It seems probable that Hipparchus was struck by the deviation of his seasonal year from the Babylonian year, which was greater than $365\frac{1}{4}$ days and that he came to explain it by showing that they were two different things, viz., return to the same equinoctial points and return to the same stars. Thus he was led to his most important discovery, that of the progress of the stars relative to the equinoxes in the direction of the signs by a rotation of the celestial sphere about the poles of the ecliptic. This phenomenon, usually called the *precession*, may also be described as a retrogression of the equinoctial points. It appears in the tables of the Chaldeans in such a way that at different times different longitudes have been adopted as zero points; hence the Chaldeans are sometimes claimed to have been the true discoverers, from whom Hipparchus borrowed his knowledge. There can be no doubt, however, that it was Hipparchus who recognized it as a continuous regular progress; he derived its amount from a comparison of earlier Alexandrian observations with his own. Ptolemy tells the story in this way: 'In his treatise *On the Change of the Solstices and the Equinoxes*, Hipparchus, by exact comparison of observed lunar eclipses of his own time with others which had been observed by Timocharis in earlier times, arrives at the result that in his own time Spica preceded the autumn equinox by 6° and in Timocharis' time by 8°.'[64] In the middle of an eclipse the moon stands exactly opposite the sun, and the longitude of the sun can be derived from its declination, i.e. by measuring its altitude at midday; so by measuring the distance of Spica from the eclipsed moon, its longitude, i.e. its distance from the equinox, can be derived. 'And also for the other stars which he compared he shows that they have proceeded by the same amount in the direction of the zodiacal signs.'

From this change of 2° in an interval of 169 years a yearly variation of 45″ is derived. The curious thing is that this value does not occur with Hipparchus (i.e. it is not in Ptolemy's book) but that the latter quotes from his treatise *On the Length of the Year* thus: 'When by this reason the solstices and equinoxes in one year are regressing at least $\frac{1}{100}$ degree, they must have regressed at least 3° in 300 years.'[65] Then this value of 1° in 100 years, i.e. of 36″ per year (14″ too small), without the words 'at least', is used by Ptolemy farther on as the value derived by Hipparchus. That the displacement of the stars took place parallel to the ecliptic was shown by a comparison of the declinations of 18 stars measured formerly by Aristyllus and Timocharis and later by Hipparchus, as communicated by Ptolemy: at one side of the celestial sphere they had increased, by moving toward the north; at the other side they had decreased, by moving toward the south, with a maximum amount of 1°. Hipparchus concluded that the stars moved regularly about the poles of the ecliptic, or rather, according to the title of his writing, that

the equinoctial points, with the equator attached to them, moved regularly back along the ecliptic.

The inequality of the four seasons, in which the sun completes the four quadrants of the ecliptic, was already well known to Callippus, whose values were given earlier. Ptolemy ascribes more accurate values to Hipparchus: $94\frac{1}{2}$ days for the spring, $92\frac{1}{2}$ days for the summer, so that $178\frac{1}{4}$ days remain for the half-year between autumn and spring equinox. These are very nearly equal to the values involved in the Chaldean tables: 94.50, 92.73 and 178.03 (cf. p. 74). Whether he borrowed them from Babylon or derived them from his own observations is uncertain; in any case he made observations of equinoxes and solstices, of which Ptolemy has communicated a small number. His great merit, however, consists in his theoretical explanation of this inequality, by means of an eccentric circle which the sun describes about the earth.

According to their essential nature and the need for harmony, it is assumed that the circular orbits of the heavenly bodies are performed quite uniformly. It is because the earth has its place outside the centre that we see the sun's velocity unequal, regularly increasing and decreasing between a largest value in the perigee and a smallest in the apogee. How far the earth stands outside the centre of the circle—its eccentricity—and in what direction, Hipparchus could easily derive from the length of the seasons by means of simple relations between lines and arcs in a circle (as shown in fig. 9), which form a first beginning of trigonometry. A first table of chords was ascribed to Hipparchus.

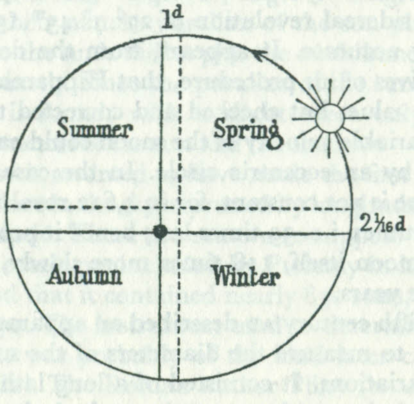


Fig. 9

His result was an eccentricity of $\frac{1}{24}$ of the radius, with the apogee in the direction of longitude $65\frac{1}{2}$°. In this explanation of the irregularity of the phenomena by means of the spatial structure of the orbits, we see the Greek mind, with its faculty for abstraction and its command of geometry, in its full strength.

The knowledge of the moon's motion, that had previously received little attention in Greece, increased considerably as a result of Hipparchus' work. Several eclipses between 146 and 135 BC were observed by him. Ptolemy says that, by comparing them with earlier Chaldean eclipses, Hipparchus derived more accurate periods than those which 'still earlier astronomers' had at their disposal. It was supposed that these words referred to Babylonian astronomers; but since we know that exactly the same values were used in contemporary Chaldean tables and that they must therefore have been known still earlier, it is assumed that Hipparchus acquired his knowledge of the periods from Babylon. It is quite credible, though we have no exact details, that in these centuries there was a certain intellectual intercourse between Babylon and the Hellenistic centres of learning. The eclipses show that the return of the moon to the same node (called 'return of latitude') and the return to greatest velocity (called 'return of anomaly') take place in periods different from the return to the same star. Instead of the saros period of the Babylonian astronomers ($6{,}585\frac{1}{3}$ days $=223$ synodic periods $=239$ returns of anomaly $=242$ returns of latitude $=241$ revolutions in longitude $+10\frac{2}{3}°$), Hipparchus introduced a much longer interval of time: 126,007 days $+1$ hour $=4{,}267$ synodic periods $=4{,}573$ returns of anomaly $=4{,}612$ revolutions minus $7\frac{1}{2}°$ $=$ nearly 345 revolutions of the sun; moreover 5,458 synodic periods are 5,923 returns of latitude. They afford a synodic period of $29^{d}\ 12^{h}\ 44^{m}\ 3.3^{s}$ (only 0.4^{s} too large) and a sidereal revolution of $27^{d}\ 7^{h}\ 43^{m}\ 13.1^{s}$ (only 1.7^{s} too large), both very accurate. It appears, from the detailed description which Ptolemy gives of his procedure, that Hipparchus did not simply copy Babylonian values but checked and corrected them by a careful discussion. The variable velocity of the moon could easily be explained, as with the sun, by an eccentric circle. In the case of the moon the direction of apogee is not constant, for in 4,612 revolutions of the moon it is passed 4,573 times, i.e. 39 times less; hence it proceeds in the same direction as the moon itself, 118 times more slowly, and completes a revolution in nine years.

Proclus in the fifth century AD described an apparatus through which Hipparchus tried to measure the diameters of the sun and the moon, and even their variations. It consisted of a long lath, provided at one end with a vertical plate with an opening to look through, at the other end a movable plate with two openings at such a distance that when the sun was low the upper and lower edges of the disc were just covered by them. Of its use and of the results nothing is known.

Eclipses were used by Hipparchus for yet other purposes. During a solar eclipse (probably in the year 129 BC),[66] which had been total at the Hellespont, only four-fifths of the sun was obscured at Alexandria. Since the distance between these places, expressed in the earth's radius, could be computed, Hipparchus was able to derive the parallax of the moon, hence its distance from the earth; he found it variable between 62 and 74 radii of the earth.

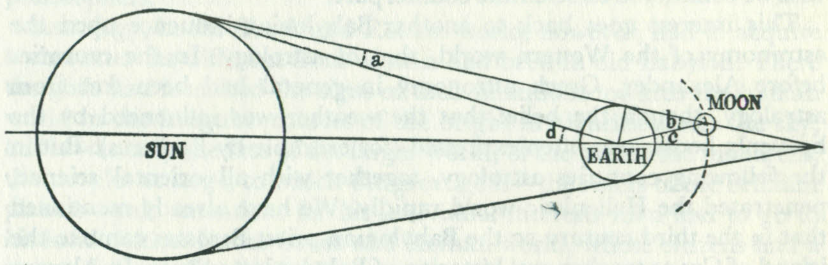


Fig. 10. Shadow Cone of the Earth

Another ingenious method of determining the moon's parallax was ascribed to him: from the measurement of the size of the earth's shadow where it is traversed by the moon. From fig. 10, representing the plane section of the shadow cone and the spherical bodies, we can see at once a simple relation of the angles designated by letters. In the triangle moon-earth-sun, angle a + angle b = angle c + angle d, or, calling them by their names: parallax of the sun plus parallax of the moon are equal to the sun's radius plus the semidiameter of the shadow, as seen from the earth. Since the parallax of the sun is very small, the moon's parallax is found, with a slight error only, by adding the apparent semidiameters of the sun and the shadow.

Hipparchus also is assumed to have made the first catalogue of fixed stars, with their place in the sky expressed by longitude and latitude with respect to the ecliptic. There are reasons to suppose that it formed the main part of the catalogue included by Ptolemy in his own work three centuries later and that it contained nearly 850 stars, to which Ptolemy added another 170. The instrument used to determine the positions is not mentioned; it was probably of the kind later called an 'armillar sphere' or 'armilla'. The Roman author Pliny (*c.* AD 70) gives as the reason why he undertook this work that Hipparchus 'discovered a new star, and another one that originated at that time', and for this reason counted them and determined their positions. This short sentence does not allow us to decide whether during Hipparchus' life a 'nova' really did appear, or whether he saw the appearance and disappearance of a variable star, such as Mira in the Whale. It might also be that his work resulted simply from an increasing interest in the stars and constellations at that time. This interest, as a social phenomenon, manifested itself in his critical commentary on the poetical description of the starry heavens by Aratus and in the fact that this work is the only one among Hipparchus' writings that has been preserved. The exact star catalogue may thus be considered its scientific counterpart.

This interest goes back to another Babylonian influence upon the astronomy of the Western world, that of astrology. In the centuries before Alexander, Greek astronomy in general had been free from astrology, though the belief that the weather was influenced by the heavenly bodies was often expressed (for example by Eudoxus). But in the following centuries astrology, together with all oriental science, penetrated the Hellenistic world rapidly. We have already mentioned that in the third century BC the Babylonian priest Berossus came to the island of Cos as teacher and historian of Babylonian culture. In Alexandria, about 160 BC, a treatise of astrology appeared, with the names of Nachepso and Petosiris as the alleged authors, supposedly two Egyptian priests of earlier times; all later books on astrology have borrowed freely from this early source. The philosophic school of the Stoics contributed to the spreading of astrological belief; it fitted well into their doctrine of the unity of all mankind with the universe. Thus Posidonius, already mentioned as author, scientist and philosopher of renown—whose works, however, are all lost with the exception of some fragments preserved by others—taught a universal sympathy, i.e. consonance and fellow-feeling, between the earthly and the heavenly worlds. His, too, was the well-known exposition of the difference between astronomical and physical science: for the astronomer every explanation is valid that saves (i.e. represents) the phenomena, whereas the physicist must deduce the truth, explaining them from the first causes and working forces. So, if it does not ascertain the physical truth, what is the use of astronomy, i.e. of the astronomical computation of the course of the stars? It has to serve a higher purpose—the prediction of human destiny.

Astrology, as an ever-spreading mode of thinking, permeated first the Hellenistic and then the Roman world, where it found fertile soil. Especially was this so when Rome, after conquering all the kingdoms of ancient culture, and swollen with the immense riches captured from all their treasuries, was afflicted by a century of cruel civil wars that imperilled the life and prosperity of every citizen. 'The collapse of the great Hellenistic monarchies under the pressure of Rome and the ruin of the flourishing Greek cities under the Roman domination, the exhausting wars, the repeated cruel and bloody social revolutions, the general

misery of the times and the growing oppression of both rich and poor in the Hellenistic East'—this was how the historian M. I. Rostovtzeff[67] described the conditions which brought about the sudden bankruptcy of science and learning in the first century BC. Astrology now became an integral part of the world conception and of the culture of antiquity, not only among the masses in the form of crude superstition and a belief in soothsaying, but also as a theoretical doctrine of scientists and philosophers.

