the method of fluxions, but that in order not to make the understanding too difficult by using an unfamiliar form of analysis he transferred them into the well-known forms of classic geometry. Perhaps it is more right to say that it is the spirit of the method of fluxions which pervades his geometry, the idea of considering quantities and motions not as definite values but in the process of originating, changing, or disappearing. This essence of the new fluxion method, our modern calculus, is most clearly elucidated in some sentences at the close of Section I: "by the ultimate ratio of evanescent quantities is to be understood the ratio of the quantities not before they vanish, nor afterwards, but with which they vanish. In like manner the first ratio of nascent quantities is that with which they begin to be. . . For these ultimate ratios with which quantities vanish are not truly the ratios of ultimate quantities [called indivisibles], but limits towards which the ratio of quantities decreasing without limit do always converge; and to which they approach nearer than by any given difference. . . Therefore if in what follows . . . I should happen to mention quantities as least, or evanescent, or ultimate, you are not to suppose that quantities of any determinate magnitude are meant, but such as are conceived to be always diminished without end."

Thus in his demonstrations Newton makes use of geometrical figures of straight lines, of triangles, rectangles of finite size; but then he lets the number of such parts be augmented and their size diminished *in infinitum,* to fit a curved orbit and a continually working force; and he shows that then the demonstration holds. It is for this reason that Book I opens with a first section on limiting properties and values in the case of evanescence of the quantities considered, and demonstrates that the versed sines in curves lines are as the squares of the chords, the tangents, and the arcs, and that the spaces which a body describes by a finite force urging it are as the squares of the times.

II

The first proposition serving to build a bridge from Kepler's empirical laws of planetary motions to the acting forces deals with the law of areas. If a revolving body is subject to a centripetal force directed to a fixed point, the areas described by radii drawn to that point will be proportional to the times in which they are described. For the demonstration Newton makes use of finite time intervals, after each of which the force gives a finite impulse to the body towards the centre; so the

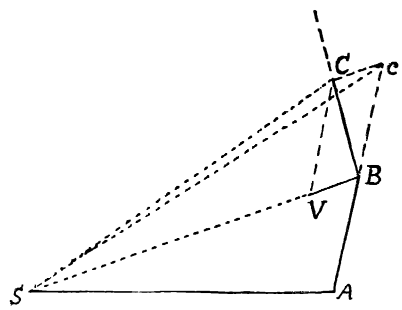

FIGURE 1

motion remains the same during an interval (from A to B in Figure 1) and then at B is changed suddenly by an additional motion BV; in the second interval, instead of continuing the motion along Bc = AB, the body follows the resulting path BC. Then, geometrically, the areas SAB and SBc are equal; the areas SBc and SBC are equal because the impulse BV was directed toward S; so the area SBC is equal to SAB. This holds for every further interval. Every next triangular area is equal to the preceding one, and they will lie in the same plane. "Now let the number of those triangles be augmented, and their breadth diminished *in infinitum;* and their ultimate perimeter will be a curved line." The impulses, having become continually smaller and more numerous, in the limiting case constitute a continually acting force, always directed to the centre S, and the areas remain proportional to the times.

In the same way the reverse is demonstrated. When a body moves in a plane curved orbit in such a way that the radii drawn to a point describe areas proportional to the times, it is urged by a centripetal force directed to that point. If in Figure 1 area SBC = area SAB, then area SBC = area SBc, hence Cc is parallel to BV, *i.e.,* the impulse by the force is directed along BS, toward the point S.

Thus Kepler's law of equal areas proves that the planets in their orbits are moved by a centripetal force directed to the sun.

