conclusion—must have been at least six times farther away than the moon. This was sufficient to demolish entirely, in a publication on the comet of 1577, Aristotle's theory that the comets were fiery phenomena in sub-lunar regions, the upper layers of the atmosphere. Tycho has elevated the comets to the rank of celestial bodies; they belong to the realm of stars, and their orbits in space must be determined by astronomers. That he himself, by supposing them to be inclined circles, was quite mistaken cannot surprise us.

Far more extensive than this sporadic cometary work were the observations made of the planets. These had been from the beginning the chief object of Tycho's astrologically directed mind, and they constitute the chief mass of the work at Uraniborg. The treatment and discussion of these materials, in order to derive the orbits of the planets, lay ahead as his main task, to which all the preceding results were only the preliminaries.

But now the practical conditions of his life became an impediment. In 1588 his patron King Frederic died, and the government first came into the hands of a body of high noblemen, guardians of his successor, and then of the young prince himself. They were not so well disposed towards him. Tycho had been repeatedly in conflict with tenants and with others, because he tried to spend as much money as possible on his astronomical work, and the higher courts of justice had often put him in the wrong. His proud and haughty bearing had brought him many enemies among his fellow-nobles and the high officials. Some of his former prebends now being taken from him, he felt aggrieved and left Denmark in 1597. After some wanderings he found a new patron in the Emperor Rudolf II, who himself, however, was always in financial difficulties. Tycho settled in 1599 in Prague, in the emperor's residence, where with his assistant Longomontanus he continued his observations with such instruments as he had been able to carry with him from Hven. But his strength was broken; in October 1601 he died, leaving all his observations in the hands of Johannes Kepler, who had been his assistant for computations in the last years.

CHAPTER 21

THE REFORM OF THE CALENDAR

The calendar serves for the regulation of time in social life. Like all social regulations, it had been a religious concern from early times. With the rise of Christianity in the Roman Empire, which as a universal religion was the expression of new and deeper social relations than were the old tribal and state religions, the religious festivals too acquired a new character. They lost their direct close connection with the pattern of work in its yearly cycle. As a means of regulating the Christian festivals, the calendar was now a Church affair. In the first place came the fixing of the dates of Easter, with the other connected movable festivals; for Christianity, Easter no longer meant the spring offerings of the new harvest but the yearly commemoration of the crucifixion and resurrection of Christ.

According to the Gospels the resurrection had taken place on the Sunday after the Sabbath following the Jewish Passover that fell at the full moon, the fifteenth of the first month, Nisan. Therefore, the commemoration must take place on the first Sunday after the first full moon of spring, i.e. after the vernal equinox, March 21. So astronomy was needed to fix the date, since the announcement of the date of a festival that was to be celebrated simultaneously in all the churches of the Orient and the Occident could not wait until the full moon had been observed. It must be fixed long beforehand, through knowledge of the theory. But not too difficult a theory, for the rules for computing the date had to be manageable by well-instructed priests, by means of cycles and periods. Exact astronomical computation, with inclusion of all irregularities of the moon, had to be avoided.

The large irregular changes in the date of Easter are due to the adaptation of the lunar phenomena to the Roman solar calendar. With a lunar calendar, Easter Sunday certainly would not have a fixed date, since the seven-day weeks roll on independent of sun and moon; but this date could not vary more than between one and seven days after full moon; the adaptation to the solar calendar brings the difficulties and irregularities. Of course, the 19-year cycle, the common multiple of month and year, had to be used, with easy computing directions, manageable by simple minds. But if we consider that the regular alternation of 29 and 30 days goes wrong after a few years and that, by the intercalation of one day every fourth year, the regularity is also disturbed, it is clear how many difficulties, shown by much internal strife, the Church encountered in the first centuries of its existence when fixing the Easter dates.

The happy solution was the work of one Dionysius, archivist in the Pope's service, who, to distinguish himself from the great Church Father, called himself *Exiguus*, the 'little one'. In a report of about AD 520, in which at the same time he introduced the counting of the years since the birth of Christ instead of numbering them in relation to the Roman emperor eras, he fitted the Easter regulations used in the Orient into the Roman calendar. If we omit some minor complications, it comes down to this. In every year the 12 lunar months are taken alternately as of 29 and 30 days, hence in total, 354 days; but their real duration is 354.367 days. In 19 years we are then $19 \times 0.367 = 7$ days short. In this cycle a thirteenth month is intercalated seven times; if they are all taken to have 30 days—that is, on the average, half a day too much—we are $3\frac{1}{2}$ days ahead. Moreover, the Julian leap years add $4\frac{3}{4}$ days in a 19-year cycle, so that the shortness of 7 days is amply compensated. Indeed $19 \times 354 + 7 \times 30 + 4\frac{3}{4} = 6{,}940\frac{3}{4}$ days, whereas $19 \times 365\frac{1}{4} = 6{,}939\frac{3}{4}$ days. There remains one day's difference; it was smuggled away at the end of each cycle, under name of *saltus lunae*. All through the Middle Ages it remained a matter for surprise—because this human regulation was supposed to be a sacred fact of nature—that the moon made a jump every 19 years.

In the extensive medieval literature and the tiresome practice of this *computus*—then the name for Easter reckoning—many technical terms came into use which survive in the almanacs of our own day. The 'epact' of a year is the number of days by which the age of the moon at a given date is greater than at the same date in the first year of the 19-year cycle. It increases by 11 (i.e. 365–354) each succeeding year, or decreases by 19, because 30 is subtracted when it exceeds 30. In the succeeding years of a cycle it is 0, 11, 22, 3, 14, 25, 6, etc. The values 365 and 354 in reality are 365.25 and 354.37; these errors are compensated one day in the cycle, the same one day mentioned above; it lies hidden here, because the epact is 18 for the last year of the cycle, where we do not pay attention to the fact that $18 + 11$ being 29 does not return 0. The epact thus depends on the series number of the year within the 19-year cycle, called the 'golden number'; it is one more than the remainder after division of the year by 19 (the first year has the remainder 0). In the first year of the cycle the new moon falls on March 22nd; hence for every year the epact indicates the age of the moon on

March 22nd; the full moon, with age 14, falls in the first year on April 5th, and in each next succeeding year 11 days later or 19 days earlier.

To know which day of the week falls on such a date, the 'solar cycle' of $4 \times 7 = 28$ years is used, after which the series of names returns. Every next year this weekday advances by one and after a leap year by two. If we give to the days of the year the seven letters A (for January 1st), B, C to G, then the letter falling on Sunday is called the 'Dominical Letter'; it goes back by one every year, by two every leap year (after February). Knowing this letter, we find the weekday for the Easter full moon, hence the date of Easter Sunday.

Here in order to show modern progress in mathematical control of the world we may remark that Gauss in 1800 condensed this entire complicated Easter computation into a couple of simple formulae which give the result in a few minutes. Dividing the year by 19, by 4 and by 7, the remainders are called a, b, and c; put $19a + 15 =$ multiple of $30 + d$ $(d < 30)$; put $2b + 4c + 6d =$ multiple of $7 + e$ $(e < 7)$. Then the date of Easter is March 22nd $+ d + e$.

Conversely, the culture and mode of thinking in the Middle Ages can be measured by the fact that such a simple piece of arithmetic, by clothing it in the garment of self-made shapes, presented itself to mankind as another mysterious world with curious rules dominating the rites of the Church and the life of man. With these shapes and these rules, however, the Church succeeded in carrying on and fulfilling its task of fixing the festivals during the darkest ages of the decline of science.

When in the later medieval centuries science rose again, it was soon perceived that calendar and computus no longer agreed with reality. The Julian year of $365\frac{1}{4}$ days was 0.00780 too large; the difference amounted to one day every 128 years, so that about 1300 the vernal equinox fell on the 13th instead of the 21st of March. Moreover, the 235 months of a 19-year cycle, assumed to be $6{,}939\frac{3}{4}$ days, in reality were 6,939.69 days; and owing to this difference, now, after 950 years = 50 cycles, full moon came three days earlier than was computed. So the Easter computation was completely wrong, and Easter was celebrated on wrong days. Roger Bacon had already pointed out how shameful for the Christian Church it was that flesh was eaten in Lent, a cause for derision to Jews and Mohammedans. But the inner quarrels of the Church in the succeeding centuries prevented any measure being taken.

We mentioned above that in the fifteenth century Pope Sixtus IV invited Regiomontanus to Rome for this purpose; but, because of his early death, matters remained as they were. For the Lateran Council (1512–17) Paul of Middelburg, Bishop of Fossombrone, who had written a great work on calendar reform, asked the advice of Copernicus, who, however, had to answer that new data were not yet avail-able to secure an accurate new regulation. But his own work soon supplied the deficiency; with the new data from *De Revolutionibus* and from Reinhold's tables, the reform could now be carried out. The Council of Trent (1545–63) charged the Pope to make the necessary regulations and in 1582, under Gregory XIII, it was effected.

First, the big errors that had gradually accumulated—that the equinox now came 10 days too early and the assumed moon came three days after the real moon—had to be removed by introducing sudden jumps. To keep the calendar correct in the future, new regulations had to be made. It was known that the true length of the tropical year was $\frac{1}{128}$ day less than the Julian period of $365\frac{1}{4}$ days. It was known also that the 235 months of the 19-year cycle, the basis of medieval Easter-reckoning, were $\frac{1}{17}$ day shorter than 19 years, so that after every 317 years the full moon had to be taken one day earlier.

The rather simple modification now made in the calendar was an ingenious invention of the librarian Aloysius Lilius (Giglio), offered after his death by his brother to the ecclesiastical commission installed for the reform of the calendar. It was approved and praised by the astronomical experts and at once accepted by the commission. It consisted, firstly, in the omission of three leap years every 400 years, which came down to assuming an error of $\frac{1}{133}$ instead of $\frac{1}{128}$ of a day in the Julian year; it was done by omitting the secular years that are not multiples of 400. Secondly, in keeping the 19-year cycle and having the time of full and new moon pushed back one day 8 times in 2,500 years; this should be done in the years 1800, 2100, and every 300 years to 3900, then 4300, and then again every 300 years.

The consequence of all these jumps is that at the secular years the calendar quantities undergo sudden changes: the epact decreases by 1 in 1700, 1900, 2200, 2300, 2500, and increases by 1 in 2400.

The constants in Gauss's formulae now also change considerably by the jumps at the transition; the values 15 in *d*, 6 in *e*, holding for the eighteenth century, had to be replaced by 23 and 4 during the nineteenth century and by 24 and 5 during the twentieth century.

The leading astronomer of the commission, Father Clavius, published a book to explain the principle and practice of the new calendar to the world. If the calendar reform had come before the Reformation, nothing would have stood in the way of its general acceptance. But now, in a time of great religious strife, it was established by the Pope and a College of Cardinals who had no authority in Protestant countries. Still worse, in the papal bull wherein it was proclaimed, the Pope 'ordered' the princes and republics to introduce the new calendar. Whereas it was done immediately in Spain, France and Poland, the Protestant princes and countries refused. At a later date Kepler, by

very able arguments in a booklet, in the form of a discourse, tried to convince his co-religionists of the necessity for its adoption; but his purity of doctrine was suspect anyhow. When the question became urgent in 1700 in Germany, a way out was found by introducing the New Style in a somewhat different way, exhibiting and emphasizing independence from Rome. This was done by computing Easter after the real moon, not after cycles, so that Easter was sometimes a week different in Catholic and Protestant regions. In the course of the eighteenth century however the Gregorian calendar was introduced everywhere in Protestant Europe (in England in 1752).

Russia followed in the twentieth century, and the Gregorian calendar was adhered to throughout the world, though for ritual purposes local and national calendars remained in use. There is now under discussion a project for calendar reform, proceeding from the idea that in our machine age a mechanized calendar belongs to a mechanized world. In view, however, of the value attached by leading social circles to ancient traditions and customs, it seems unlikely that such a change will soon take place.

CHAPTER 22

THE STRUGGLE OVER THE WORLD SYSTEM

HOWEVER highly Copernicus was esteemed among astronomers for renovating astronomy in its fundamental numerical data, his heliocentric world system with a moving earth did not find much approval. The objections hampering its acceptance were of two kinds, the theological, arising from the authority of the Bible, and the physical, from the authority of Aristotle's doctrine, corresponding to everyday experience.

The theological difficulty held more weight for Protestants than for Catholics. Cardinals and bishops had encouraged Copernicus to publish his work; one Pope had benevolently listened to an exposition of the new theory, and another had accepted the dedication of his book. On the other hand the Protestant leaders, Luther and Melanchthon, sharply rejected it. Martin Luther, in one of his table-talks in 1539, said: 'That fool will reverse the entire Ars Astronomiae; but, according to the Scripture, Joshua bade the sun and not the earth to stand still.'[103] Melanchthon in 1550 quoted the Psalms and Ecclesiastes: the earth stands eternally, the sun rises and sets; and he added: 'Fortified by these divine testimonies, we will cling to the truth.'[104] For Protestantism the strict literal validity of the Bible was the basis of faith, whereas the Catholic Church claimed the right of interpretation. Under Paul III the general trend of the Church might seem to be hesitating and inclined to reconciliation and to the bridging-over of differences, in order to restore unity by concessions to the new idea. But conditions changed. In the second half of the century the controversy between the parties became more definite, the fight became fiercer and irreconcilable, the world became harsher. The establishment of rigorous doctrine by the Jesuit Order drove out the sunny philosophy of the Renaissance. At the Council of Trent the Church organized itself into a solid militant power, giving strict attention to any deviation from the established doctrine; thus it regained more and more of the ground previously lost.

