were scrupulous people, they took care to check the positions of the planets by their own observations and to complete them by their independent results.

In their theoretical ideas, however, they did not go beyond antiquity. They often were not content with Ptolemy's theory; but when they deviated, it was from preference for Aristotle. Thâbit ibn Qurra is reported to have assigned to each planet a space between two eccentric spheres. Ibn al-Haitham ('Alhazen'), known by his work on the refraction of light, had 47 spheres, all turning in a different way about and within one another. Ptolemy's conception of circles and centres, existing in fancy only, was not concrete enough for them.

In the twelfth century Aristotle's philosophy was diligently studied and developed by Moslem thinkers, especially in Spain. The most famous among them, Muḥammad ibn Rushd ('Averroës'), used it as the foundation for a pantheistic philosophy which spread through Europe and was condemned as a dangerous heresy by the Church. He and his followers thought that circular motion about a centre was possible only when a solid body, like the earth, occupied that centre. The Jewish scholar Moses ben Maimon ('Maimonides'), as well as the Moroccan astronomer Al-Biṭrûjî ('Alpetragius'), rejected the epicycle theory; the latter considered the motion of the sun, the moon and the planets as a lagging behind the daily rotation and so came back to the ancient ideas of Plato.

So there was a brilliant rise in Arabian astronomy, but no significant progress. After some centuries it died down. This indeed was the case with Mohammedan culture as a whole. A mighty impulse of conquest, borne by great religious enthusiasm, had built a world empire in which trade and crafts under social and economic prosperity engendered a special civilization. But an impulse towards continual progress was lacking; minds were dominated by a quiet fatalism. Then came devastation by the Mongol inroads from the steppes of Asia; for in their irresistible attacks the Mongols razed towns, exterminated the inhabitants, destroyed the irrigation works and thereby turned flourishing, thickly-populated regions into lifeless deserts. The power and the flower of Islam declined, and along with them its culture and its astronomy. Of the rich libraries little was saved; no Arabian manuscript is known to exist today of the tables of Al-Khwârizmî. The importance of Arabian astronomy lay in the fact that it preserved the science of antiquity in translations, commentaries, interpretations and new observations and handed it down to the Christian world. Thus it considerably influenced the first rise of astronomy in medieval Europe.

PART TWO

ASTRONOMY IN REVOLUTION

CHAPTER 16

DARK EUROPE

In the ninth and tenth centuries, when under Islam commerce and trade, culture and science attained their highest flowering, Europe was a barbaric land, sunk in utter impotence. The Arabs controlled the Mediterranean; they sacked the city of Rome in AD 846, and occupied not only Spain and Sicily but also Provence, whence they raided France. From the north the Vikings came looting and conquering; from the east the Magyars invaded and devastated Western Europe. Economic life had dwindled to a miserable agricultural activity carried on by ignorant serfs, who were ruled by lords and priests hardly less ignorant than themselves. A scanty knowledge of Latin and of some Latin authors maintained itself with difficulty in some few monasteries. Latin, a living language down to the seventh century, had now become a clerical language, restricted to the Church. 'For centuries following,' said Pirenne, 'there was to be no science but within the Church.'[87]

Yet the connection of man with heaven and science was not lost altogether. In some monasteries a limited knowledge of constellations was preserved because their rise in the night indicated the hour for religious services. The rules for fixing the date of Easter formed part of the Christian doctrine; some ecclesiastics were therefore needed who could understand something of the course of the stars and who were able to make computations according to precept. As a thin rivulet of science, *computus* (i.e. computation, which in this context always means calendar-computation) ran through these centuries. At an even earlier date the English monk Bede, surnamed 'Venerabilis' (died AD 735), had computed lists of Easter dates; his writings show that he was acquainted with Pliny and Seneca, and also with the sphericity of the earth. The mysterious variations of the date of Easter were considered by the monk Notker (of the St Gall monastery, a renowned centre of studies) as part of the general domination of the heavenly bodies over earthly life.

About the year 1000 the European world began to recover. The Northmen and the Magyars were repelled, Christianized and absorbed into the community of European culture. The Italian seaports conquered their Mediterranean trade routes and became centres of flourishing commerce. Towns developed as market places, or around the castle, or where the trade routes between Italy and the North intersected. At newly-founded monasteries, where productive labour was combined with intellectual study, a more profound cultivation of the Christian doctrine and a stricter pattern of life developed. In the eleventh century, under the leadership of Hildebrand (afterward Pope Gregory VII), the Church grew into a well-organized clerical hierarchy, directed from Rome. The Papacy took its place as a spiritual power parallel to the temporal power of feudal lords, the kings and emperors. It assumed the spiritual leadership of Christian Europe and felt strong enough to take the offensive against Islam in protection of Christendom. These Crusades brought the rude and primitive world of knights and monks into contact with the refined, but already declining, Arabian culture.

European Christendom now began to raise itself spiritually with the help of Arabic science, chiefly imported from Moslem Spain. Even before this time the learned Gerbert, afterward Pope Sylvester (99–1004), is reported to have made inquiries at Barcelona about books on astrology; he is credited with the authorship of a book on the astrolabe. A century later, Athelhard of Bath personally went to Spain to study Arabic wisdom at its source, which he extolled and explained after his return. In 1126 he edited a Latin translation of the astronomical tables of Al-Khwarîzîm, in Maslama's version, thus proceeding from the first instead of from the later more highly developed forms of Arabic astronomy. He also translated the astrological work of Albumazar. Such a translation generally amounted to a difficult search for, and a forging of, more or less appropriate terms. The first tables of European astronomy bear witness to the awkwardness of the tiro; Arabic words appear untranslated in the Latin text or in the headings of the tables, e.g. 'inventio elgeib per arcum' (the finding of the sine for the arc).[88] Often we come across Latin words which are quite different from the more precise terms adopted later, thus, e.g. 'obliquatio' for what was afterward called 'declination' of the sun. Some Arabic words have remained in use as technical terms, e.g. 'azimuth', 'zenith', 'nadir'. In the same way the names of some stars, which simply designated certain objects or parts of the body in Arabic, became proper names in European astronomy, e.g. Betelgeuse ('the giant's shoulder' —bat al-dshauzâ'), and Algol (the 'monster's head'—ra's al-ghûl).

A number of other translators of Arabic science now came forward. The most famous among them was Gerhard of Cremona (1114–87), who went to Toledo to search for Ptolemy's work and there found a wealth of books not only by Arabian authors but also by the scholars of antiquity. He set himself to edit Latin translations, first of the

Almagest (1175)—a translation made shortly before in Sicily, direct from the Greek, was little known—and then also of Euclid, Galen, Aristotle, Archimedes and many others. Universities were now founded in Bologna, Paris and Oxford, where the new sciences were taught and began to pervade men's minds. But not without opposition, for at first Aristotle was banned at the University of Paris. Irresistibly, however, the new ideas spread all over the European world which had now outgrown the primitive agrarian economy and its spiritual counterpart, the primitive cosmology.

At the end of the twelfth century the consolidation of European Christendom was accomplished. The Church had by now become the leading spiritual power in the feudal society of princes and knights, of abbeys and monasteries, of peasants and urban citizens. All over Western Europe the spires of Gothic cathedrals soared, symbols of burgher freedom in ecclesiastical garb; and at the courts the poetry of chivalry flourished. The clergy constituted the intellectual class in this simple society of agriculture, craft and commerce, and performed the social functions of spiritual, administrative and scientific leadership.

Under Innocent III, the Papacy rose to the summit of its power and became a universal monarchy of Christendom, commanding and deposing kings and emperors. The Franciscan and Dominican orders were founded as the strong moral and intellectual militia of the Church. Through their preaching and propaganda, soon reinforced by persecution and inquisition, all dissenting ideas were opposed and suppressed, and even forcibly exterminated; in this way, unity of doctrine was established. It was from these circles that the students and exponents of science emerged.

At the beginning of the thirteenth century the work of translation was finished; it was followed by assimilation, critical discussion and independent creative work. The study of astronomy as a special doctrine was mainly restricted to 'sphaerica', the doctrine of the celestial sphere and of the phenomena resulting from its daily rotation. A textbook on this subject, written by Johannes de Sacrobosco (John of Holywood), who died in Paris in 1256, was still widely used three centuries later. A new astronomical world-picture arose, which formed part of the general conception of the world, framed by the scholars of the thirteenth century.

The ablest and most original thinker and teacher of this century was the Dominican friar Albertus Magnus (1193–1280), who in his writings expounded the conceptions and doctrines of Aristotle and also referred to Ptolemy as an astronomer and astrologer. Even greater fame was won by his fellow-friar and pupil, Thomas Aquinas (1225–74), a less original but more methodical mind. In his works he united Aristotle's cosmology with the doctrine of the Church into one system of thought which, under the name of Scholasticism, was to dominate the minds of men for many centuries. By their side stood the Franciscan friar Roger Bacon, who also praised Aristotle as the source of wisdom, and sharply criticized his own age and its learning. He made fantastic references to future machines, and recommended experience, *experimentatio*, as the true method for the acquisition of knowledge; on this account he has often been acclaimed as a precursor of the later principle of inductive science.

Thus the astronomical world-picture in Europe had risen to the level of Greek antiquity again. About the spherical earth in the centre of the world the planets and stars move in the celestial sphere. That is how we find it described in Dante's *Divina Commedia*, in which he placed Hell in the deepest depths, that is, at the centre of the earth. With his companion, the poet descended to the centre, where Lucifer was undergoing his punishment, and then on the opposite side ascended to the earth's surface again. There, under a milder sky—which Columbus later on believed he had found in the mild climate of the West Indies—he saw four brilliant stars, symbols of the four principal virtues, which later commentators, probably wrongly, identified with the not very bright stars of the Southern Cross. He then climbed the mountain of Purgatory, from which the planetary spheres were ascended.

It must be added, however, that the world concept of those days was not confined to the sober structure of Aristotle's cosmos. It was pervaded and dominated by astrology. The dominion of the stars over terrestrial events caused medieval man to look upon astronomy as the supreme doctrine of the world. The belief in occult forces and in magic was universal then, even among the most famous scholars. Their knowledge of the secrets of nature made them magicians; the study of nature in its early stages was intimately associated with magic, and all great scholars, Ptolemy and Galen as well as Avicenna, Gerbert and Albertus Magnus, were regarded by contemporaries and posterity as sorcerers and miracle-workers. So little was known of the laws of nature that the whole of creation appeared as a miracle, a world of wonder in which everything was possible or might be found true, and in which naïve credulity accepted all so-called 'facts'. Along with the science of antiquity its superstitions were taken over, and among them astrology appeared as the all-embracing doctrine of the world.

This also applies to the three great thirteenth-century thinkers referred to above, whose views about astrology were substantially similar: the stars rule the earthly bodies, God rules the lower creatures by means of higher beings, and all things on the earth by means of the celestial spheres. Thus the movement of the stars dominates life on earth, and the conjunctions of the planets disturb the regular order of

events. But for man this is no ineluctable fate; he is not entirely subject to it. His will is free, because his soul, as a higher being, proceeds from the Supreme Being. It is only when he fails to offer resistance that he is carried along by nature, and thus is swept away by the power of the stars. Horoscopes were therefore useful as warnings, and prognostications had to be kept within general terms; hence severe censure of astrological fortune-tellers was imperative. After having thus reconciled itself with the Church, astrology could maintain itself through the following centuries, notwithstanding occasional scepticism and criticism from opponents.

CHAPTER 17

THE RENASCENCE OF SCIENCE

In the fourteenth century the dream of the Church as a universal monarchy of European Christendom faded. The medieval world had developed into new forms. In all countries the towns had grown up as centres of crafts and commerce; the urban dwellers, increasing in prosperity and power, emerged as the class that more and more determined the aspect of society. Kings, supported by the financial power of the burgher class, established centralized state powers with laymen as civil officials. In continual struggles, the powers of the kings of France, England and Castile increased, and they superseded the power of the popes, who were often little more than dependent bishops of Rome. Secular power superseded ecclesiastical power. The clergy were no longer the spiritual leaders; intellectual leadership of society passed into the hands of the laity. Interest in science increased among the urban dwellers, who were eager for knowledge with which to promote the development of trade, and desirous of raising their status as masters of the new world. Their sons attended the universities, which were steadily growing in number, studied Roman Law, and became officials and advisers of the princes, assisting them in undermining feudal law and feudal society by means of this new judicial doctrine.