Astrology, in the Greek and Roman world, however, had to acquire a character far different from what it had been in old Babylon. There the gods had inscribed the signs of their intentions towards the human world in the irregular courses of the bright luminaries across the sky; but it was only the fate of the larger world, of the states, the monarchs, the peoples at large, to which the great gods—the rulers of the brilliant planets—paid attention: for his individual interests man had to go to his local deities. In the Greek and Roman world, whose citizens had a strong feeling of individuality, astrology had to acquire a more individual character; it had to interest itself in the personal lot of everybody. Moreover, now that the planets had become world bodies describing their orbits in space, calculable by theory, their character had changed; their course was not a sign but a cause of the happenings on earth. The life of every man, like meteorological and political phenomena, was subject to the stars. So the horoscope—which deduces the life-course of a person from the position of the stars at the moment of his birth—became the chief purpose and content of astrological practice. The positions of the planets relative to one another and to the stars, their risings and settings, the position of the constellation relative to the horizon, were the most important data. The oftener the supposition of simple relations was put to shame by the events, the more complicated and arbitrary became the rules and directions. As a result of the mutual cultural influences we now see personal horoscopes appearing sometimes in the Chaldean cuneiform inscriptions.

Chiefly because of this universal spread of the astrological concept of life, astronomy in antiquity stood in the centre of public interest. It was at the time the only knowledge deserving the name of science; and it was the most practical science, more than just a basis for the calendar. Through the intimate connection of the stars and human life, astronomy became man's most important form of knowledge. Allusions to celestial phenomena are common and numerous in the works of Roman poets and prose writers, which cannot be well understood without knowledge of astronomy. Moreover, there was an extensive popular astronomical literature, most of which had since been lost. Eudoxus is said to be the first to have given a detailed description of the stars and the constellations. It formed the basis of a great poem, *On the Phenomena* by Aratus, who lived (about 270 BC) at the court of the Macedonian King Antigonus Gonatus, himself a pupil of the Stoics. This poem was famous throughout antiquity; it was read in the schools and was the source of all the mythological tales concerning the heroes and animals represented in the constellations, which could be found in our books on astronomy up to the nineteenth century. From the great mathematician Euclid a work has been preserved that gives the mathematical theory to Aratus' poetry: a lucid exposition of the circles on the celestial sphere, its rotation, the ensuing phenomena of rising and setting—in short, all that afterwards was called 'sphaerica', the theory of the (celestial) sphere. The universal esteem for Aratus' poem explains why Hipparchus found it worth while to give a detailed criticism, with corrections, of the poem as well as its source, Eudoxus.

Many other popular astronomical works are known from these and later centuries: a Latin poem of Manilius (about AD 10) on the stars, highly praised as a literary work; and still later a poetical description of the heavens by Hyginus. Then there is a thorough work by Geminus (about 70 BC) on the whole of astronomy, that provides us with many valuable details on its history; and a work of Cleomedes (contemporary of the Emperor Augustus) entitled *Circle Theory of the Celestial Phenomena*. The number and spread of such books indicate how strongly astronomy as a living science was rooted in the society of the time.

Besides such books, there existed, for the use of astrological predictions, almanacs with the positions of the planets computed ahead, fragments of which have been found in Egyptian papyri. Further, celestial globes are mentioned as an aid to picturing the heavens and for instruction; for example, an Archimedes globe, brought to Rome by the consul Marcellus, after the conquest of Syracuse. On such globes the stars themselves were often omitted and only the figures of the constellations were depicted, naturally, since the astrologically important effects, as a rule, came from the constellations and not from the separate stars. The Farnese Atlas may serve as an example; in it is pictured a statue of Atlas, the giant, bearing the celestial globe on his neck; an engraving made from it in the eighteenth century by Martin Foulkes, and published in Bentley's edition of Manilius' poem, has been reproduced here on a reduced scale (see plate 2). Attention may be drawn to the four horns on the head of the dog, probably representing the fiery beams emanating from the Dog-Star Sirius. Mechanical models of the world system, a kind of orrery, seem also to have been constructed; but what they represented is an already more highly-developed world structure.

CHAPTER 13

THE EPICYCLE THEORY

THE epicycle theory offered the first satisfactory explanation for the irregular course of the planets. The planet was assumed to describe a circle (epicycle), the centre of which described a larger circle about the earth, which occupied the centre of the universe. This theory was a natural sequel to Heraclides' proposition that Venus and Mercury describe circles about the sun. Whilst they are seen to oscillate from side to side of the sun, the sun carries them along in its yearly course; seen from the earth, they must alternately go a long way in the same direction with the sun, but more rapidly, and a short way back in the opposite direction. Their apparent motion relative to the stars then has the same irregular character as that shown by the other planets. Since it is composed of two regular circular motions, it is plausible to conceive the motion of the other planets (Mars, Jupiter and Saturn) as a combination of two circles also; a larger circle (deferent, leading circle) about the earth as centre, along which the centre of the smaller circle (the epicycle) moves. This centre is a void point here, whereas for Venus and Mercury it was occupied by the sun.

Two points of revolution are thus needed to express the planet's motion. We have to bear in mind here that in Greek science the epicycle was held to be attached to the radius of the large circle; in revolving, its lowest point (nearest to the earth) always remained the lowest point. The planet's time of revolution along the epicycle is always reckoned from the lowest or the highest point until the same point is reached again; this is the synodic period of the planet. Its passage through the lowest point is the middle of the retrograde motion. For Venus and Mercury it is the inferior conjunction with the sun; for the other planets it is the opposition to the sun.

The epicycle theory offered a far simpler and more accurate representation of the variable course of the planets than did the rotating spheres of Eudoxus and Aristotle. Moreover it explained their variable brightness as a result of their varying distances from the earth. These distances could be computed easily from the sizes of the circles. The relative size of the epicycle and the deferent for Venus and Mercury follows from their greatest elongation to the right-hand or the left-hand side of the sun; from 46° for Venus and 22° for Mercury, we find the ratio of the radii 0.72 and 0.37. For the other planets this elongation must be taken relative to the invisible, regularly progressing epicycle centre. For Mars 42° is found; then the epicycle radius is 0.67 times the radius of the great circle, and the distances of Mars at the highest and lowest points have a ratio of 1.67 to 0.33. This explains its great variations in brilliancy.

It has often been asked why the Greek astronomers, having become acquainted with the heliocentric world system of Aristarchus, went back to the more primitive geocentric system of the epicycles. We surely cannot find the reason in what Paul Tannery said, that ancient Greece in these centuries lacked the genius for complete renovation;[68] in this respect the Greeks have certainly shown their superiority. The reason must be, first, that the epicycle theory was the most natural course of development for Greek science. The epicycle theory *was* the renovation wanted. The heliocentric world structure devised by Aristarchus was a fantastic stroke of genius, not a necessary consequence of facts. What bore it out was the bodily size of the sun; though it was argued on the other hand that in the human body the heart, the seat of life, was also outside the bodily centre of mass.

A new world structure had to be a theory of the planetary motions. The motions of the planets at the time were known only in rough outline; they had first to be observed accurately and represented in the most natural way, which was effected in fact by the epicycle theory; it constituted the direct geometrical representation of the visible phenomena.

Secondly, social influences probably also played a role, especially the general belief in astrology, which was the practical application of science. Astrology did not need theories on the physical nature of the celestial bodies, but only practical tables for computing their apparent motions. It was not merely indifferent but flatly hostile to physical structures which might disturb the primitive belief that the stars in their courses pronounced the fate of human beings. So it is easy to understand the saying of Cleanthes, the leader of the Stoics, that 'it was the duty of the Greeks to indict Aristarchus of Samos on the charge of impiety for putting the Hearth of the Universe in motion' (i.e. the earth).[69] The epicycle theory, representing the appearances, had to take preference over Aristarchus' world structure.

The epicycle theory must have originated, perhaps gradually, in the third century BC. The first certain report on it is connected with the name of the great mathematician Apollonius of Perga (about 230 BC), the founder of the theory of conic sections. Ptolemy hands down one of

his geometrical propositions which derives the stations of the planets. The Chaldean astronomers had carefully noted these stations and in their tables had computed them as important basic elements of knowledge of the course of the planets. Greek theory had to show that it was equally or better able to solve the same problem, to foretell the time and place of the stations. Apollonius did so by reducing the problem to geometrical levels, drawing a line from the earth which intersects the epicycle in such a way that the sections had a definite ratio. Because of its importance, the demonstration is given in Appendix B.

In the next century Hipparchus occupied himself with the epicycle theory and gave it its classical form. He demonstrated that the motion in an epicycle, described in the same period but in opposite direction with the concentric circle which is the orbit of its centre, is identical with the motion along an eccentric circle. This can at once be seen in fig. 11, where the points 1, 2, 3 and 4 occupy a circle shifted upward. Both models, therefore, can be used to represent the variable velocity along the ecliptic which the sun shows and which the planets also show besides their oscillations. So it was natural to render these oscillations by epicycles and to choose the eccentric circle for the variable velocity along the ecliptic.

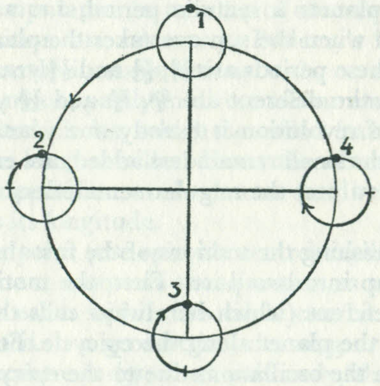


Fig. 11

According to Ptolemy, Hipparchus had indeed recognized that the retrogradations of the planets were different at opposite sides of the ecliptic; hence the motions were more irregular than had been previously assumed. From a somewhat obscure statement by Pliny, two centuries later—that the farthest apogee of Saturn is situated in the Scorpion, of Jupiter in the Virgin, of Mars in the Lion—it appears that in an earlier time, possibly in Hipparchus' own time, there was also some qualitative knowledge of these irregularities. But, Ptolemy says, he did not have at his disposal a sufficient number of observations from predecessors to work out a numerical theory completely. For that reason he restricted himself to assembling new data: 'a man who in the whole field of mathematics had reached such a profoundness and love of truth'[70] could not content himself with giving a theory in general terms only. He had to determine the numerical values in the orbital motions from the phenomena and to show that they could be adequately represented by uniformly described circles. This, however, was not possible from the available data.

This was the work of Ptolemy himself, who thus brought the epicycle theory to completion. He says that he first corrected the planetary periods of Hipparchus by means of his own observations, and found them to be as follows:

Saturn	57 s.p. = 59 y. + $1^3/_4$ d. = 2 r. + 1° 43′
Jupiter	65 s.p. = 71 y. + $^{49}/_{10}$ d. = 6 r. − 4° 50′
Mars	37 s.p. = 79 y. + $3^{13}/_{60}$ d. = 42 r. + 3° 10′
Venus	5 s.p. = 8 y. − $2^3/_{10}$ d. = 8 r. − 2° 15′
Mercury	145 s.p. = 46 y. + $1^1/_{30}$ d. = 46 r. + 1°

(s.p. = synodic periods; y. = years; d. = days; r. = revolutions)

These are the same multiples as used by the Babylonians. For the first three named planets a synodic period, i.e. a revolution in the epicycle, has passed when the sun overtakes the planet; neglecting the small remainders, these periods are $\frac{59}{57}$, $\frac{71}{65}$ and $\frac{79}{37}$ years, and the periods of revolution along the deferent are $\frac{59}{2}$, $\frac{71}{6}$ and $\frac{79}{42}$ years. For the two others the period of revolution is exactly one year. As to exactitude, these periods, with the small remainders added, are entirely comparable with the Chaldean values; the angular remainders may be inaccurate up to half a degree.

Ptolemy, in establishing the motions of the first three planets, had to split the problem up into two parts. First, the motion of the epicycle centre along the deferent (which he always calls the 'excentre'), and then the motion of the planet along the epicycle. For the first purpose he had to eliminate the oscillations due to the epicycle and to observe the planet when it was seen exactly in the same direction as the epicycle centre, i.e. when it stood in front of this point. How could he know that? The basic principle of the epicycle theory is that the radius of the epicycle, which connects its centre with the planet, turns uniformly in the same stretch of time in which the sun describes its circle and thereby has always the same direction in space as has the radius in the solar orbit. Hence the planet will stand exactly before the centre (as fig. 12 shows) when its longitude is 180° different from the sun's longitude as seen from the centre of its orbit. In other words, the planet stands in

opposition not to the real sun but to the 'mean sun', which performs its yearly course exactly uniformly; where the real sun is seen from the earth does not matter. Then the observed longitude of the planet is the desired longitude of the epicycle centre.