For Tycho Brahe the theological argument was also important. An opinion contrary to the Bible, which in Protestant Denmark would certainly produce troublesome quarrels that would hamper his work of

renovating practical astronomy, could not attract him. But the physical objections were decisive. First, the clumsy and heavy material earth cannot be a rapidly moving celestial body. Moreover, if the earth describes a yearly circle, this must appear in an apparent yearly circle of the stars, which, as a kind of resting epicycle, we call a yearly parallax. Copernicus had said that it would be too small to be perceptible; but Tycho was now able to measure positions and changes of position down to 2′. With a parallax of 2′ the distance of the stars would surpass by a hundred times the dimensions of the planetary system. Such an enormous void between the farthest planet, Saturn, and the sphere of the stars would be entirely useless. These difficulties had induced Tycho —in 1583, he said—to devise another world system, having with Copernicus the same advantage of not needing epicycles for all the planets, and avoiding the difficulties resulting from the earth's motion. In Tycho's system sun and moon described circles about the resting earth; the sun was the centre of all the planets' orbits and carried them along with her in her yearly revolution. The celestial sphere carried all along in its daily rotation. It is clear that in this system the motions of the celestial bodies relative to the earth would agree exactly with the system of Copernicus.

In a regular correspondence between Tycho and the Cassel astronomer Rothmann, who ably defended the Copernican system, the arguments for and against each of the theories were discussed. Tell me, Tycho wrote in 1589, if the earth rotates so rapidly, how can a ball falling from a high tower hit exactly the point below? And do you think it probable that Saturn is 700 times more distant from the sphere of the fixed stars than from the sun? A star of the third magnitude—for which he assumed an apparent size of 1′—in that case would be as large as the earth's orbit. Tycho also mentioned that he made experiments on a rapidly sailing ship, by dropping objects from the top of the mast; they did not fall down precisely below but farther back.

The universities were dominated in apparently unassailable strength by Aristotle in philosophy and by Ptolemy in astronomy. Yet Aristotle's theory of motion was such an artificial doctrine that only after replacing it by more natural views directly connected with everyday appearances could the road open to scientific progress. It was this double task that made the establishment of a science of mechanics so extremely difficult. Opposition to Aristotle's theory had already appeared in antiquity and again in the Paris school of Oresme. It ascribed the continued motion of a thrown object not to a pushing of the surrounding air but to an 'impressed force', which meant that the impetus or push once acquired was preserved until exhaustion of this impetus caused the object to fall down by its own weight. In the sixteenth century, instigated by the use of artillery in war and of simple mechanical devices in technical work, thinking on mechanics and opposition to Aristotle increased. Benedetti (in 1585) explained the acceleration of falling bodies by the weight continually adding new push to the existing impetus, and he contested Aristotle's opinion that the velocity of falling increases with weight. At the same time Guidobaldo dal Monte studied the weight, pushing bodies downwards along a circular track.

The adherents to the heliocentric system in the sixteenth century were insignificant in number. To be sure, immediately after its publication in 1543, Copernicus's book was diligently studied by the scholars, who used its numerical data for the computation of almanacs and tables, often praising them for their accordance with observations. But this did not involve acceptance of the new world system. In 1549 Melanchthon published his lectures on physics at Wittenberg as a university textbook; in this, by means of physical and theological arguments, Copernicus's system was refuted and dismissed as absurd. The wide influence of its seventeen impressions was enhanced through numerous books by dozens of other authors, all alike in praising Copernicus but rejecting or not even mentioning his theory. Here Catholic and Protestant universities were unanimous: Clavius, in 1570, in an often reprinted commentary to Sacrobosco, called Copernicus the renovator of astronomy but refuted his theory as absurd on the same grounds as had Melanchthon. Against this profusion of traditional instruction, Copernicus's book was not reprinted until 1566 (in Basel) and again in 1617.

Among the few adherents of the heliocentric system we find Thomas Digges, an Englishman, who in a book of 1576 speaks of an infinite world filled by mostly invisible stars. Another was Giordano Bruno, an enthusiastic apostle of the new doctrine, who in his travels all over Europe propagated the new world concept. But the boldness of his ideas—the doctrine of the plurality of worlds, all the fixed stars suns like our sun, surrounded by planets that are perhaps abodes of other men—tended to frighten rather than to attract more timorous minds; and his burning as a heretic, at Rome in 1600, was a warning. There were others who had sufficiently overcome prejudice to see the truth of the Copernican system: thus Benedetti, refuting the arguments against Copernicus, wrote: 'The celestial bodies have not been created to influence such a subordinate body as the water-covered earth with its animals and plants.'[105] William Gilbert, on finding the earth to be a large magnet, assumed it to have a daily rotation about the poles, which he supposed to be the magnetic poles; but he did not speak of the yearly motion. Simon Stevin in his *Wisconstighe Ghedachtenissen* ('Mathematical Memoirs'), published in 1605 at Leiden, sided entirely with Copernicus; he expounded the motions of the planets first with a resting earth,

then with a moving earth, calling the former the 'untrue' and the latter the 'true' system. Moreover, he corrected Copernicus by omitting the third motion of the earth, the conical motion of its axis, considering its constant direction in space as a consequence of its magnetic character disclosed by Gilbert. In a letter to Tycho Brahe of April 18, 1590, Rothmann made the same remark: 'I know that in this point Copernicus is very obscure and not easy to understand. . . . This can be explained much more easily, and the triple motion of the earth is not necessary; the daily and the yearly motions suffice.'[106]

Johannes Kepler (1571–1630) became a fervent adherent after hearing, as a student at Tübingen, the exposition of the heliocentric system by Maestlin, who himself, though well-disposed towards it, was reluctant to support it entirely. Then, teaching at Graz as a provincial mathematician, in his first book *Mysterium cosmographicum* (1596), Kepler came forward as a vigorous defender with an entirely new argument. In this book he gave an explanation of the structure of the planetary system—why there are six planets at just these several distances from the sun—by connecting them with the five regular polyhedrons, called the 'Platonic figures'. If a sphere is constructed upon each of the six planetary circles, we may put between each pair of successive spheres, supposed to be exactly concentric, one of the regular solids in such a way that its edges are situated on the exterior sphere and its planes are tangent to the interior sphere. The ratio of the diameters of the outer and the inner sphere in the case of a icosihedron and a dodecahedron is 1.24, in that of a cube and an octahedron 1.73; of a tetrahedron it is 3. Among the ratios of the radii of the successive planetary circles, there are two small values, 1.4–1.5 for earth-Venus and Mars-earth; two larger values, 1.8–1.9 for Venus-Mercury and Saturn-Jupiter; and one very large, 3.4, for Jupiter-Mars. Of course there cannot be exact equality because the centres of the planetary orbits are not situated in the sun, so that the eccentricities also play a part. But the concordance is too great to be due to chance alone. Hence Kepler, guided moreover by astrological ideas, placed the five solids in his succession proceeding from the centre outwards: 8-, 20-, 12-, 4-, 6-hedron, in between the six planetary spheres. The figure in his book, representing a model of this arrangement, is reproduced in fig. 27. By disclosing this secret of world structure, Kepler elevated Copernicus's theory far above the level of a debatable opinion based upon uncertain empiricism and made it a fundamental philosophical truth.

Kepler sent his cosmological work to several astronomers, among them Galileo Galilei, who since 1592 had been teaching mathematics and astronomy at Padua. In his letter of acknowledgment of August 4, 1597, Galileo wrote: 'Many years ago I came to agree with Copernicus, and from this position the causes of many natural effects have been found by me which doubtless cannot be explained by the ordinary supposition. I wrote down many reasons and arguments, and also refutations of opposite arguments, which, however, I did not venture until now to divulge, deterred by the fate of Copernicus himself, our master, who, although having won immortal fame with some few, to countless others appears . . . as an object of derision and contumely. Truly, I would venture to publish my views if more like you existed; since this is not so, I will abstain.'[107] Present yourself with your proofs, was Kepler's answer (October 13, 1597); with combined forces we must shove the cart; with your proofs you can assist your partners who now suffer

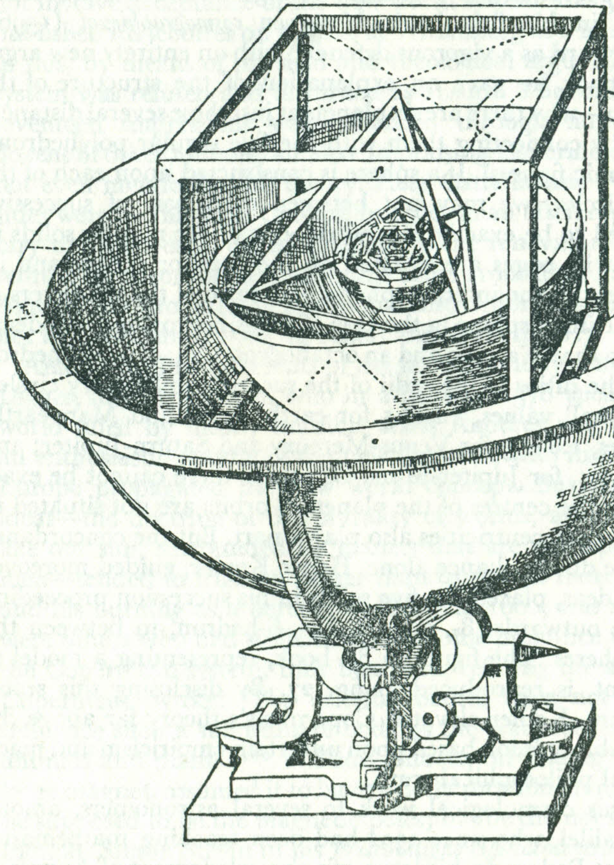


Fig. 27. Keeker's model of five regular polyhedrons between the planetary spheres

from unjust judgment. He suggested a well-directed action to create an impression of influence in order to encourage timorous minds. 'Be confident, Galilei, and proceed! If I am right, only a few of the chief mathematicians of Europe will keep aloof from us; such is the power of truth.'[108]

Galileo Galilei (1564–1642)—often called by his Christian name only—continuing in a more thorough way the first attempts of Benedetti and dal Monte, had arrived at a critical attitude toward Aristotle's theory of motion. Chiefly by clear argument, sometimes aided by experiments on swinging pendulums and on motion along an inclined plane, he gradually gained a better understanding of the laws of motion determining the phenomena of falling and thrown bodies. He could now recognize the futility of the objections against a rapid motion of the earth's surface, objections which had been brought forward by Ptolemy and repeated over and over again. He came to understand that a state of motion, just as well as a state of rest, remains when not disturbed by other influences. But his adherence to Copernicus's theory did not appear in his academic lectures; here, conforming to the imposed task, he taught the celestial sphere and the theory of Ptolemy with its arguments.

Yet, when Tycho's *Progymnasmata* appeared in 1602 with all the results on the new star of 1572, and when in 1604 another equally brilliant new star appeared in the low southern parts of Ophiuchus, he could not refrain from pointing out, in some well-attended public lectures, that they completely refuted Aristotle's doctrine of the immutability of the superlunar regions of the stars.

Then, about 1608, the first telescopes became known in Europe. We do not know the true history of the invention; apparently Zacharias Janssen, optician in the Dutch town of Middelburg, constructed one in 1604, copied from a specimen belonging to an unknown Italian—the possibility of combining lenses had already been alluded to before. Janssen, as a peddler, had shown and sold some of them at fairs in Germany and elsewhere. In 1608 Lippershey, another optician in Middelburg, originally from Wezel, offered a telescope to Prince Maurice and the States of Holland, chiefly for use in war. It was tested successfully and its author was well remunerated. The requested licence —also asked for shortly afterward by Metius at Alkmaar—was refused, because many people already knew of the telescope. Soon the report spread, and specimens were shown and sold in France. Galileo, after hearing about it, himself constructed one and offered it in 1609 to the Doge and the Signoria of Venice for use in war and navigation. Gradually improving his instruments, he directed them towards the moon and the stars. Then followed rapidly the series of wonderful discoveries that put Copernicus's doctrine in the centre of public interest. At first he communicated them only in repeatedly copied letters to his friends and colleagues. Then he published them in a little booklet, *Sidereus nuncius* ('Messenger of the Stars') which appeared in March 1610 and caused great commotion in learned circles.