The study of science was still directed to assimilating ancient knowledge and the entire culture and science of antiquity. But a new spirit arose here and there, a spirit of independent research, of new and bold ideas, and of desire for further progress. At the University of Paris, a group of philosophers, Jean Buridan, Albert of Saxony and Nicolas Oresme, precursors of the scholars of the sixteenth century, attacked Aristotle's physics, especially his theory of motion. But when shortly afterwards France was increasingly ravaged by the Hundred Years' War with England, their influence succumbed to the power of scholasticism.

It was chiefly Italy and Germany which now formed the vanguard of the revival. In Italy the urban dwellers had acquired great wealth and power during the preceding centuries because they were nearest to the sources of oriental trade. It was here that the arts and sciences began

to flourish. Germany was at the crossroads of trade between Italy and the north, and between east and west; in prosperous towns like Nuremberg, Augsburg and Cologne a strong and independent middle class ruled. Here, in the fifteenth century, great fortunes were amassed in the hands of the captains of finance, like the Fuggers and the Welsers, who became important powers in world politics. Whilst Paris lost its leading position in science, new universities were founded in Prague, Heidelberg, Vienna and Leipzig.

In the fifteenth century the sources of knowledge of antiquity began to flow more abundantly. The Byzantine Church, seeking help against Turkish aggression, came into closer contact with the Roman Church; knowledge of Greek thus spread among the Western scholars. Numerous Greek manuscripts were brought to the West, especially to Italy, where they were ardently collected and studied; they presented direct and pure texts of the ancient writers, instead of the corrupt and often unintelligible translations through Syriac and Arabic. The Western World was ripe to take possession of the entire spiritual heritage of antiquity. The spirit of humanism with a hint of paganism inspired the scholars, clerics as well as laymen, and began to supersede scholasticism. In astronomy the study of Ptolemy took a prominent place.

Two ways lay open and were followed for the study of astronomy: the collecting and studying of incorrupt manuscripts of the ancients, and the making of new observations. Observations with simple instruments, borrowed mostly from Arabic writings, had already been made in earlier centuries; the most widely used instrument was the astrolabe, many specimens of which can still be seen in our museums. More accurate instruments were used by Guillaume St Cloud, who in Paris at about 1290 measured solar altitudes, from which the latitude of his observation post and the obliquity of the ecliptic (23° 34′) could be derived; in 1284 he also observed a conjunction of Jupiter and Saturn. Paolo Toscanelli (1397–1482), afterwards geographical adviser to Columbus, systematically noted the positions of comets among the stars, in 1433, 1449, 1456 and later years. In Vienna, Georg Purbach (1423–61), who took his name from his Austrian birthplace, taught at the university after having travelled in Germany and Italy. He was first in Western Europe to expound Ptolemy's epicycle theory in a book called *New Theory of the Planets*, in which he inserted it into Aristotle's world system by separating the region of each planet from its neighbour by solid spherical shells.

To him came as pupil, afterwards as assistant, Johann Müller of Königsberg, a village in Franconia, who later called himself Johannes de Monte Regio and in astronomical literature is known as Regiomontanus. He lived from 1436 to 1476. During the years 1456–61 Purbach and he made many observations of eclipses, comets and solar altitudes, in the course of which they perceived that the Alfonsine Tables were several degrees in error. Their desire to obtain better manuscripts of Ptolemy was stimulated by a diplomatic visit to Vienna by Cardinal Bessarion, who had held high rank in the Byzantine Church. Their plan to attend him on his return to Italy was frustrated by Purbach's early death; Regiomontanus alone now accompanied the cardinal to Italy, where he learned Greek, collected and copied Greek manuscripts, and lectured on astronomy. After a short visit to Hungary, whose King Matthias Corvinus had acquired manuscripts during his wars against the Turks, Regiomontanus settled in 1471 in the town of Nuremberg. Here, in this centre of Middle European trade, flourishing commerce and handicrafts offered the most favourable opportunities for the construction of instruments as well as for the printing of books.

Indeed, the newly-invented art of printing opened up new possibilities for science. The printing of books, with careful correction of the text, put an end to the annoying evil of the numerous copying errors in manuscripts. The new process, it is true, did not yet include the printing of tables and figures. Regiomontanus had therefore to found a printing office himself and to instruct the compositors, thus acting as a pioneer of the printing trade. A circular letter from him is still extant, in which he enumerated the titles of the books he intended to print and publish. This list of 22 items, all in Latin, mostly editions of ancient astronomers and mathematicians, includes Ptolemy's *Geography* and *Astronomy* ('*Magna Compositio Ptolemaei quam vulgo vocant Almagestum, nova traductione*', i.e. in new translation); also Archimedes, Euclid, Theon, Proclus, Apollonius and others, followed by his own works, almanacs and minor writings. He began by publishing the planetary theory of his teacher Purbach and the astronomical poem by Manilius; after this, carefully computed almanacs, in Latin and in German, appeared. He won great fame with his *Ephemerides*, in which the positions of the sun, the moon and the planets had been computed for 32 years, from 1475 to 1506. Then, in 1475, the Pope summoned him to Rome to seek his advice on the urgently needed reform of the calendar. Here he died the next year. His great projects remained unfinished; the printing office was not continued, and his manuscripts were scattered. His own works were not printed until forty years later; he had not been able to accomplish the translation of Ptolemy, and it was not until 1505 that an older Latin translation was printed in Venice, while the first printed Greek edition of Ptolemy appeared in Basle as late as 1538.

It was not merely through his printing works, but even more because of his practical astronomical work, that Regiomontanus gathered around him in Nuremberg a circle of admirers and students of science,

who also provided money for the printing business. Among them were the patricians Willibald Pirkheimer and Bernhard Walther, both humanists well versed in Greek. Walther became his pupil in practical astronomy, and at his house equipped a room for mounting instruments, the first real observatory, where they made observations together. After Regiomontanus's death, Walther assiduously continued to observe the celestial bodies. By the time he died in 1504 he had made 746 measurements of solar altitudes and 615 determinations of the positions of planets, moon, and stars. It was the first uninterrupted series of observations in the new rising of European science; a century later Tycho Brahe and Kepler utilized them in their work.

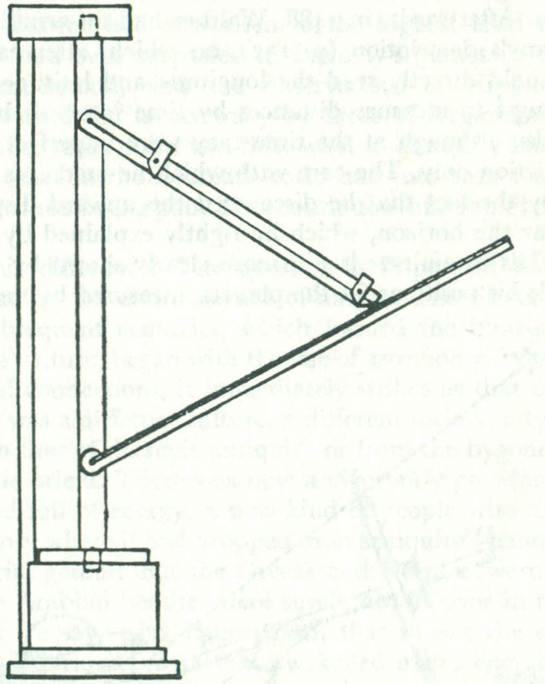


Fig. 19. Regiomontanus's three-staff

The instruments, made of wood after Regiomontanus's design, were of simple construction. First there was the so-called *Dreistab* (three-staff), also called *triquetrum*, already described by Ptolemy. It consisted of a lath about 9 feet long (with two sights to direct it towards a star) which was hinged at the top of a vertical pole; the lower end was pressed against a second lath, graduated and hinged at a lower point on the pole; the distance from this point to the lower end of the first lath indicated its inclination. This instrument was used mainly to measure midday altitudes of the sun; one inch on the divided lath corresponded to nearly half a degree. A more widely used device was the cross-staff for measuring the distance between two celestial objects (fig. 20). Along a graduated lath, which the observer took in his hand and directed at the mid-point between the two objects, a cross-lath was adapted to slide up and down, until its two ends, as seen from the lower end of the lath, coincided with the two stars. The reading of the cross-lath combined with its constant length gave the angular distance between the two stars. For several centuries the cross-staff was the most common instrument for navigators to measure the altitude of the sun or a star above the horizon. Afterwards, in 1488, Walther had an armilla made, also after Ptolemy's description (p. 152), on which, after careful adjustment, he could directly read the longitude and latitude of a planet. In an attempt to measure distances by time intervals he also made use of clocks, although at the time they were imperfect, being regulated by friction only. The care with which he made his observations is shown by the fact that he discovered the upward displacement of the sun near the horizon, which he rightly explained by atmospheric refraction. His carefulness is still more clearly shown by the accuracy he attained; his positions for the planets, measured by means of cross-

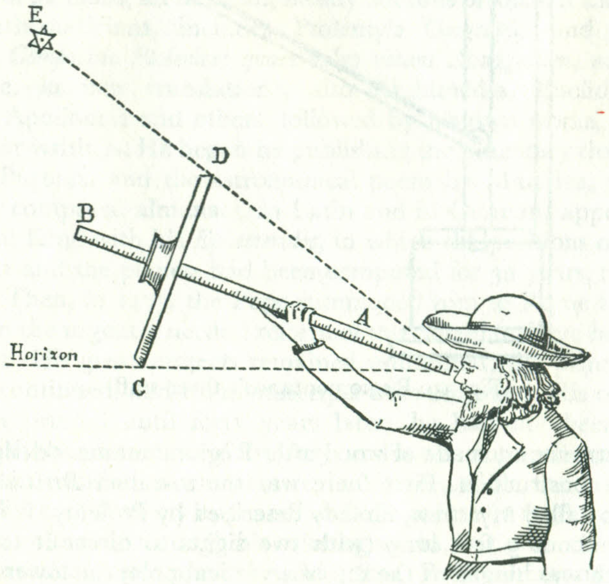


Fig. 20. The Jacob's staff

staff and armilla, had a mean error of only 5′, and the errors of his solar altitudes were usually below 1′. His younger collaborator, Johannes Schoner, who continued his work, published all these observations later.

Thus we see how, in the fifteenth century, science took a new trend. In the preceding centuries the most highly praised scientists had been scholars, not investigators. They reproduced science as they found it but did not produce new science. Books and writings, not experiments and observations, were the source of their knowledge. Now, however, a new era opened up, in which observation of new phenomena became the source of continuous scientific progress.

Science in Europe had now risen to the highest level reached in antiquity and had even surpassed it. Ptolemy's planetary theory was completely understood, and the observations of Hipparchus and Ptolemy were exceeded in accuracy by those of Regiomontanus and Walther. But the main point was this: what in antiquity had been the farthest end, what in the Arabian world had just been reached, was here in Europe the starting point for a continuous and rapidly increasing progress.

Whence this difference? The question is important not only for astronomy but for the entire science of nature. The rise of science in these and subsequent centuries, which formed the basis of modern society and its culture, began with the rise of astronomy. When we look for causes and connections, it immediately strikes us that now in this Europe there was a different culture, a different society, a type of man different from that of decrepit antiquity or from the bygone grandeur of the fatalistic orient. There was now a vigorously growing society, a burgher world full of energy, a new kind of people who took up the thread of history where it had dropped from antiquity's hands. Was it a matter of racial genius? But the Greeks and Romans were a kindred race, and the Arabian Semites were surely not inferior in talents and ability. Shall we say, with Huntington, that it was the stimulating temperate and stormy climate that awakened man's energies? In this respect Renaissance Italy cannot have been much different from Roman Italy. We shall not be far wrong if we assume that among all the interacting influences, the social structure and the forms of labour and trade played an important part.

The barbarians, the Teutonic and Sarmatic tribes who destroyed the Roman Empire and whose descendants built medieval society, had known the degradation of slavery not as a regular economic system but at most incidentally. Their sturdy working power, which found scope for initiative also in serfdom, made the economic system of feudalism the starting point of medieval peasant revolt and burgher freedom. The driving force of craft and commerce had roused them to fresh energy in developing technical methods, trade routes, the organization of the shops, and the assembling and investing of capital, thus revolutionizing society. A strong individualism pervaded the work of the artisans, the enterprises of the merchants, the ideas of the thinkers. It constituted a new culture which sought support in a revival of the antique world in order to liberate itself from the bonds of medievalism. Man's highest aspiration was directed at discarding medieval barbarism by the study of ancient culture. Renaissance to them was the enrapturing revelation of the beauty, the wisdom, the joyful happiness of antiquity, which in their eyes was the acme of human civilization, and which they passionately absorbed.