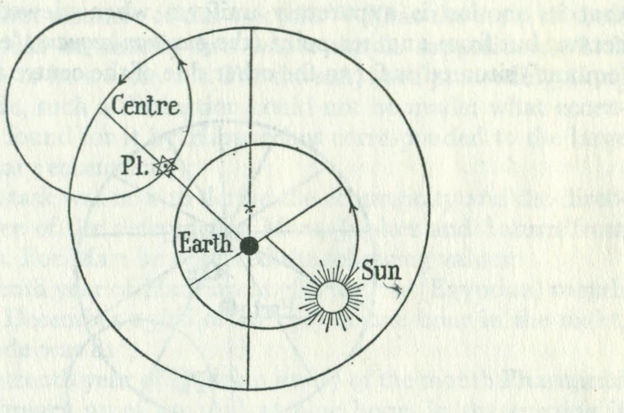


Fig. 12

For practical computation, a number of observations at consecutive days about opposition are necessary. Ptolemy's observations of the planet consisted partly in conjunctions with, or near approaches to, bright stars or the moon, partly in direct measurements with an instrument with graduated circles, which he called 'astrolabon' and which corresponds to the later armillas. From the observed longitudes the exact moment of opposition to the 'mean sun' between them can be derived, as well as its longitude.

If we know for a point (the epicycle centre) moving on the eccentric circle for three distant moments the direction as seen from the earth, the position of the earth within this circle can be determined. It can either be constructed in a geometrical drawing or computed numerically. Since the excentre is described uniformly, the three positions on the circle are known from the intervals of time. Then the problem is identical with the so-called 'Snell's problem' in geodetics: to derive the position of a station by measuring there the directions to three surrounding known stations; this problem can be solved in a direct way.

Here Ptolemy met with a difficulty. He says: 'Now we found, however, with continued exact comparison of the course given by observation and the results from combinations of these hypotheses, that the progress of the motion cannot be quite so simple. . . . The epicycles cannot have their centres proceed along such eccentric circles that, seen from the centre [of these circles], they describe equal angles in equal time. . . . But the latter bisect the distance between the point from which the motion appears to be uniform, and the centre of the ecliptic.'[71] Expressed in another way: the epicycle centre does describe an eccentric circle, i.e. a circle with its centre outside the earth, but in such a way that its motion is apparently uniform when viewed, not from this centre, but from another point (the *punctum aequans*, 'equalizing point', 'equant') situated as far to the other side of the centre as the earth is on

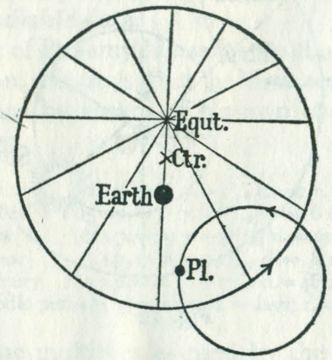


Fig. 13

this side. This means, as figure 13 shows, that in reality the excentre is not described uniformly by the epicycle centre; nearest to the equalizing point (in the apogee) it goes more slowly, at the opposite side more rapidly. Whereas the basic principle of Greek cosmology—the circular and uniform motion of all celestial bodies—is paid lip service, actually, on the pretext that the motion appears to be uniform as seen from another point, it is violated.

But in this way the phenomena offered by the planets could be represented in the most perfect manner. Ptolemy does not indicate by what argument or by what observation he arrived at this theory. He only says: 'we found that . . .' It is, however, easy to see what phenomena must have led to it. The distance of the earth from the centre of the great circle—which is its real eccentricity—determines the variable apparent size of the epicycle, visible in a variation of the oscillations of the planet to both sides of the epicycle centre. The distance of the earth from the point at which the angular motion is seen to be uniform determines with what variations of velocity the epicycle centre seems to move along the ecliptic; the precise situation of the circle and its centre in this respect is of secondary importance only. The observations must have shown that in the latter case an eccentricity is found twice as large as in

the first case; so the distance of the equalizing point from the earth is twice the distance from the centre to the earth. Or, in his own words elsewhere: 'The eccentricity deduced from the greatest deviation in the anomaly relative to the ecliptic was found to be double the amount of the eccentricity derived from the retrograde motion in the cases of the largest and smallest distance of the epicycle.'[72] That the ratio of these eccentricities should be exactly two was a simple supposition which proved, however, to be a lucky hit. For the sun, itself proceeding along its eccentric circle, such a distinction could not be made; what eccentricity had been found for it by Hipparchus corresponded to the large or double planetary eccentricity.

Ptolemy's first task was now to derive the eccentricity and the direction of the apogee of the deferent for Mars, Jupiter and Saturn from three oppositions. For Mars he observed the following values:

(1) in the fifteenth year of Hadrian at 26/27 of the (Egyptian) month Tybi (i.e. December 15/16 of AD 130) at one hour in the night, the longitude was 81°;
(2) in the nineteenth year of Hadrian at 6/7 of the month Pharmuthi (i.e. at February 21 of AD 135) at nine hours in the evening it was 148° 50′;
(3) in the second year of Antonine at 12/13 of the month Epiphy (i.e. May 27 of AD 139) at 10 hours in the evening it was 242° 34′.

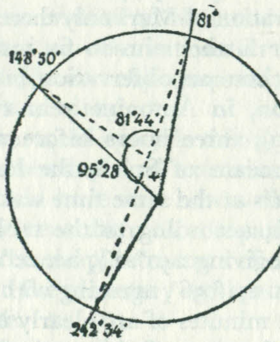


Fig. 14

From the intervals in time, after subtraction of an entire number of revolutions, the angles between the directions as seen from the equant are computed: 81° 44′ and 95° 28′; between the directions as seen from the earth these angles are 67° 50′ and 93° 44′. These data are represented in figure 14. What is required is the position of the earth within the circle. The problem cannot be solved directly; Ptolemy solved it by successive approximations. First he supposed a circle about the equant as centre. Then the problem, as stated above, could be solved directly by the mathematical means at his disposal: the propositions of Euclid and the table of chords for every arc, computed by himself, the prototype of the later sine tables. The result is the large eccentricity, half of which is the amount the circle must be displaced. He now computes how much the directions as seen from the earth are changed by this displacement, and with the corrected values the computation is repeated in a second approximation. It affords very small corrections for a third approximation, which entirely satisfies the original data. In these computations, which are communicated *in extenso*, Ptolemy applied the method of convergent approximation which played such an important role in later mathematics.

The results obtained in this way for the total eccentricity and the longitude of the apogee are: for Mars $\frac{72}{360}=0.200$ and 115° 30′; for Jupiter $\frac{33}{360}=0.092$ and 161°; for Saturn $\frac{41}{360}=0.114$ and 233°. If we compare them with what, according to modern knowledge, were the true values at the time—for Mars 0.186 and 121°; for Jupiter 0.096 and 164°; for Saturn 0.112 and 239°—it appears that his representation of the planetary orbits was highly satisfactory.

The second problem, the size of the epicycle relative to the excentre, had to be solved by observation of the planet outside its opposition, when it stands far to the side on its epicycle. For this derivation Ptolemy, curiously, gives an observation of Mars only three days after opposition (fig. 15): 'Since it is our further aim to fix numerically the relative size of the epicycle, we chose an observation made nearly three days after the third opposition, in Antonine year two at 15–16 Epiphy (i.e. May 30–31, AD 139), three hours before midnight.'[73] With the astrolabon directed by means of Spica, the longitude of Mars was found to be 241° 36′. Mars at the same time was found to stand 1° 36′ east of the moon (which, according to the tables stood at 239° 20′, corrected 40′ for parallax giving 240° 0′), hence 'also in this way a position of Mars was found at 241° 36′, agreeing with the other result'. The exact concordance of the minutes of arc clearly indicates that they are fitted on purpose; since the time of observation is given in full hours only and the moon moves 33′ per hour, any value within this range might be chosen so as to give the same minutes as the direct measurement.

With these data it is again simply a computation by geometry, consisting of a calculation of sides in a triangle with known angles—a triangle, surely, with very small angles, 2° 43′ at the earth and 1° 8′ at the epicycle centre, so that errors of measurement not greater than ¼° can strongly vitiate the result. Yet the result, 0.658 for the radius of the

epicycle, is almost exactly equal to the true value, 0.656. So it may be taken for granted that his value does not rest solely on this observation but has been derived from further observations at greater distances from the opposition. The derivation in his book, then, must be considered rather as an example to show the method used. Computed

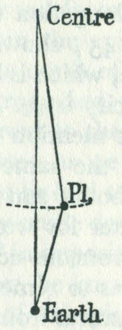


Fig. 15

with modern data, the longitude of Mars at the moment of his observation was 242° 16′, hence 0° 40′ larger. It is well known that, owing to an error of about 1° in his vernal point, all Ptolemy's longitudes are too small; for Spica the error in his catalogue was 1° 19′, so that in the measurement of Mars an error of 39′ was made. For Jupiter and Saturn his derivation was based on observations far from the opposition; the result ($\frac{23}{120}=0.192$ and $\frac{13}{120}=0.108$) compares well with the values from modern data of 0.192 and 0.105.

With these three planets the epicycle theory appears in its full power. If the ancient astronomers had had extensive series of observations at their disposal, they would have seen how well a computation with this theory was able to represent them. A comparison with modern theory may convince us that this simple structure of circles in space excellently represents the details in the planetary motions which formerly looked so capricious and intricate.

With Venus and Mercury things were not so simple. Here not the epicycle but the deferent is performed in pace with the sun. The epicycle theory does not say, however, as Heraclides did, that the sun is the centre of their movements but demands solely that the epicycle centre revolves in the same period of time as the sun and that its radius remains always parallel to the radius of the sun in its circle. Whereas the greatest elongations of these planets east and west of the sun are different and variable in amount, this must be represented theoretically by uniformly described concentric epicycles.

For Venus the differences are small; the great elongation from the sun is always between 45° 53′ and 46° 43′. Here a circle with its centre near to the sun fits excellently as epicycle. In order to derive the figure of the deferent, Ptolemy made use exclusively of a number of greatest elongations to both sides of the sun. He first derived its apogee at 55° of longitude. In a direction 90° different—at a longitude of 325°—the greatest eastern and western elongation were found to be 48° 20′ and 43° 35′. Their difference, 4° 45′, shows that the epicycle centre is 2° 22½′ ahead of its mean place, which is identical with the place of the mean sun; hence the eccentricity is $\frac{1}{24}$. Apparently, Ptolemy did not perceive—at least he does not mention it anywhere—that this is the same eccentricity and nearly the same apogee as he had formerly derived for the sun, so that the bodily sun itself occupies the centre of the Venus epicycle. If it is a matter for wonder that the observations of Venus here give the same erroneous eccentricity as those of the sun (the real value was $\frac{1}{30}$), one has to remember that Ptolemy's observations were elongations, i.e. distances from the real sun, from which the longitudes of Venus were computed by means of reductions taken from solar tables based on the erroneous eccentricity of $\frac{1}{24}$; so that he got as 'result' what he had put in under another name.

To derive the size of the epicycle, Ptolemy took a greatest elongation at the apogee and one at the perigee of the deferent; he found 44° 48′ and 47° 20′, from which the two distances 0.705 and 0.735 can be computed. Their mean, 0.720, is the radius of the epicycle; half their difference expressed as a fraction, $\frac{1}{48}$, of the radius is the eccentricity of the observing point, the earth. It is half the amount of the total eccentricity $\frac{1}{24}$ just found; hence the Venus deferent has an equant. It is a remarkable fact that in its character of Venus deferent the solar circle exhibits an equant, that could not be derived from itself. Since Ptolemy was not aware of their identity, this fact had to be rediscovered afterward by Kepler. There are, however, many points of doubt in the data used.