On the moon he saw the border line of the illuminated part irregularly broken. In the dark part he saw, near the border line, isolated light patches, which, as the moon waxed, grew larger and merged with the illuminated part; clearly they were mountain tops (pl. 4). So the moon was not crystalline but, like the earth, an uneven broken surface with mountains and valleys; the circular walls, however, showed a different structure. The planets in the telescope looked different from the stars; they were pale discs with enlarged surfaces, whereas the stars remained strongly sparkling points, only appearing brighter than with the naked eye. Nebulous patches and the Milky Way appeared to consist of a number of small stars; in the Pleiades he counted more than 40 stars, and everywhere between the known stars smaller ones which were invisible to the naked eye were now seen. Looking at Jupiter, he perceived on January 7, 1610, three small stars, on January 13th one more; they accompanied Jupiter in its progress, but every other night in a different formation, moving to and fro. In a letter of January 30th, Galileo called them new planets revolving about a larger planet; in honour of the Grand Duke of Tuscany he named them 'Medicean stars'. That these bodies clearly described orbits about Jupiter as their centre showed that the earth cannot be the centre of all movements and, therefore, gave support to the system of Copernicus.

A great sensation was caused by these discoveries, admiration and joy on the part of friends and partisans, scholars as well as laymen, but still more doubt and opposition, increasing to hostility, among the dominant authorities of learning. The professors of philosophy, attacked in their own empire—the University of Padua was known as a stronghold of Aristotelianism—announced a sharp refutation. In a number of writings the existence of the new wandering stars was refuted on logical arguments; since Aristotle had not mentioned them, it could not be true. When Galileo in a public disputation tried to convince his Paduan colleagues by showing the stars, they would not look in his telescope, maintaining that it was appearance only and therefore illusion. As to the two leading astronomers of Italy, old Clavius dismissed the matter with a disclaiming joke; the other, Magini at Bologna, an esteemed correspondent of Tycho and Kepler, jealous of his younger colleague, wrote letters in which he expressed his doubts and declared the observations to be deceit or self-deceit. When Galileo, visiting Bologna in March, met a number of professors at Magini's and showed them Jupiter

through his telescope, nearly all of them declared that they saw nothing of the new stars; this negative result was given wide publicity. It was not simply bias through unwillingness; it was also the real difficulty of seeing things through a telescope for people entirely untrained. The first telescopes were very primitive, the lenses were badly figured, the images had irregular coloured borders, and all in all they were not comparable even to a modern opera glass. In September Galileo wrote to Clavius, who had not yet succeeded in seeing them, that the instrument should be fixed, because when held in the hand its vibrations made the new stars invisible. The discoveries which opened the road to the modern world system were not fully ripened fruits, easily picked up after gently shaking the tree. They were the fruits of strenuous exertion and extreme care in observing and in asserting what was seen, of laborious and often enervating struggle, first against the inner doubts and then against the obstinate, often perfidious, defence of an antiquated doctrine that still powerfully dominated men's minds.

Kepler at Prague gave Galileo enthusiastic support. Immediately on receiving the *Nuncius* he expressed his ideas thereon in a printed open letter. There he said that the idea of constructing a telescope for studying the celestial luminaries had never occurred to him, because he supposed that the thick blue air would prevent the details of the very remote stars from being seen; but now Galileo had shown that space was filled with a thin and harmless substance only. He pointed out—what his friend Pistorius had already maintained—that more accurate measuring with instruments would now be possible. He dealt with the consequences of the new discoveries by indulging in fantasies of possible inhabitants on the moon and of the phenomena visible on Jupiter with its four moons. He had always shrunk from Bruno's 'terrible philosophy' of an infinite world with an endless number of other solar systems; but now Galileo's discovery of innumerable small stars showed that our sun was a unique body, surpassing all the others in luminous power. Such were his thoughts on the new discoveries.

Yet, though he had full confidence in Galileo's observations, he himself had not seen Jupiter's companions, and with his friends he was uneasy about the tidings from Bologna. Kepler was not a good observer; his sight was bad and he was awkward with instruments; so he never tried to make a telescope himself. The Dutch telescopes found in Germany did not show the Medicean stars; they magnified too little. According to Galileo, magnification of twenty or thirty times was necessary. The grinding of good lenses was a difficult art. In September for the first time Kepler succeeded in seeing the new stars through a better telescope belonging to a visitor. But finding the laws of their motion seemed to him very difficult, almost impossible. In the next months they were seen in France and also in England (by Harriot). The observers of the Jesuit College at Rome at last also succeeded in improving their telescopes, and in December Clavius sent word to Galileo—who in the meantime had settled in Florence, in the service of the Grand Duke—that he and his colleagues were convinced of the existence of the new planets. His younger colleague, Father Grienberger, in January 1611 wrote to Galileo: 'Things so hard to believe as what you assert neither can nor should be believed lightly; I know how difficult it is to dismiss opinions sustained for many centuries by the authority of so many scholars. And surely, if I had not seen, so far as the instruments allowed, these wonders with my own eyes. . . . I do not know whether I would have consented to your arguments.'[109]

In the meantime other discoveries had followed. Hidden in a letter-puzzle—to make his priority secure—Galileo ascertained in July 1610 that Saturn was treble, touched at either side by a smaller resting globe; to us this appearance testifies to the low quality of the first telescopes. In the same way he announced in December that Venus imitates the figures of the moon. Some followers of Copernicus had predicted it; others, Kepler among them, believed that the planets partly radiated their own light or were saturated by absorbed sunlight. In his letters Galileo emphasized that this result—the darkness of the planetary bodies showing the similar character of the planets and the earth—confirmed the truth of Copernicus's theory that the earth is a planet.

In order to win over the influential Fathers of the Roman College to his views by discussion and demonstration with his instruments, Galileo went to Rome in March 1611. Welcomed as an honoured guest, he gave lectures and held discussions, and in an assembly was abundantly praised for his discoveries. In a report of April 1611, to Cardinal Bellarmin of the Holy Office, Clavius and his colleagues confirmed the truth of Galileo's discoveries. But they did not confirm Galileo's theory; it seemed to them more probable that in the crystalline body of the moon there were denser and lighter parts than that its surface was unequal; others thought differently, but as yet there was no certainty. The General of the Jesuit Order had instructed members as far as possible to retain and to defend Aristotle's doctrine. The confirmation related only to the observed facts; the report did not discuss the explanation by the heliocentric system. Though, shortly before his death, Clavius wrote that now the astronomers should see by what structure of the celestial spheres these phenomena could be explained, this did not imply the acceptance of Galileo's theory. Indeed, a strict demonstration of the earth's motion could be given neither by the mountains of the moon, nor by the Jupiter satellites, nor by the crescent figure of Venus.

Galileo did not remain the only traveller on these new paths. Telescopes had come into many hands and were diligently used to study the sky; they brought a wide conviction of the reality of the new phenomena and led to fresh discoveries. This, however, implied much contention over the priority of the new discoveries and over their interpretation. Galileo naturally considered this field to be his special domain; but others, also naturally, tried to get their share of the fame.

In a calendar published in 1611, and afterward in a book published in 1614 on the 'World of Jupiter' (*Mundus Jovialis*), Simon Marius (Mayer) of Anspach tells that in 1609 he came into possession of a Dutch telescope, directed it at the stars and gradually became conscious of the importance of what he saw; that he had discovered the satellites of Jupiter independently and began to observe them somewhat later than Galileo. The latter afterward hotly inveighed against Marius, accused him of plagiarism, and imputed that he had not really seen the new stars. Independent discovery, of course, could not be proved; but from his later observations Marius had succeeded in deriving better values than Galileo for the periods of revolution and the other elements of their orbits.

The discovery of sunspots occurred at the same time. In former times it had happened that an exceptionally large spot had been perceived on a hazy sun or in a solar image projected by a small opening. But now regular detection and observation became possible, at first by directing the telescope upon the sun when much dimmed by morning or evening haze. The first public announcement came from Johann Fabricius of Emden, the son of Kepler's friend David Fabricius, in the summer of 1611. Galileo had already shown them during his visit to Rome in the spring of the same year. The researches of Chr. Scheiner, SJ, at Ingolstadt, aided by his pupil Cysat, began at the same time; they were published the next year. He had been warned by his Superior not to put trust in his observations, because nothing of it existed in Aristotle; so he had to publish his work anonymously, and in order to avoid conflict with Aristotle he explained the spots as small dark bodies circulating about the sun. Like Fabricius, Galileo declared the spots to be parts of the sun itself, proving its axial rotation. In criticisms of Scheiner's publications, made in his book, *History and Demonstrations Concerning the Solar Spots*, distributed by his friends in Rome in 1613, Galileo strongly attacked the entire doctrine of Aristotle.

The scene of the struggle had now changed; the subject of the contest was not the observations but the interpretations, not the practice but the theory. After his return from the triumphs at Rome, Galileo gradually perceived that the climate had changed. His friends, who had previously greeted his discoveries with enthusiasm, now became cautious and advised him to be content with the victories won—Campanella only, writing from his prison, strongly urged him to stand firm in the defence of Copernicus. Galileo himself considered his discoveries with the telescope only as aids to the real goal, the proof of the truth of the heliocentric theory.

The fight now concentrated upon the Copernican system, i.e. on the movement of the earth. Galileo believed he had found a direct proof of this movement in the phenomena of the tides. Kepler had supposed them to be an effect of the moon, but Galileo tried to explain them by inequalities in the velocity of the earth's surface. This velocity is a combination of the daily and the yearly revolutions; on the night side of the earth their velocities combine, and on the day side toward the sun they subtract. This, he said, makes the water of the oceans oscillate. Neglecting the obvious dependence of the tides on the moon, he thought to find in the tides a proof of the earth's motion. It was not convincing, because according to his theory high and low tide should occur only once each day.

The real fight took place on the field of theology. Ignorant monks preached in the churches against the new theory as a heretical doctrine contrary to the Bible; secret denunciations against Galileo arrived at the Holy Office. Galileo on his side dealt with the Bible texts in some letters which were widely distributed and interpreted them according to the new views—not always successfully, when he tried to make the biblical texts proofs for the Copernican system. He found supporters among the clerics themselves; a Carmelite monk, Foscarini, in a printed letter to the General of his Order, defended the Copernican system with strong arguments.

Galileo again travelled to Rome, first to clear himself from the charge of being a heretic; indeed, he was a devout Catholic, who always in his writings professed that he obeyed what the Church in its deeper divine insight should declare to be truth, though at the same time demanding that the theologians, before deciding, should acquire an exact knowledge of Copernicus's work. The justification succeeded well enough; more difficult was his second task, to convince the influential persons and the authoritative cardinals. He argued that there could be no conflict between revealed and natural truth; so when the latter had been clearly demonstrated by the facts and could not be changed by any interpretation, the former, the interpretation of the Holy Scripture, had to adapt itself. But this logic did not impress the theologians, who were firm in the ecclesiastical doctrine and for whom the results of the scientists carried no conviction. For them the issue was, that if the Church did not take a firm stand by a clear verdict an unbridled discussion by clerics and laymen on the interpretation of Bible texts would arise, and this was incompatible with its rigid discipline of doctrine. So the result

was not in doubt; the Church had to take its stand against the motion of the earth. Characteristic was the sloppy and inaccurate rendering of Copernicus's doctrine in the report of the doctors of theology and the verdict of the court (February 25, 1616). As regards the first thesis, that 'the sun is centre of the world and entirely unmoved as to its place', they proclaimed that it was foolish and philosophically absurd and formally heretical. As regards the second thesis, that 'the earth is not centre of the world and not unmoved but moves relatively to itself also in daily motion', philosophically the same conclusion holds, and theologically it is at least an error of faith. This wording suffices to show the ignorance or carelessness of the judges. Galileo was summoned before Cardinal Bellarmin. He was informed that the doctrine of the earth's motion must not be taught nor held to be true—it might only be dealt with as a mathematical hypothesis—and he was admonished to abandon it. To which Galileo submitted. On the basis of this verdict, the Congregation of the Index resolved, March 5, 1616, that the books of Copernicus and all other books teaching the same ideas should be suspended and forbidden until they were amended.

Now that it was no longer permissible publicly to defend the heliocentric world system of Copernicus, Galileo concentrated his researches and criticism on Aristotle's philosophy of nature. At the same time he tried repeatedly, but ever in vain, to have the prohibition repealed by the next Pope, Urban VIII, who, when a cardinal, had shown friendly interest in his work. At last it became too much for him and he found a way to meet, as he supposed, the objections by giving to his ideas the form of a discourse in which the old doctrine imposed by the Church was formally put in the right but in such a way that the arguments of the new doctrine could be exhibited in full. His literary ability and dramatic talent made this work, which appeared in 1632 in Italian, *Dialogo sopra i due massimi systemi del mondo* ('Dialogue on the Two Principal World Systems'), an excellent popular work which completely finished Aristotle's doctrines of motion and of physics.