But their minds were too full of tension to remain content with the antique pattern. Once they had completely assimilated it, it became the basis for further development. The same spirit of adventure and daring that incited man to discover new worlds across the oceans, also drove them to make discoveries in science. Thus they outstripped antiquity. To these men this was no break with antiquity, but its continuation. The spirit of the Renaissance, pervading at the dawn of the new century became manifest in a renewal of science.

The principal science to which this applied, and, properly speaking, the only science worthy of the name, was astronomy. Strong social needs put it into the foreground of public interest. First of all, there were the demands of chronology. The calendar was in disorder; the ancient intercalation rules had not been sufficiently accurate, and the discrepancies had become unduly large. The vernal equinox fell on March 11th instead of on March 21st, and the full moons came three days too early; thus the computation of Easter Day was all wrong. Regiomontanus had been obliged to inform the Pope that new observations would be needed to provide a reliable basis for improved rules.

Next came the needs of navigation, which on the wide oceans made more severe demands than did ancient Mediterranean traffic. The stars, sun and moon were now needed to find the position of a ship by the determination of geographical latitude and longitude. The Iberian peoples, first the Catalans and then the Portuguese and the Spaniards, as seafarers and discoverers, opened up the routes across the oceans. They made use of the astronomical knowledge handed down from Arabian science, chiefly by Jewish scholars. In the fourteenth century King Pedro III of Catalonia had astronomical tables made by the Jewish astronomer Jacub Carsono. The cross-staff was said to have been invented—or at least introduced—at the same time by Levi ben Gerson from Provence. Zacuto, who taught at Salamanca University,

published a 'perpetual almanac' in Hebrew. One of his pupils, José Vizinho, court physician to King João II of Portugal, was a member of the 'Junta dos Mathematicos', the scientific body for navigation. For the use of the Portuguese navigators this Junta published a *regimento* with instructions on the manipulation of instruments, and a nautical yearbook with directions for computation. Thus Arabian astronomy contributed to the discoveries of the Portuguese. Wherever they went on their voyages to the south and to India, their pilots measured solar altitudes with the astrolabe in order to deduce their latitude by means of the solar declination in their tables—they evidently did not use the cross-staff. At this time the works of the German astronomers also began to be used, especially Regiomontanus's *Ephemerides*. Columbus, on his voyages, used these as well as Zacuto's almanac; and it was the lunar eclipse of February 29, 1504, predicted therein, which he used in a perilous situation to make the aborigines pliable. Probably Martin Behaim from Nuremberg, geographer and merchant in the Azores, contributed, as a member of the Junta, to the introduction of the results of German astronomy into Portugal.

Conversely, the practice of navigation now greatly stimulated the interest in astronomy. The sailors now became acquainted with the sphericity of the earth as a personal experience. Along with the unknown regions of the terrestrial globe, the unknown parts of the starry sphere were opened up to man's view. European navigators perceived the unknown stars and constellations in the southern sky with amazement. From one of the early Portuguese voyages along the African coast, at the Gambia river in 1454, Cadamosto wrote: 'since I still see the north polar star I cannot yet perceive the southern pole star itself, but the constellation I behold in the south is the Southern Wain.'[89] This belief in the symmetry of the stellar arrangement about the two celestial poles was a curious belief of the first explorers. Probably what Cadamosto saw was a combination of the two bright Centaurus stars (α and β) with the same quadrangle of stars which Christian zeal afterward regarded as the Southern Cross, and in which Vespucci on his travels south imagined that he recognized Dante's 'four brilliant stars'. Certainly, astronomy in those days was not a theoretical doctrine but a living science.

But interest in astrology was more universal than these practical activities. The medieval doctrine, that from the stars emanated the forces that determined events on earth, was now spread by almanacs through all classes of the population. Such almanacs, as well as single calendar sheets, printed in large numbers, contained besides the celestial phenomena, the eclipses and conjunctions, also predictions of the weather, of natural catastrophes, and of favourable and unfavourable times for various activities, even bloodletting and haircutting. Regiomontanus wrote a book on medical astrology, containing similar instructions.

Attention was directed particularly to the conjunctions of the two mighty planets Jupiter and Saturn, which take place every 20 years, each succeeding occurrence at a longitude 117° back. So they occur alternately in three zodiacal signs in trigonal position, until, slowly advancing, after two centuries the next triad of signs becomes the scene of the occurrences. In 1488 a famous astrologer, Johannes Lichtenberger, wrote: 'Attention must be paid to the important constellation of the weighty planets Jupiter and Saturn, whose conjunction and coincidence threaten terrible things and announce many future calamities . . . and to this terrible conjunction the horrible house of the ill-fated Scorpion has been assigned.'[90] This refers to the conjunction of 1484, which was assumed to extend its influence over twenty years. It was followed in 1504 by a conjunction in Cancer and in 1524 by one in the Fishes. The belief that great floods would then devastate the earth brought panic to Europe; 133 writings on this theme by 53 authors are known. The times were full of terror and fright; only an isolated few among the scholars, such as the Italian humanist Pico della Mirandola, combated astrology as a superstition. In a preface to a book by Lichtenberger, printed at Wittenberg, Martin Luther wrote: 'The signs in heaven and on earth are surely not lacking; they are God's and the angels' work, warn and threaten the godless lords and countries, and have significance.'[91]

The sixteenth century was an age of great social and spiritual revolutions. What had developed gradually in preceding centuries, the growing forces of burgherdom, the rise of monarchical power, the contest within and the opposition against the Church, all resulted in a rupture with the past. The opening-up of the entire earth by the navigators liberated human minds from the old narrow world conceptions but also, owing to the influx of American gold and silver, caused a rise in prices and general impoverishment, which exploded in revolts of exasperated peasants and urban artisans. New conceptions of life expressed themselves in new religious systems, in Lutheranism, in Calvinism, and in the renovation of the Catholic Church. Their struggle was mingled in a chaotic way with the fight between the princes, the nobles and the burghers in a series of political and religious wars. The quiet certitude which the Church formerly could guarantee through its unchallenged authority had vanished; the basis of life was shattered. In these unstable and turbulent earthly conditions, the bond linking everything on earth to the stars with their fixed and calculable course offered to man his only refuge. Stronger than ever before grew the need for, and the confidence in, astrology; more than ever it occupied the

thoughts of everyone. The princes employed 'mathematicians', chiefly as astrological advisers (the word *mathematica* at that time meant 'astrology'). Municipal and provincial councils, too, appointed mathematicians, not only to teach the sons of wealthy citizens and nobles in the schools but also to compile almanacs for peasants and citizens, containing prognostications on weather and on political prospects.

Astronomy in the fifteenth and sixteenth centuries stood in the centre of practical life and occupied man's attention more than any other science. Thoughts dwelt mainly on the stars; any ingenuity and scientific initiative in the human mind were directed primarily to the science of the celestial bodies. So it is not surprising that in those days people no longer confined themselves to respectfully repeating the views of the ancients, but struck out upon new tracks.

CHAPTER 18

COPERNICUS

THE new world system, as it appeared briefly in ancient Greece, had been developed by different individuals in two distinct stages: first the rotation of the earth, as an explanation of the apparent daily rotation of the celestial sphere, and second the yearly orbit of the earth. But the old theory, based on the positive premise of the immobility of the earth, had an inherent unity and was seen by everyone as a coherent whole. This probably was the reason why the new world system, in replacing the old picture, sprang up complete in its entirety. This was the work of Nicolas Copernicus (plate 3).

Copernicus (Niklas Koppernigk), born in 1473 at Thorn (now Polish: Torun), was descended from a family of German colonists, who a century before had been called into the country by the Polish king. German immigrants had already settled in these eastern regions, first under the rule of the Knights of the German Order, then under the Polish kings. They had founded a number of prosperous towns, such as Danzig, Thorn and Cracow, which became flourishing centres of trade and commerce and seats of urban culture. Such pioneers, as we remarked in a former case, are more open-minded, less prejudiced, and less bound to tradition than people staying on in their old homes.

Having become acquainted with astronomy in its close connection with astrology at the University of Cracow, Koppernigk was sent to Italy in 1496 for juristic studies and again in 1501 for medical studies. Here, while studying astronomy with Domenico Maria Novarra, a native of Ferrara, he came into close contact with the strongly pulsating life of the Renaissance. He studied Greek and became acquainted with what Greek and Latin sources reported on the dissentient opinions of ancient philosophers. These reports doubtless awakened or strengthened his ideas of a different world structure. When, later, he was dedicating his work to Pope Paul III, he mentioned the following as the sources which gave him the courage to work out his new theory: Nicetas, who according to Cicero made the earth move; Philolaus the Pythagorean, who made the earth describe a daily orbit around the central fire; and Heraclides and Ecphantus, who made it rotate about its axis.

Having returned in 1503 to Poland, where his uncle, the Bishop of Ermland, had procured him a seat in the Chapter of Frauenberg, he soon, probably about 1512, formulated his ideas in a *Commentariolus* ('short comment') which he sent to a number of friends and astronomers. Concisely and emphatically, the pillars of the new world system were erected in seven theses: (1) There is no single centre for all celestial orbs or spheres. (2) The earth's centre is not the centre of the world but only of gravity and of the lunar orbit. (3) All orbs encircle the sun, which as it were stands in the midst of all, so that the centre of the world is situated about the sun. (4) The relation of the distances of sun and earth to the height of the firmament is smaller than that of the earth's semi-diameter to the solar distance, so that its ratio to the altitude of the firmament is imperceptible. (5) What appear as motions in the firmament are not due to it but to the earth; hence the earth with its closed elements rotates in daily motion between its invariable poles, whereas the firmament is immobile, and the last heaven is permanent. (6) What appears to us a motion in relation to the sun is not due to the sun itself but to the earth, with which we are revolving just as any other planet; so the earth is carried along by several movements. (7) What shows itself in the planets as retrogradation and progress does not come from their own but from the earth's part; its single motion therefore suffices to [explain] many different phenomena.

Thus he was already sure of the basis of his new world system. But his main task occupied the years to come, in his leisure moments between official duties, participation in the administration and political direction of the diocese. That task consisted in obtaining by observation an exact numerical derivation of the orbits of the planets, as the basis for the computing in advance of future phenomena. In this way the practical needs of astronomers could be satisfied, now that the Alfonsine Tables were increasingly in error. In this way, too, his work could replace that of Ptolemy. However, when he had finished all this research, he hesitated to publish it for many years. Though influential friends among the clergy—Tiedeman Giese, Bishop of Kulm, and Cardinal Schoenberg of Capua—strongly encouraged him, he delayed publication because he foresaw the opposition due to prejudice, and his placid mind shrank from strife. Lack of fervent zeal against the new Lutheran doctrines had already made him suspect by the clerical zealots. He overcame his hesitation only when a young mathematician from Wittenberg, Georg Joachim, called 'Rheticus' (i.e. a native of the Grisons), visited him in 1539 to learn about his theory, and had given it an enthusiastic review in a *Narratio prima* ('First Communication'), about 1540.

He entrusted the manuscript, *De revolutionibus, libri VI* ('Six Books on the Revolutions'),* to Rheticus to have it printed in Nuremberg; it appeared in 1543, the year of his death. To this first publication, Osiander, a Lutheran minister in that town, who supervised the printing, had added—probably to meet the opposition of the Wittenberg theologians—an anonymous preface, entitled 'On the Hypotheses of this Work'. Also the extension of the title into *De revolutionibus orbium coelestium* ('On the Revolutions of the Celestial Orbs') could convey the idea that the earth was not necessarily included. Not until many years later did it become known that these additions were not by Copernicus himself.

In this work of Copernicus we may note three different aspects, distinguishing it from Ptolemy and from other books on astronomy written since Ptolemy. These aspects are the heliocentric world structure, the introduction of new numerical values, and a new mechanism to represent the details of the planetary motions.