Some further conclusions may be drawn. If in the limiting case a continuous orbit is described through the points ABC in the figure, then drawing the chord AC it appears that the versed sine belonging to the arcs AB and BC is half the distance BV, hence proportional to the force in B. In the simple case of circular motions the versed sines are equal to the squares of the arcs divided by the diameter, hence the forces will be as the squares of the arcs described in the same time divided by the radii of the circles. Since the arc in unit time is the velocity, the equation derived by Huygens for the centrifugal force comes out: that the centripetal forces are as the squares of the velocities divided by the radii; or, as the radii divided by the squares of the periodic times. Kepler had discovered and expressed in his third law that the periodic times of the planets were as the 3/2th power of the distance. So the velocities, in the simplified case of circular orbits, vary inversely as the square roots of the radii, hence the forces acting upon such planets are as the inverse squares of the distance. Thus the law of centripetal force toward the sun is derived from Kepler's third law.

III

In a more general way, however, this law can be derived from the laws of the elliptic motion of a single planet. Newton first derives a general expression for the centripetal force in a curved orbit. The deviation of the body from the tangent, in the direction of the centre, is as the force and as the square of the time; the time is given by the area of the sector described; hence the force is as the deviation from the tangent divided by the square of the area. Newton expresses it by saying that the force is inversely as a solid, the product of two line squares divided by a line being of the dimension of the cube of a length.

In proposition XI the computation is made for an elliptical orbit in which the centripetal force is directed to the focus. It is a computation

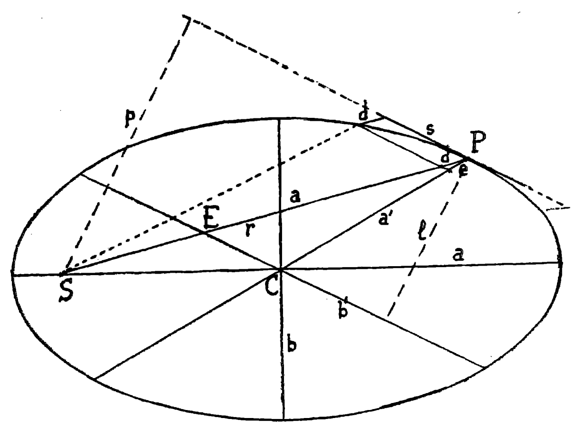

FIGURE 2

indeed, where by making use of the geometrical properties of the ellipse and the lengths of lines therein, a simple result comes out. Drawing in Figure 2 from the planet's place P the conjugate diameters $2a'$ and $2b'$, drawing the perpendiculars l and p, remembering that $PE = a$, denoting the planet's motion by s and its deviation from the tangent toward the focus, S, by d, we have to make use of the following properties and equations: (1) $l \times b' = a \times b$; (2) the deviation d toward S stands to the deviation e toward the centre C as $a : a'$; (3) the product $2a' \times e$ stands to the square of s as the square of a' stands to the square of b'; (4) the area described is $\frac{1}{2} s \times p$ where $p : r = l : a$. Then the expression of the force is given by

$$\frac{d}{s^2p^2} = (\text{by 4, above})\ \frac{da^2}{s^2l^2r^2} = (\text{by 2})\ \frac{ea^3}{a's^2l^2r^2} = (\text{by 3})$$

$$\frac{a^3}{2b'^2l^2r^2} = (\text{by 1})\ \frac{a^3}{2a^2b^2r^2} = \frac{1}{2\mathrm{L}r^2}$$

where L stands for the latus rectum b^2/a. Thus the law of inverse squares follows from the elliptic motion about the focus as the centre of the areas proportional to the times.

In modern treatment usually the elements of the elliptic motion, semi-axes and period of revolution, are at once introduced; the area described in time t, $\frac{1}{2} sp = \pi abt/\mathrm{T}$, and the deviation from the tangent is $\frac{1}{2}gt^2$, if g is the acceleration of the force, hence the expression for the force used above is

$$gT^2/8\pi^2a^2b^2,$$

and for g we find

$$(4\pi^2a^3/T^2) \times (1/r^2),$$

where in the factor before $1/r^2$ is expressed (because of Kepler's third law) the independence of the force from whatever planet we take.