Because of the light it shed on the questions vehemently disputed at the same time, the book was hailed enthusiastically by his friends and all modern-minded scholars. But now the Church authorities (although the *imprimatur* had been given) were roused to repulse the attack. They did not let themselves be deceived by the outer form of submission to theological authority; the meaning behind the words was clear enough. Personal influences also made themselves felt; Jesuit scholars, stung by Galileo's severe criticism of their writings, which offended the scholarly pride of the entire Order, worked against him and succeeded in making the Pope Galileo's personal enemy. So he was summoned before the Inquisition. Formally Galileo was right, because it was permissible publicly to discuss the Copernican doctrine as a hypothesis; yet he was condemned for disobedience, probably on a forged document, and was compelled in 1635 solemnly to abjure the heliocentric doctrine. A modern Catholic astronomer, Professor Joseph Plassmann at Münster in 1898, called this event 'the most fatal mistake that has ever been made by the Church authorities against science'.[110]

In the seventeenth century many writings appeared for and against the Copernican system; those against mostly adhered to the Tychonian system. But doubt and opposition among scientists were lessening. The ecclesiastical ban could not hold up the progress of science. It only made it more difficult for Catholic scientists to accept or to publish new ideas. Even in 1762 a new edition of Newton's major work by two expert Minorite Fathers was preceded by the declaration that they considered the theory expounded as a hypothesis only and that they adhered to the Church's verdict. But in the eighteenth century real opposition ceased, and the new view penetrated widely among the people. In 1744, for the first time, Galileo's *Dialogue* was allowed to be printed, though with all the 'corrections' prescribed by the Inquisition. After many attempts, the ban was finally lifted in 1822; and after 1835 Copernicus, Kepler and Galileo no longer appeared on the index of prohibited books. So the attempt of the Church to arrest the progress of science by her authority had resulted in an acknowledged defeat.

CHAPTER 23

KEPLER

By his first book, *Mysterium cosmographicum*, Kepler had caught the attention of Tycho Brahe, because of its independent thinking, its astronomical knowledge, and the author's clearly apparent ability and perseverance in computing. Tycho made approaches, but their personal contact became possible only after Tycho had settled in Prague, when, at the same time, through a general expulsion of the Protestants from Styria, Kepler was forced to seek a living elsewhere. Tycho saw to it that in 1601 Kepler was appointed 'Imperial Mathematician' in the service of Rudolf II, with the assignment to assist in the reduction of the observations of the planets and the construction of new planetary tables. After Tycho's death, when the work had scarcely started, after settling certain difficulties with the heirs, he inherited Tycho's observations as well as the task of finishing the work.

In Kepler, a man of another generation, the character of a new century emerged. Astrology as the doctrine of world unity dominated his mind, not as a fearful spying upon the stars to discover human destinies but as a passionate desire to penetrate into the secret of this unity. As with many of his contemporaries, the spirit of research lived in him, the intense curiosity, the desire to unravel the secrets of nature. Not in the sober sense of modern research into nature but rather in the sense of what we call mysticism. He had the intuitive feeling that the entire universe was a miracle, materially and spiritually connected with the equally miraculous creations of the human mind, geometry, the theory of numbers, music—all of which was already apparent in his first book. His astrology bore a more modern character than did that of his predecessors; he derides the belief that events might depend on the names given to the constellations in ancient times; but he did assume that conjunctions of the planets influenced earthly events, just as the sun and the moon do. He was in the first place a physical thinker; in all phenomena he looked for causal connections, sometimes surprising us by modern views, sometimes entirely in error. The previous generation had asked of any phenomenon: What does it mean? The new generation asked: What is it and what is its cause?

In a booklet on astronomical optics (*Astronomiae pars optica*) in 1604 he explained refraction in a way somewhat different from Tycho, through the transition of the light-rays from the thin ether into the air itself, so that it must be present at all altitudes, increasing from the zenith, and must be equal for all celestial luminaries. He did not succeed in finding the law of refraction by experiment; this had to wait for Snell at Leiden (1620). But with his results he was able to construct a table better than Tycho's.

He considered the moon, as also did Bruno and Gilbert, as similar to the earth, dark and having high mountains—this was before the invention of the telescope. He spoke of the penumbra at lunar eclipses and explained the ruddy light of the totally eclipsed moon through refraction of the sun's light when passing through the earth's atmosphere. He also defended Maestlin's explanation of the pale illumination of the lunar disc beside the crescent—the old moon in the arms of the young—as light reflected upon the moon by the sunlit earth. The brilliant new star mentioned above, which, after a conjunction of Jupiter and Saturn, soon accompanied by Mars, appeared in October 1604 in the 'fiery' sign of Sagittarius, led Kepler to publish in 1606, after the star had faded and disappeared, a booklet in which he discussed its physical characteristics as well as its astrological significance. Here he compared the scintillation of the stars with the sparkling of a diamond when twirled; a better explanation, that it was due to the undulating motion of the air, had already been suggested by the renowned classicist J. J. Scaliger.

All these topics were accessory occupations; they are important because they indicate the tendencies of his thinking on the celestial bodies. His chief work during these years consisted of computations from Tycho's observations of the planets. From 1601 onward he was occupied with the planet Mars, which Longomontanus had previously found too difficult an object. Kepler began by deriving from the observations a list giving accurate values of the moment, the longitude and the latitude of all the oppositions since 1580. Here he at once went his own way. Copernicus, following Ptolemy, had assumed the centre of the earth's orbit to be the centre of reference for all planetary orbits, and Tycho had adhered to it in deriving oppositions to the mean sun. Kepler in his first work had already designated the real sun itself as the natural centre of the planetary system; so he judged that opposition to the real and not to the mean sun should be derived and discussed. This was his first important modification and simplification of former methods of treatment.

The oppositions deduced from Tycho's observations, completed by those observed by himself and by his friend David Fabricius at Emden in the years 1602 and 1604, are contained in the accompanying list.[111]

OPPOSITIONS OF MARS

Date	*Time*	*Longitude*	*Latitude*	*Longitude Computed*	*Difference*	*Latitude Computed*
1580, Nov. 18	1h. 31m.	66° 28′ 35″	+1° 40′	66° 28′ 44″	−0′ 99″	+1° 45½′
1582, Dec. 28	3h. 58m.	106° 55′ 30″	+4° 6′	106° 57′ 4″	−1′ 34″	+4° 3⅓′
1585, Jan. 30	19h. 14m.	141° 36′ 10″	+4° 32⅙′	141° 37′ 46″	−1′ 36″	+4° 30½′
1587, Mar. 6	7h. 23m.	175° 43′	+3° 41′	175° 43′ 16″	−0′ 16″	+3° 37′
1589, Apl. 14	6h. 23m.	214° 24′	+1° 12¾′	214° 26′ 12″	−2′ 12″	+1° 5⅓′
1591, June 8	7h. 43m.	266° 43′	−4° 0′	266° 43′ 51″	−0′ 51″	−3° 59⅙′
1593, Aug. 25	17h. 27m.	342° 16′	−6° 2′	342° 16′ 42″	−0′ 42″	−6° 3¾′
1595, Oct. 31	0h. 39m.	47° 31′ 40″	+0° 8′	47° 31′ 54″	−0′ 14″	+0° 5⅕′
1597, Dec. 13	15h. 44m.	92° 28′	+3° 33′	92° 28′ 3″	−0′ 3″	+3° 20′
1600, Jan. 18	14h. 2m.	128° 38′	+4° 30⅝′	128° 38′ 18″	−0′ 18″	+4° 30¼′
1602, Feb. 20	14h. 13m.	162° 27′	+4° 10′	162° 25′ 13″	+1′ 47″	+4° 7⅖′
1604, Mar. 28	16h. 23m.	198° 37′ 10″	+2° 26′	198° 36′ 43″	+0′ 27″	+2° 18⅖′

The variations in angular velocity, which is great in the region of Aquarius (at longitude 330°) and small near the Lion (at longitude 150°), are conspicuous at first sight. Ptolemy had represented this variation by introducing a *punctum equans* and placing the centre of the circle midway between it and the earth (here to be replaced by the sun). Kepler wished to test whether this bisection exactly represented Tycho's results. Ptolemy, in computing the orbit, had needed three oppositions; since Kepler had to determine one additional unknown, viz., the ratio of division of the total eccentricity by the circle's centre, he had to use four data. For four chosen moments (the oppositions of 1587, 1591, 1593, 1595) he knew the directions as seen from the sun and also the directions as seen from the *punctum equans*, since the latter increased proportionally to the elapsed time. The problem of finding from these data the direction of the line of apsides (longitude of aphelion) and the two distances from the circle's centre to the sun and to the equant could not be solved by a direct method. Kepler had to solve it by trying various suppositions, in successive approximations. 'If this cumbersome mode of working displeases you,' he says to the reader, 'you may rightly pity me, who had to apply it at least seventy times with great loss of time; so you will not wonder that the fifth year is already passing since I began with Mars. . . . Acute geometers equal to Vieta may show that my method is not at the level of art. . . . May they solve the problem geometrically. For me it suffices that . . . to find the way out of this labyrinth I had, instead of the torch of geometry, an artless thread guiding me to the exit.[112]

The result of these computations was that the total eccentricity amounted to 0.18564 times the radius, that the sun was 0.11332, and the equant was 0.07232, distant from the centre, whereas the longitude of aphelion (for 1587) was 148° 48′ 55″. Modern theory shows that the two distances should be approximately $\frac{9}{16}$ and $\frac{7}{16}$ of the total eccentricity. How exactly these elements represent the data may be seen from the list on page 237, in which the remaining differences between observation and computation are given in the sixth column. 'So I state that the places of opposition are rendered by this computation with the same exactness as Tycho's sextant observations, which, through the considerable diameter of Mars and the insufficiently known refraction and parallax, are affected by some uncertainty, surely as much as 2 minutes (prime).'[113]

Thus Chapter 18 of his book closes. Then Chapter 19 begins with the words: 'Whoever would think it possible? This hypothesis so well in accordance with the oppositions, yet is wrong.'[114] Ptolemy was right in bisecting the eccentricity. This was apparent at once when Kepler computed the real distances by means of the observed latitudes. Computing, on the other hand, the oppositions in the case of a bisection of the total eccentricity, he found for 1582 a longitude of $107° 4\frac{3}{4}'$, deviating nearly $8'$ from the previous computation and $9'$ from observation.

'From this so small deviation of $8'$ the reason why Ptolemy could be content with bisecting the eccentricity is apparent . . . Ptolemy did not claim to reach down beyond a limit of accuracy of $\frac{1}{6}°$ or $10'$. . . . It behoves us, to whom by divine benevolence such a very careful observer as Tycho Brahe has been given, in whose observations an error of $8'$ of Ptolemy's computation could be disclosed, to recognize this boon of God with thankful mind and use it by exerting ourselves in working out the true form of celestial motions. . . . Thus these single eight minutes indicate to us the road towards the renovation of the entire astronomy; they afforded the materials for a large part of this work.'[115]

This was the enigma he had to solve: the unequal division of the eccentricity could not be true, though it rendered all longitudes in a perfect way; under the name of 'vicarious hypothesis' he used it later on to compute for any moment the longitude of Mars as seen from the sun. Moreover, the equal division did not represent the oppositions. In this dilemma, for the first time, the trigonometric determination of distance was employed and remained the backbone of his work. In the triangle sun-Mars-earth the direction of each side was known: earth-sun through the observations of the sun (rendered by Tycho's tables), earth-Mars through the observations of Mars, and sun-Mars through the vicarious hypothesis. From the angles known, the ratio of the sides, i.e. of the distances could then be computed. By selecting observations separated by exact multiples of a year, so that the distance earth-sun was the same, the variations of the distance sun-Mars were found. Taking observations with intervals of a full number of revolutions of Mars, the variations of the distances earth-sun afforded the figure of the earth's orbit.

Kepler now turned first of all to a closer examination of the latter.

It was necessary, not only because its uncertainties could spoil the exactness of his computations of Mars, but also because, with Ptolemy and Tycho, the earth had no equant, whereas as a planet it should not be different from the others. By applying the trigonometric method to observations where Mars occupied the same place, he found an eccentricity of the earth's orbit of 0.01837 (the five decimals are not an indication of precision but a consequence of Kepler's always taking the radius 100,00 instead of writing decimals). Since the fluctuations in angular velocity had provided Tycho with an eccentricity of 0.03584, twice as large, it appeared that the earth too had an equant. This enabled him to construct tables giving exact distances and longitudes of the sun. Now computing the distances of Mars to the sun accurately, he found the greatest value, in aphelion, to be 1.6678 and the smallest value, in perihelion, 1.3850 radii of the earth's orbit. Then the radius of Mars's orbit is 1.5264, and the distance of the sun from its centre is $0.1414 : 1.5264 = 0.0926$, essentially the exact half of the total eccentricity 0.18564 derived from its motion.

Thus it was established from observations of Mars that for this planet and for the earth the total eccentricity between the sun and the equant was divided exactly into halves by the centre of the orbit. Kepler, however, as a physical thinker, could not reconcile himself to the idea that a void point should be able to govern the planet's motion. In aphelion the planet proceeded more slowly, in perihelion more rapidly, because farther from the *punctum equans*. In the same ratio it was in the first case farther from, in the second case nearer to, the sun. So the sun regulated the velocity of the planet; the velocity was in exactly inverse proportion to the distance. The logic of the case was now at once obvious to Kepler. According to the lever principle, a planet at greater distance is moved by the solar force with greater difficulty, hence needs more time to describe a certain arc. This physical explanation shows, said Kepler, why he was right to relate all motions to the bodily sun instead of to a void centre of the earth's orbit, and at the same time why Tycho was wrong in having the heavy sun describe an orbit about the earth. Now the importance of the sun became even greater than it had been for Copernicus; it was now not only the source of light and heat for the entire planetary system, but the source of force also. Light and force, both immaterial, expand in space, but in a different way. Since light expands over spherical surfaces, going upward and downward and to all sides, it decreases with the second power of distance. Solar force, on the contrary, expands along circles in the ecliptic, not upwards and downwards, driving the planets through the zodiac in longitude only, hence decreases with the distance itself. One can understand this solar force by assuming that the sun rotates about an axis and thus draws along the planets in the same direction, more slowly as their distance increases. As to the nature of this force, Kepler pointed out that magnetism, which is a directing force, operates as if the magnet consisted of threads or fibres. The sun, too, did not attract the planets (if it did, they would fall into the sun) but directed their course through a sideways force, as though it consisted of annular magnetic fibres. This is more than an analogy, for Gilbert had discovered that the earth, which in the same way directs the moon in its orbit, is a magnet.