Firstly he sets forth the new basis of the world system, and in the First Book provides the arguments against Ptolemy. He explains that the world is spherical and that the earth, too, has the shape of a sphere. The mobility of a sphere consists in its turning around an axis in a circle which has no beginning and no end. The celestial bodies exhibit various different motions, yet their motions must be circular and uniform, or consist of circular motions; for only thereby does what was before return in fixed periods. Since reason refuses to accept irregularities in what is arranged in the best order, we have to assume that uniform motions appear to us irregular because of a difference of poles or because the earth is not in the centre of the circles. Most authors, surely, assume the earth to be at rest in the middle of the world and regard other views as ridiculous; but on closer consideration it is seen that the question is not yet decided. For every observed change originates either from the motion of the object, or of the observer, or of both. If a motion of the earth is assumed, this must appear, although in opposite direction, in all that is outside, as if things filed past her; and this holds especially for the daily motion. Since heaven contains all, it is not conceivable that motion should not be attributed to what is contained therein rather than to what contains all. If then someone should deny that the earth occupies the centre of the world but admits to her a distance not large enough to be measured against the sphere of the fixed stars, but comparable to the orbits of the sun and the planets, he could perhaps indicate the cause for the apparent irregularities in the different motions as due to another centre than the earth's.

The ancient philosophers tried to show that the earth rests at the centre chiefly because all heavy matter tends to move to the centre of

* Revolutions means here circular movements.

the world and remains there at rest. According to Aristotle, earth and water move downwards, air and fire upwards, whereas the heavenly bodies revolve in circles. A rotation of the earth, Ptolemy says, would be contrary to this, and by so violent a retraction everything would be torn asunder. Objects falling down vertically would not reach their intended place because it would have been carried ahead from under them with great velocity; and we would see clouds and everything suspended in the air always moving westward.

If, however, one assumes a rotation of the earth, he certainly will say that it is a natural and not a violent motion; and what happens by nature is contrary to the effect of outer violence, and remains in perfect order. Why did Ptolemy not fear the same thing with heaven, which, according to him, has to rotate at a much more tremendous speed

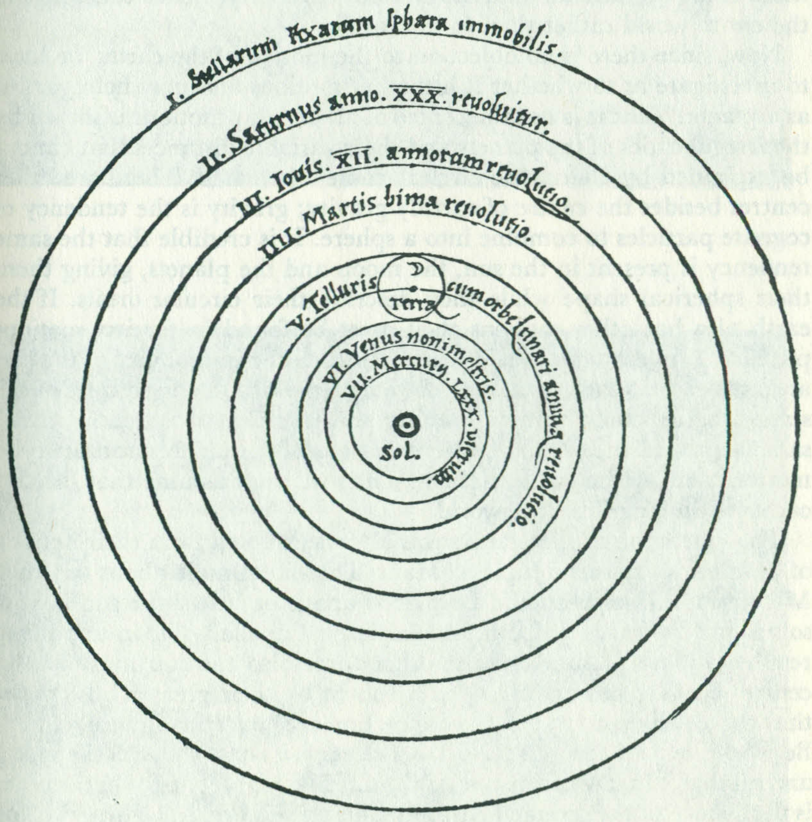


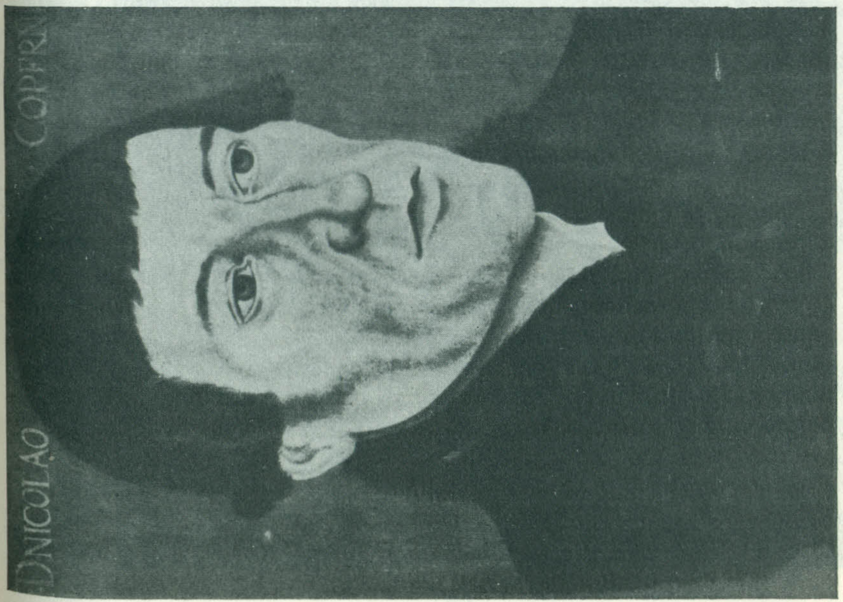
Fig. 21. The Solar System according to Copernicus

because it is larger than the earth? Since the earth is a globe enclosed between its poles, why not attribute to it the motion that is natural to a sphere rather than to assume that the entire world, of which the boundaries are unknown, is moving? A large part of the air, in which the clouds are floating, is drawn along with the earth, so that to us they appear to be at rest; whereas the remote realms of the air, in which the comets are seen, are free from this motion. The motion of falling or rising things is always a double one relative to the universe, composed of linear and circular motions. To a simple body when it is in its place belongs a simple motion, and this is the circular motion through which it appears to be at rest. When it is removed from its natural place, this is against the natural order and gives rise to an accelerated linear motion. To this must be added that the state of immobility is considered to be more noble and divine than unrest and variability; hence it belongs to the entire world rather than to the earth.

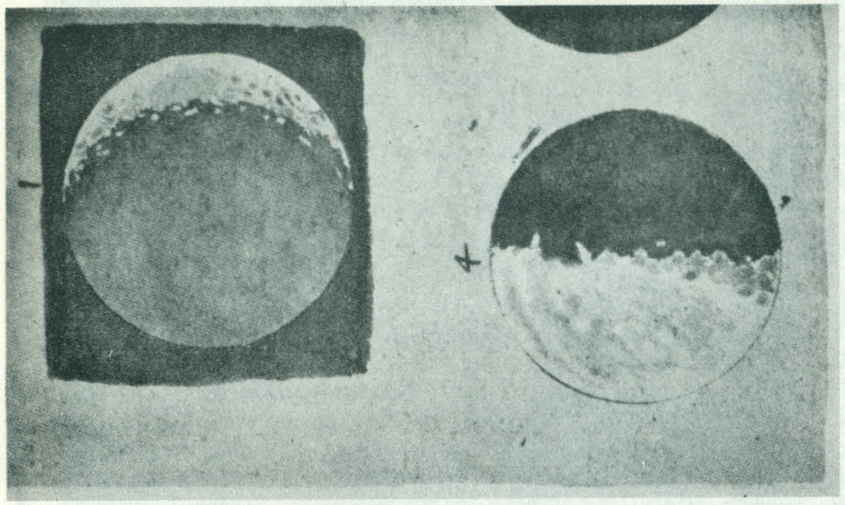
Now, since there is no objection to the motion of the earth, we have to investigate as to whether it has other motions and may be regarded as a planet. That it is not the centre of all circular motions is shown by the irregularities of the planets and their variable distances that cannot be explained by concentric circles around the earth. There are other centres besides the centre of earthly gravity; gravity is the tendency of cognate particles to combine into a sphere. It is credible that the same tendency is present in the sun, the moon and the planets, giving them their spherical shape while they describe their circular orbits. If the earth also has other motions they must be found in foreign motions presenting a yearly period. When we assume the immobility of the sun and transfer its yearly motion to the earth, the risings and settings of the stars, whereby they become evening and morning stars, follow in the same way, and the oscillations and stations of the planets appear to be motions lent by the earth to them. Then we must assume that the sun occupies the middle of the world.

The succession of the planets was always assumed from their periods of revolution: Saturn, Jupiter, Mars. The old contest about whether Mercury and Venus should be placed above or below the sun is now solved in this way, that both, as Martianus Capella wrote in antiquity, revolve about the sun. For the other three also the sun must be the centre, because in opposition, as is shown by their greater brightness, they come nearer to us, and in conjunction are fainter and more remote. Between these two groups the orb of the earth is situated, with the moon and all that is below the moon. At the outside, highest and most remote, is the sphere of the fixed stars, immobile and so large that the dimension of the earth's orbit is negligible against it. Then follow Saturn, Jupiter and Mars, finishing their orbits in 30, 12 and 2 years respec-

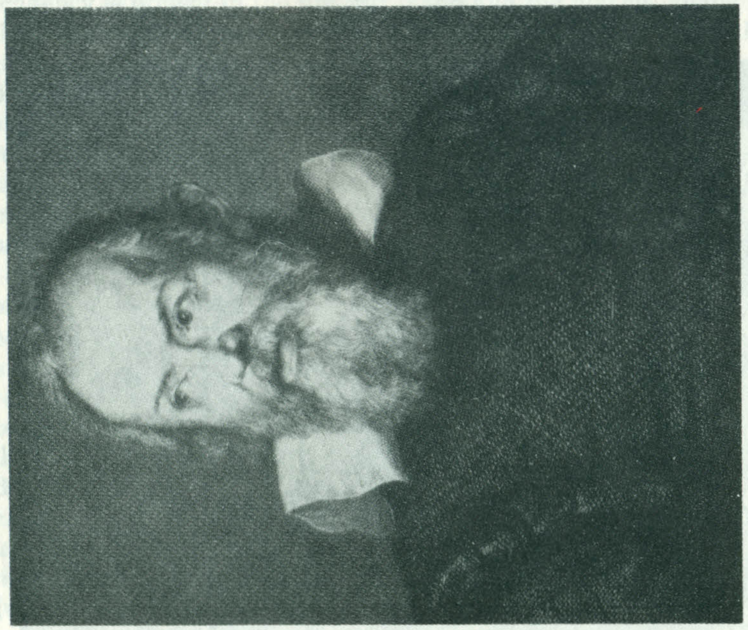
Tycho Brahe (p. 204)



3. Copernicus (p. 188)



Two drawings of the moon from Galileo's Day Book (p. 228)



4. Galileo (p. 227)

tively; then comes the earth with 1 year; then Venus with 7 months and Mercury with 80 days. In the middle of them all stands the sun. 'Who in this most beautiful temple would put this lamp at a better place than from where it can illuminate them all? Thus, indeed, the sun, sitting as on a royal throne, leads the surrounding family of stars.' [92]

These are the arguments (given mostly in his own words but much shortened) which Copernicus adduced for his new system. His arguments against Ptolemy were mostly philosophical, belonging to modes of thinking handed down from the ancient world. There remains some vagueness in the exposition of his system, as appears from the use of the Latin term *orbes* (of which the English equivalent is 'orbs'). This name, as is seen in the first thesis of the *Commentariolus*, sometimes means spheres with the sun as centre; by their rotation they carry the planets attached to them around in circular orbits. Sometimes the name means the orbits themselves, especially when numerical dimensions are derived.