Newton restricts himself here to demonstrating the geometric proportionalities; but he extends his work by deriving important general propositions on curvilinear motion and treating a number of other cases. In the case of an elliptic motion about the centre of the ellipse, the treatment makes use partly of the same relations as used above: (1) and (3) hold in the same way, for (2) we have the deviation e replacing d, and for (4) we now have the area $\frac{1}{2} l \times s$. Then the expressions for the force is

$$\frac{e}{s^2l^2} = (\text{by 3})\ \frac{a'}{2b'^2l^2} = (\text{by 1})\ \frac{a'}{2a^2b^2}.$$

So the centripetal force towards the centre of the ellipse is directly as the distance of the body from the centre.

Since the ellipse is only one of the conic sections, Newton of course treats also the other ones. For the force directed to the centre only a few additional words are needed to show that the same demonstration holds, and that in the parabola there will be a constant force, representing Galileo's case, and in the hyperbola there is a repulsive force varying as the distance. For the force directed to the focus the problem could with the same brevity be reduced to that of the ellipse; ". . . but because of the dignity of the problem and its use in what follows, I shall confirm the other cases by particular demonstrations." Thus he gives a separate demonstration that for a hyperbolic and for a parabolic motion the force directed toward the focus is as the inverse square of the distance. Since the importance of the conic sections now comes forward so strongly Newton treats them at length in a wealth of demonstrations and constructions.

IV

These mathematical propositions find their application in the third book, where first the "rules for reasoning in philosophy" are given. We find there as the first ones: "(1) We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances; (2) therefore to the same natural effects we must, as far as possible, assign the same causes." Then follow the "phenomena," which restrict themselves to the Kepler's third law connecting periods and distances for the Jupiter satellites, for the Saturnian satellites, and for the planets, and to Kepler's second law of areas holding for the moon. Then from these phenomena it is deduced that they all are drawn to their central bodies by forces that are inversely as the squares of the distances. For the moon a numerical computation is made: its distance 60 semi-diameters of the earth, its time of revolution $27^d\,7^h\,43^m$, the circumference of the earth 123,249,600 Paris feet; then deprived of its velocity it would descend towards the earth in one minute 15-1/12 Paris feet. If, then, this force increases, in approaching down to the surface of the earth 60×60 times, such a falling body at the surface of the earth ought to descend in one second the same 15-1/12 Paris feet. Huygens had deduced from the length of a pendulum oscillating seconds, which he found to be 3 Paris feet $8\frac{1}{2}$ lines, the space described by a falling body (through multiplication by $\frac{1}{2}\pi^2$) to be 15 feet 1 inch 1-7/9 line. "And therefore the force by which the moon is retained in its orbit becomes, at the very surface of the earth, equal to the force of gravity which we observe in heavy bodies there. And therefore (by Rule 1 and 2) the force by which the moon is retained in its orbit is that very same force which we commonly call gravity; for, were gravity another force different from that, then bodies descending to the earth with the joint impulse of both forces would fall with a double velocity. . ." It is well known that when first conceiving this idea in 1666, Newton made the computation with too small a value of the circumference of the earth, and only in later years could repeat it with the right values. He now strengthens his argument by presenting it in a "Scholium" in another way: supposing the earth to have more moons just as Jupiter and Saturn have, connected by Kepler's third law, and one to move near the surface, grazing the mountain tops, then it would exhibit a centripetal force obliging it to descend just the same amount in a second as a free falling body at the same mountain top; hence these forces must be identical. "Because otherwise this little moon at the top of a mountain must either be without gravity, or fall twice as swiftly as heavy bodies are wont to do." And then the conclusion is made: "The force which retains the celestial bodies in their orbits has been hitherto called centripetal force; but it being now made plain that it can be no other than a gravitating force, we shall hereafter call it gravity. For the cause of that centripetal force which retains the moon in its orbit will extend itself to all the planets."