With Mercury the differences and difficulties were far greater; the greatest elongations vary between 17° and 28°. So Ptolemy did not succeed in establishing a satisfactory theory. To a considerable degree this was due to the difficulty of observing the planet; it is visible only in strong twilight near the horizon, at a great distance from other bright stars, which hampers exact observation. So the time and, because of its rapid motion, the position of its greatest elongation were difficult to establish. Another source of trouble lay in the theory adopted; circular orbits were not likely to express the asymmetric oscillations of the planet. Thus Ptolemy came to assume an oval orbit for the epicycle centre, with the earth on the major axis outside the centre. Such an oval

can be produced by a combination of circles, namely, by making the centre of the deferent describe a small circle in the opposite direction twice during one revolution. Some chief features of the motion of Mercury could indeed be represented in this way, but in a less accurate and more complicated way than with the other planets. It shows the many-sided possibilities of the epicycle theory; but Mercury certainly was too difficult an object for satisfactory treatment.

Finally, Ptolemy included in the epicycle theory the deviations of the planets to the south or the north of the ecliptic, expressed in their latitudes. It was done by introducing, for Mars, Jupiter and Saturn, small inclinations of the deferent to the ecliptic and of the epicycle to the deferent. He had no reason—as we have—to suppose the epicycle to be parallel to the ecliptic; so he had to determine two inclinations for each planet. He used some few crude data only: from a latitude of Jupiter of 1° in conjunction and 2° in opposition he found the inclinations to be 1½° and 2½°. By observation, he knew that the planet's deviation in latitude on the nearest point of the epicycle, at one side of the ecliptic was to the north, at the opposite side to the south. According to Greek theory, this deviation should remain the same in going around the ecliptic, because the epicycle was assumed to be fixed to the radius of the deferent. Ptolemy therefore had to correct it by a special contrivance: the nearest point of the epicycle revolves along a small vertical circle in one period of revolution, so that it is lifted up and down, oscillating between the extreme northern and southern deviation. To the already complicated structure of the orbits of Venus and Mercury he added oscillations up and down, two of the epicycle relative to the deferent and one of the deferent relative to the ecliptic, all directed by small vertical circles. And for those who should think these mechanisms too complicated for the celestial bodies he added some philosophical consolations: 'Let nobody, looking at the imperfection of our human contrivances, regard the hypotheses here proposed as too artificial. We must not compare human beings with things divine. . . . What more dissimilar than creatures who may be disturbed by any trifle and beings that will never be disturbed, not even by themselves? . . . The simplicity itself of the celestial processes should not be judged according to what is held simple among men. . . . For if we should look at it from this human point of view, nothing of all that happens in the celestial realms would appear to us to be simple at all, not even the very immutability of the first [i.e. daily] rotation of heavens, because for us human beings this very unchangeableness, eternal as it is, is not only difficult, but entirely impossible. But in our judgment we have to proceed from the immutability of the beings revolving in heaven itself and of their motions: for from this point of view they would all appear to be simple, and even simple in a higher degree than what is regarded on earth as such, because no trouble and no pains can be imagined with regard to their wanderings.'[74] It must be added that afterwards, in tables for practical use, he considerably simplified the structure of the orbits.

Chaldean astronomy did not bother about the latitude of the planets. The Chaldean tables dealt with longitudes, in progress and retrogradation; the ecliptic as central circle of all planetary orbits seems to have remained unnoticed. Latitude occurs only for the moon, because here it is necessary for the eclipses, and for the exact computation of the crescent. In this point the superiority of Greek over Babylonian astronomy is manifest; it saw the celestial luminaries as bodies having definite orbits in space. The epicycle theory, in the definite form given by Ptolemy, stands out as the most mature product of ancient astronomy.

CHAPTER 14

THE CLOSE OF ANTIQUITY

At the commencement of our era, all civilized peoples of antiquity living around the Mediterranean, the ancient world sea, were assembled in the Roman Empire. A lively sea traffic connected them into one economic unit; from the opposite shore and from the conquered East riches and foodstuffs flowed to privileged Italy, especially to the ruling capital, Rome. The peoples who lived beyond the frontiers, barbarian tribes in Europe and Africa and the Asiatic empires in the Orient, had to be repelled by strong armies in continual frontier warfare.

Inside the enormous realm peace reigned under the emperors of the first two centuries, interrupted only once by a contest between a couple of generals over the emperorship. The growth of agriculture, trade and commerce, extending ever more evenly over the provinces, led to the spread of intellectual culture; originating from the eastern lands that remained Greek in language and Hellenistic in character, it now reached the rustic Western conquerors. Here it remained an imitation; in the field of natural sciences and art the Romans produced little that was original.

This holds for astronomy too. The contributions of the Romans and their subjects are soon enumerated. Cleomedes—his name is Greek—has already been mentioned for his manual of astronomy; the simultaneous visibility of the sun and the eclipsed moon opposite above the horizon was explained by him in the right way: refraction of the rays of light near the horizon. Ptolemy used observations of occultations of stars by the moon made originally by Menelaus at Rome in AD 92 and by Agrippa in Bithynia in AD 98; the first name, Menelaus, is also Greek. From Plutarch, the author of the famous *Lives*, in the second century AD, we have a dialogue *On the Face in the Moon*, wherein the moon is described as an earthlike body, with mountains and depths casting shadows, a view far more modern than that of Aristotle. Not as an extension but as an application of science, Julius Caesar's calendar reform must be noted here. To do away with the confusion caused by arbitrary changes, all relation to the moon and all intercalation of months was abolished. The historian Suetonius described the situation in these words: 'Then, turning his attention to the reorganization of the state, he reformed the calendar, which the negligence of the pontiffs had long since so disordered, through their privilege of adding months or days at pleasure, that the harvest festivals did not come in summer, nor those of the vintage in the autumn [consequently, objects for offerings were lacking]. And he adjusted the year to the sun's course by making it 365 days, abolishing the intercalary month, and adding one day every fourth year.'[75] What constituted the character strength of the Romans, their sense of social-political organization, created a mode of time-reckoning destined to dominate the entire future civilized world.

The nature of the Roman Empire, however, gave a new and special character to the scientific work of these times. In former centuries the rise of trade and commerce and the rivalry of the small states and towns in Hellas and the Orient had awakened creative initiative in the growth of new ideas. Now that civilized mankind had become an all-embracing unity, the character of scientific work also tended towards an all-embracing unity. The study of nature became the assembling of all knowledge into one body of science. Instead of the fresh ingenuity of original thinkers, came the all-encompassing learning of the compilers. Instead of the ebullition of new ideas, came the organization of all that past centuries had wrought—often, of course, elucidated by genuine ideas—into encyclopaedic works. Scientifically the centuries of the Roman emperors were the centuries of the great collective works: of Strabo, and later of Ptolemy, on geography; of Pliny on natural history; of Galen on medicine. They constituted the completion of ancient science.

It was the same with astronomy. As a compendium of Greek astronomy there appeared Claudius Ptolemy's *Thirteen Books of the Mathematical Composition* (*Matematikè Suntaxis*). He lived at the time of the emperors Hadrian and Antonine, a contemporary of Plutarch. His years of birth and death are unknown; observations by him are known from the years AD 127 to 151. He lived in Alexandria and belonged entirely to the Greek cultural world. His work was far more than a compilation of former knowledge. Ptolemy was no compiler but a scientific investigator himself; with Hipparchus he was the greatest astronomer of antiquity. He improved and extended the theories of his predecessors; he added to science through his own observations and explanations. We have already seen how he brought the epicycle theory to completion by giving it a precise form and numerical values.

Ptolemy's work is a manual of the entire astronomy of the time. It is true that he deals with the celestial sphere only—the stars, the sun, the moon and the planets—and does not speak of comets. Those, he considered, did not belong to astronomy; although the philosopher Seneca

(about AD 70) in a much-quoted sentence spoke of them as celestial bodies, whose far-stretched orbits, on which they were mostly invisible, would certainly be discovered in later times, Ptolemy sided with Aristotle, who considered them to be earthly phenomena in the higher realms of the air.

Also in harmony with Aristotle is the basic structure of the universe which Ptolemy expounded in his first chapters. The heavenly vault is a sphere, and as a sphere it rotates about its axis; the earth, too, is a sphere and occupies the centre of the celestial sphere; as to its size, the earth is as a point relative to this sphere, and it has no motion to change its place. The arguments are the same as Aristotle's; moreover—without even mentioning Aristarchus or Heraclides—he argued against those who had expressed an opposite opinion:

'Some philosophers think that nothing prevents them from assuming that the heaven is resting and that the earth in nearly a day rotates from west to east. . . . As to the phenomena of the stars, nothing would prevent this by its greater simplicity from being true; but they did not perceive how very ridiculous this would be with regard to the phenomena around us and in the air. In that case, contrary to their nature, the finest and lightest element [the ether?] would not move at all or not differently from those of opposite nature—whereas the bodies consisting of atmospheric particles show a tendency to more rapid motion than the earthly matter. Moreover, the coarse-grained heaviest bodies would have a proper strong and rapid uniform motion—whereas, as everyone knows, heavy earthly things hardly can be put in motion. If we should concede this, they certainly would have to admit that a rotation of the earth would be more violent than all motions that take place on her, and in a short time would have so rapid a reaction that everything not fixed to her would appear to have only one single motion, contrary to hers. And we would never see a cloud, or anything that flies or is thrown, move towards the east, because the earth would outdo them in the motion towards the east, so that all the others, outdistanced, would appear to move towards the west.

'If they should say that the air is carried away with equal velocity, then also the earthly bodies in it would be seen to lag behind. Or if carried away by the air as if firmly attached to it, they would never be seen to move forward or backward, and all that flies or is thrown would have to stay at its place without being able to leave it; as if the motion of the earth would deprive them of any ability to move, be it slow or quick.'[76] These are the arguments which Ptolemy advanced for his geocentric world system.

In a geometrical introduction a table of chords for angles increasing by half a degree is first computed and presented; the proposition used, on quadrangles in a circle, still appears in our modern textbooks as 'Ptolemy's theorem'. Since the Greek numerical system did not know of decimal fractions, the chords are given in sexagesimals, with the diameter taken as 120 units, a token of Babylonian influence (so for an arc of 90° the chord is given as 84, 51, 10, i.e. reduced to our notation, 0.707107). Throughout the work fractions in the length of lines are given in sixtieths.

Then propositions on plane and spherical triangles are derived which are needed farther on. Connected with them, quantities on the rotating celestial sphere are computed: viz. the time and duration of the rising of the different zodiacal signs, as well as the inclination of the ecliptic to the horizon, the meridian, and other vertical circles—all necessary in astronomical and astrological computations. Because they depend on the latitude of the place of observation, they are given for standard latitudes specified by the maximum length of daylight (from sunrise to sunset): from 12 hours at the equator, over $12\frac{1}{4}$, $12\frac{1}{2}$, $12\frac{3}{4}$, 13 hours, etc. (corresponding to latitudes of 4° 15′, 8° 25′, 12° 30′, 16° 27′, etc.), increasing up to 23 and 24 hours (at 66° and 66° 8′ 40″ of latitude).

Then the sun's motion is dealt with. The relevant quantities are the obliquity of the ecliptic, the length of the year, and the eccentricity of the sun's circular orbit. Ptolemy describes two instruments used to determine the obliquity; one is a graduated circle on a pedestal, within which a smaller circle with notches and index can be turned, so that, by means of the shadow, the meridian altitude of the sun can be read; a picture of this instrument was given later on by Proclus. The other is a graduated quadrant, serving the same purpose. He found the difference between the meridian altitudes of the summer and winter solstices always to be between 47° 40′ and 47° 45′, and he states that it is nearly the same value as that found by Eratosthenes and used by Hipparchus: $\frac{11}{83}$ of the circumference. Half of it, 23° $51\frac{1}{3}$′, is the value adopted for the obliquity of the ecliptic.

As to the orbit of the sun, he first mentions that Hipparchus stated the intervals between spring equinox, summer solstice and autumn equinox to be $94\frac{1}{2}$ and $92\frac{1}{2}$ days, and had derived from them an eccentricity of $\frac{1}{24}$. 'We, too, came to the result that these values are nearly the same today. . . . For we found the same intervals from exactly observed equinoxes and an equally exactly computed summer solstice in the 463rd year after Alexander's death' (i.e. AD 139–140).[77] These are AD 139, September 26, one hour after sunrise; 140, March 22, one hour after noon ($178\frac{1}{4}$ days later); and 140, June 25, two hours in the morning. 'The last interval is $94\frac{1}{2}$ days; for the interval from this solstice to the next autumn equinox there remain $92\frac{1}{2}$ days.' So he derives an eccentricity of $\frac{1}{24}$ and an apogee at 65° 30′, both identical

with Hipparchus' values. In reality, at his time the intervals were, according to modern data, 93.9 and 92.6 days.