From these speculations it appears that Kepler was not simply an astronomical computer; what mattered to him was an understanding of the physical nature of things. His speculations are similar to the ideas that developed later in the seventeenth century. They were the product of new inquisitive impulses, far superior to the sterile scholasticism of the preceding century that still dominated the academic chairs. Because he spoke in terms of a force proceeding from the sun, he has sometimes—wrongly as we see—been called a precursor of Newton. Rather he was a precursor of the natural philosophy of the seventeenth century; what appears in Descartes's vortex theory as a vague philosophical speculation, with Kepler has the freshness of direct conclusions imposed by the facts of experience.

It is certainly true that Kepler, in trying to explain the remaining problems by referring to the planets as if they played an active part—speaking of the 'spirit' or 'essence' of the planet that has to pay attention to the apparent size of the sun—also became vague and contradictory. But it is more important that he had developed a new method of computation. The time spent by the planet on a small arc of the orbit, inverse to the velocity, was proportional to the distance from the sun; so, to have the total time for a longer arc, all the intervening distances had to be summarized. This was a problem of integration: 'If we do not take the sum total of all of them, of which the number is infinite, we cannot indicate the time for each of them.'[116] He first solved the problem by numerical summation; but then for the sum total of the distances he substituted the area between the limiting radii; though this was not exactly the same thing, it fitted well. The area could easily be computed as a circular sector diminished by a triangle. This was formulated afterward as Kepler's second law, the '*law of areas*': the radius vector describes equal areas in equal times.

Thus the elements of the orbit and the method of computation were known, and in aphelion and perihelion, as well as at 90° distance, the computed longitudes were right. But in the octants, at 45° distance from these points, the old difference of 8′ again appeared, too large to be attributed to errors of Tycho's observation. To solve this riddle, he again applied his trigonometric method to derive the distance of Mars

from the sun directly from observation. At longitudes 44°, 185°, 158° he found 1.4775, 1.6310, 1.66255. Computation by means of the elements found above afforded 1.48539, 1.63883, 1.66605. Observation thus showed the distances of Mars from the sun to be smaller than that which followed from the circular orbit. Sideways the planet took its course, not on, but within, the circle described through aphelion and perihelion. 'The matter is obviously this: *the planetary orbit is no circle*; to both sides it goes inward and then outward until in the perihelion the circle is reached again. Such a figure is called an oval.'[117]

Thus for the first time, and forever, the principle that for a thousand years had been accepted by all astronomers as the basis of astronomy had been destroyed, that the circle was the natural and true orbit of heavenly bodies. The exactness of Tycho's observations had shown it to be incorrect. As the first triumph of the empirical study of nature, Kepler's result stands at the entrance of modern scientific research.

His difficulties, however, had not yet come to an end. In attempting to give a physical explanation for the deviation from the circle, by proposing a propelling force of the sun variable with distance and a kind of active epicyclic motion of the planet, he was led into a year's futile computing. David Fabricius of Emden, with whom he exchanged letters and whom he praised as the best observer since Tycho's death, warned him that his computation did not tally with observation. At last he perceived that the lateral compression of the orbit, 'the maximal breadth of the deficient sickle, 0.00429 times the radius', was exactly half the square of the eccentricity ($0.0926^2 = 0.00857$). Then 'it was as if I awoke from sleep and saw a new light'.[118] Now it was clear that the orbit must be an ellipse, with the sun in one of the foci; in a nearly circular ellipse the oblateness is half the square of the eccentricity. In this result, usually called 'Kepler's first law', the *true elliptic figure* of the planetary orbits was disclosed for the first time.

The last chapters of his book are devoted to the remaining task of deriving the position of the plane of the orbit. He found the longitude of the ascending node to be 46° 46⅓′ and the inclination 1° 50′ 25″. The latitudes observed in the different oppositions are, as is shown in the list on page 237, well represented by these elements. The remaining deviations are larger than for the longitudes, owing mostly to uncertainties in refraction and parallax. Of the accessory oscillations, which Copernicus had assumed, nothing could be perceived. Kepler pointed out that a parallax of Mars larger than some few minutes would spoil the concordance; hence the parallax of the much more remote sun must certainly be below 1′.

Most of this work had been done in the first years at Prague, under the constant pressure of financial and other worries, of delicate health, of difficulties as to the disposal of the observations, and with many other interruptions. In 1604 his results were ready, and in 1605 he could present the manuscript to Emperor Rudolf. It was not until 1609 that it appeared, the delay being due chiefly to the lack of money in the imperial treasury. The title *Astronomia nova, aitiologetos seu physica coelestis* ('New Astronomy, causally explained, or Celestial Physics'), preceding the title of the contents: 'De motu stellae Martis' ('On the Motion of the Star Mars'), indicated that he was aware that astronomy was put upon a new basis by this work.

In Tycho Brahe's and Kepler's work the new method of scientific research is embodied—the method of collecting data from experiment and observation, and from them deriving rules and laws which form the body of science. They were not the only ones; at the turn of the century, in every field of knowledge, investigators appeared: Simon Stevin and Galileo studied mechanics, the laws of equilibrium and motion, Gilbert studied magnetism, Vesalius studied anatomy, Van Helmont studied chemistry, and Clusius studied botany. In all of them—few in number—a new spirit forged its way as they replaced the old belief in authority by their own experimental research.

Kepler's work did not end with his book on Mars; it had only begun. His task, handed down from Tycho, was the construction and publication of the new planetary tables, the 'Rudolphine Tables'. By his work on Mars he had found the key, the knowledge of the rather simple laws of planetary motion. He now had to apply them to all the other planets. In Linz, the capital of Upper Austria, where he had accepted the job of provincial mathematician, he published in 1618 his *Epitome astronomiae Copernicanae* ('Survey of Copernican Astronomy'), in which for the first time the structure of the solar system was correctly expounded. In it the true orbits of Mercury and Venus were given as entirely regular ellipses, with the following numerical elements: for Mercury, eccentricity 0.210 and aphelion at 255°: for Venus, 0.00604 and 302°. Gone were all the complications which Ptolemy thought necessary and which Copernicus had taken from him. Like the other planets, their orbits were fixed, with constant inclination to the ecliptic, without additional oscillations. How Kepler derived them from the observations is nowhere explained. In a letter of May 5, 1616, to his former teacher, Maestlin, he merely says: 'In the summer of 1614 the theory of Venus followed, in the winter of 1615 that of Mercury; they are in no way peculiar as compared with Saturn, Jupiter, and Mars; I did it by means of a large orbit of the earth and a simple eccentric orbit just like that of Mars.'[119]

The *Epitome* constituted the first complete manual of astronomy constructed after the new principles. It deals, first of all, with spherical astronomy, the shape and the size of the earth and its place in the

universe; then come the stars, of which he says that it is uncertain whether they are at equal or different distances, though their realm seems to be infinite; of these the sun is one, and, because of its nearness, appears particularly bright. Kepler next deals with the atmosphere, refraction, the twilight and the twinkling of the stars. Speaking of the daily rotation of the earth, he argues that nature, to attain its end, always chooses the simplest way. Then he discusses the risings and settings of the celestial bodies, the years and the days, the seasons and the climates. The second part, published in 1620–21, expounds the theory, the *physica coelestis*, explaining the motion of the planets by the same physical principles as in the Mars book. These physical explanations he considered equally as important as the numerical elements of the orbits; or rather, as more essential, because they disclose the causes. All this is given in the form of questions and answers, not as Galileo did, for the dramatic power of a discourse, but after the manner of a catechism, in which the question as a heading summarizes what is expounded in the answer. In a letter to the Estates of Upper Austria he indicated that this book was an explanation to the Rudolphine Tables with which he was occupied.

He was occupied with this work for many years, chiefly because lack of money in the imperial treasury prevented him from appointing computers; at last he found in Jacob Bartsch, afterwards his son-in-law, a devoted assistant in his computations. In 1627 the tables appeared; he had had to contribute to the cost of printing from his own small possessions. On account of their excellent basis, they superseded all former and also contemporary less perfect tables, like the one constructed by the Middelburg minister, Philip Lansbergen; and they dominated the field of practical astronomy throughout the seventeenth century. His theoretical work, on the contrary, penetrated but slowly. Notwithstanding their friendly correspondence, Galileo, most curiously, remained ignorant of the laws discovered by Kepler and in his *Dialogo* in 1632 wrote that the true figure of the planetary orbits was unknown.

Through all those years, in between the practical computations, Kepler worked at a second task imposed not by heritage or office but by his own craving for knowledge and understanding. What had inspired him in the work of his youth, what had driven him toward Tycho, persisted with him as his deepest longing: to disclose the secret of the world-structure, to penetrate into the thoughts which the Creator had followed in the creation of the world. In his work *Harmonice mundi* ('The Harmony of the World') he connected the planetary motions with all fields of abstraction and harmony; with geometrical figures, with the relations of numbers, with musical harmonies. But among all these fantastic relations we find one precious discovery, afterwards always cited as *Kepler's third law*—also mentioned in his *Epitome*: for all the planets the squares of the periods of revolution are proportional to the cubes of their mean distances from the sun. For himself, all this was the consummation of what during all his life had been the radiant goal of his efforts and pains: 'I write my book to be read, either by present-day or by future readers—what does it matter? It may wait a hundred years for its reader, since God himself has been waiting 6,000 years for one who penetrated his work.'[120]

We read this proud pronouncement with a smile of admiration, knowing that later science has accepted and preserved from the entire work on 'The Harmony' only that one page containing the third law. Must we say, then, that all other work done by Kepler in this book was a waste of time? To perform great things, man has to set himself even greater aims. The lasting fruit can grow only in a larger organic structure, first living, afterwards withering to dry straw. The strong impulses to work and struggle which man receives from his world are transformed in him into objectives and tasks largely determined by the world concept of his time. Through his lifework then there runs, as the fulfilment of the ideas absorbed in his youth, a unity of purpose which makes it a harmonious entity. But later generations—different persons with different aims in a changed world—take from it only what may serve them, discarding the framework. Thus what inspired and proved the triumph of the earlier precursor often appears to those who follow to be superfluous or a false direction. In later centuries, when scientific research took on more the character of the routine work along fixed tracks, this may be less visible. In this time of renovation, discovery and transition, Kepler's work shows better than any other the relation between general and personal elements in the growth of science.

CHAPTER 24

MECHANICS AND PHILOSOPHY

The social storms of the sixteenth century had ceased to rage, and a new social order had settled down. The power of urban craft had declined, as new industrial settlements with their factories grew up mostly in new sites free from the restraints of the guilds. Commerce with distant continents expanded in old and new centres and became a force in society. The need of centralized power found its political expression in royal absolutism, suppressing both urban particularism and the quarrels of the nobility. Foremost in these developments were France, Holland and England. In France, at that time the most populous, the strongest and wealthiest country of Europe, royal power attained its greatest brilliance under Louis XIV. Holland, dominated by a class of rich merchants, was temporarily powerful through large trading profits, its flourishing economy producing strong spiritual impulses. England, a rising country of merchants and citizens, had first to experience a period of civil war, in which the ambitious royal power was repelled by parliament, before it could unleash its forces. On the other hand, Germany, cut off as it was from the new world trade—as also was Italy, where an earlier culture had gradually stagnated and declined—had no part in this progress; plundered and devastated by its neighbours, it was thrown back several centuries in development.

The science of nature now began to present itself consciously as a means to improve the life and to increase the prosperity of man. What formerly had been called 'science' was not directed to the material enrichment of human life. Neither the doctrines of Aristotle and Plato nor the superstitions of medieval times could serve this end. A true knowledge of nature based on experience and experiment was needed. The useful arts now entered into the view of science; what previously had been a secret tradition of craft came to light in scientific publications, often by the craftsmen themselves, as in the art of metal mining and working by Georg Agricola in Germany and in the art of pottery by Bernard Palissy in France. 'In the later Middle Ages technical inventions had been remarkably frequent and their cumulative effect was now such as to raise visions of the possibility of a radical transformation of the conditions of human life,' thus Farrington in his study on Bacon.[121]

Experience and experiment became the basis of science, and its direct purpose was often to improve technical processes and industrial methods. The scientists of the seventeenth century applied themselves energetically to the use of tools and mechanisms; they were skilful workers, constructors and inventors. Or, to express it with more accuracy, many people full of curiosity about things around them made experiments with self-made apparatus, devised and constructed tools and made discoveries. These were mostly wealthy citizens, gentlemen or skilled artisans; sometimes professors or officials in the service of the princes were among them. They were afterwards mentioned in the history books as the scholars of that time, whereas most of the occupants of academic chairs, who were then called the 'scholars', are now forgotten.