Now the consequences are drawn. The moons of Jupiter gravitate towards Jupiter, the planets towards the sun. A power of gravity is tending to all the planets; Jupiter also gravitates towards its satellites, the earth towards the moon, all the planets gravitate towards one another. Since all the heavy bodies descend to the earth from equal heights in equal times (Newton tested it by pendulum experiments) their weights are proportional to their quantities of matter. "Forces which equally accelerate unequal bodies, must be as those bodies; that is to say, the weights of the planets towards the sun must be as their quantities of matter." Newton takes into consideration the possibility that it could be otherwise: "if some of these bodies [Jupiter's satellites] were more strongly attracted to the sun in proportion to their quantity of matter than others, the motions of the satellites would be disturbed by that inequality of attraction." The weights of bodies towards different planets, hence the quantities of matter in the several planets, computed from the distances and periodic times of revolving bodies, are found to be 1 for the sun, 1/1067 for Jupiter, 1/3021 for Saturn, 1/169282 for the earth (from a solar parallax of 10″.5). "The force of gravity towards any whole planet arises from, and is compounded of, the forces of gravity towards all its parts . . . If it is objected that . . . all bodies with us must gravitate one towards another, whereas no such gravitation anywhere appears, I answer that . . . the gravitation towards them must be far less than to fall under the observation of the senses." "The force of gravity towards the several equal parts of any body is inversely as the square of the distance from the particles."

V

The resulting principle that gravity is a universal force, exerted as an attraction from every body and every particle of a body, makes a reconsideration of some of the arguments necessary. The mathematical treatment was already given in the further sections of Book I.

Newton here always speaks of attractions. In the Scholium at the close of Section XI we read: "I here use the word attraction in general for any endeavor whatever, made by bodies to approach each other, whether that endeavor arise from the action of the bodies themselves, as tending to each other or agitating each other by spirits emitted; or whether it arises from the action of the ether or of the air, or of any medium whatever, whether corporeal or incorporeal . . ." After the mathematical investigation of the forces and the physical comparison with the phenomena of Nature "we argue more safely concerning the physical species, causes and properties of the forces. Let us see, then, with what forces spherical bodies . . . must act upon one another; and what kind of motions will follow from them."

Thus the attraction of extensive bodies is treated in Section XII. In the former demonstrations the centripetal force was directed to a point, and it was tacitly assumed that the centre of the solar or the planet's globe should be that point. But in reality the forces are attractions proceeding from the separate particles, and then we have to find the sum of them. It is known that Newton struggled a long time with this problem, and considered his work achieved only when he had solved it. This solution consists in the demonstration that a spherical surface layer does not exert any attraction upon a corpuscle inside, and attracts a corpuscle outside with a force inversely proportional to the square of its distance from the centre. The same holds for complete spheres if every spherical layer within is homogeneous. So it was legitimate to simplify the attraction of a celestial body as if it were condensed in its centre. A large number of demonstrations for the attraction of spherical as well as non-spherical bodies under different laws of attraction is added.

A second point is the mutual attraction of bodies. "I have hitherto been treating of the attraction of bodies towards an immovable centre," thus Section XI of the first Book begins, "though very probably there is no such thing existent in nature. For attractions are made towards bodies, and the actions of the bodies attracted and attracting are always reciprocal and equal . . . both revolve about a common centre of gravity." By comparison with the fictitious case of a body P revolving about a fixed body S at their total distance he shows that the two bodies describe concentric similar ellipses about their common centre of gravity, with areas proportional to the times, and in a periodic time $\sqrt{S}/\sqrt{(S+P)}$ relative to the case of comparison. In modern times, after the discovery of double stars, the latter result is more commonly expressed by saying that the periodic time is the same as if in the comparison case the sum total of the two masses were collected in the attracting centre.

Now the sun cannot be the immovable centre of the world any more. It has to abdicate as such in favor of the common centre of gravity of the entire system; this centre chiefly depends on Jupiter, and in a smaller degree on Saturn, and is situated near the surface of the sun; the sun itself is moving about this point in a somewhat irregular way.