The same equinoxes are used to derive the length of a year by comparing them with Hipparchus' values for 147 and 146 BC, 285 years earlier. He finds the interval to be $70\frac{3}{10}$ days more than 285 × 365 days; since 285 × $\frac{1}{4}$ days is $71\frac{1}{4}$ days, he concludes 'that in 300 years the return of the sun to the vernal equinox takes place nearly a day earlier than would correspond to a year of $365\frac{1}{4}$ days'.[78] And again he states his complete accord with Hipparchus' length of the year, 365 days 5 hours 55.2 minutes. This length, however, was seven minutes too great; hence his interval surely was one day too long. The moments of his equinoxes, when computed from modern data, are in fact found to have taken place one day earlier than he reports as his observational result.

The length of the year or, rather, the difference between the return to the same star (the sidereal year), and the return to the equinoxes (the tropical year) is directly connected with the precession. Whereas other authors do not mention this discovery by Hipparchus, Ptolemy understands its importance and confirms its amount. 'In comparing the distance of the stars to the solstices and equinoxes with those observed and noted by Hipparchus, we also found that a corresponding progression in the direction of the signs had taken place.'[79] Comparing an observation of Regulus made by himself (AD 139) with one by Hipparchus, he finds that it had proceeded 2° 40′ in the intervening 265 years, hence 1° per 100 years. Then he compares a number of observations of occultations or conjunctions of different stars (Pleiades, Spica, β Scorpii) with the moon, made by Timocharis in Alexandria, with analogous observations made by Menelaus at Rome and Agrippa in Bithynia, and again he derives an increase in longitude of 1° in 100 years. Yet we know that the amount given is far too small and that its real value is 1° in 72 years; the real displacement since Hipparchus was 1° greater and since Timocharis it was $1\frac{1}{2}$° greater.

Because of these contradictions, modern astronomers have often severely criticized Ptolemy, claiming that not only was he so possessed by blind faith in his great predecessor that, without criticism, he adopted his values, but that for this purpose he even fabricated or fashioned and doctored his own observational results, i.e., he falsified them to make them agree. In Delambre's great work, *Histoire de l'astronomie ancienne* (1817), we read: 'Did Ptolemy himself make observations? Are not those which he says he made but computations from his tables and examples for the purpose of understanding his theories better?' Farther on he adds: 'As to the main question, we cannot see how to decide it. It seems hard to deny absolutely that Ptolemy made observations himself. . . . If, as he says, he had in his possession observations in greater number, we may reproach him that he did not communicate them and that nowhere does he tell what might be the possible error of his solar, lunar, and planetary tables. An astronomer who today acted in this way would certainly inspire no confidence at all. But he was alone; he had no judges and no rival. For a long time he has been admired on his own word.'[80] It is clear, however, that it is not right to judge Ptolemy's work by the usages and standards of modern science. The scientific outlook in antiquity was different from ours; there was no regular experimental research with acknowledged standards of judgment; observational results were not considered documents. Ptolemy's work was essentially theoretical; his aim was to develop and expound a geometrical picture of the world. Observation and theory, as we saw with Aristarchus, were at that time differently related to each other. Observation was simply an extension of experience, finding out where the celestial body happened to be. Theory was the new wonderful view of the world and the deeper insight into its structure; it was philosophy inquiring into the essence of things. The data used were instances or specimens, chiefly, but not necessarily, taken from observation with all its uncertainties, not intended as important new knowledge but often simple verifications, easily accepted, of respected earlier knowledge.

In the case of the precession, moreover, the sources of real error must not be overlooked. In Ptolemy's observation of Regulus, the setting sun was first compared with the moon, and the low moon was later compared with the stars; the effect of refraction must have been that too small a longitude of the star was found relative to the sun. If the sun's longitude was taken from the tables, their errors due to errors in the equinoxes were transferred to the result. In deriving the longitude of stars from occultations by the moon, the tables and parallaxes of the moon had to be used, which contained a considerable number of errors. With the time roughly noted, the velocity of the lunar motion, 1° per 2 hours, allows large adaptations of the moon's assumed position to the expected value. Many of Ptolemy's surprising data might be due to such causes.

Another set of data is worth mentioning here. In order to show that the precession is really a motion parallel to the ecliptic, so that on one side of the celestial sphere the stars move to the north, on the opposite side to the south, Ptolemy states the declinations (the distance in latitude from the equator) for a number of stars, as measured by Timocharis and Aristyllus, by Hipparchus and by himself. If from the extents of displacement in declination we now compute the value of the precession in longitude along the ecliptic, we find 46″ per year, 1° in 78 years, not so very different from the true value. The measurements, not vitiated by the errors and complications introduced by solar and lunar tables,

appear to be good and reliable, with a mean error of not more than 8′. Ptolemy, however, did not possess the trigonometrical formulas to make such a computation.

After the sun, Ptolemy deals with the moon. Here he does not content himself with confirming Hipparchus' results; he goes new ways of his own. First he checks and corrects its motions. As to the mean daily motion in longitude, expressed in sexagesimals (separated as usual by commas) 13°, 10, 34, 58, 33, 30, 30, and the motion relative to the sun, 12°, 11, 26, 41, 20, 17, 59, corresponding to a synodic period of $29^d\ 12^h\ 44^m\ 3\frac{1}{2}^s$ and to a tropical year of $365^d\ 5^h\ 55^m$ he finds that no correction is needed. To find the return to the apogee (the 'anomalistic period'), he made use of three Babylonian lunar eclipses from 721 and 720 BC and compared them with three observed by himself in AD 133, 134 and 136.

To find the period of return to the same node, he compared an eclipse from 491 BC with another in AD 125, selected in such a way that all the other determining quantities were the same in both cases. Thus he found a correction of 0° 17′ in 854 years for Hipparchus' return to apogee, which was based on the ratio 269 : 251, and a correction of 0° 9′ in 615 years for Hipparchus' return to the node. The values in degrees and its sexagesimals per day became in the first case 13; 3, 53, 56, 17, 51, 59, in the second case 13; 13, 45, 39, 48, 56, 37. They correspond to a backward motion of the node in a period of 6,796.26 days and a forward motion of the apogee in a period of 3,231.62 days. In all these daily motions the two last sexagesimals are not warranted.

The basic orbit of the moon, in this way, is a circle inclined to the ecliptic, with the points of intersection—the nodes—uniformly regressing in a period of 6,796 days, i.e. a good 18 years. To represent the variations in velocity between apogee and perigee, an eccentric circle is not used (for reasons given below), but an epicycle. By taking the motion along the epicycle a little bit slower (6′ 41″ a day) than the motion of the epicycle, a regular progression of the apogee in longitude is obtained. The radius of the epicycle, corresponding to the eccentricity in an eccentric orbit, was found from the eclipses to be $\frac{1}{11.49} =$ 0.087, producing a maximum deviation of 5° 1′ from uniform motion.

All these results are based upon lunar eclipses; the time of mid-eclipse determines the position of the moon far more accurately than could any direct measurement. Ptolemy, however, did not content himself with them; he wished to know the moon's position at other parts of its course. So he constructed an instrument which he called 'astrolabon', which has no connection with what in later times was called an 'astrolabe' but is identical with the later *sphaera armillaris* or, in short, *armilla.* He gives an extensive description of the instrument, which, because of its importance for astronomy, is reproduced in figure 16. Two solidly connected rings represent the ecliptic and, perpendicular to it, the colure, i.e. the circle through the summer and winter points of the ecliptic and the poles of equator and ecliptic. An inner circle can turn around two pins at the poles of the ecliptic; its position, a longitude, is read on the graduated circle. Another graduated circle sliding along it within, and provided with sights, enables the observer to read the latitude of the star towards which they are pointed. This system of rings must be placed in such a way that its circles coincide with the circles at the celestial sphere. For this purpose it can revolve about two pins which are fixed in the colure ring at the poles and are attached to a fixed ring representing the meridian.

First of all, therefore, the meridian and the poles are set in the right position by placing the pedestal correctly. After the colure ring has been set at the longitude of the sun, the system of rings is revolved until the sights are pointed at the sun. Thus the position of the circles is the same as at the sphere, and the longitude and latitude of any star can be read by pointing the sights at the star. Instead of the sun, a well-known star

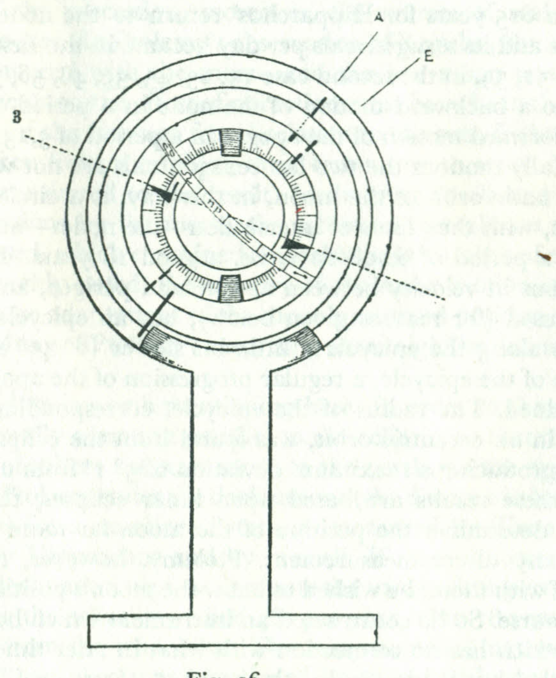


Fig. 16

can also be used to put the circles into the right position. In later manuscripts a picture of this instrument was given, with all its circles—the ecliptic of course excepted—placed in one vertical plane. To distinguish their functions, one must look at the connecting pins.

This instrument, afterwards also used to determine the positions of the fixed stars, served Ptolemy to measure repeatedly in the daytime the longitude of the moon relative to the sun. Then it was found that the measurements did not agree with theory. At first and last quarter, the maximum deviations from the uniform course were not $5^\circ 1'$, as at full moon, but $7^\circ 40'$.

It was for the purpose of representing and explaining this 'second anomaly' of the moon (in modern times called 'evection'), which depends on the position relative to the sun, that an epicycle for the first anomaly had been introduced. The mechanism devised by Ptolemy makes its distance from the earth alternately smaller and larger, at the quarter moons one and a half times larger than at full moon and new moon. Actually it comes down to the epicycle's centre describing in a monthly period not a circle but an oval; both the maximum and the minimum distance occur twice a month, and the great axis turns slowly, always being directed to the sun. Formally, to keep in line with the rule of Greek astronomy that all motions must be circular, Ptolemy assumes the deferent circle itself to revolve in the opposite direction, so that the epicycle's centre in its monthly revolution twice meets with both the apogee and the perigee. The eccentricity of the lunar deferent determines the ratio (the ratio of $7^\circ 40'$ and $5^\circ 1'$) of the distance in perigee and apogee at $1 - 0.21$ to $1 + 0.21$; the values themselves depend on the size of the epicycle, 0.106 times the radius of the deferent (see fig. 17).

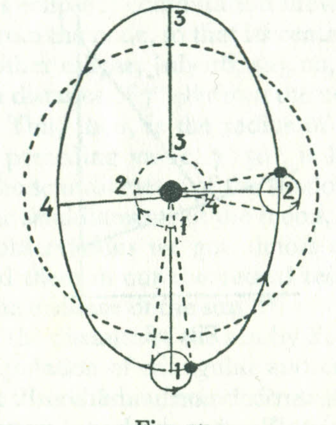


Fig. 17

Thus in an ingenious way the two irregularities in the moon's motion are represented by a system of circular motions, but with the result that the distance of the moon itself from the earth may change between $1.21 + 0.11$ and $0.79 - 0.11$, so that its apparent size also would change in the same ratio, i.e. as 33 to 17. Now everyone who by chance looks at the moon can state, without needing a measuring instrument, that this is not true; that the moon is not sometimes (at first and last quarter) double the size of the full moon. So the theory certainly cannot represent the real motion through space. It is said that this does not matter if only the apparent, observed motions are well rendered; the space structure is only a formal means to represent the visible course. This, however, is not true in the case of the moon; the parallaxes, which affect the visible position of the moon, would also be variable in the same unacceptable ratio.