At the beginning of the century lived Cornelis Drebbel, of the Dutch town of Alkmaar, inventor of European fame, constructor of many remarkable instruments, who navigated a submarine under the Thames and for this purpose prepared oxygen from nitre; he applied his chemical findings to a new and profitable method of dyeing cloth. Christiaan Huygens, whose great talent for mechanics had been encouraged in his youth, with his brother Constantijn was engaged for many years in grinding lenses for telescopes. Newton, as a boy a tinkerer at home, in later life with great patience ground a metal mirror for his telescope when the specialist workers could not satisfy him. Leeuwenhoek discovered his 'little animals' with small lenses made by himself. When Huygens had to outline a plan of projects for the recently installed Paris Academy, he proposed, as one of the items, the investigation of the motive power of gunpowder and of steam for use in machines. The seventeenth century was the century of mechanics, i.e. the science of forces and motions, in the same sense that the sixteenth century might be called the century of astronomy, the eighteenth century of calorics and the nineteenth century that of electricity—thus expressing which realm of knowledge at each period made the most notable progress.

This trend of thought and this tendency to construct mechanical instruments induced the minds of men to recognize such mechanisms in nature also. Nature, hitherto a world of wonder and a realm of incomprehensible and often mystical powers, now became a mechanical structure. The celestial bodies were supposed to be carried along by a moving fluid filling endless space. The ancient atomic theory of Democritus and Epicurus was revived by Pierre Gassend (mostly quoted Gassendi): all matter consists of small particles that by their mutual action produce all natural phenomena. In explaining the phenomena

of light, Newton considered light rays as streams of small corpuscles ejected by luminous bodies, whereas Huygens spoke of a space-filling ether whose particles, by pushing one another, propagate the light waves.

New philosophical principles now arose in opposition to the traditional doctrine of Aristotle. Two outstanding names, Bacon and Descartes, proclaimed the new methods of science. Francis Bacon (Baco of Verulam) in his *Novum Organon* in 1620 set forth with resolute clarity that experiment and research are the sole bases of science and philosophy. He emphasized that science has to serve practical life. What in former centuries had been called science was mostly sterile; philosophy stood outside the real life of man, studying as it did books and theories instead of the reality of nature. Knowledge is power; man has to make himself master of nature, not by magic, as medieval ignorance supposed, but by experiment. The investigation of nature in England since that time has always been called 'experimental philosophy'. Bacon did not invent it or bring it to prominence. Many others, such as Gilbert, Galileo and Kepler, had already applied the same principles and in detailed scientific knowledge Bacon was their inferior. But as he expressed the new principle in the most general and precise way it has in England always been called by his name.

In his utopian tale *Nova Atlantis* Bacon described an ideal state, where a ruling community of scholars, investigators, travellers and experimenters was living and working together in laboratory and garden, performing practical experiments and technical inventions. They discussed philosophy, all of them filled with an eager desire to find the truth about nature and make it serve a happier life. In every utopia of the time, as for instance, in the *Civitas solis* of the gifted rebellious monk Tomaso Campanella, the necessity for a firm knowledge of nature as the basis of trade and labour was emphasized. It was a trend of thought characteristic of the beginning of the seventeenth century; in Thomas More's *Utopia*, a century earlier, nothing is to be found of such ideas.

René Descartes, in his works published between 1637 and 1643, placed the principle of critical doubt in opposition to the belief in traditional authority: one's own thinking alone could be trusted. The spiritual slavery of the belief in authority must give way to the spiritual liberty of free thinking. Thus his principle is seen to be opposed to Bacon's; not experiment but thinking is the source and warrant of truth. Pure reasoning is the sole basis of certainty. Thinking must be the source of all truth as the necessary result of the single principle. 'I will explain the results by their causes, and not the causes by their results.'[122]

This basic idea was also applied to the construction of a new world system, which, contrary to Aristotle's structure of philosophical abstractions, presented an intelligible mechanical picture of the world. In this theory the universe was filled with a thin fluid, consisting of fine dust, produced from the particles by their impacts; it was rotating in whorls, *vortices* (*tourbillons*), around the sun, and, of course, in the same manner, far away around the stars. This rotation carried the planets along in their orbits; around the earth and Jupiter smaller vortices produced the revolutions of their moons. As to the comets, there remained for them the wide spaces outside Saturn's orbit, where they roamed, now approaching, now moving off between the stellar vortices. Not wishing to be disturbed in his philosophical studies by the Church—Galileo's trial had just taken place—Descartes artfully explained that according to his theory the earth may be said to be at rest, i.e. relative to the streaming fluid, its surroundings, just as a floating ship carried by the streams is at rest relative to the water.

The attempt by pure reasoning to drive the phenomena from primary causes, gave rise, of course, to many fantastic explanations and results refuted by later science. 'Descartes', said Bailly, the later historian of astronomy, 'dealt with nature as if it did not yet exist and had to be constructed. Bacon considered it as a vast edifice to be invaded and decomposed to discover its structure and arrive at its foundations. Also Bacon's philosophy, restricted to facts, still subsists, whereas Descartes's doctrine, too much subjected to imagination, has perished. Bacon had the greater wisdom, Descartes the greater boldness, but by this boldness he has served the human spirit well.'[123]

The contrast between the two philosophers corresponds to a contrast of wider scope, in attitude and life-system, between the two peoples. In England people in medieval times had already acquired a high degree of personal liberty and independence, certainly owing to their insular security which precluded the necessity of an armed state power. Personal initiative was not hampered by command and prescript from a ruling power. Thus the mode of action in trade as well as in research was direct practice; man was trying and experimenting, doing things in his own way. On the Continent people were more constricted, oppressed and hampered by powerful authority and, through old and new dependencies, were prevented from practical action in their own way. So the new ideas had to remain in the field of thought and, by consistent thinking, to be perfected into complete theoretical systems. Thus, philosophically, England came to be the country of empiricism, France and the Continent the country of rationalism.

Gradually Descartes began to oust Aristotle from the academic chairs which felt a need for complete philosophical systems. At the end

of the seventeenth and the beginning of the eighteenth centuries, the vortices had penetrated most manuals of physics; in the Jesuit schools of the southern countries, however, Aristotle maintained his position during the eighteenth century. Astronomically Descartes's theory was not on a par with the science of the time; for Kepler's laws there was neither room nor explanation in the vortices. That these laws were also entirely unknown to Galileo, who assumed the planetary orbits to be circles, has been mentioned above. The young English clergyman, Jeremiah Horrox, who died at the age of twenty-two, was among the few who knew them; having first followed Lansbergen, he soon became an ardent admirer of Kepler. He was the first to explain the greatest inequality in the moon's course by the elliptic figure of its orbit. Kepler's Rudolphine Tables were universally used; but even at the end of the century Cassini and La Hire tried to find other explanations for the irregularities in the planetary movements.

This cannot surprise us if we consider that even the heliocentric doctrine of Copernicus was not widely accepted without difficulty. It is true that the physical objections gradually lost their importance. In 1638 Galileo's *Discourses on Two New Sciences, of Mechanics and of Motions* had been published in Holland. Blind and broken in health, in the last years of his life he had dictated the work to his pupil Viviani. In this compendium of his lifelong researches on the motion of bodies the foundations of the renovated science of mechanics have been laid down. Though Copernicus and his theory are not mentioned therein, it established a firm theoretical basis for the motion of the earth. Moreover, experiments were made by Gassend in 1640 to demonstrate the preservation of uniform motion, e.g. through balls thrown up vertically by a rapid rider, which fell back into his hands.

The theological difficulty, however, was more strongly felt, since in Catholic countries authors on astronomy had to avoid any conflict with the Church. The most learned and best known among the opponents was J. B. Riccioli, professor at the Jesuit College at Bologna, who made experiments to see whether bodies falling from a high tower arrived exactly below the starting point. In 1651 he published his *Almagestum novum*—the name indicates a modernized Ptolemy, though he seemed to prefer the Tychonic system—a large collection of astronomical facts and opinions, intended to be a refutation of Galileo's *Dialogue*: not a hard task when the other side was gagged. He enumerated and discussed 49 arguments in favour of Copernicus's theory and 77 arguments against; so Copernicus was defeated by a majority. But such artfulness in arguing could not actually impede the progress of science, though in Italy the chilling hand of clerical threat made scientific discussion well-nigh impossible and continually prevented able astronomers, such as Borelli and Montanari, from publishing and uttering their real opinions. In France, because the power of the Church was less dominant, conditions were better. In 1665 the French astronomer Auzout, in a letter to an influential prelate, agreed that the hypothesis of Copernicus was neither absurd nor false philosophically and that the Scriptures were not intended to instruct us in the principles of physics and astronomy, which are as useless for the life hereafter as they are useful for the life here; so he demanded more freedom for the scientists—of course, without avail.[124] In the next century, in the northern countries, the new truth gradually began to spread among wider circles of the population.

There now arose a certain organization in science. In the preceding centuries single individuals had stood out from among their contemporaries through special knowledge and predilection for the study of nature. In the seventeenth century a keen interest in the surrounding world developed among a numerous class of well-to-do and educated citizens; it appeared as an ardent curiosity and desire for knowledge which they felt to be salutary for society. They contacted one another, and through extensive correspondence, in which we find the germs of many new views, they discussed their opinions and discoveries. Before long they assembled in regular meetings; soon they got support and protection from the princes and acquired the official status of an 'Academy', somewhat on the lines described by Bacon in his *New Atlantis*, though with much less power and influence. A first organization had been formed in this way in England in 1645, meeting at first in secret because, being mostly royalists, they could not expect sympathy from the Puritan government. Afterwards, in 1662, through a royal charter, they became the 'Royal Society'. Lectures were given and communications were made at their meetings, new discoveries and ideas were discussed, letters from foreign scholars were read and experiments were performed. The Secretary, Oldenburg, from the German trading town Bremen, through his extensive correspondence with numerous European scholars, for a long time acted as a kind of central office for science. For many years Robert Hooke, a keen and versatile scientist, was appointed at a small salary to demonstrate at every session a new and interesting experiment. Among his papers a document was found describing the business and purpose of the Royal Society: 'To improve the knowledge of natural things, and all useful Arts, Manufactures, Mechanick practices, Engynes and Inventions by Experiments (not meddling with Divinity, Metaphysics, Moralls, Politicks, Grammar, Rhetorick or Logick).'[125] For the publication of all this work the *Philosophical Transactions* was founded in 1666, and

throughout the following centuries they remained a most important scientific review.

In France, too, naturalists and scientists were already meeting in regular assemblies before Minister Colbert gave them official status in 1666 as the 'Académie des Sciences'. There was a curious difference between the two academies, typical of the different conditions in the two countries. This was clearly pointed out by Voltaire in the next century and was described in a later *History of the Royal Society* in these words: 'The members of the French Institute receive a yearly stipend; the Fellows of the Royal Society pay an annual sum for the support of their Institution and the advancement of science. It would be repugnant to the feelings of Englishmen to submit to the regulations of the [French] Institute, which require that official addresses, and the names of candidates for admission should be approved by the Government before the former are delivered or the latter elected.'[126] In France the Academicians received salaries, called *pensions*, from the king. Louis XIV felt himself to be the great European monarch, who extended his influence far beyond the frontiers of his country; he awarded 'pensions' to foreign scholars, and he tried to attract to Paris the most famous among them to enhance the splendour of his reign. Ole Römer came from Denmark and Dominico Cassini from Italy to take their seats beside the French astronomers Auzout and Picard. When Picard had come to inspect the ruins of Tycho's Uraniborg, Römer made his acquaintance and accompanied him to Paris. Cassini came to direct the building of an observatory, which was also to serve as the home of the Academy. Discussions took place at the sessions and experiments were made; when a comet appeared it was jointly observed and papers on its nature were read. The reports published in the newly-created *Journal des Savants*, often of a rather primitive character, sometimes presented important new ideas.

The example of the two prominent kingdoms was imitated in other countries. In Florence even earlier, in 1657, a dozen or so naturalists, mostly pupils or admirers of Galileo, the Grand Duke himself among them, had united into an 'Academia del Cimento', which in its assemblies initiated a systematic series of experiments on problems of physics; but when they were informed that the high Church authorities disapproved of such activities, they had to stop their work ten years later. In an impoverished and divided Germany, where petty princes tried to emulate the brilliant court at Versailles, learned societies were founded in different towns. The most important was the Academy at Berlin, due mainly to the personality of its founder, Georg Wilhelm Leibniz, a versatile scholar, an important philosopher and a mathematician of genius, who tried to realize his ideal of a republic of scientists in this entirely inadequate milieu. Also in the particularistic Netherlands every town of importance in the next century had its 'learned society'.

Thus the increasing thirst for knowledge in the rising middle class everywhere in Europe laid the foundations for this organization of science.