More important, fundamentally, is the mutual attraction of the planets.

VI

Newton, by proceeding from Kepler's empirical laws, derived the universal law of mutual attraction, dominating the motions of the celestial bodies. Thus Kepler's laws, from empirical results based upon observation, hence partaking in the uncertainty unavoidable in any empirical rule, were elevated to the rank of absolute truths, necessary consequences of a fundamental law, rigidly holding, without any uncertainty. But at the same time this fundamental law of universal attraction asserted that Kepler's laws could not be strictly correct, that they were only approximations holding for the case that only two bodies existed, the sun and the planet; or, more correctly, that all the planets had masses infinitely small.

Though this apparent contradiction—that the foundation of the structure was destroyed by the structure itself—could not seriously disturb Newton in his derivation, we perceive it in the way in which in his discussion he first has to minimize the deviation. "But the actions of the planets one upon another are so very small, that they may be neglected" (in Prop. 13 of the third Book). Even in the case of the moon, where in Prop. 3 he enounces that the force by which it is retained in its orbit is inversely as the square of its distance from the earth, he says that this is evident "from the very slow motion of the moon's apogee; which, in every single revolution amounting but to 3° 3′ forwards, may be neglected."

He does not restrict himself to assertions only. In the first Book, after having derived the centripetal force for a fixed ellipse he treats the case of a body moving in a curve that itself revolves about the centre of force. Because, compared with the fixed ellipse, the body has to perform an additional transverse motion, it has, in order to arrive at the same distance, to perform an additional deviation towards the centre, requiring an additional force. Newton demonstrates in Prop. 44 that this additional force varies as the inverse cube of the distance. For the case of orbits approaching very near to circles he is able to deduce a number of numerical corollaries. If the centripetal force varies as the $(n-3)$th power of the distance, the descent of the body from the upper apse to the lower apse will take place over an angle $180°/\sqrt{n}$; hence for $n=4$ over 90° (elliptic motion about the centre), for $n=1$ over 180° (elliptic motion about the focus), for $n=0$ there is no apse, and the motion takes place, as was demonstrated in Prop. 9 already, in a logarithmic spiral. If the centripetal force is a composite function, consisting of an inverse square power diminished by a force direct as the distance, $(1/r^2 - cr)$, the body will return to the same apse after $360° \sqrt{(1-c)/(1-4c)}$; for $c = 1/357.45$ we find that in each revolution the apse will go forward $1° 31' 28''$. "The apse of the moon is about twice as swift." Here Clairaut's later result is forestalled that simple theory gives only half the motion of the moon's apogee.

The perturbations of the moon due to the solar attraction of course offer the first main problems. Newton has treated them in his first Book as a number of corollaries to Proposition 66, dealing with the problem of three bodies in a general way; and then again in his third Book, where especially the retrogression of the nodes is derived. The perturbations of the planets are so much smaller that they hardly are mentioned, or only as insignificant. "It is true," Newton says, "that the action of Jupiter on Saturn is not to be neglected . . . the gravity of Saturn towards Jupiter will be to the gravity of Saturn towards the sun as . . . 1 to about 211. And hence arises a perturbation of the orbit of Saturn in every conjunction of this planet with Jupiter, so sensible, that astronomers are puzzled with it. As the planet is differently situated in these conjunctions, its eccentricity is sometimes augmented, sometimes diminished; its aphelion is sometimes carried forwards, sometimes backwards, and its mean motion is by turns accelerated and retarded; yet the whole error in its motion about the sun . . . may be almost avoided (except in the mean motion) by placing the lower focus of its orbit in the common centre of gravity of Jupiter and the sun . . . and therefore that error, when it is greatest, scarcely exceeds two minutes yearly . . ." The perturbation of Jupiter by Saturn is much less. "The perturbations of the other orbits are yet far less, except that the orbit of the earth is sensibly disturbed by the moon," since it revolves monthly about the common centre of gravity. How the insignificance of the perturbations occupies his mind, compared with the effects of the solar attraction, appears where again in the next proposition (14) he asserts: "The aphelions and nodes of the orbits of the planets are fixed. . . It is true that some inequalities may arise from the mutual actions of the planets and comets in their revolutions; but these will be so small, that they may be here passed by." And reversing the usual argument he concludes: "the fixed stars are immovable, seeing they keep the same position to the aphelions and nodes of the planets." And concerning the planets near the sun he says in the Scholium added: "their aphelions and nodes must be fixed, except so far as they are disturbed by the actions of Jupiter and Saturn and other higher bodies." He derives that their aphelions move forward a little as the 3/2th power of their distances from the sun; so that, if for Mars the aphelion is carried forwards $33' 20''$ in a century, this displacement for the earth, for Venus and for Mercury will be