Ptolemy then describes an instrument for measuring the parallax of the moon (fig. 18). It is an instrument for measuring in the meridian the distance of an object from zenith: an inclined lath, its upper end hinged at a vertical pole so that it can describe a vertical plane, is directed toward the moon by two sights. Its inclination is not read on a circular arc but on a graduated rod supporting its lower end, hinged at a lower point of the pole, and so representing the chord of the arc between the moon and the zenith. Ptolemy first used it to determine the meridian altitude of the moon at its maximum distance north of the ecliptic, which gave the inclination of the moon's orbit to the ecliptic as 5° 0′. Then, to find the parallax of the moon, he measured its zenith distance when, at its highest point, the parallax is negligibly small, and again at its lowest point, when the parallax is great; because the orbit

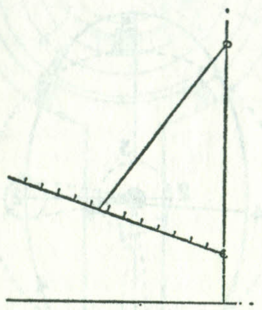


Fig. 18

is assumed to be symmetrical north and south of the equator, the parallax can be deduced. 'From a number of parallax observations in

such positions made by us, we will communicate one to show the course of the computation and to derive the further consequence.'[81] This is an observation made on October 1 of AD 135, which afforded a parallax of 1° 7′ at zenith distance 49° 48′, i.e. a distance of $39\frac{3}{4}$ radii of the earth. The cusps of the orbital oval then are at a distance of 59 radii and its flat sides at $38\frac{43}{60}$; these values are the foundations on which his tables of the parallax are based. A modern reader will ask with surprise whether all his other observations for parallax, not communicated, could agree, and why he chose just this one. But the ancient philosopher rather will be exalted to see how such sublime and intricate things as the distance and motion of the moon can be treated by such simple and strict calculation. The extreme values of parallax appearing in his tables are now 53′ 34″ and 63′ 51″ (highest and lowest points of the epicycle) at the cusps (full and new moon) and 79′ and 104′ at the flat sides (in the quarter). The range of these numbers, of course, far exceeds the real extremes of the moon's parallax. In this theoretical structure Ptolemy became a victim of his principle of explaining all the peculiarities in the moon's course by a system of uniformly described circles.

The derivation of the dimensions of the celestial bodies and of the distance of the sun was the task that remained. By using a diopter instrument like that of Hipparchus he found that the diameter of the sun appeared to be constant and equal to the full moon at its greatest distance. This means that annular solar eclipses cannot occur. For the determination of diameters, he thought this method not sufficiently accurate; so he now proceeded to derive them by theory, by means of the known elements of the orbits. He took two lunar eclipses observed at Babylon, so chosen that the moon was at its greatest distance from the earth. One was the eclipse of April 22, 621 BC; one-fourth of the moon's diameter was eclipsed; computation showed the moon to be at a distance of 9° 20′ from the node, so that its centre was 48′ 30″ north of the ecliptic. At the other eclipse, July 16, 523 BC, half its diameter was eclipsed; and, with a distance of 7° 48′ from the node, the latitude of its centre was 40′ 40″. This, then, is the radius of the shadow, and the difference from the preceding value, 7′ 50″, is half the radius of the lunar disc. 'Hence the semidiameter of the shadow is only slightly (4″) less than $2\frac{3}{5}$ times the semidiameter of the moon, 15′ 40″. Since from a number of similar observations we got almost concordant numerical results, we have used them in our theoretical researches on eclipses as well as in deriving the distance of the sun.'[82]

This derivation of the distance of the sun by Ptolemy, which appears as an intricate manipulation of triangular and circular sections of the spherical bodies and their shadow, comes down to the same relation as that derived by Hipparchus mentioned earlier (p. 129), but it is now used in another way: the sum total of the radii of the shadow and the sun, 40′ 40″ and 15′ 40″, diminished by the lunar parallax at the greatest distance of the full moon, 53′ 34″, leaves 2′ 46″ as the remainder for the solar parallax, so the sun's distance is 1,210 times the radius of the earth. It implies that the diameter of the sun is $5\frac{1}{2}$ times greater, its volume 170 times greater, than that of the earth. For the moon they are $3\frac{2}{5}$ times and 39 times smaller.

For a modern reader this entire derivation is illusory. The rough statements that one-fourth or one-half of the lunar diameter was obscured—rough inevitably because the border of the shadow is ill-defined—may be several minutes in error, and then the solar parallax may vanish completely. For Ptolemy, however, the matter must have been quite different. The character and purpose of the derivation are theoretical, an exposition of the geometrical connections of the phenomena and quantities. As such, it may be said that this derivation also exhibits the high point of view of Greek astronomy. That the astronomer is able, in theory, to derive the distance of the sun from observation of lunar eclipses shows how far man has proceeded in broad insight into world structure.

The sixth book of Ptolemy's work is devoted to the computation of the solar and lunar eclipses on the basis of this theory of the sun and the moon. In this fundamentally different treatment of eclipses as compared with the Chaldean method, we see reflected the different character of both astronomies. In Babylon the continuous series of eclipses was constructed as a totality, completed by an equally continuous, though irregular, row of eclipse indices. In Ptolemy's treatment every eclipse is computed individually, on the basis of tables from which the longitude differences, the anomalistic arcs, and the distances to the nodes can be derived for any moment; a special table indicates the distance to the node corresponding to eclipses of 1, 2, etc., digits' magnitude. The great errors in his lunar parallaxes mentioned above do no harm here, because the eclipse phenomena occur in the vicinity of full moon and new moon.

After sun and moon, he dealt with the stars. Ptolemy's work contains the first published catalogue of fixed stars, consisting of 1,022 stars, for which the co-ordinates relative to the ecliptic, longitude and latitude, are given, as well as the brightness, here indicated by the word 'magnitude'. The stars are arranged in constellations (part of the stars, called 'formless', stand outside) and are described by the parts or limbs they occupy.

The degrees of brightness are given as first, second, third, sixth magnitude, while for some stars the words 'greater' or 'smaller' are

added for greater precision; these designations have been maintained throughout subsequent centuries. The longitudes generally are about 1° too small; this has often been explained by Ptolemy having borrowed his catalogue from Hipparchus and having corrected the longitudes for precession by a correction of 1° too small, 2° 40′ instead of 3° 40′. As stated above, however, it is quite possible that his precession rests on observations made by himself. It has been conjectured that a list of a good 800 stars observed by Hipparchus were taken over by him without new measurements and that he added 170 mostly smaller stars. This is presumed on the grounds that the latitudes are mostly given in sixths of a degree, but for some stars in fourths of a degree; these sixths and fourths must therefore have corresponded to the graduation of their instruments. So far, this cannot be decided. The accidental errors are, of course, greater than these units of circle reading; comparison with modern data shows that the longitudes have a mean error of 35′, the latitudes of 22′.

For six stars Ptolemy adds a remark on their colour; he calls them *hypokirros*, i.e. yellowish. They are Aldebaran, Betelgeuse, Arcturus, Antares, Pollux and Sirius; the first five of these are the first magnitude stars now called 'red' or 'reddish'. In this 'red' the real colouring is exaggerated, just as Mars may appear to us fiery red, though as seen in a telescope it is only yellowish. So Ptolemy's description is more correct, and he must have seen them as we do, with the exception, however, of Sirius, which we know as bluish-white. That Ptolemy described it as having a reddish colour has always caused surprise in later centuries, and modern authors have often concluded that Sirius must have changed its colour since antiquity. That it was not simply a copying error may be inferred from the fact that in Roman literature *rubra canicula* is often spoken of, the red Dog Star, the fiery, burning star that brings the heat of summer. In a cuneiform text the star Kak-si-di is mentioned, that rises in the late autumn evenings and 'shines like copper'.[83] Such a catastrophic change however, from a red to a blue star is entirely excluded by modern astrophysics. Moreover, we find in the astronomical poem of Manilius a line on Sirius reading: 'Since it is standing far away, it throws cold rays from its azure-blue face.' The most probable explanation may be that Sirius, already visible when just rising, is coloured by the long distance which its rays must travel through the atmosphere; this holds good especially for Egypt, where it was observed only at its heliacal rising as a reddened star close to the horizon. Moreover, the colour attributed to the stars and planets by Roman authors often indicates their astrological, rather than their physical, character; thus was Saturn referred to as 'black'.

After the star catalogue Ptolemy gives a detailed description of the

Milky Way, which in all later centuries up to the nineteenth has not been repeated or improved; only in the second half of that century was it surpassed by more careful researches. And then an entire chapter is devoted to the construction of a celestial globe to depict the catalogued stars. 'For the background we choose a darker hue as corresponds not to the sky at daytime but more to the dark at night.'[84] He describes in detail how, by means of graduated rings, the stars are inserted as points according to their longitude and latitude. 'Finally for the yellow and otherwise coloured stars we put up their special colour in such measure as corresponds to the magnitudes of the stars.' The figures of the constellations are indicated by faintly visible lines only and not by striking colours; the Milky Way also is depicted with its bright parts and its gaps. So must Ptolemy's celestial globe have surpassed all later ones in precise representation of the sky. He used it to study and read, for any place of observation, the phenomena and positions of the stars relative to the circles, and especially to the horizon, i.e. their risings and settings.

The remaining and most important part of Ptolemy's work, the last five books, is occupied by the planets, in an exposition of the epicycle theory of their orbits, as explained above. Tables are given for all the movements in deferent and epicycle, allowing rapid computation of their longitude; in the last book tables are also given for deriving the latitudes. Finally, Ptolemy treated the computation of special positions and conspicuous phenomena such as the stations, the heliacal risings and settings, and the greatest elongations of Mercury and Venus, which stood at the origin of astronomy and played a major role in Babylon. The elaborate theory representing all irregularities is able to derive them in full detail; here Greek theory had surpassed the Chaldean rows of numbers. Tables are given of the elongations in the stations for consecutive values of the longitude, also tables of the greatest elongations of Venus and Mercury and of the heliacal risings and settings for the first point of every zodiacal sign.

Thus in this great manual of ancient astronomy, the *Mathematikè Suntaxis*, the world of heavenly bodies stands before us as a universe geometrically portrayed. It is a picture of eternal continuous motion in circular orbits, obeying determinate laws, a picture full of simple harmony, a 'cosmos', i.e. an ornament. It is expounded in a straight progression of exact demonstrations, without disturbing irregularities. Data are so selected or fashioned with admissible limits that the demonstrations tally exactly, and no incidental deviations—needing explanation, which might leave a certain feeling of doubt to disturb the harmony of the construction—distract the reader's attention. Moreover, the book, the work of the author, had to conform to a standard of perfection

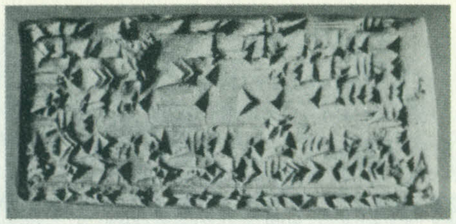
in form and workmanship. Mental work in those days stood not so far from handicraft; just as the craftsman carefully removed from his product any rough irregularity which might disturb the eye in enjoying the pure harmony of form and line, so the piece of mental work had to captivate the eye and mind by the pure presentation of the universe in its mathematical image.

Geometry occupied a paramount place in Greek culture as the only abstract exact science of the visible world. Because of its rigidly logical structure, proceeding from axiom and proposition to proposition, it stood out as a miracle of the human mind, a monument of abstract truths, outside the material world and, notwithstanding its visibility to the eye, entirely spiritual. The small particle of utility in its origin in Egyptian geodesy hardly carried weight against Euclid's great theoretical structure. Certainly the *sphaerica*, the theory of the sphere and its circles, found a wide practical application in astronomy in the description of the celestial sphere, and the risings and settings of the stars, and so was used and taught during all later centuries. But spherics was only a small part of geometry. The entire science of lines and angles, of triangles, circles and other figures, with their relations and properties, was a purely theoretical doctrine, studied and cultivated for its intrinsic beauty.