The new theory was not the result of experience or observation; it did not contain new empirical facts that would compel man to relinquish the old concepts. Observations can only show motions of other bodies relative to the earth. These were the same in both world systems. What gave strength to the new system was its simplicity and harmony. With its simple basis, however, its consequences were enormous, even beyond the vision of its author. In their full extent they were only drawn by his later followers. Whereas Copernicus spoke of the celestial sphere as a real object, immovable and containing all, its original function was dissolved into nothing since the stars at rest in space no longer needed to be connected by a material sphere. The unfathomable depths of space were opened to man. At the same time—a still heavier demand on man's imagination—the most solid basis of his existence, the fixed earth beneath his feet, was drawn along in whirling motion with furious velocity, unimaginable, unacceptable, contrary to the most direct and certain experience. The doctrine of Copernicus meant a complete upheaval in man's world conception, which, as the new truth spread, was to determine modern thinking ever after.

The renovation of cosmic theory might have consisted in a simple transcription of Ptolemy's system, copying all numbers and measures but giving them new names: what was called the 'deferent' of Mars or Jupiter was now called its 'real orbit'. But Copernicus at the same time revised Ptolemy's numbers and measures—this was the second aspect of his work. He used new observations, mostly made by himself to derive the orbits for the present time with more accurate periods of revolution, and he computed new tables. Thus he produced a new manual of astronomy suited to replace Ptolemy in every respect.

Ptolemy's theory, moreover, could not satisfy him in the explanation of the variable velocity of the planets. His fundamental principle was that all motions of the celestial bodies consist of uniformly described circles. He thought it inadmissible to disregard this principle, as is actually done with the assumption of a *punctum aequans*. In criticizing this theory, he said: 'It is certain that the uniform motion of the epicycle must take place relative to the centre of its deferent and the revolution of the planet relative to the line through this and the centre of the epicycle. Here, however [in the old theory], they allow that a circular motion may take place uniformly about a foreign and not its own centre. . . . This and similar things have induced us to consider the mobility of the earth and along such other ways that the uniformity and the principles of science are preserved and the reason for the apparent inequality is rendered in a more constant way.'[93]

This then was the third aspect in which his work deviated from Ptolemy, a new and most ingenious mechanism to replace the equant. He faced the problem that in aphelion and perihelion (in 1 and 3 in the figure) the distances to the sun should be affected by the single

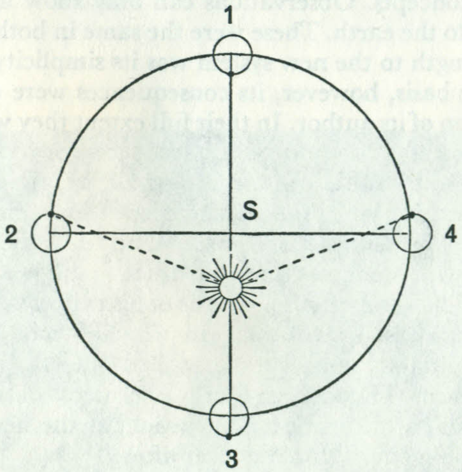


Fig. 22

eccentricity *e*, whereas at 90° anomaly (in 2 and 4) the direction sun – planet should be affected by the double or total eccentricity 2*e*. He solved it by making the distance of the sun from the centre of the circle $1\frac{1}{2}e$ and in addition having the planet in the same period describe a small epicycle of radius $\frac{1}{2}e$. Then in aphelion and perihelion, as fig. 22 shows, the two effects of eccentricity and epicycle subtract, whereas in the sideways positions they add.

Following this principle Copernicus made a new computation of Ptolemy's data for the oppositions of Saturn, Jupiter and Mars, as well as of his own observations of three oppositions of each of them, made between 1512 and 1529. In accordance with Ptolemy, it was not the opposition to the real sun but to the mean sun that was used. In Ptolemy's system this was unavoidable, as we saw above; in the new system it was not. It means that not the sun itself but the centre of the earth's orbit is taken as centre of the world, around which the planets describe their orbits; it is this centre that occupies the point *S* in our figure. It seems as if the proud proclamation of the sun's kingship in the first chapter was forgotten. The computations were made in the same way as with Ptolemy, by successive approximations; first the small epicycle was neglected, then from the resulting distance taken to be 2*e* the fourth part was taken as radius of the epicycle, and the observed longitudes were corrected for its effect. For the ancient data he arrived, of course, at the same numerical results as Ptolemy, only expressed in a different way: for eccentricity and epicycle radius he found for Saturn 0.0854 and 0.0285; for Jupiter, 0.0687 and 0.0229; for Mars, 0.1500 and 0.0500; and the longitudes of the aphelion (former apogee) were also identical.

From his new observations he found for Saturn the eccentricity 0.0854, the epicycle 0.0285, in total 0.1139, exactly corresponding with Ptolemy's $6\frac{5}{6}$ sixtieths = 0.1139; the longitude of aphelion 240° 21′ had increased by 14° in the fourteen intervening centuries. For Jupiter he was led astray by an error in computation;[94] so he assumed Ptolemy's value for the eccentricity and derived therewith the longitude of aphelion 159°, which had advanced $4\frac{1}{2}$° since Ptolemy. For Mars he found the eccentricity 0.1460 instead of 0.1500, which would have fitted Ptolemy's 0.200, and the aphelion advanced by 10° 50′. 'The distance of the centres of the orbits we found to be 0.0040 smaller than he did. Not that Ptolemy or ourselves have made an error, but as a clear argument that the centre of the orbit of the earth has come nearer to that of Mars, whereas the sun has remained immobile.'[95] If we consider that this difference corresponds to 14′ only in the first opposition, it seems questionable whether he did not put too much confidence in the reality of such small differences.

For Venus and Mercury, now called the 'inferior planets', the new world system could have removed many of Ptolemy's difficulties simply by having the planet describe a smaller eccentric circle within the larger circle of the earth. But first, by his insistence upon the centre of the earth's orbit instead of the sun itself as the world centre, Copernicus made his task more difficult and his system more complicated than was necessary. He followed Ptolemy so closely that his exposition often seems to be a copy of his predecessor's in somewhat altered language. Moreover, it was difficult to transform a centric epicycle into an eccentric orbit. He made the centre of the orbit of Venus describe a small circle in half a year; and the many irregularities inserted by Ptolemy into both the epicycle and the deferent of Mercury in the new theory also made Mercury's orbit in space dependent on the motion of the earth. That the other planets in their true course should depend on the earth makes a strange impression; here something remained of the old geocentric ideas.

To derive exact results for the orbits of these planets, just as for the superior ones, new observations besides the data from antiquity were necessary. For Venus he could make use of his own observations; but for Mercury he did not succeed. 'Indeed, this way of investigating the course of this star was shown to us by the ancients, supported, however, by a clearer sky, since the Nile, it is said, does not breed such vapours as the Vistula does here. Nature denies this facility to us, who live in a harsher country, where tranquillity [we might expect here the word transparency] of the air is rarer, and moreover, owing to the more inclined position of the celestial sphere, Mercury can be seen more rarely . . . so this star has caused us much trouble and labour in investigating its wanderings. Therefore we have borrowed three positions from those observed carefully in Nuremberg.'[96] Of these observations, one was made by Walther in 1491 and two by Schoner in 1504.

In respect of the inclinations of the planetary orbits, Copernicus may seem to have been at an advantage with Ptolemy in having to determine one inclination only for each planet. Since, however, he took over the data laid down by Ptolemy in his tables, there was no other way for him than to assume it to be variable. Thus he said that the latitude 'changes most where the planets, near opposition to the sun, show to the approaching earth a larger deviation in latitude than in other positions of the earth. . . . This difference is greater than would be required simply by approaching or retreating from the earth. From this we recognize that the inclinations of their orbits are not fixed but vary by some oscillating movements related to motions of the earth's orbit.'[97] With Venus and Mercury matters were still more complicated, as they had also been for Ptolemy. In his theory of the latitudes of the planets, Copernicus adopted Ptolemy's theory almost literally, with all its inadequacies, only expressing it in a different way in order to adapt them to the heliocentric basis. All this was a consequence of his respect for his great predecessor and his unjustified trust in Ptolemy's observational data and theoretical deductions.

For the moon, however, he could, by his system of circular motions, free Ptolemy's theory from the defect that the moon in the quarters

should be at times twice as near to the earth as at full moon. To explain his 'second inequality', Ptolemy had made the entire lunar epicycle approach to and recede from the earth. Copernicus achieved the same purpose by having the moon twice a month describe a small circle, whereby the distance of the moon changed by $\frac{1}{10}$ only.

There were more complications in his world structure. The precession was given by Ptolemy as 1° in 100 years and this was derived from the change of stellar longitudes since Hipparchus. In later times a more rapid displacement had always been found, and Copernicus assumed, as did the Arabian astronomers, a *trepidatio*, an alternation of a slower and a faster precession. Like Thâbit, he connected this alternation with the gradual decrease of the obliquity of the ecliptic, clearly shown by the measurements since antiquity (Ptolemy 23° 51⅓′; Arzachel, 23° 34′; his own measures in 1525, 23° 28½′), which, according to these data, should even be variable itself. He combined them all into an oscillating movement of the earth's axis of rotation, to which he had already given a movement. The ancient idea that in the case of an orbital revolution of a body some fixed connection with the centre must be assumed implied that, in such a simple case of movement, the axis is always kept inclined in the same way towards the central body. To explain the fact that the earth's axis kept its direction in space, Copernicus had to give it a yearly conical motion, such as we often see with a rapidly rotating top—he called it the third motion of the earth. The precession could now find a simple explanation if the period of the conical motion was taken somewhat smaller than the orbital period, so that each year there remained the small precessional displacement of the axis. Copernicus now superimposed on this system another oscillation in a period of 3,400 years to make up for the variations in the precession and the obliquity. The result was that the length of the tropical year now showed periodical variations. A second consequence was that Copernicus, through lack of a regularly moving equinox, had to count his longitudes of the stars from an arbitrarily chosen star (he took the first star of the Ram: γ Arietis) and to reduce Ptolemy's catalogue to this zero point.

The same over-confidence in the observations and data from antiquity brought about another complication. Since Ptolemy gave an eccentricity of the solar orbit of $\frac{1}{24}$, and later authors found $\frac{1}{30}$ and different apogees, Copernicus assumed them to be irregularly variable. He represented them by supposing that the centre of the earth's orbit in the same 3,400 years describes, relative to the sun, a small circle with a radius of 0.0048, making the eccentricity vary between 0.0414 and 0.0318 and the apogee oscillate by 7½° to both sides. 'But if someone should suppose that the centre of the yearly orbit is fixed as centre of the world and that the sun is mobile by two movements similar and equal to those which we have derived for the centre of the eccentric circle, everything would appear as before, with the same numbers and the same demonstration. So there remains some doubt as to which of them is the centre of the world, for which reason we expressed ourselves from the beginning in an ambiguous way on whether it is situated within or about the sun.'[98]

Thus, the new world structure, notwithstanding its simplicity in broad outline, was still extremely complicated in the details. This, on first impression, gives to Copernicus's book a strange and ambiguous character. In the first chapters a new world system is proclaimed and explained which subverted the foundations of astronomy, brought about a revolution in science and in world concept, and for many centuries made the name of Copernicus a war cry and a banner in the struggle for enlightenment and spiritual freedom. Then, on studying the later chapters, we feel completely transferred into the world of antiquity; on every page his treatment shows an almost timidly close adaptation to Ptolemy's example. Nowhere the breath of a new era, nowhere the proud daring of a renovator, nowhere the symptoms of a new spirit of scientific research!

In reality, however, the contrast is not so great. The first chapters also breathe the spirit of antiquity. We have already seen this in his arguments on the earth's motion; they belong entirely to ancient philosophy. Copernicus did not consider his work as a break with the ancient world concept but as its continuation, and he appealed to ancient precursors. Through the desire to lean upon venerable authorities, the struggle between adherents and opponents in the years that followed was carried on under the names of 'Pythagorean' and 'Ptolemaic' world systems respectively. It all remained within the realm of ancient science; Copernicus was wholly a child of the Renaissance.

The astronomers of the sixteenth century regarded the addition of all those complicated circular motions as a refinement of the old theory. Copernicus was highly esteemed among them as one of the foremost astronomers, the man who had improved and replaced Ptolemy. This, however, was only on account of the details and the improved numerical values; his heliocentric system was considered an ingenious theory but was not accepted as truth. His numerical values were the basis of new astronomical tables computed by the Wittenberg mathematician Erasmus Reinhold. These tables, which Reinhold, in honour of his patron the Duke of Prussia, called the 'Prutenic Tables', soon superseded the Alfonsine tables in use until that time.