17′ 40″, 10′ 53″, and 4′ 16″. "But these motions are so inconsiderable, that we have neglected them in this Proposition."

This is all that Newton had to say on the perturbations of the planets. His was the great task of establishing the force of universal gravitation as the cause of the laws of planetary motion, thereby at the same time explaining the orbits of the comets—to which a large remaining part of Book III is devoted—the precession of the equinoxes, the tides of the oceans, and, in first indications, the irregularities of the moon. The perturbations of the planetary motions he had to leave to his successors.

VII

When Newton's *Principia* was published in 1687 the vortex theory of Descartes was universally acepted as the explanation of the planetary motions. Compared with old Aristotle who still in the first half of the 17th century dominated the academic chairs, the philosophy of Descartes was a considerable advance. Through the common experience of phenomena of floating objects carried by a stream and of dead leaves rushed by the wind, the idea that the planets were carried along by a light space-filling fluid circulating about the sun—and about the earth and the planets too—looked highly plausible. Huygens in a treatise "On the cause of gravity" had tried to explain gravity by applying his results on centrifugal force upon this revolving world aether. So Newton had to devote considerable time and space to the study of the motion of fluids and of bodies contained in them, which occupies his second Book. This Book contains far more than was necessary for a criticism of the prevailing ideas; but it makes the criticism of the vortex theory the more stringent. What is expressed in the concluding General Scholium of the entire work in the mild sentence: "The hypothesis of vortices is pressed with many difficulties," is demonstrated strictly and in detail in the end of the second Book: "Hence it is manifest that the planets are not carried round in corporeal vortices." The criticism was crushing, embodying the progress, in half a century, from vague philosophical talk to the clear mathematical and scientific demonstration.

So it might be expected that Newton's theory immediately, as by storm, would have conquered the minds of the fellow-scientists and replaced the old ideas. Nothing of the sort happened. It is true that in England he soon found admirers and adherents, who spread his new philosophy by smuggling it in the form of footnotes into a generally used textbook on Cartesian physics. But on the continent during half a century it remained unappreciated. The great physicists there knew his work and admired its mathematical demonstrations but did not see what it meant; its entire spirit was foreign to them. As Huygens wrote in 1691 in a reprint of his former treatise: "I had not thought of this regular decrease of gravity, namely that it is as the inverse square of the distance; this is a new and highly remarkable property of gravity, of which it is worthwhile to investigate the cause." Here it is evident that for him the cause of the planetary motions was not at all cleared up by Newton. The pressure of some world-filling medium against objects as applied in the vortex theory was an understandable cause of their moving; so Newton's law may show a way to clear up the character of this medium and its pressure, but that is all. And Newton's opinion that all particles, even deep inside the earth, should attract each other, looks quite absurd to Huygens because he thinks he sees clearly "that the cause of such an attraction cannot be explained by any principle of mechanics or by the laws of motion." And further on, speaking of gravity, he says "It would be otherwise if we should consider gravity an inherent property of the corporeal matter. But I do not believe that Newton thinks of that, for such a supposition would lead us too far astray from mathematical and mechanical principles." And Leibniz in a letter to Huygens still more clearly gives expression to their reluctance: "It seems that to him [Newton] gravity is nothing but a certain immaterial and inexplicable power (vertu), whereas you explain it very well through mechanical laws."