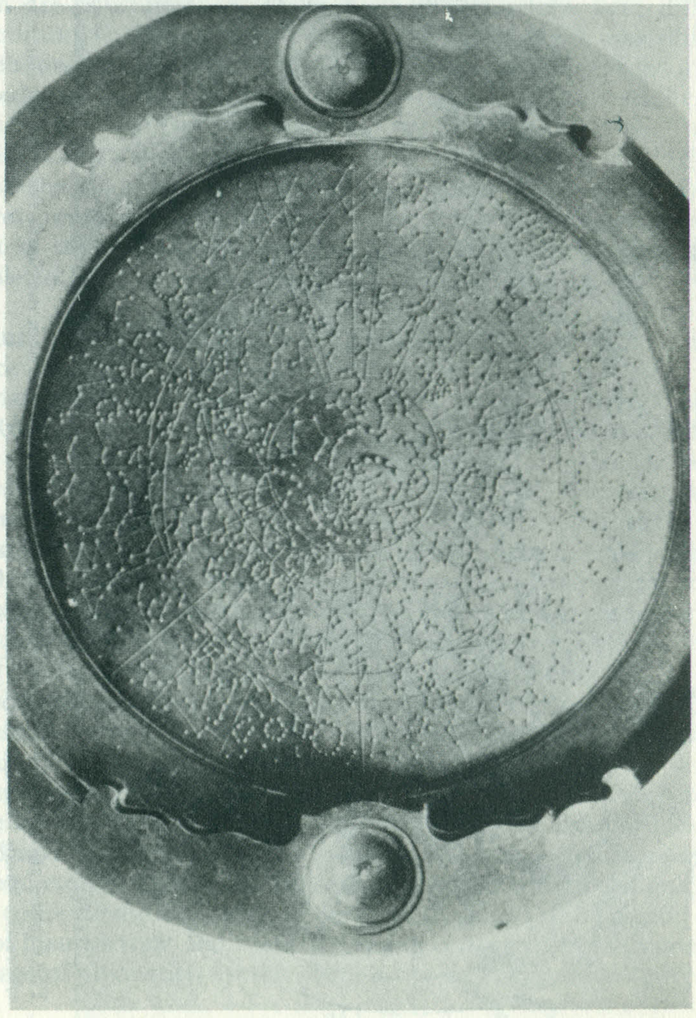
Now, however, came the science of the planetary motions, the work of Ptolemy, as a practical embodiment of the theory. What otherwise would have been imagined truths, existing in fantasy only, here became reality in the structure of the universe. Here it acquired form and specific value and size. In the world of planets the circles moved, distances stretched and shrank, angles widened and dwindled, and triangles changed their form, in endless stately progression. If we call Greek astronomy the oldest, indeed the only real, natural science of antiquity, we must add that it was geometry materialized; the only field, truly, where geometry could materialize. Whereas outside this world of astronomy the practice for students of geometry would have been restricted to an idle working with imaginary self-constructed figures. In astronomy they found a realm, the only one but the grandest, where their figures were real things, where they had definite form and dimension, where they lived their own life, where they had meaning and content: the orbits of the heavenly luminaries. Thus the *Mathematical Composition* was a pageant of geometry, a celebration of the profoundest creation of human mind in a representation of the universe. Can we wonder that Ptolemy, in a four-line motto preceding his work, says that 'in studying the convoluted orbits of the stars my feet do not touch the earth, and, seated at the table of Zeus himself, I am nurtured with celestial ambrosia'?

His task, however, was not finished. Rising from the table of Zeus, he had to enter the council room of the gods, to hear how they make known their ordinances to the mortals. It was not disinterested eagerness for knowledge of the heavenly motions that incited him, as well as his predecessors and contemporaries. The knowledge of these motions was a means toward the higher purpose of practical knowledge of the future happenings on earth, of human life and destiny. Hence the thirteen books are followed by four more books, which were mostly edited separately and acquired the name *Tetrabiblos*. In modern times attempts have often been made, by doubting the authenticity of his authorship, to absolve the great astronomer from the blame of having believed in that astrological superstition. But in his purely astronomical work also we can find a statement of this kind: at the end of the eighth book, speaking of heliacal risings and settings, he says that their influences upon the weather are not invariable but depend also on the oppositions to the sun and the position of the moon.

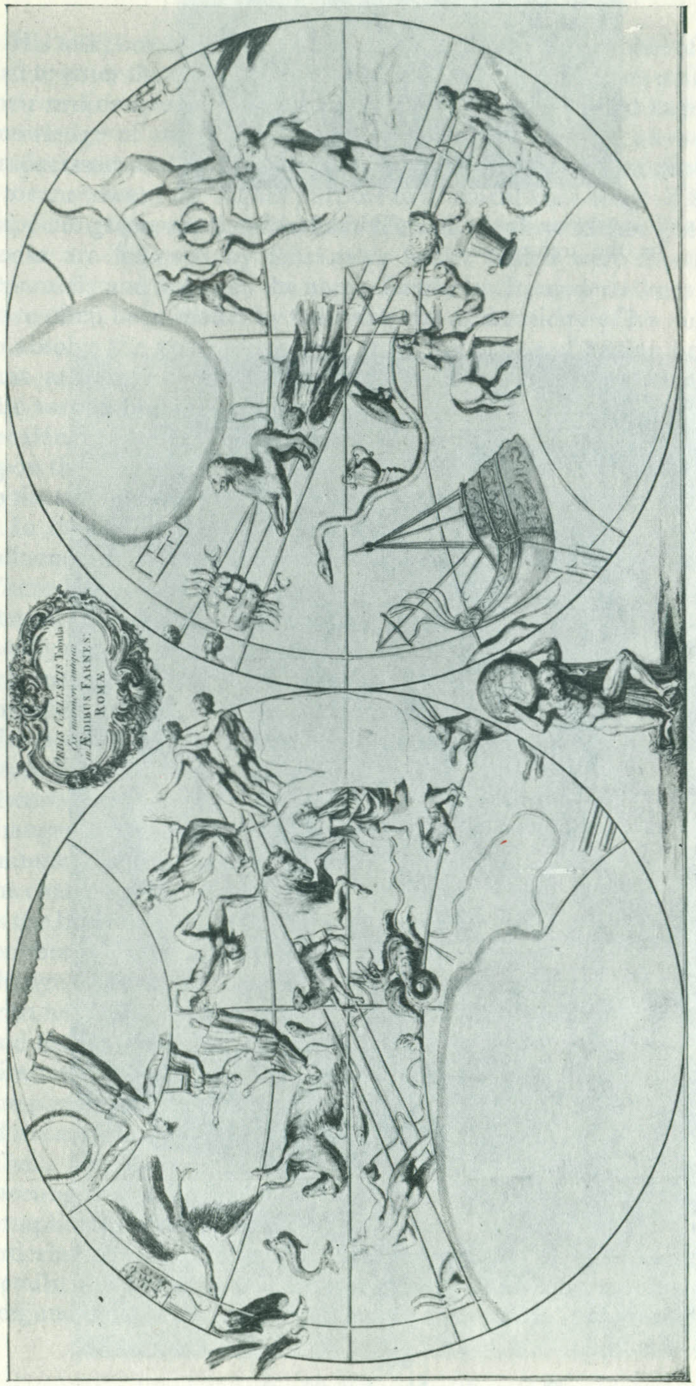
In the four books, however, Ptolemy gives a general theory of the influence of the celestial bodies upon earthly happenings and on man. There he says: 'That a certain power emanates and spreads from the eternal world of the ether upon all that surrounds the earth and that is totally subject to change; that the first sublunar elements, fire and air, are encompassed and changed by the motions in the ether; and that they then encompass and change all else, too, earth and water and the plants and animals therein—this is entirely clear to everybody and needs little comment. For the sun, with its surroundings, in some ways always imposes its order upon everything on earth, not only by the changes accompanying the seasons of the year, the procreation of the animals, the fruit bearing of the plants, the flowing of the waters, and the changes of the bodies, but also by its daily revolutions. . . . The moon, as the body nearest to the earth, bestows her effluence upon the earth; for most things, animate and inanimate, are sympathetic to her and change together with her, the rivers increase and decrease their streams with her light, the seas turn their own tides with her rising and setting, and the plants and the animals in whole or in some part become full or diminish together with her. . . . Also the passages of the fixed stars and the planets give abundant presages of hot, windy and snowy conditions of the surroundings, by which also all that lives on earth is conditioned. Then, too, their positions [aspects] relative to one another, by the meeting and mingling of their dispensations, bring about many and complicated changes. For though the sun's power prevails in the general ordering of quality, the other [bodies] can add to or retract from it in details; the moon does it more obviously and continuously, as in her conjunctions, at quarter or full moon; the stars do it at greater intervals



1. The Assyrian Tablet, K725, British Museum (see page 40 for the transcription)



Chinese Star Map (pp. 88–90)



2. The Constellations on the Farnese Atlas (the blank spaces, other than the South Pole, are the damaged Ursa areas and the places where the hands hold the globe) (p. 132)

and more obscurely, as in their appearances, occultations and approaches.' Of course, other causes are also at work on man: descent, place of birth, nationality, rearing; moreover, lack of ability and knowledge often makes predictions fail. But 'it would not be fitting to dismiss all prognostication because it can sometimes be mistaken, for we do not discredit the art of the pilot for its many errors. . . . Nor should we gropingly in human fashion demand everything of the art but rather join in the appreciation of its beauty.'[85]

Then detailed expositions follow, first on the characters and working powers of each of the planets, then on the fixed stars and zodiacal signs which are concordant or discordant in their effects with different planets, and, further, on what countries are affiliated with these different planets and stand under their influence. In subsequent books the method and basic predictions are explained, first for the weather and then, by means of the horoscopes, for men, not only as to quality and events of body and soul but also as to their material fortunes and happiness. Since the fundamental data—the positions of the celestial bodies at every moment—are provided by astronomical computations laid down in the former books, science also in its practical aims is now completed.

Thus the work of Ptolemy stands before us, a great monument to the science of antiquity. During his life and work, under the emperors Hadrian and Antonine, undisturbed peace and prosperity reigned in the Roman Empire. It was as if, at long last, after all the previous strife and chaos of wars of conquest and civil war, civilized mankind, now united, had entered on an era of sunny peace, of harmonious culture and of tranquil security. But this society was rotten to the core. The sunset glow of antiquity shed its last radiance upon a worn-out world. Before long, towards the end of the second century, the storms broke loose that within the space of a single century were to undermine the power of the world empire and, after another century, were to lay waste the foundations of the ancient world and its culture.

The totality of the causes of this downfall cannot be discussed here; there is, moreover, much diversity of opinion on the connection between its various aspects and phenomena. First, there was the fact that its labour system was based on slavery, which like a cancer corrupted the entire life-system of antiquity and which, when the wars of conquest ended and the supply of slaves was checked, resulted in a primitive serfdom. Another important cause was the disappearance of gold and silver coin—by the exhaustion of the Spanish mines and by the importation of luxuries from India—with ensuing commercial paralysis and a stoppage of production for sale, with a retreat to primitive agriculture for private use. It brought about a great decline in the financial power of the state. Then came a terrible pestilence, brought back in AD 188 by the armies from Asia, with a toll of 2,000 dead per day for Rome alone. It swept over all countries, was rampant for many years and shattered the military power of the Empire. Now the barbarian tribes pressed in and, after devastating raids, settled in the depopulated areas. Mention has also been made of the extermination of the ruling classes, the nobility and the propertied urban citizens, by the peasant armies and their soldier-emperors. Finally, after a century of continuous decline there followed the establishment of a despotic government of officials, until, another century later, the Western Empire was entirely conquered by armed Teutonic peoples.

In this chaos of devastation and utter ruin of ancient society the culture of antiquity also collapsed. In the masses, devoid of hope and future, a new life-conception arose which turned away from the actual world and, while striving to make it tolerable through mutual succour and philanthropy, found comfort and refuge in the belief of a better life hereafter. Christianity increasingly superseded the old religions, fought successfully against the pagan systems of philosophy, and, with the withering of the old state power, took its place as the social and spiritual organization of society.

In this new world concept, concealed from the wise but revealed to the simple-hearted, there was no longer room for the most highly developed science of antiquity. Its cosmologic world scheme was superseded by the original biblical teaching of the flat earth, which, indeed, fitted in with the return to the primitive modes of production—agriculture for home and for tribute and tithes. Whereas the Christian authors of the second and third centuries were well versed in their polemics against the pagan philosophers, the ideas of the Church Fathers in the next centuries became more primitive. Lactantius ridiculed the doctrine of a spherical earth, and Cosmas Indicopleustes, the traveller to India, described a square, flat earth in the form of the poetic effusion in the Book of Ecclesiastes.