CHAPTER 25

THE TELESCOPE

The progress of astronomy in the seventeenth and eighteenth centuries was due in the first place to the new instrument, the telescope, which was now at the disposal of the astronomers. Its discovery had been by chance, nothing more than a marvellous plaything. Galileo had no real knowledge of how it worked. Kepler was the first, in 1611, in a booklet *Dioptrical Researches*, to give a theory of the course of the light rays through the lenses and of the formation of an image. He discussed different ways of combining lenses, by placing them one behind the other into an optical system. Among them was not only the combination of a convex object lens and a concave ocular, realized in Galileo's telescope, but also a combination of a convex objective and a smaller convex ocular. Since then the latter has always been called the 'Kepler' telescope, although he never tried actually to make it.

Such a telescope was impracticable in ordinary life because it reversed the images and showed people upside down. In astronomy it came into use during the following twenty years, whether by practical trial or guided by Kepler's theory, we do not know. Neither do we know with certainty who was the inventor. In 1655 Hans Zachariassen, the son of Zacharias Janssen, made a statement to an official fact-finding commission that, together with his father, he had constructed in about 1619 a *lange buyse* ('long tube'); it is supposed that this may have meant a Kepler telescope, because here the focus is situated between the lenses, hence the length is the sum total of the focal distances, whereas the ocular of the Galileo telescope is situated between the objective and its focus. Fontana said in 1646 that he had already made such a telescope in 1608; and Christoph Scheiner mentioned in 1630 that he had been using this type of telescope for many years to project the sun's image upon a screen to observe the spots. In 1645 Father Schyrrle of the Rheita monastery described how by means of Kepler's construction the stars were visible far more sharply over a larger field of view. The advantage of Kepler's over the Dutch construction is here stated exactly; it is the large field of view. The light rays coming from a star to which the telescope is not exactly pointed, passing obliquely through the tube, the practical ability that made observations under such conditions possible. These excessive dimensions were abandoned later on, especially after Newton's researches had shown that the chief cause of the unsatisfactory images was not the figure of the lenses but the chromatic dispersion of the glass, which persisted with even the greatest focal length.

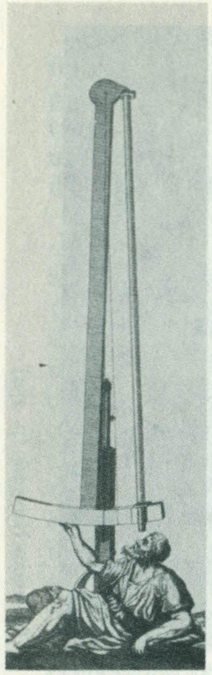
Telescopes could be used for more purposes than discovering new celestial bodies; the pioneer work of Galileo and his contemporaries had to be extended in various directions. Fontana at Naples, between 1630 and 1646, made many observations of the planets and still more of the satellites of Jupiter. Their eclipses, when immersed in Jupiter's shadow cone, were observed by Hodierna in 1652, as well as the passages of their shadows over the disc of the planet. On the basis of a number of observations, Borelli in 1665 gave a theory of their motions; and a more complete theory with tables of their movements was given by Cassini in 1668. They were based chiefly on observations of the eclipses in times of easy visibility, before and after Jupiter's oppositions. When the observations were continued at Paris, it was found that the moments of eclipse of the first satellite, the most rapid one, did not fit when Jupiter was near conjunction with the sun; they were more than 10 minutes late. The Danish astronomer Ole Römer, being in Paris in 1675, gave an explanation of this difference by means of the finite velocity of light: when Jupiter is at its greatest distance from the earth, the events are observed with greater delay. From it he derived the fact that the light takes 11 minutes to cover the distance from sun to earth.

Several observers in the seventeenth century directed their telescopes at different planets, and sometimes they thought they perceived irregular figures or spots. In his Saturn book Huygens gave a drawing of Jupiter with two equatorial streaks clearly depicted, and one of Mars with one dark band. His diary from later years contains sketches of Mars, on which some of the spots discovered later could be recognized, so that they allowed the determination of the rotation period. Cassini, with his better telescopes, in 1663 determined the rapid rotation of Jupiter (as 9 hours 56 minutes) by means of small irregularities in its equatorial bands, and in the next years found 24 hours 40 minutes for the rotation period of Mars. These discoveries were the utmost limit of what could be obtained with the rather primitive instruments of the time; interspersed were abundant announcements of pretended discoveries.

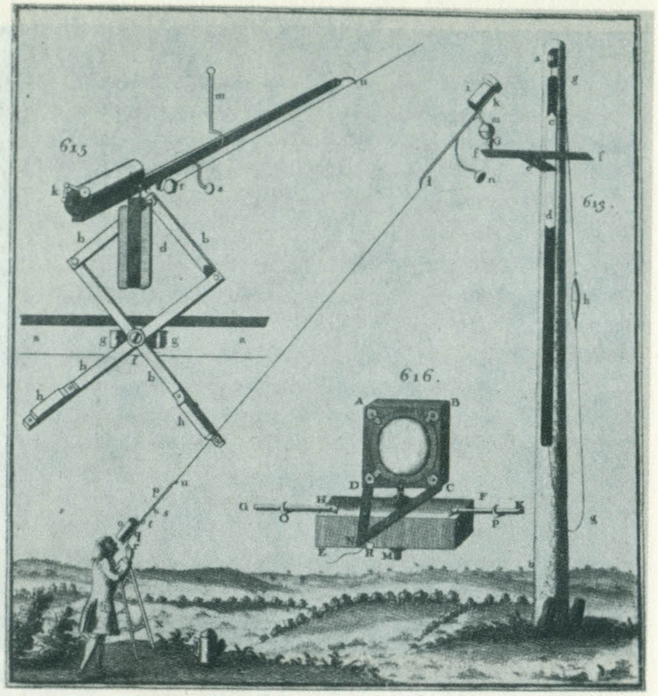
The most promising object for study with the new instrumental aids was the moon. After the first discoveries of Galileo it must have seemed an alluring task to make an exact picture or map of the moon. Here was another world, a counterpart of the earth, but easier to picture. The interiors of the far continents on our earth were inaccessible; only the coasts could be explored, so that the making of a complete map was

Christiaan Huygens (p. 254)

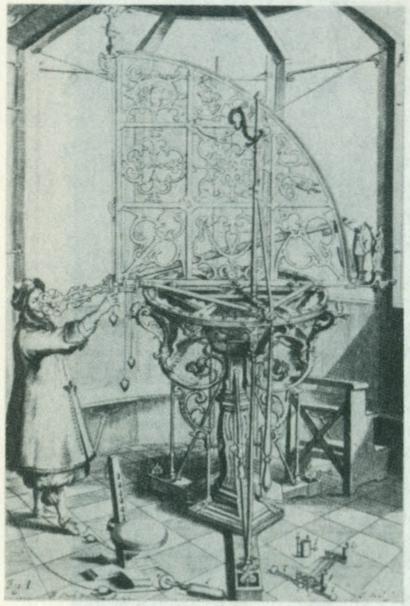
5. Johannes Kepler (p 225)



6. Picard's Zenithsector (p. 259)



Huygens' tubeless telescope (p. 255)



Hevelius and his Quadrant (p. 259)

Hevelius' map of the full moon (p. 257)

impossible. The lunar world, on the other hand, was entirely open to our view—of course, only the near side—so that astronomers could map it completely. The first work of this kind appeared about 1630, from the hand of the now nearly forgotten Belgian mathematician and cosmographer, M. F. van Langeren (Langrenus). More fame was won by the great atlas of the moon by Johann Hewelke (1611–87), better known as Hevelius, the Latinized name on his publications. Hewelke, patrician of Danzig, having studied at Leiden University, after his return installed an observatory on his house, where in 1641 he began to observe the moon regularly. His *Selenographia*, published in 1647, consisted of a number of self-engraved drawings and maps, on which the different features—the dark and bright regions, the mountains, the circular walls and craters—were inserted according to direct observation and were provided with names largely taken from earthly geography. Of these, only some few names of mountain ridges (Alps, Apennines, Caucasus) have been preserved in modern selenography. For all the other objects they have been discarded and replaced by another system of names devised by Riccioli in 1651 and inserted in a lunar map drawn by his pupil Grimaldi. For the mountains they used the names of famous astronomers and mathematicians; for the dark plains, regarded as seas, they took fancy character names, with geographical or meteorological meaning; hence we call some of the finest and largest ring mountains by the names Tycho, Plato, Aristarchus, and some plains are named Mare Serenitatis (Sea of Serenity) and Oceanus Procellarum (Ocean of Tempests).

The deeper thought underlying this study of the moon and the planets was conveyed in this question: If they are similar to the earth, might they not also be inhabited by living and intelligent beings? Kepler had formerly written a book entitled *Somnium*, published posthumously, the dream of a fantastic voyage to the moon; on this romantic basis a scientifically well-founded account was given, embroidered as an exercise of wit, of the celestial phenomena and living conditions on the moon. It was part of the world conception of the seventeenth-century scientists that they tried to figure out an adequate picture of the living conditions on the planets considered to be inhabited. Thus Huygens wrote a *Cosmotheoros*, published in 1698 after his death, containing reflections on the conditions and the living beings on other worlds. 'It is hardly possible,' he said, 'that an adherent of Copernicus should not at times imagine that it seems not unreasonable to admit that, like our globe, the other planets are also not devoid of vegetation and ornament, nor, perhaps of inhabitants.'[129] It is not necessary to assume that the equipment of the other planets is fundamentally different from what we know on earth. 'There appear on the surface of Jupiter certain bands darker than the rest of the disc, and they do not always preserve the same form; that is proper to clouds. . . . On Mars clouds have not yet been observed, since the planet appears so much smaller . . . since, however, it is certain that the earth and Jupiter have clouds and water, it can hardly be doubted that they are found also on the surface of the other planets.' As to the animals, 'there is no reason why their mode of sustenance and of multiplying themselves on the planets should not resemble what they are here, since all animals on this earth . . . follow the same law of nature'.[130] If there are intelligent beings, the rules of thinking and of geometry must be the same as for us.

Great popularity was won by Fontenelle's *Entretiens sur la pluralité des mondes* ('Conversations on the Plurality of Worlds') 1686. Here the planetary system of Copernicus, the inhabitants of the moon and the planets, the vortices and the comets all are treated in the light, courtly style of the reign of the Roi Soleil. 'You will not be surprised,' the author says to the Marquise, 'to hear that the moon is another earth and appears to be inhabited.' Farther on he says of the moons of Jupiter 'that they are not less worthy of being inhabited, though they have the misfortune to be subject to, and to revolve about, another more important planet.'[131] But what about the theological difficulty—which he treats at the close of the preface—that people on the moon cannot be descendants of Adam? 'The difficulty is made by those who are pleased to place men upon the moon. I do not place them there; I place only inhabitants there that are not men . . . you will see it is impossible that there are men there, according to my idea of the boundless diversity which nature must have put into its works.'

The fact that in the Kepler telescopes a real image is formed in the focal plane, which, as by a magnifying glass, is looked at through the ocular placed behind the focus, acquired a fundamental importance for astronomy. An object situated in the focal plane, be it a metal sheet, ring, or wire, is seen sharply in focus together with the image of the celestial object; by comparing them, small distances or sizes can be measured. Huygens described in 1659 how he determined the diameter of a planetary disc by exactly covering it with a metal strip in the focus. The French astronomer Auzout in 1667 described an improvement on this method by putting two parallel wires in the focal plane, one of which could be moved by means of a screw. This was the first specimen of a filar micrometer, which in later centuries developed into an ever more perfect measuring apparatus. That a young English astronomer, William Gascoigne, afterwards killed in the civil war, in 1640 had already used the same device for measuring planetary diameters was discovered much later in his manuscripts.

However important for human knowledge the discoveries with telescopes have been, the application of telescopes to measuring instruments has been far more important for science. A proposal by Morin, a notorious astrologer of Paris, in 1634, to use a Galileo telescope as a measuring instrument, of course, was impracticable and useless; only the Kepler telescope could open new ways. By bringing the image of a star as seen through the enlarging ocular exactly at the intersection of two cross-wires, the position of the telescope can be fixed far more accurately than formerly. Thus the position of a star could be determined with far greater precision. Jean Picard, the most diligent and capable of practical astronomers, in 1667 was the first to introduce this method into astronomy. His object was a new accurate determination of the dimensions of the spherical earth by measuring a meridian arc. A new method had been devised by Snellius at Leiden: a large distance on earth was derived by means of triangulation from a small, accurately measurable base-line. He had applied it to the distance between the towns of Alkmaar and Bergen-op-Zoom, separated by broad waters: and in 1617 he described it in a booklet bearing the adequate title *Eratosthenes Batavus*. Picard, following this method, measured a series of triangles with a base line in northern France; the difference in latitude between the northern and southern ends was determined by measuring at both points the meridian zenith distances of a number of stars. His instrument was a circular arc, 10 feet in radius (hence $1' = 1$ mm.), extending only some few degrees, and was provided with a telescope instead of with sights. The added figure, reproduced in plate 6, characterizing the spirit of the time, shows the observer in the guise of a philosopher of antiquity. The individual results did not deviate by more than $5''$ from the mean, testifying to the accuracy reached and the reliability of the result, 57,057 toises for one degree of the meridian. This, however, was a favourable case, owing to the relative character of the determinations. With other determinations of stellar positions, Picard found differences up to $10''$ or $15''$ in the declination, apparently dependent on the season; their origin was not detected until many years afterwards.