CHAPTER 19

ASTRONOMICAL COMPUTING

THE growth of science consists not only in the development of ideas and theoretical explanations but also in the improvement of the practical working methods. The practical work of the astronomer is twofold—observation and computation. From the numbers read on the instruments, which are the direct results of observation, the desired values of the astronomical quantities must be derived by computation. Thus in the fifteenth and sixteenth centuries the construction of the mathematical apparatus was as important a part of scientific progress as was the construction of the technical apparatus, the instruments for observation.

Greek geometry had already taught how from given lines and angles other lines and angles could be computed. For practical use Ptolemy had computed and inserted in his great work a table of chords. For all angles or arcs increasing by $\frac{1}{2}°$ from 0° to 180°, the length of the chord was given in sexagesimals of the diameter. Since the sine of an angle is half the chord belonging to the double angle, his table corresponds to a table of sines for angles increasing by $\frac{1}{4}°$ from 0° to 90°. The Arabian astronomers then derived practical relations between sides and angles in a triangle which we know as 'trigonometric formulae'.

Computation means practical handling of numbers and numerals. The Greek system of numerals was highly unsuitable for this purpose. They consisted of 27 letters of the alphbet, the first 9 for the units from 1 to 9, the following for the tenths from 10 to 90, the remainder for the hundreds from 100 to 900, simply placed one after the other; for the numbers from 1,000 to 1,000,000 the same letters were primed. These numerals could be used to write down the numbers, but they gave no aid in computing, i.e. in addition, subtraction, multiplication or division of large numbers. Decimals were unknown; some simple fractions like $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{4}$, often occurred and had their special signs. Far more practical was the Babylonian sexagesimal system, where the units in which the successive numbers from 1 to 59 were expressed were the successive powers of $\frac{1}{60}$. Therefore Ptolemy gave the chords of his table in sexagesimal parts of the radius; three numbers secured an accuracy of $1 : 60^3 = \frac{1}{216000}$.

Europe in the Middle Ages had inherited the clumsy Roman system of numerals, so little suited to practical computing that in commerce a counting-frame or abacus always had to be used. Then gradually the Indian system of numerals, with place value and zero for the open places, penetrated from the Arabic world into Europe. The Italian merchants in their commerce with the Orient came to appreciate this system, though at first it was distrusted by less skilled dealers as a kind of unfair secret art. In the beginning of the thirteenth century a manual, *Liber abaci*, appeared, written by the much-travelled Leonardo of Pisa, afterwards called 'Fibonacci', that gave instructions for computing with arabic numerals and was used for many centuries. The translators of Arabian astronomical works of course copied the tables with their arabic numerals, though at the very first, in the manuscripts of Athelhard's translation, they were changed to Roman numerals.[99] Gradually the arabic numerals spread over Europe, drawn in different shapes (as may be seen on the figure of an astrolabe of the year 1547) before they got their modern forms. In the fifteenth century German commercial clerks went to Venice to learn the new calculation and the new ('Italian') book-keeping. In the sixteenth century division of numbers was still deemed such a difficult operation that Melanchthon at the University of Wittenberg had to give special lectures on the subject. In the book of Copernicus all numbers in the text are given in Roman numerals.

With the first rise of astronomy in Western Europe, Purbach began with the computation of a more extensive and accurate table of sines for angles increasing by 10′; the sines are given in seven figures, with the radius put at 6 million. Regiomontanus expanded it by means of interpolation into a table with the angles increasing by 1′, so that for angles measured (in degrees and minutes) the sine could simply be copied. He also computed a table of tangents, for every full degree, to be used for measurements with the cross-staff. In a work on plane and spherical trigonometry, not printed until 1561, long after his death, he derived many formulae for the computation of lines and angles.

Though the extension and accuracy of these tables could suffice for the astronomical practice of that time, mathematicians continued to make them more perfect, thus anticipating future application. Rheticus (1514–76) began in 1540 with the computation of sines, tangents and secants in 15 figures (in printing afterwards cut to 10) for angles increasing by 10″. At his death the tables were not yet ready; his disciple Otho had to finish them, publishing them in 1596. They served as the basis for many later tables; other computers later on corrected some remaining errors.

Theory was further developed at the same time. The system of

relations between the goniometric functions and of the formulae for computation of all the elements of triangles was built up into a complete trigonometry. This was done by Rheticus, and then in a more complete way it was perfected by the acute mathematician Vieta (1540–1603). So it became a handy and usable apparatus with which the astronomers could work. Most important were the complicated formulae for spherical triangles to derive angles and distances at the celestial sphere. Formerly the problem had been met by dividing oblique-angles into right-angled triangles and applying their simpler relations repeatedly. Now the more general formulae were adapted to universal use.

The first problem for which they were needed was given by the old practice of measuring the sun's altitude to find the time. In the thirteenth century Sacrobosco had provided a method, borrowed from the Arabian astronomers, of reading the desired values from curves engraved on the back of the astrolabe. But this method was not exact and could be used only because of the small demand for accuracy. Celestial globes also were used to solve the spherical triangles; but the accuracy in reading the quantities on globes was limited. Walther often fixed the exact moment of a phenomenon or an observation by measuring the sun's altitude; so he could not do without exact formulae.

There were other applications of trigonometry. Because the positions of the planets relative to the ecliptic were needed, they were expressed in ecliptical co-ordinates, longitude and latitude; so the same system was used for the stars. These co-ordinates could be read from armillas—complicated structures of many rings turning one within another—which with some difficulty had to be brought into the right position. So the results were less accurate than would have been the case with a simpler structure. In the sixteenth century the conviction grew that it was better to measure equatorial co-ordinates, right ascension and declination, with a more simply constructed instrument, which allowed greater accuracy. Longitude and latitude then could be derived from them by means of trigonometric formulae. Trigonometry in this way led to the practice of measuring such quantities as were directly, and therefore accurately, measurable and deducing the other quantities by computation.

To determine the position of a planet, the simplest way was to measure its distance from bright stars, e.g. with the cross-staff, as had been done by Regiomontanus, Walther, and probably also by Copernicus. Two such distances fix the position of the planet. The problem of how to deduce the longitude and latitude of the planet from the measured distances and the position of the stars was again a problem of spherical trigonometry; for the astronomers of that time it was usual practice. Another problem to be solved by trigonometry was to find the difference in right ascension of two stars with known declinations, by measuring their distance from each other. Direct measurement of right ascension was not feasible, whereas the declinations of the stars could be directly determined by measuring their altitude in the meridian when they were highest, and subtracting the altitude of the equator. Thus trigonometric formulae combined with tables of the sines and other goniometric functions became the most important auxiliary apparatus for sixteenth-century astronomers.

In the same sixteenth century the art of computing by means of arabic numerals found its completion in the introduction of decimal fractions. The chief impulse came from Simon Stevin, a native of the Flemish town of Bruges, who is mostly known as 'the inventor' of this system of numeration. In his book *De Thiende* ('The Tenth') written in Dutch, and appearing in 1585, he explained how all computations may easily be performed 'by means of entire numbers without fractions'; hence he considered the numerals after the decimal point as integers, counting the successive powers of $\frac{1}{10}$ as units. Now the road was open not only to all kinds of astronomical computing but also to the practical handling of approximate and irrational numbers.

Of course, the practical use of the formulae in astronomy demanded long and tedious computations. The invention of logarithms, at the beginning of the next century, brought an enormous saving of time. Its principle is well known; if we append to the numbers of a geometric series a corresponding arithmetical series (hence, beside 10, 100, 1,000, 10,000 . . . or beside 2, 4, 8, 16 . . . we place 1, 2, 3, 4, . . . which now are called their 'logarithms'), any multiplication is now simplified into an addition, any division into a subtraction ($8 \times 4 = 32$ is reduced to $3 + 2 = 5$). The problem was which numbers had to be put, as their logarithms, beside 11, 12, 13. About 1580 this idea had arisen in the mind of Joost Bürgi, assistant at the Cassel observatory, but he did not grasp its great importance; it was not published, and only much later did a first table of logarithms computed by him come to light. The credit for giving them practical introduction to science falls to the Scottish scientist, John Napier, or Neper, as he wrote his name in Latin. His tables, published in 1614, had not yet the necessary practical utility, because they were not adapted to the decimal system. That was given to them by Henry Briggs, mathematician at Oxford, who came to Napier to propose a better arrangement, especially by the introduction of 10 as the basic number. In 1618 Briggs published the first, still incomplete, table of logarithms based on the decimal system, in 8 decimals. They were completed and published in 1628 by Adriaan Vlacq, bookseller at Gouda in Holland, assisted by his friend, the skilful computer, Ezekiel de Decker. These tables, with the logarithms given

to 10 decimals, were the source of many tables published afterwards. Already Neper's first tables contained also the logarithms of the goniometric functions, which constitute an integral part of all later tables. In later times computers and computing offices have made new and still more accurate computations of all the logarithms, for some small prime numbers up to 64 decimals.

Logarithmic computation as an indispensable aid found its way into all practical sciences. But more than any other science, astronomy has profited by this invention. By executing the long and tedious computations of the logarithmic tables, the first inventors and computers, so to speak, lengthened the life of all later astronomers. They made possible researches which, because of the immense computation work involved, could not have been performed without this aid.

CHAPTER 20

TYCHO BRAHE

One of the many persons who in the sixteenth century ardently devoted themselves to the new study of nature was the Danish nobleman Tyge (latinized into Tycho) Brahe (1546–1601). As a youth, sent to Leipzig for juridical study, his passion for astronomy was shown in his secret nightly studies and observations. Like most of his contemporaries, he was deeply convinced of the truth of the astrological doctrines, and he often computed horoscopes. But to him this doctrine meant more than making prognostications. The most prominent thinkers of these times, in the confusing chaos of social and political strife and the dark uncertainty of future and fate, sought support in their confidence in an inner connection and harmony of the entire world. The stars, they said, ruled the earth, and the course of the stars is dominated by eternal laws. Our knowledge, however, of this connection and of the course of the stars was insufficient. The belief in an indissoluble bond between the precarious happenings on earth and the regularity of the stars was the guiding principle in their researches. There was a better astrology than the practice of prognostics; but it was still in its infancy, and only by careful study could it develop from this primitive condition into reliable science. This science, being knowledge of the inner unity of the world, would give man power over his fate.

This was also Tycho Brahe's world concept. He expressed it in later years in a public lecture in Latin (then the international language of scholars) given in 1574 at the University of Copenhagen. Its title, *De disciplinis mathematicis*, though dealing entirely with astrology, shows how in his view, which conformed to the views of the time, astrology was the main practical object of mathematical discipline.

In this lecture he said: 'To deny the forces and the influence of the stars is to undervalue firstly the divine wisdom and providence and moreover to contradict evident experience. For what could be thought more unjust and foolish about God than that He should have made this large and admirable scenery of the skies and so many brilliant stars to no use or purpose—whereas no man makes even his least work without a certain aim. That we may measure our years and months and days by

the sky as by a perpetual and indefatigable clock does not sufficiently explain the use and purpose of the celestial machine; for what it does for measuring the time depends solely on the course of the big luminaries, and on the daily rotation. What purpose, then, do these five other planets revolving in different orbits serve? . . . Has God made such a wonderful work of art, such an instrument, for no end or use? . . If, therefore, the celestial bodies are placed by God in such way as they stand in their signs, they must of necessity have a meaning, especially for mankind, on behalf of whom they have chiefly been created . . .