So progress in astronomy after Ptolemy was out of the question. Works on astronomy in these centuries are summaries and comments, giving explanations of the classical works, including Ptolemy. They are of value to us because they often give details from works themselves lost. Such commentators are not investigators but scholars; they were praised among their contemporaries not for what they did but for what they knew. Among them was Proclus, surnamed Diadochus (the successor), who worked in the fifth century in Athens as the last of the pagan philosophers; he wrote a useful commentary on the more elementary parts of Ptolemy. Shortly afterwards there was Simplicius, famous

commentator on Aristotle, who, when expelled by the Byzantine Church in AD 529, found a temporary refuge in Persia. He mentions that his teacher Ammonius of Alexandria had made observations of Arcturus by means of an astrolabe. In these commentaries the most simple principles form the main contents; the scientific level reached by Ptolemy was no more. The astronomical part of the works of Isodorus, Archbishop of Seville in the seventh century, highly praised for his learning, wherein it was said that the stars receive their light from the sun and that the moon revolves in 8 years, Mercury in 20, the sun in 19 and Mars or the Evening Star in 15, shows to how low a level astronomical knowledge in Western Europe had fallen.

But in the next century a new power—Islam—appeared in the Near East, and an offshoot of ancient astronomy sprang up in those parts.

CHAPTER 15

ARABIAN ASTRONOMY

IN the middle of the seventh century AD the Arabs broke out of their deserts and conquered the surrounding civilized countries, first Egypt and Syria, which then were parts of the Byzantine Empire, and then Mesopotamia, the prosperous nucleus of the New-Persian Empire. This was a repetition of analogous former expansions, such as the Semitic conquest of Babylonia after 2000 BC, and the later Aramaic invasion of Syria. We do not know exactly the cause of such migrations; often climatic events such as severe droughts were the underlying factor. In the most highly developed commercial and trading centres, in the Hejaz, influenced by the Jewish and Christian religions, Mohammed had founded and propagated his doctrine that was to bind the ever-divided and interwarring tribes into one powerful unity—the doctrine of Islam, the brotherhood of all the faithful, before which all the old ties of tribe and family had to give way.

The Arabs, a people of strong and hardened men, camel-tending nomads, proud warriors and robbers, imaginative possessors of a flowery language and of rich poetry, now became the masters of Near-Asia. With their fresh primeval force and the intensive new ideology of Islam, the Arabs gave a powerful impulse to the economic and cultural development of this part of the world, bringing it to a rich flowering. Through their conquests they created a world empire extending from the Atlantic coast in Spain and Morocco to India and the mid-Asian steppes. Trade and commerce unfolded here, bringing far distant regions within one large economic unit. As a religious community the entire realm remained a cultural unit even when politically split up into a number of independent sultanates. In all these countries, mostly in the fertile plains of Mesopotamia, Syria, Egypt and Andalusia, artistic handicrafts developed in a number of new flourishing towns. Commerce, carrying the products of China and India to the West, of Byzantium and Europe to the East, brought about an interchange of culture and science. Powerful rulers, first the caliphs at Baghdad, later the sultans of smaller countries like Egypt, and still later the Turk and Mongol conquerors from Central Asia, became promoters and protectors of the arts and science.

It was the concern for the security of their own lives and future that aroused in sagacious princes a direct interest in science. Medicine to maintain health and life, astronomy to ascertain the future and their destiny, held a first place for them. Mohammedan scientists—we call them Arabians although by birth they were Syrians, Persians, Jews and, later, natives of other countries—were most famous as physicians and astronomers, and in this connection they cultivated mathematics, chemistry and philosophy. Mostly these sciences were combined; physicians, as in later centuries in Europe, had to know astrology in order to find the propitious time for different treatments. Moreover, in the Mohammedan world there was a direct need for astronomical knowledge; for the pure moon-calendar observation of the moon was necessary. A precise timekeeping by means of water-clocks and sun-dials had to indicate the times prescribed for prayer; because when praying the face had to be directed towards Mecca, astronomical experts had to indicate that direction in the mosques throughout the world.

In the first flourish of the Baghdad caliphate in the eighth century, ancient science made its entry. Primary knowledge was borrowed from the Nestorian Christians, who had found in Persia a refuge from the persecutions of the Byzantine Church and had founded schools there. What was borrowed from India was more important. After the conquests of Alexander and of the later Macedonian rulers of Bactria, Greek science had grown an offshoot in India. Under the Gupta dynasty in Hindustan (about AD 400–650) there arose a literature of mathematical and astronomical writings, called 'Siddhantas', proceeding from different authors, amongst whom Brahmagupta is the best known. In these works one meets the Greek world picture: the spherical earth and the epicyclic orbits of the planets, less detailed in comparison to Ptolemy and without the equant. Sometimes even a rotation of the earth is mentioned.

From India this influence now turned back to the west. It is mentioned that in 773 there appeared, before the caliph Al Mansur, a man from India who was acquainted with the stars and could calculate eclipses. Whereupon the caliph ordered the translation of the Indian books. The first astronomical tables were published in the next century by Muḥammad ibn-Mûsâ al-Khwârizmî. That these tables had been translated from Indian originals was evident since all data were given for the meridian of Udshain, the capital of one of the states in central India and seat of an observatory. Theory was lacking in these works; they consisted only of numerical tables with instructions for use, intended for calendar and astrological purposes. They introduced a valuable innovation into western arithmetics: the Indian system of the old, less perfect, but more handy tables of Al-Khwârizmî were re-edited by the Spanish astronomer Maslama ibn Ahmed (died 1008), transferred to the meridian of Cordoba, and completed by an excerpt from Al-Battânî.

From the same time (the tenth century) 'Abd al-Raḥmân ibn 'Umar called Al-Ṣûfî (i.e. 'the wise') must be mentioned, because in a book on the fixed stars, where he takes the longitudes from Ptolemy, increasing them by 12° 42′, he gives the magnitudes of the stars after careful observations by himself. So his work is a valuable independent source of knowledge of their earlier brightness. At the same time in Egypt, protected by Sultan Al-Hakim of the Fatimid dynasty, the astronomer Ibn Junis not only published new tables (the 'Hakemite Tables') with theory and computing methods, but also communicated a large number of observations of eclipses, conjunctions and altitudes, partly taken from the records of former Arabian observers as early as 829, and partly from his own work in Cairo 977–1007. About the same time the Turkish prince Sharaf Al-Dawla ordered the construction of an observatory, in his garden at Baghdad, to be provided with many new instruments. Here a number of astronomers, among whom Abu'l Wefa is best known, made observations of equinoxes, solstices, and the obliquity of the ecliptic.

Astronomy now makes its appearance in the more distant realms of Islamic culture. Spain in the eleventh and twelfth centuries, under the Cordoba caliphate, had a high tide of culture, of arts and sciences. Among the Spanish astronomers of this time, Ibn al-Zarqâla ('Arzachel', 1029–87) was prominent; he published the *Toledo Tables* with a description of the instruments and their use, especially the astrolabe. He made many observations, from which he deduced a solar apogee at 77° 50′, smaller and less accurate than Al Battani's value. At a somewhat later date lived Jâbir ibn Aflaḥ (sometimes mistaken for Gabir ibn Haijan, the alchemist) whose astronomical work was translated into Latin and often published in medieval Europe. In the thirteenth century the power of Islam in Spain came to an end. The religious zeal of the Christian Castilians—in need, moreover, of southern winter pastures for profitable sheep-breeding, to produce the fine wool for Flemish cloth—conquered the fertile Andalusian plains with the brilliant capitals Cordoba and Seville, thus putting an end to the flowering of Arabic horticulture.

The work of the Castilian King Alfonso X, surnamed 'The Wise', may be considered as a last offshoot of Arabian astronomy in Spain. He assembled around him a number of astronomers to construct new astronomical tables; these 'Alfonsine Tables' (1252) were in use for three centuries, up to the middle of the sixteenth century. The leader of this group was the Jewish scholar, Isaac ben Said; it is said that, owing

to the traditions of Jewish jubilee years, the period of precession in the tables was assumed to be 49,000 years and the period of trepidation, 7,000 years.

In the same century the Mongol ruler, Hulagu il Khan, a grandson of Genghis Khan, founded an observatory at Maraga (near Tabriz in Persia), under the direction of his counsellor Nâṣir Al-Dîn al-Ṭûsî (died 1274), an able astronomer from Khurasan. At great expense a library of 400,000 manuscripts was installed and many instruments, partly of new design; among them was a great quadrant of 10 feet radius, constructed by Al-'Urḍî. After a dozen years of assiduous observation of the planets by Nasir ud-din and his assistants, they were able to construct the 'Ilkhanic Tables'.

Another two centuries later astronomy sprang up in Samarkand. Ulugh Beg, a grandson of the Mongol conqueror Tamerlane, during his father's reign founded there a richly endowed observatory, where he himself took part in the observations. One of the instruments was part of a circle, probably a quadrant, 60 feet in radius, placed vertically in the meridian between masonry, in order to determine accurate altitudes of the sun. Ulugh Beg also determined the positions of the stars; he was the only oriental astronomer known not to have copied Ptolemy after correcting the longitudes for precession; he founded them upon his own observations. His catalogue of stars observed in 1420–37, first became known in Europe a century afterwards and was not printed until 1665, when it had already been surpassed by European catalogues.

Some of the most important names in Arabian astronomy have been mentioned here. In his list of astronomers and mathematicians, H. Suter gives nearly four hundred names, all of whom were praised by contemporaries and successors as great scholars. There are many among them whose chief merit lay elsewhere, in the realm, for example, of medicine or chemistry. Ibn Sînâ ('Avicenna', 980–1037), famous as a physician and philosopher, also made observations, wrote on astronomy and edited a compendium of Ptolemy; in those days it was possible to master many sciences. But it cannot be assumed that to be a scholar always implies being an investigator. Arabian scientists certainly observed diligently; they constructed new instruments, and in astronomy they seem to have displayed more practical activity than did the Greeks. Also, the accuracy of their work often surpassed the results of antiquity. Their aim, however, was not to further the progress of science—this idea was lacking throughout—but to continue and to verify the work of their predecessors. The continual compiling of new tables did not necessarily mean—in fact seldom meant—progress in more precise values. The tables were needed for astrological purposes. Hence the astronomers had to compute and publish them ever anew; and if they were scrupulous people, they took care to check the positions of the planets by their own observations and to complete them by their independent results.

In their theoretical ideas, however, they did not go beyond antiquity. They often were not content with Ptolemy's theory; but when they deviated, it was from preference for Aristotle. Thâbit ibn Qurra is reported to have assigned to each planet a space between two eccentric spheres. Ibn al-Haitham ('Alhazen'), known by his work on the refraction of light, had 47 spheres, all turning in a different way about and within one another. Ptolemy's conception of circles and centres, existing in fancy only, was not concrete enough for them.

In the twelfth century Aristotle's philosophy was diligently studied and developed by Moslem thinkers, especially in Spain. The most famous among them, Muḥammad ibn Rushd ('Averroës'), used it as the foundation for a pantheistic philosophy which spread through Europe and was condemned as a dangerous heresy by the Church. He and his followers thought that circular motion about a centre was possible only when a solid body, like the earth, occupied that centre. The Jewish scholar Moses ben Maimon ('Maimonides'), as well as the Moroccan astronomer Al-Biṭrûjî ('Alpetragius'), rejected the epicycle theory; the latter considered the motion of the sun, the moon and the planets as a lagging behind the daily rotation and so came back to the ancient ideas of Plato.

So there was a brilliant rise in Arabian astronomy, but no significant progress. After some centuries it died down. This indeed was the case with Mohammedan culture as a whole. A mighty impulse of conquest, borne by great religious enthusiasm, had built a world empire in which trade and crafts under social and economic prosperity engendered a special civilization. But an impulse towards continual progress was lacking; minds were dominated by a quiet fatalism. Then came devastation by the Mongol inroads from the steppes of Asia; for in their irresistible attacks the Mongols razed towns, exterminated the inhabitants, destroyed the irrigation works and thereby turned flourishing, thickly-populated regions into lifeless deserts. The power and the flower of Islam declined, and along with them its culture and its astronomy. Of the rich libraries little was saved; no Arabian manuscript is known to exist today of the tables of Al-Khwârizmî. The importance of Arabian astronomy lay in the fact that it preserved the science of antiquity in translations, commentaries, interpretations and new observations and handed it down to the Christian world. Thus it considerably influenced the first rise of astronomy in medieval Europe.

PART TWO

ASTRONOMY IN REVOLUTION