The curious thing is that in what here appears as a fundamental difference of philosophical opinion, Newton himself agreed with them. We quoted already some of his sentences about the causes of gravity and revolution. In a letter to Bentley he wrote: "That gravity should be innate, inherent and essential to matter, so that one body may act upon another at a distance through a vacuum, without the mediation of anything else . . . is to me so great an absurdity . . ." Notwithstanding the often quoted proud statement in his concluding chapter: "Hypotheses non fingo," he was not different from his contemporaries in wanting and fancying explanations of the phenomena through understandable causes. In private letters to his friend, Robert Boyle, he tries to explain the cause of gravity by means of a fine elastic "aetherial substance" filling up all space between the minute particles of the bodies. But also in the *Principia* itself this point of view is exhibited clearly enough. "Hitherto we have explained the phenomena . . . by the power of gravity, but have not yet assigned the cause of this power. This is certain, that it must proceed from a cause that penetrates to the very centres of the sun and planets. . . But hitherto I have not been able to discover the cause of those properties of gravity from phenomena, and I frame no hypotheses; for whatever is not deduced from the phenomena is to be called an hypothesis; and hypotheses, whether metaphysical or physical, whether of occult qualities or mechanical, have no place in experimental philosophy. . . And to us it is enough that gravity does really exist, and acts according to the laws which we have explained, and abundantly serves to account for all the motions of the celestial bodies."

Thus the merit and the greatness of Newton's work seems to be his resignation, his restriction to computation of effects by mathematical formulas. Certainly it was this that secured its victory. People could work with his theory. When he himself, and, following his traces, first English and then French theorists succeeded in computing and predicting ever more phenomena by means of the law of gravitation, when theory went from triumph to triumph, the old vortices, unable to produce anything of the kind, fell flat, disappeared and were forgotten. Since the middle of the 18th century the truth of Newton's theory was universally recognized. Truth, a modern writer says, is what you can work with. And now gradually there came another fundamental scientific creed. Gravitation did not have or want a cause; it was itself a cause. Though Newton had vehemently rejected the idea that he should believe in "action at a distance," scientists in the next century ever more considered forces acting at a distance a sufficient explanation of motions, and proclaimed them the real, essential, and sufficient causes of the observed phenomena, introduced wherever, as in electrical or magnetic phenomena, attraction or repulsion was observed.

In reality, when we say that gravity is the cause of bodies falling, we explain nothing. Gravity is a name, a word, that does not contain more than do the phenomena themselves; in calling them by this name we express their general character. By expressing the general character by a law, a mathematical formula, we are able to predict, to compute the motions of all falling or thrown bodies. The law embodies them all; it is the abstract concept our mind constructed out of the phenomena. Newton's work consisted in that he explained the celestial motions by extending earthly gravity over all bodies of the universe and finding its dependence on distance. So it is not, as it might seem at first sight, an attempt to explain an unknown by reducing it to another unknown. The "explanation" consists in that Newton's gravitation embodies a far wider field of phenomena into one common rule. Thus our image of the world is simplified; an endless multitude of the most diverse experiences in heaven and on earth, in the past and the present (and the future), is condensed in a single law, in a formula from which they all can be computed.

It is needless to look for further causes or to ask what gravitation is. Gravitation as a special something pulling at the bodies and steering them through space has no more separate existence than Snell's law of refraction as a somewhere given command to the light rays how they have to run through the different media. If they are called "causes" of which the phenomena are the "effects," it must be borne in mind that to modern science cause means the short summary or compendium, effect means the diverse multitude of phenomena. In this sense, then, Newton's gravitation is rightly called the cause of the heavenly