'But no less do they [who deny the influence of the stars] violate clear evidence which for educated people of sane judgment it is not suitable to contradict. Who does not perceive that the difference in quality between the four seasons is caused by the rising and the retreating of the sun and its ordinary course through the twelve parts of the zodiac? We see, too, that, with the waxing of the moon, everything cognate to her nature—such as the brains of living beings, the marrow in the bones and the trees, the flesh in the lobsters and shells and many things more—increase also; but when she wanes, these also diminish. In the same way the flux and reflux of the wide ocean is affixed as with a chain to the moon's motion, so that immediately, with the rising of the moon, the sea also begins to rise. . . . These and many kindred phenomena are known even to uneducated people. For sailors and peasants, by numerous experiences, have noted the risings and settings of certain stars, from which they can foresee and predict yearly recurring storms. The scholars, however, who are trained in this abstract science, deduce the influences of the configurations of the other wandering stars: with one another or with the Luminaries, or with the fixed stars. They have observed that the condition of the air in the four seasons of special years is affected by them in various ways. So it has been perceived that conjunctions of Mars and Venus in apt parts of the sky raise rains, showers and sometimes thunderstorms. That important conjunctions of the big planets cause vast changes in this lower world has often been shown by experience. Thus in 1593, when a great conjunction of Jupiter and Saturn took place in the first part of the Lion, near to the nebulous stars in Cancer, which Ptolemy calls the smoky and pestilent ones, did not the pestilence which swept over the whole of Europe in the years that followed, and caused innumerable people to perish, confirm the influence of the stars by a very certain fact?'[100]

Then, after dealing with the opinions of those opposed to astrology, Tycho proceeded: 'But we, on the contrary, hold that the sky operates not only on the atmosphere but also directly upon man himself. Because man consists of the elements and is made out of earth, it is necessary that he be subjected to the same conditions as the matter of which he consists. Since, furthermore, the air which we inhale and by which we are fed no less than by food and drink, is affected in a different way by the influence of the sky, as has been shown above, it is unavoidable that we should at the same time be affected by it in different ways. I leave aside what must be easily clear to every mind, that man by some hidden cause lives and is fed by the sky itself in a still higher degree than by air, or water, or by any other low elementary thing, and acquires an incredible affinity to the related stars; so that the ancient philosophers, among them Hipparchus, according to Pliny's testimony, were not wrong when they said that our spirit is a part of heaven itself.'[101]

Here all attention was directed to the stars; but in his own mind this was part of the general doctrine of the unity of the entire world, a doctrine that stimulated him to scientific research. In observing the conjunction of Jupiter and Saturn in 1563, he perceived that the Alfonsine Tables gave its time of occurrence a month wrong, and the Prutenic Tables some days wrong. How, in such a case, was a reliable judgment on the connection with earthly events possible? In the first place, a better knowledge of the planetary motions was necessary. It could be secured only by new and better observations, for which improved instruments were needed. When he visited Augsburg in 1569, he devised for the brothers Paul and Johann B. Hainzel, members of the town council and devotees of astronomy, a vertical wooden quadrant 19 feet in radius, suspended by the centre, that could be read at a plumb line to 10″. It has been used for useful measurements, but in the long run it was too unwieldly for regular work. For himself, for use on his travels, he had a lighter instrument made, which he called a 'sextant'; it was a sector of 30° with two radial diopter arms, one fixed, the other movable, to measure the distance between two stars by looking from the centre through the sights of both diopters at the same time. To read the graduation in minutes, he devised a contrivance by means of transversals, rows of ten points going up and down at constant distances between an outer and an inner circle of the border (fig. 23). This method of division he later applied to all his instruments. Having thus travelled through Europe, he returned to Denmark and lived on his uncle's estate, occupying himself chiefly with chemical experiments.

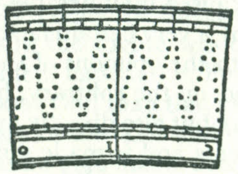


Fig. 23

Then it happened that in the evening of November 11, 1572, returning from the laboratory, his eye was caught by a brilliant star right overhead in the constellation Cassiopeia, which he had never seen there. It was a new star, a *nova*, which, as was soon ascertained, had appeared certainly after November 1st (probably between November 2nd and 6th). It was brighter than any other star; it equalled Venus in brilliancy and was even visible by day. This wonderful phenomenon stirred the entire world, first of the scholars, but also of the common people. What did it mean? This was the question posed by everyone. Beza, the friend of Calvin, supposed it to be a second star of Bethlehem, announcing a second coming of Christ on earth. Others discussed the calamities it would bring, its nature, and its place in the universe. Did it belong to the stars? Aristotle had established that in the world of stars everything was eternal and invariable. Or did it belong to the sublunar world of the variable earthly elements, and was it perhaps a singular comet condensed from fiery vapours?

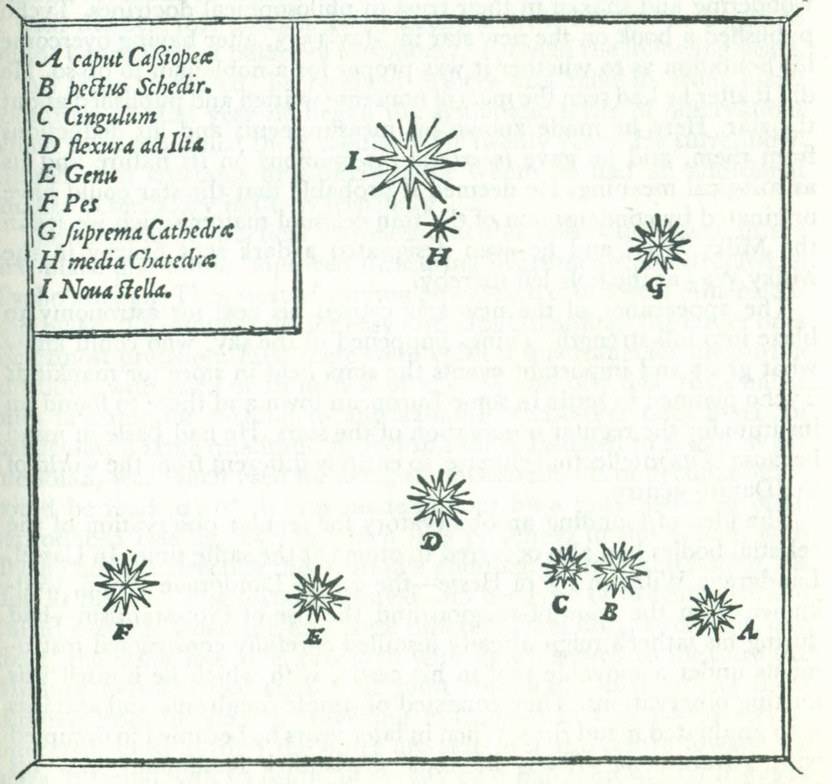


Fig. 24. The new star in Cassiopeia, from Tycho Brahe, *De Nova Stella*.

Tycho, full of the same questions, immediately began to observe the new star by measuring repeatedly its distance from the pole star and the nearby stars of Cassiopeia, both when this constellation stood near the zenith and when, twelve hours later, it stood low in the north beneath the pole. If the star had been at the same distance as the moon and so had a parallax of one degree, it would have appeared in the second case one degree lower relative to the other stars of Cassiopeia. But the distances in both cases, at high and at low altitude, were always found exactly equal, with only a few minutes of uncertainty. So it was proved that the new star had no perceptible parallax and was at a far greater distance than the moon. This demonstrated that, contrary to Aristotle's doctrine, changes did occur in the world of the stars, the realm of the ether.

Soon the star began to decline in brightness, more so the next year, while at the same time its colour changed to yellow and then to red, until at last it became colourless. It disappeared in 1574, leaving people wondering and shaken in their trust in philosophical doctrines. Tycho published a book on the new star in May 1573, after having overcome his hesitation as to whether it was proper for a nobleman to do so. He did it after he had seen the mass of nonsense written and published about the star. Here he made known his measurements and his deductions from them, and he gave *in extenso* his opinions on its nature and its astrological meaning. He deemed it probable that the star could have originated by condensation of the thin celestial matter which we see in the Milky Way, and he even designated a dark spot nearby in the Milky Way as the hole left thereby.

The appearance of the new star caused his zeal for astronomy to blaze into full strength. Things happened in the sky; who could know what great and important events the stars held in store for mankind? Tycho planned to settle in some European town and there to found an institute for the regular observation of the stars. He had Basle in mind because of its intellectual climate, so entirely different from the world of the Danish gentry.

The idea of founding an observatory for regular observation of the celestial bodies had also occurred to others at the same time. In Cassel, Landgrave Wilhelm IV of Hesse—the son of Landgrave Philip, well-known from the wars of religion and the rise of Protestantism—had during his father's reign already installed carefully constructed instruments under a movable roof in his castle, with which he himself was making observations. They consisted of simple quadrants and sextants with graduated metal rims. When in later years he became too occupied with government affairs, he took Christoffel Rothmann into his service as an observer. Later also Joost Bürgi, a talented Swiss and a

skilful mechanic, was engaged, who applied himself diligently to the improvement of clocks as aids in observation. Tycho, in his travels through Germany on the lookout for a future site, visited Cassel in 1575, and this visit proved highly stimulating to both sides. There was a considerable increase in the intensity of observing the sun, the planets and the stars at Cassel in the next few years. Here the first efforts were made to introduce time as an astronomical measure for differences in right ascension; from Cassel came the first West-European catalogue of stars based on new measurements.

Another consequence of this visit was of still more lasting value. The Landgrave wrote to his colleague King Frederic of Denmark that an important and famous personality was likely to be lost to the honour and fame of the Danish kingdom. The King, himself interested in the sciences, who had already endowed Tycho for his studies, then offered him the small island of Hven, near Copenhagen in the Sound, as a fief with all revenues, in order to build an observatory there and to provide it with the best instruments. Here Tycho settled; in 1576 he began to build 'Uraniborg', a palace of astronomy, where in the following years, as his fame increased, he received princes and scholars from many countries. In that year he began the systematic series of observations which were continued there regularly for twenty years. He surrounded himself with assistants and disciples for whom he had an additional smaller observatory built, 'Stjerneborg'.

The renewal of practical astronomy at this observatory was in the first place due to the improved measuring instruments, constructed to Tycho's design. They were of varying types, each of different dimensions and make, the smaller ones for easy and rapid handling, the larger ones for utmost precision. First there were vertical quadrants for measuring altitudes, with radii of 16 inches, of 2, of $5\frac{1}{2}$ and of 7 feet, the smaller ones movable for turning to any azimuth, the larger ones fixed. In particular, a large quadrant, $6\frac{3}{4}$ feet in radius, fastened to a wall in the meridian, was much used for accurate measurements of declination; it could be read to 10″. It was made famous by a great painting often reproduced (see fig. 25), in which it is presented in full action, with Tycho himself directing, his assistants pointing, looking and noting, his dog at his feet. The sextants formed another type of instrument (also called sextants when the arcs were different from 60°): two arms, one fixed, the other movable, provided with sights, served to measure the distance between two stars. The most commonly used among them differed from his smaller travelling instrument in that two observers operated, looking from the outer arc side over the centre toward the stars. It rested with its centre of gravity on a ball-joint so that it was in equilibrium in any direction of the line connecting the stars; it had a

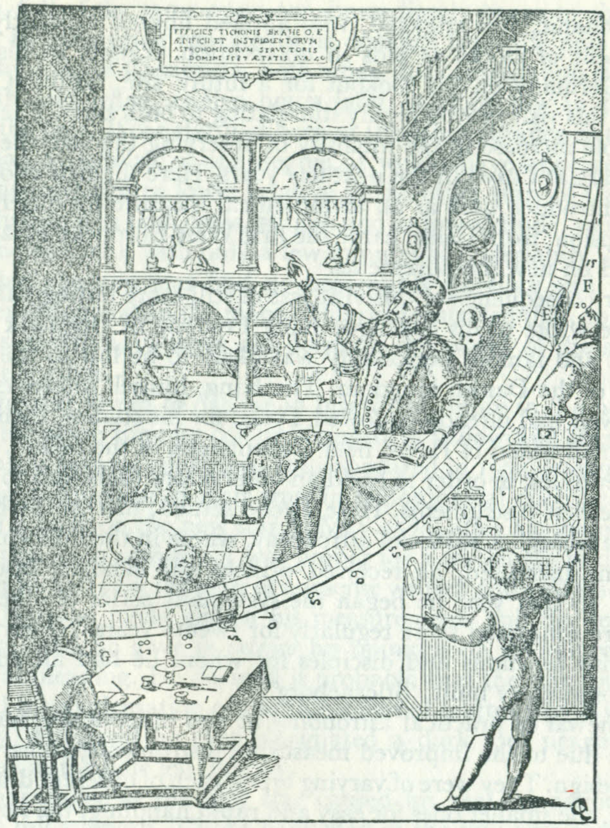


Fig. 25. Tycho Brahe and his great quadrant

radius of 5½ feet. There were armillas too, but of much simpler and stabler construction, without ecliptical rings, to read equatorial co-ordinates in a more reliable way. The rings were set in the correct position by means of the right ascension of a known star; and then the right ascension of all other stars and planets could be read. The ultimate in simplification was reached in a polar axis resting above and below in masonry, adjustable by screws, to which only a declination circle of 9½ feet in diameter, provided with sights, was attached. It could turn through all hour angles, which were then read on a somewhat larger half-equatorial circle situated on the northern side below the horizon.