The new method of astronomical observation did not find general approbation. Hevelius of Danzig was working by Tycho's method. With extreme care he himself constructed accurate measuring instruments (quadrants and sextants); after the fashion of his time, he made them into show pieces of fine workmanship and installed them in his observatory (pl. 6). With his sharp vision—he could see stars of the seventh magnitude with the naked eye—he succeeded in attaining an accuracy of $1'$ and even less, thus surpassing Tycho. He measured meridian altitudes with a quadrant of 5 feet radius and distances between planets and stars with a 6 foot sextant. His observations numbered thousands, but unhappily a large part was lost through a fire that destroyed his house and his instruments. He also prepared a celestial atlas, published in 1690 after his death, in which he introduced a number of new constellations made up of small stars in the blank spaces between the ancient asterisms such as the Hounds, the Lizard, the Shield of Sobieski, the Unicorn, the Sextant, the Fox, the Lynx.

He had a sharp dispute with Hooke in England concerning the best method of observing. Hooke asserted that observation with sights only, without telescope attached, could not give sufficient accuracy. Thereupon, Edmund Halley (1656–1742), who had already used a sextant with telescopes when observing a number of southern stars at St Helena in 1676, was sent from London to Danzig in 1679 to observe with Hewelke a number of identical stars, each with his own instrument. It then appeared that the differences between their results was mostly a matter of seconds only, never reaching one minute—a proof of Hewelke's perfection in the art of observing. His precision was surely the utmost attainable without telescopes, but there was no future in it. The same precision was easily reached by the use of telescopes on the instruments and could be raised by further improvements in the instruments and methods.

From now on telescopes became a permanent part of astronomical equipment. In England they were used by Flamsteed in his first observations of 1676 in the newly-founded Greenwich Observatory. These telescopes, as may be seen in old pictures, were long narrow tubes; the objective was a small lens about one or two inches in diameter. No great brightness, but only a strong magnification was needed to point the stars with greater precision. Even with these modest objectives, the stars became brighter in relation to the background and some of them became visible in the daytime. In 1634 Morin had already described enthusiastically how with his telescope he could follow Venus for many hours after sunrise. Picard, in 1669, by observing the passage of Arcturus through the meridian in the daytime, opened up new possibilities of astronomical measurement.

CHAPTER 26

NEWTON

The concept of attraction was not introduced for the first time by Newton. Copernicus had already spoken of the mutual attraction of the parts of the earth as the cause of its spherical shape; he assumed this faculty to be present in other celestial bodies too, causing their particles to be compressed into a sphere. Kepler, too, had spoken of gravity as a tendency of cognate bodies to approach and join one another. To him the tides were a proof that the moon exerted an attraction upon the water of the earth: 'if the earth ceased to attract the waters, all the sea water would be drawn upward and would flow to the moon'.[132] He compared gravity with magnetism: 'the earth draws along the bodies flying in the air, because they are chained to her as though by a magnetic force, just as if there existed a contact between them.'[133] This attraction had nothing to do with orbital motion; the sun, as quoted above, did not exert an attractive force upon the planets but a directive force, dragging the planets along with its rotation. Gravity and orbital motion were two different and entirely separate fields.

Nor did the seventeenth century see any connection between the vortices, which moved the planets in their circles, and gravity, working at the surface of the earth and doubtless also at the surface of the sun and the other planets. Huygens made an attempt to establish such a connection in a lecture held, in 1669, at the Paris Academy, 'On the Cause of Gravity'. Whereas Descartes had assumed that the ethereal fluid, by rotating uniformly about a certain axis through the earth, carried the moon along, Huygens made the thin fluid matter in rapid rotation move in all directions about the earth's surface. As a consequence of their centrifugal force directed outward, i.e. upward, the fine particles pressed down the larger particles of the coarse-grained visible matter, which did not participate in the rotations. This origin of gravity implied that the thin fluid matter passed freely through all heavy objects and filled the space between their particles. The velocity of this whirling motion had to be 17 times greater than the velocity of the equator, because, with a rotation of the earth 17 times more rapid than the actual one, the objects at the equator would lose their gravity.

The actual progress of science, however, went in exactly the opposite direction, not in explaining gravity by circular orbits but in explaining circular orbits by gravity. The development of the fundamental principles of mechanics had made this possible. Galileo had explained the constant velocity of a horizontal movement in the absence of friction by pointing out that such movement was part of a circular orbit about the earth's centre, which had always been considered uniform by nature. He had not been able to overcome this conception; but his researches had so perfectly cleared the way that pupils and younger scientists, like Cavalieri (1632) and Torricelli (1644), could express the 'principle of inertia' in modern form: when acting forces are absent, the motion is rectilinear with constant velocity. Then the next step was the realization that a circular orbit is not simply a natural motion—as all the preceding centuries had supposed—but a complex enforced motion. A circular motion is the result of a force directed towards the centre, continuously preventing the body from following the rectilinear motion along the tangent. This tendency to follow the tangent and move with increasing rapidity away from the centre was observed as a 'centrifugal force', a tension in the string when an object is swung around. In his work on the Jupiter satellites, Borelli in 1665 had expressed himself in this way: that the centrifugal force of the orbital motion was exactly in equilibrium with the attractive force of Jupiter. The complete theory of the centrifugal force was given by Huygens in 1673 in his work *Horologium oscillatorium*, in which he, in connection with his invention of the pendulum clock, treated a number of related mathematical and mechanical problems. He deduced that the centrifugal force is proportional to the square of the velocity and to the inverse of the radius of the circle.

So the idea became dominant that an attraction directed toward the centre of their orbit works upon the planets and the moon. It might be expected that this force decreases with increasing distance; but in what ratio? The answer to this question was given by Newton (plate 7).

Isaac Newton, a farmer's son from the hamlet of Woolsthorpe, in Lincolnshire, born in 1642, went to study in Cambridge in 1661. When the university was closed for a couple of years because of a pestilence in the town, he returned in 1665 to his native village. Here he made his first studies in what were to become the most important subjects of his later work: mathematics (the theory of fluxions), optics (the discovery that common light is composed of numerous kinds of simple light, all of different colours and refrangibility), and gravitation. The falling of bodies toward the earth caught his attention (the anecdote relates that, seeing an apple fall from the tree, he began to ponder over the cause of this falling) and raised the question as to what height gravity extended. To the moon perhaps? If so, could gravity be the force that kept the moon

in its circular orbit? To settle it, he had to know in what ratio gravity decreases with distance from the earth. For this problem Kepler's third law could give a valuable indication. According to this law, a four times larger circular orbit has an eight times larger period, hence a two times smaller velocity; therefore, the centrifugal force, according to Huygens's formula, is 16 times smaller. Generally in such a planetary system the centrifugal force must be as the inverse square of the distance. Gravity compensating it must vary in the same ratio.

The moon's distance being 60 times the earth's radius, its gravity must be 3,600 times smaller than that of a stone falling on the earth's surface, or, as it was sometimes expressed, the moon falls in a minute as far as a stone falls in a second. Newton, in making the computation, assumed an arc of one degree on earth to be 60 miles, as given in a sailor's manual, the only book at hand—even today an English nautical mile is always taken to equal one minute of arc. Assuming this to be the usual 'Statute mile' of 5,280 feet, equal to 4,954 Paris feet, he computed the moon's acceleration per second to be 0.0073 feet, per minute 26.3 feet. Through Galileo's experiments, however, afterwards repeated more accurately by others, the acceleration of freely falling bodies per second was known to be 30 feet. The two values are of the same order of magnitude, but the difference, one-eighth of the amount, is too great to be acceptable. Disappointed, the story runs, he abandoned his apparently so brilliant idea. In the years that followed he occupied himself with optical and mathematical studies.

He could have used a better value, because Snellius's result, which gave, for an arc of 1°, a length of a good 69 English miles, could already be found in English books. It was confirmed by the more extensive and accurate determination of Picard in France, published in 1671, giving, for 1°, 57,065 toises or 69 English miles. Performed with this value, the new computation gave complete agreement. Thus the law of gravitational attraction, decreasing as the inverse square of the distance, was established.

Newton was not the only man to formulate this law of variation of force with distance. Part of his mathematical deductions were found in Huygens's work published in 1673. Robert Hooke, that acute and versatile but jealous scientist, asserted afterward that he had known the law for a long time—which was quite possible—and even that Newton had got the idea from him. Probably Hooke, by facing him with the problem of what the orbit of a body would be, if affected by such an attractive force, was a strong factor in drawing Newton's attention to this matter. But he himself could do nothing with the mere idea. Halley and Wren discussed the same questions, without being able to solve them. What was necessary was to demonstrate all arguments and derive all consequences of this law for the celestial orbits with exact mathematics. Newton was the only man able to do so by means of the mathematical methods he himself had constructed.

In 1684 the theory was ready in its main part; and in 1685, by solving the problem of the attraction of a solid sphere and demonstrating that it was exactly equal to the attraction by its mass if concentrated in the centre, he removed the last difficulty in the argument. Another year of severest mental exertion was needed, in which he was so entirely absorbed by his problems that dinner and sleep often were neglected and his health was badly shaken; the many anecdotes about his absent-mindedness relate to this period. Then the first part could be presented to the Royal Society in 1686. That the manuscript was not buried for a long time in its archives, was due to the unremitting care of his friend Halley, at that time assistant secretary (called 'Clerk') of the Society, who procured money for its printing, partly from his own pocket. In 1687 the work appeared under the title *Philosophiae naturalis principia mathematica* ('Mathematical Principles of Natural Philosophy').

The title of the book expresses how it could lay down new foundations for astronomy. 'Natural philosophy' was in England the name for scientific research; why mathematical principles were needed he explained in Book III, which bears the special title 'The System of the World'. There he said: 'Upon this subject I had, indeed, composed the third book in a popular method, that it might be read by many; but afterwards, considering that such as had not sufficiently entered into the principles could not easily discern the strength of the consequences, nor lay aside the prejudices to which they had been many years accustomed, therefore, to prevent the disputes which might be raised upon such accounts, I chose to reduce the substance of this book into the form of propositions (in the mathematical way), which should be read by those only who had first made themselves masters of the principles established in the preceding books.'[134] This is understandable when we consider that Newton was extremely sensitive to criticism, which, often based on shaky foundations, was set against results on which he had pondered carefully and profoundly; often he postponed publication of his results to avoid unpleasant polemics. The mathematical demonstration convinced the well-instructed and deterred the ignorant. It was at the same time that Spinoza expounded his philosophy in the mathematical form of propositions and demonstrations.

The contents of the first two Books, indeed, consist of mathematics; it is geometry applied to the motion of bodies, i.e. what we call 'theoretical mechanics'. In his preface Newton said: 'Therefore geometry is founded in mechanical practice, and is nothing but that part of universal mechanics which accurately proposes and demonstrates the art of

measuring. But since the manual arts are chiefly employed in the moving of bodies, it happens that geometry is commonly referred to their magnitude and mechanics to their motion. In this sense rational mechanics will be the science of motions resulting from any forces whatsoever, and of the forces required to produce any motions, accurately proposed and demonstrated.'[135] Rational mechanics was the discipline needed to unite earthly and celestial motions into one system. Earthly motions were ruled by Galileo's laws of falling and gravity; celestial motions were ruled by Kepler's laws of planetary orbits. To connect them, Newton, as the founder of the new science, completing the work of Galileo and Huygens, began by stating its principles in the form of 'Definitions' and 'Axioms, or Laws of Motion'.

(1) Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it. (2) The change of motion is proportional to the motive force impressed; and is made in the direction of the line in which that force is impressed. (3) To every action there is always opposed an equal reaction; or, the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.[136] The concept of mass was introduced as 'the quantity of matter arising from its density and bulk conjointly'; 'the quantity of motion arises from the celerity multiplied by the quantity of matter; and the motive force arises from the accelerative force multiplied by the same quantity of matter.' Mass and weight were sharply distinguished. Hence it is that, near the surface of the earth, where the accelerative gravity, or force productive of gravity, in all bodies is the same, the motive gravity or the weight is as the body; but if we should ascend to higher regions, where the accelerative gravity is less, the weight would be equally diminished, and would always be as the product of the body, by the accelerative gravity.[137]

Because the chief aim is the treatment of the freely moving heavenly bodies, centripetal forces were introduced directly under the definitions. 'A centripetal force is that by which bodies are drawn or impelled, or any way tend, towards a point as to a centre. . . . Of this sort is gravity . . . and that force, whatever it is, by which the planets are continually drawn aside from the rectilinear motions, which otherwise they would pursue, and made to revolve in curvilinear orbits. . . . They all endeavour to recede from the centres of their orbits; and were it not for the opposition of a contrary force which restrains them to, and detains them in their orbits, which I therefore call centripetal, would fly off in right lines, with a uniform motion.' Then, after mentioning a projectile shot from a mountain horizontally with sufficient velocity, which would go round the earth in an orbit, he proceeded: '. . . the moon also, either by the force of gravity, if it is endued with gravity, or by any other force,