All these instruments were distinguished by special precautions to secure the utmost precision. Transverse rows of points, as described above, served to read the smallest subdivisions. It was half a century afterwards that Pierre Vernier invented the method of an auxiliary

arc (for instance, divided into 10 equal parts), which in the next centuries was applied to all astronomical instruments. In some countries it was wrongly called 'nonius' after the Portuguese astronomer, Pedro Nunez, whose invention was based on a different principle. For exact pointing to a star, Tycho had devised a special kind of pinnules which he applied to all his instruments. At the great quadrant, for instance, a

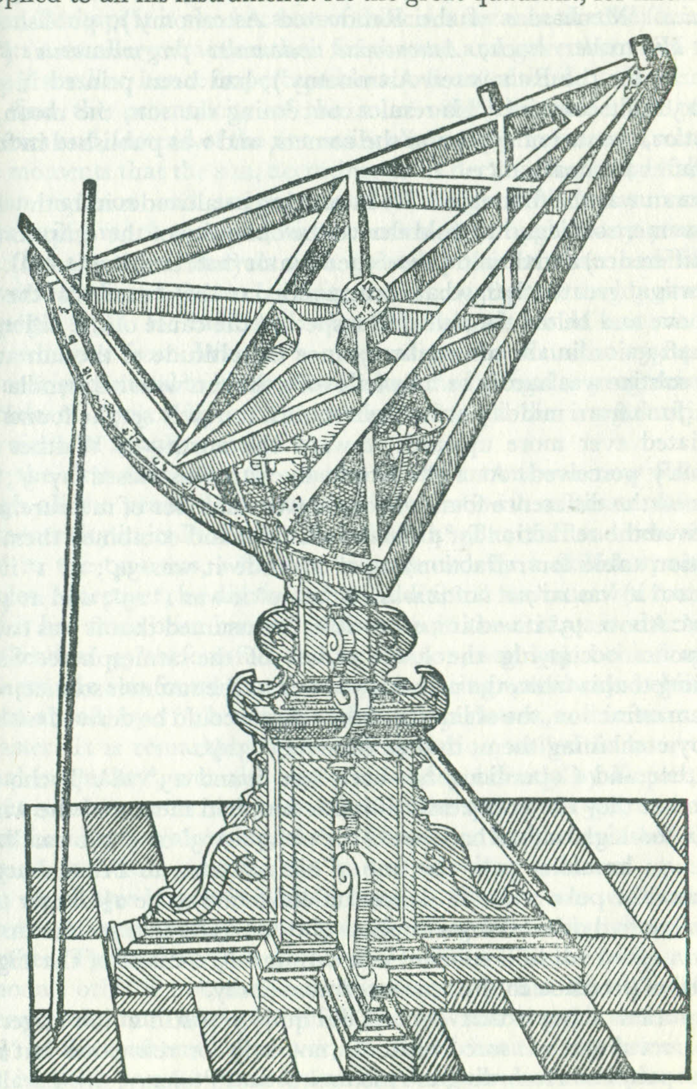


Fig. 26. Tycho's great sextant

cylindrical peg was put in the centre; before the eye was a metal plate with two horizontal slits, one above the other at a distance exactly equal to the diameter of the peg. If the eye, looking alternately through the slits, saw equal parts of the star project above and below the peg, the star was exactly pointed. Tycho gave detailed descriptions and pictures of all these instruments in a book *Astronomiae instauratae mechanica* ('Mechanics of the Renovated Astronomy'), published in 1598. His other book, *Astronomiae instauratae progymnasmata* ('Preliminaries for the Renovated Astronomy'), had been printed in parts since 1588; it contained his results concerning the sun, the moon, the fixed stars, the star of 1572, and the comets, and was published in full in 1602, after his death.

The sun was the first object. By measuring its altitudes in both winter and summer solstices, he could derive the obliquity of the ecliptic (half their difference) and the altitude of the equator (half their sum total). The latter was 4′ greater than what corresponded to the altitudes of the pole star above and below the pole. He suspected the cause of the difference to be refraction in the atmosphere, since the altitude of the sun at the winter solstice was hardly 11°. By following the sun with his armilla on a day in June from midday at its highest until it nearly set, he found that it deviated ever more upwards, toward the north—as Walther had previously perceived. At 11° altitude the sun was raised by 9′; this explained the difference found. From a great number of measurements he derived the refraction for different altitudes and combined them in a correction table for refraction; at 0° altitude it was 34′; at 1° it was 26′; at 10° it was 10′; at 20° it was 4½′; at 30° it was 1′ 25″; and at 40° it was 10″. Above 45° it was imperceptible; he assumed that it was caused by vapours occupying the lower strata of the atmosphere. Since, according to this table, the pole star, as well as the summer solstice, were free from refraction, the obliquity of the ecliptic could be derived without error by combining them; thus he found 23° 31½′.

Walther and Copernicus, however, had found 23° 28′. Tycho suspected that they too, as a result of refraction, had measured the winter altitude too high and so had found the obliquity 3′ or 4′ too small. To make sure, he afterwards sent one of his assistants to Frauenburg to determine the polar altitude. It was indeed found to be 2¼′ larger than Copernicus had derived from his solar altitudes. Here we see the careful attention given to differences of a few minutes, a mark of the higher standard of precision in the renovated astronomy.

The curious thing is that Tycho's obliquity was still 2′ too large. He had corrected his measured altitudes not only for refraction but also for solar parallax. According to Aristarchus and Ptolemy, this parallax, $\frac{1}{19}$ of the lunar parallax, was 3′; and, adopting and applying this value,

Tycho got, for the solar altitude at summer solstice, a value 1½′ too large. Apparently no doubts caused him to distrust this value—certainly it would have been difficult to make a new determination—and he simply says: 'this value appears by such subtle investigation by the Ancients to have been transmitted to us with great certainty.'[102] Consequently, many quantities determined by him are some few minutes in error. Perhaps it is also because of this circumstance that he found for the stars a smaller refraction than for the sun and gave them in a table as all 4½′ smaller, hence imperceptible above 20° altitude.

To find the eccentricity of the solar orbit, he made use of the equinoxes and, instead of the not easily observable moments of solstices, of the moments that the sun, according to its declination, stood halfway between equinox and summer solstice. Because his measurements, besides being more accurate, were continued through many years, he could find the time intervals with greater precision than had former observers. He found the eccentricity to be 0.03584 and the longitude of the apogee 95½°. He indicated why Copernicus's result was made in error, whereas from Walther's observations he found 0.03584 and 94⅓°, nearly identical to his results. He assumed a regular progress of the longitude of apogee by 1° 15′ per century; all the irregularities of Copernicus he simply omitted.

For the first time also the course of the moon was now followed through all the years. This provided him first with accurate values of the eccentricity in its Ptolemaic variations: 4° 58′ at full and new moon, 7° 28′ in the quarters, which he represented by a skilful system of epicycles. Moreover, he discovered an additional 'variation' (a name by which it has since been known): the moon in the octants is alternately 40½′ ahead at 45° before full and new moon, and 40½′ behind at 45° past them. He also found that the moon in spring was always behind, in autumn ahead, by 11′; hence in winter it was slower, and in summer it was faster. It is remarkable that this deviation, called in later times 'annual inequality', was also perceived independently by Kepler, and without measurements. A moon eclipse in the spring of 1598, computed by him for a Styrian almanac, came 1½ hours too late; when asked about the cause, he wrote that the sun had a retarding influence upon the moon, which was largest in winter, because then the sun was nearer to the earth. An explanation entirely conforming to modern views!

Further, Tycho derived from his observations that the inclination of the moon's orbit is not, as formerly assumed, 5° but that it oscillates between a minimum of 4° 58½′ at full and new moon and a maximum of 5° 17½′ at the quarters. Moreover, he found that the regression of the nodes in their 19-year period was not uniform but took place more rapidly at full and new moon, more slowly at the quarters. He could reproduce these variations by assuming that the pole of the moon's orbit, which in 19 years describes a circle of 5° 8′ radius about the pole of the ecliptic, also describes a small circle of 9½′ radius in half a year.

In deriving the position of the new star of 1572 by means of distances from the known stars of Cassiopeia, Tycho had perceived how the rough values of their co-ordinates prevented the accuracy of his measurements from showing to full advantage. He wanted accurate co-ordinates of the fixed stars; he needed them because he continually determined the positions of the moon and the planets by measuring their distances from the stars. The declinations of the stars could be found directly by measuring their meridian altitude with the great quadrant. The right ascensions had to be read from an armilla or computed with trigonometric formulae from distances measured with a sextant. Differences in right ascension were thus obtained; the right ascensions themselves had to be counted from the equinox, an invisible point of the sphere, ascertainable only by means of the sun. From its declination, measured when it was in the vicinity of the equinox, its distance to that point could be computed; if at the same time the difference in right ascension between the sun and a star had been measured, then the right ascension of the star itself was found. Since stars are not visible in the daytime, they cannot be compared directly with the sun; so the ancients used the moon as an intermediary. Tycho used the planet Venus, which moves much more slowly. In the years 1582–88, when Venus was visible by day, a large number of measurements were made of its distance from the low-standing sun and at night of its distance from the selected star. To eliminate refraction influences, an evening and a morning observation with equal altitudes of the sun were always combined. These precautions were successful; the resulting 15 values of the right ascension of the star (the brightness of the Ram, α Arietis) did not differ more than 40″, less than what could have been expected even with such careful observations. Then, by means of a large number of oft-repeated measurements, the right ascension and declination of 21 principal stars were determined; their mean error, as is found through comparison with modern data, was less than 40″. In order to ascertain whether the arc of his sextant was the exact length, he added all the consecutive differences of right ascension around the celestial sphere to see whether their sum total was 360°. The difference was only a modest number of seconds. This of course was pure chance; the sum total of a number of values, each uncertain by 20″ or 30″, can easily by mere chance be many minutes in error. Possibly Tycho had selected a number of best-fitting values, considering them to be the most reliable. The positions of the bulk of the fainter, less-often observed stars of his catalogue, number-

ing 788, are, of course, less accurate; their mean error, by comparison with modern values, appears to be about 1′.

Tycho's catalogue of stars was the first complete modern catalogue. It replaced Hipparchus's and Ptolemy's works, far exceeding them in accuracy. As the supreme result of the most perfect care, ingenuity and perseverance during many years, it marked the beginning of a new era of practical astronomy. For more than a century it remained the constantly-consulted source of star positions. Though the equatorial co-ordinates—right ascension and declination—had been directly measured, the catalogue, adhering to old custom, gave the ecliptical longitude and latitude computed from them. The stars were indicated, as they were with Ptolemy, by the parts or the limbs of the constellations. It was afterwards, in 1603, that Bayer, publishing under the name *Uranometria* a celestial atlas with all Tycho's stars, added Greek letters to the prominent stars; these letters, an easy designation, have since come into general use (plate 15).

Comparing his longitudes of the stars with the dates from antiquity and from the last century, Tycho derived an accurate value of the precession, 51″ per year. He assumed it to be entirely uniform; he did not speak of the 'trepidation' that had given so much trouble to Copernicus; thus this phantom disappeared from astronomy. Another phenomenon drew his attention. When he compared his stellar latitudes with those of antiquity he could ascertain that in the region of the Twins they had increased by 15′ to 20′ and diminished by the same amount in the region of the Eagle. Since the obliquity of the ecliptic was known to have decreased by the same amount, it was now clear that it was the ecliptic that had changed its position, whereas the equator there (at 90° and 270° of longitude) had kept its position relative to the stars. In this way new foundations were laid for practical astronomy with an accuracy hitherto unknown.

No less important were the observations of comets made at Uraniborg. When a comet appeared, its position was determined as often as possible, mostly by measuring the distance from prominent stars. The bright comet of 1577 was measured mostly with the cross-staff, since the new instruments were not yet ready; a modern computation of its orbit shows that Tycho's positions had a mean error of 4′. For the comet of 1585, measured with his better instruments, this error was 1′ only. One of his chief objects was to find the parallax by determining its position among the stars when it was low in the sky and comparing it with that at a great altitude. If the comet had been at an equal or lesser distance from us than the moon—as he had supposed in his youth—great differences, up to 1°, due to parallax, would have appeared. But he found no trace of parallax; the comet—according to his cautious